文章编号: 1000-0887(2007) 08-0901-11

圆柱贮箱液体非线性晃动的多维模态分析方法*

余延生[1], 马兴瑞[2], 王本利[1]

(1. 哈尔滨工业大学 卫星技术研究所, 哈尔滨 150001;
(2. 中国航天科技集团, 北京 100830)

(我刊编委马兴瑞来稿)

摘要: 将 Faltinsen 等提出的多维模态理论应用到求解圆柱贮箱液体非线性晃动问题中• 根据 Narimanov-Moiseiev 的三阶渐近假设关系, 通过选取主导模态以及确定它们的阶次关系, 将一般形式的无穷维模态系统降为五维渐近模态系统, 即描述自由液面波高的广义坐标之间相互耦合的二阶非线性常微分方程组• 通过对这个模态系统的数值积分, 得到了与以前的理论分析和实验结果相吻合的非线性现象• 研究结果表明, 多维模态方法是用来求解液体非线性晃动动力学的一个很好的工具• 在我们的下一步工作中, 将继续发展这种方法, 用来研究更为复杂的晃动问题•

关 键 词: 圆柱贮箱; 非线性自由晃动; 多维模态方法; 渐近模态系统; 耗散效应
中图分类号: O353.1 文献标识码: A

引 言

部分充于光滑运动贮箱中的液体将相对于其平衡自由液面做大幅波运动, 即液体晃动• 在这种情况下, 工程中常用的刚化液体模型和线性晃动理论[1]都将不再适用• 已有很多研究者采用数值模拟的方法来分析贮箱中的液体运动, 包括 FDM[2]、FEM[3-4]、BEM[5]、ALE[6-8]等• 虽然数值方法在处理诸如大幅晃动、碎浪等解析方法难以处理的问题时非常有效, 但是由于它们不能进行长时间的模拟以及不能用来描述液体稳态波行为, 因此虽然烦琐并且只能针对一些简单形状贮箱, 但是发展解析或半解析的模态方法还是必要的•

液体非线性晃动的模态方法都是将未知的自由液面波高函数和液体速度势函数通过固有模态展开为广义 Fourier 级数, 然后将它们代入到原始的自由边界值问题或其等价变分问题中, 得到广义坐标相互耦合的无穷维数非线性常微分方程组, 即模态系统• Narimanov[9]大概是第一个用来研究如何得到模态系统的人, 他假设主导模态和次模态之间为三阶渐近关系, 这与后来 Moiseiev[10]提出的理论一致• 文献[11]至文献[13]在他们的研究中应用了 Narimanov-Moiseiev 三阶渐近假设, 所得结果与实验吻合得非常好• 另外一个得到模态系统的方法是由 Miles[14-15]提出的平均化方法, 其基于压力积分变分原理• 与 Narimanov-Moiseiev 的渐近假设不

* 收稿日期: 2006-05-15; 修订日期: 2007-03-29
基金项目: 国防十五预研资助项目(41320020301)
作者简介: 余延生(1980—), 男, 安徽安庆人, 博士生(联系人. E-mail: hitxiaobai@163.com).

同, 这种方法只考虑主导模态的作用, 将渐近模态系统降为主导模态慢变系数相耦合的 Hamilton 系统.

但是上面这些方法要么需要大量烦琐的公式推导(随模态维数的增加而成指数增长), 要么在某些情况下不能用来解释实验中观察到的非线性现象(由于忽略了次模态的影响). 最近, Faltinsen 以及他的合作者[16-17]提出了多维模态方法, 用来分析二维和三维矩形贮箱中的液体非线性晃动问题. 在我们看来, 这种方法是到目前为止用来求解液体非线性晃动动力学最好的工具, 可看作是数值方法和低维模态方法之间的桥梁.

在这篇文章中, 我们首次将多维模态方法应用到圆柱贮箱液体非线性晃动问题中. 相对于矩形贮箱, 关于圆柱贮箱液体非线性晃动的理论研究非常有限. 其中一个原因是对于圆柱贮箱, 固有晃动模态为 Bessel 函数, 与矩形贮箱的三角函数相比, 其公式推导要烦琐得多.

1 一般形式模态系统

1.1 问题描述

我们考虑刚性运动贮箱中无粘、无旋、不可压液体的波运动. 令 $O'XYZ$ 为绝对坐标系, $Oxyz$ 为与贮箱固连的运动坐标系且其 Oxy 平面与未扰动自由液面相互重合, 原点 O 位于贮箱中心. $Oxyz$ 系相对 $O'XYZ$ 系以速度 $\boldsymbol{v}_0(t)=(v_{01},v_{02},v_{03})$、角速度 $\omega(t)=(\omega_1,\omega_2,\omega_3)$ 运动, 则描述箱中液体非线性晃动的自由边界值问题为

$$\nabla^2\Phi=0,\qquad V(t)\text{ 中},\tag{1}$$

$$\frac{\partial\Phi}{\partial\nu}=\boldsymbol{v}_0\cdot\nu+(\omega\times\boldsymbol{r})\cdot\nu,\qquad \Sigma(t)\text{ 上},\tag{2}$$

$$\frac{\partial\Phi}{\partial\nu}=\boldsymbol{v}_0\cdot\nu+(\omega\times\boldsymbol{r})\cdot\nu+\frac{\eta_t}{\sqrt{1+\eta_x^2+\eta_y^2}},\qquad S(t)\text{ 上},\tag{3}$$

$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}(\nabla\Phi)^2-\nabla\Phi\cdot(\boldsymbol{v}_0+\omega\times\boldsymbol{r})+U=0,\qquad S(t)\text{ 上},\tag{4}$$

$$\int_{V(t)}\mathrm{d}V=\text{const},\tag{5}$$

其中, $V(t)$ 为液体区域, $\Sigma(t)$ 为与液体接触的贮箱壁表面, $S(t)$ 为自由液面, $\nu=(\nu_1,\nu_2,\nu_3)$ 为 $V(t)$ 边界的外法线方向, $\boldsymbol{r}$ 为流体质点相对于 O 点的位置向量, $U(x,y,z,t)$ 为重力势能, $\Phi(x,y,z,t)$ 为液体绝对速度势, $\eta(x,y,t)$ 为自由液面波高函数.

为了得到上述自由边界值问题的唯一解必须定义初始条件, 它们确定了初始液体形状和速度势

$$\eta(x,y,t_0)=\eta_0(x,y),\tag{6}$$

$$\Phi(x,y,z,t_0)=\Phi_0(x,y,z),\tag{7}$$

其中, $\eta_0(x,y)$ 和 $\Phi_0(x,y,z)$ 为给定函数.

1.2 一般形式模态系统

对联合模态方法和压力积分变分原理, Faltinsen 等人[16]得到了一般形式的模态系统. 然而, 在这里为了描述圆柱贮箱液体晃动的方便, 其模态系统须在柱坐标系中表达.

将初始边界值问题(1)至边界值问题(7)的解表示为:

$$\Phi(x,y,z,t)=\boldsymbol{v}_0\cdot\boldsymbol{r}+\omega\cdot\Omega+\sum_{n=1}^{\infty}R_n(t)\,\varphi_n(x,y,z),\tag{8}$$

$$\eta(x,y,t)=\sum_{i=1}^{\infty}\beta_i(t)\eta_i(x,y), \tag{9}$$

其中，$\varphi_n(x,y,z)$ 为一满足 Laplace 方程和零值 Neumann 固壁边界条件的完备谐函数系，而 $\eta_i(x,y)$ 为一满足体积守恒条件 $\int_{S_0}\eta_i(x,y)\mathrm{d}S=0$ 的完备正交函数系，S_0 为静止状态液体自由液面，$\beta_i(t)$ 和 $R_n(t)$ 称为广义坐标或模态函数，$\Omega(x,y,z,t)$ 称为 Stokes-Zhukovsky 势函数，定义为下列 Neumann 边界值问题的解：

$$\nabla^2\Omega=0,\qquad V(t)\text{ 中}, \tag{10}$$

$$\frac{\partial\Omega}{\partial\nu}=\boldsymbol{r}\times\nu,\qquad \Sigma(t)+S(t)\text{ 上}. \tag{11}$$

则模态函数 $\beta_i(t)$ 和 $R_n(t)$ 相互耦合的无穷维非线性常微分方程组，即模态系统为

$$\dot{A}_n-\sum_k A_{nk}R_k=0,\qquad n=1,2,\cdots, \tag{12}$$

$$\begin{aligned}&\sum_n\dot{R}_n\frac{\partial A_n}{\partial\beta_i}+\frac{1}{2}\sum_n\sum_k R_nR_k\frac{\partial A_{nk}}{\partial\beta_i}+\omega_1\frac{\partial\chi_1}{\partial\beta_i}+\omega_2\frac{\partial\chi_2}{\partial\beta_i}+\omega_3\frac{\partial\chi_3}{\partial\beta_i}+\\&\quad\dot\omega_1\frac{\partial\gamma_1}{\partial\beta_i}+\dot\omega_2\frac{\partial\gamma_2}{\partial\beta_i}+\dot\omega_3\frac{\partial\gamma_3}{\partial\beta_i}+(\dot v_{01}-g_1+\omega_2v_{03}-\omega_3v_{02})\lambda_{i1}+\\&\quad(\dot v_{02}-g_2+\omega_3v_{01}-\omega_1v_{03})\lambda_{i2}+(\dot v_{03}-g_3+\omega_1v_{02}-\omega_2v_{01})\lambda_{i3}\beta_i-\\&\quad\frac{\mathrm{d}}{\mathrm{d}t}\left(\omega_1\frac{\partial\chi_1}{\partial\beta_i}+\omega_2\frac{\partial\chi_2}{\partial\beta_i}+\omega_3\frac{\partial\chi_3}{\partial\beta_i}\right)-\frac{1}{2}\omega_1^2\frac{\partial J_{11}}{\partial\beta_i}-\frac{1}{2}\omega_2^2\frac{\partial J_{22}}{\partial\beta_i}-\\&\quad\frac{1}{2}\omega_3^2\frac{\partial J_{33}}{\partial\beta_i}-\omega_1\omega_2\frac{\partial J_{12}}{\partial\beta_i}-\omega_1\omega_3\frac{\partial J_{13}}{\partial\beta_i}-\omega_2\omega_3\frac{\partial J_{23}}{\partial\beta_i}=0,\qquad i=1,2,\cdots,\end{aligned} \tag{13}$$

其中，$\boldsymbol{g}=(g_1,g_2,g_3)$ 为重力加速度向量。

为了描述圆柱贮箱中的液体晃动问题，定义与 $Oxyz$ 系固连的圆柱坐标系 $Or\theta z$，在此坐标系下，方程(12)和方程(13)中的系数表达式如下：

$$\begin{cases}A_n=\rho\int_{V(t)}\varphi_n\mathrm{d}V,\quad A_{nk}=\rho\int_{V(t)}\nabla\varphi_n\cdot\nabla\varphi_k\mathrm{d}V,\\ \lambda_{i1}=\rho\int_{S_0}r\cos\theta\,\eta_i\mathrm{d}S,\quad \lambda_{i2}=\rho\int_{S_0}r\sin\theta\,\eta_i\mathrm{d}S,\quad \lambda_{i3}=\rho\int_{S_0}\eta_i^2\mathrm{d}S,\\ \chi_i=\rho\int_{V(t)}\frac{\partial\Omega_i}{\partial t}\mathrm{d}V,\quad \gamma_i=\rho\int_{V(t)}\Omega_i\mathrm{d}V,\quad J_{ij}=\rho\int_{S+\Sigma}\Omega_i\frac{\partial\Omega_j}{\partial\nu}\mathrm{d}S,\end{cases} \tag{14}$$

其中 ρ 为液体密度。所有这些系数在 $V(t)$ 上积分，因而它们仅为 $\beta_i(t)$ 的函数。

1.3 固有模态

模态 $\varphi_n(x,y,z)$ 和 $\eta_i(x,y)$ 通常选择为下列谱问题的解：

$$\nabla^2\varphi_n=0,\qquad V_0\text{ 中}, \tag{15}$$

$$\frac{\partial\varphi_n}{\partial\nu}=0,\qquad \Sigma_0\text{ 上}, \tag{16}$$

$$\frac{\partial\varphi_n}{\partial\nu}=\mu_n\varphi_n,\qquad S_0\text{ 上}, \tag{17}$$

$$\eta_n(x,y)=\varphi_n(x,y,0), \tag{18}$$

其中，V_0 为静止状态流体区域，Σ_0 为位于静止自由液面下贮箱湿润内壁，液体晃动固有频率与特征值的关系为 $\sigma_n^2=\mu_ng$。

对于半径为 a、平均充液高度为 h 的圆柱贮箱,其解为

$$\mu_{mn} = \kappa_{mn}\tanh\kappa_{mn}h, \tag{19}$$

$$\eta_{mn}(r,\theta) = J_m(\kappa_{mn}r)\begin{cases}\cos(m\theta),\\ \sin(m\theta),\end{cases} \tag{20}$$

$$\varphi_{mn}(r,\theta,z) = J_m(\kappa_{mn}r)\frac{\cosh(\kappa_{mn}(z+h))}{\cosh(\kappa_{mn}h)}\begin{cases}\cos(m\theta)\\ \sin(m\theta)\end{cases} = \eta_{mn}(r,\theta)\frac{\cosh(\kappa_{mn}(z+h))}{\cosh(\kappa_{mn}h)}, \tag{21}$$

其中, κ_{mn} 满足 $J'_m(\kappa_{mn}a) = 0$•

2 自由晃动渐近模态系统

对于液体自由晃动情形,无穷维模态系统具有下列形式:

$$\dot{A}_n - \sum_k A_{nk}R_k = 0, \qquad n = 1, 2, \cdots, \tag{22}$$

$$\sum_n \dot{R}_n\frac{\partial A_n}{\partial \beta_i} + \frac{1}{2}\sum_n\sum_k R_nR_k\frac{\partial A_{nk}}{\partial \beta_i} + g\lambda_{i3}\beta_i = 0, \qquad i = 1, 2, \cdots, \tag{23}$$

此模态系统将根据Narimanov-Moiseiev 三阶渐近假设降为有限维形式•

2.1 主导模态及其阶次关系

根据线性晃动理论,第一阶反对称晃动模态 $J_1(\kappa_{11}r)\cos\theta$ 或 $J_1(\kappa_{11}r)\sin\theta$ 在液体运动中为能量最大模态,这就意味着对于非线性晃动情形,这两阶模态必为主导模态,根据 Narimanov-Moiseiev 三阶渐近假设,其对应的模态函数满足阶次关系 $O(\beta_1) = O(\beta_2) = \varepsilon^{1/3}$,其中 ε 为小参数• 此外,对于非线性晃动,由于自由液面的非线性耦合,还将叠加有其它次要模态,且其中模态 $J_0(\kappa_{01}r)$、$J_2(\kappa_{21}r)\cos2\theta$ 和 $J_2(\kappa_{21}r)\sin2\theta$ 为幅值最大模态,高于其它次要模态至少一个量级(但比主模态低一个量级)•

因而对液体的描述将保留五阶模态(包括两阶主模态和三阶次模态)• 但是可以看出,每个模态的完整描述需要 3 个指标,但是这种指标不适合于后面的理论推导,因此作下述规定将它们定义为单指标:

$$\begin{cases}\eta_1 = J_1(\kappa_1r)\cos\theta, \quad \eta_2 = J_1(\kappa_2r)\sin\theta, \quad \eta_3 = J_0(\kappa_3r),\\ \eta_4 = J_1(\kappa_4r)\cos2\theta, \quad \eta_5 = J_2(\kappa_5r)\sin2\theta,\end{cases} \tag{24}$$

其中,

$$\kappa_1 = \kappa_2 = \kappa_{11} = 1.841\,2/a, \quad \kappa_3 = \kappa_{01} = 3.831\,7/a, \quad \kappa_4 = \kappa_5 = \kappa_{21} = 3.054\,2/a• \tag{25}$$

它们对应模态函数的阶次关系为:

$$O(\beta_1) = O(\beta_2) = \varepsilon^{1/3}, \quad O(\beta_3) = O(\beta_4) = O(\beta_5) = \varepsilon^{2/3}; \tag{26}$$

忽略阶次高于 $\varepsilon^{2/3}$ 的模态•

2.2 渐近模态系统

为了得到渐近模态系统,首先需要计算系数 λ_{i3}、A_n 和 A_{nk}•

通过计算 λ_{ij},得

$$\lambda_{13} = \lambda_{23} = \rho\pi E_{14}, \quad \lambda_{33} = 2\rho\pi E_{16}, \quad \lambda_{43} = \lambda_{53} = \rho\pi E_{19}, \tag{27}$$

其中系数 E 列于附录•

系数 A_n 和 A_{nk} 需要在瞬时流体体积 $V(t)$ 上进行积分，因此为了计算方便，将 $V(t)$ 划分为静止状态流体体积 V_0 和扰动流体体积 $V_\delta(t)$。此外，$V_\delta(t)$ 上的积分可表达为 β_i 的 Taylor 级数。

经过烦琐的计算，A_n 展开到关于 β_i 的三阶多项式

$$\frac{A_1}{\rho\pi} = E_{14}\beta_1 + \frac{1}{2}\mu_1(2E_1\beta_1\beta_3 + E_2(\beta_1\beta_4 + \beta_2\beta_5)) + \frac{1}{6}\kappa_1^2\left(\frac{3}{4}E_5(\beta_1^3 + \beta_1\beta_2^2) + 3E_6\beta_1\beta_3^2 + \frac{3}{2}E_7(\beta_1\beta_4^2 + \beta_1\beta_5^2) + 3E_8(\beta_1\beta_3\beta_4 + \beta_2\beta_3\beta_5)\right), \tag{28a}$$

$$\frac{A_2}{\rho\pi} = E_{14}\beta_2 + \frac{1}{2}\mu_1(2E_1\beta_2\beta_3 + E_2(\beta_1\beta_5 - \beta_2\beta_4)) + \frac{1}{6}\kappa_1^2\left(\frac{3}{4}E_5(\beta_2^3 + \beta_1^2\beta_2) + 3E_6\beta_2\beta_3^2 + \frac{3}{2}E_7(\beta_2\beta_4^2 + \beta_2\beta_5^2) + 3E_8(\beta_1\beta_3\beta_5 - \beta_2\beta_3\beta_4)\right), \tag{28b}$$

$$\frac{A_3}{\rho\pi} = 2E_{16}\beta_3 + \frac{1}{2}\mu_3(E_1(\beta_1^2 + \beta_2^2) + 2E_3\beta_3^2 + E_4(\beta_4^2 + \beta_5^2)) + \frac{1}{6}\kappa_3^2\left(3E_3(\beta_1^2\beta_3 + \beta_2^2\beta_3) + \frac{3}{2}E_8(\beta_1^2\beta_4 - \beta_2^2\beta_4 + 2\beta_1\beta_2\beta_5) + 2E_9\beta_3^3 + 3E_{10}(\beta_3\beta_4^2 + \beta_3\beta_5^2)\right), \tag{28c}$$

$$\frac{A_4}{\rho\pi} = E_{19}\beta_4 + \frac{1}{2}\mu_4\left(\frac{1}{2}E_2(\beta_1^2 - \beta_2^2) + 2E_4\beta_3\beta_4\right) + \frac{1}{6}\kappa_4^2\left(\frac{3}{2}E_7(\beta_1^2\beta_4 + \beta_2^2\beta_4) + \frac{3}{2}E_8(\beta_1^2\beta_3 - \beta_2^2\beta_3) + 3E_{10}\beta_3^2\beta_4 + \frac{3}{4}E_{11}(\beta_4^3 + \beta_4\beta_5^2)\right), \tag{28d}$$

$$\frac{A_5}{\rho\pi} = E_{19}\beta_5 + \frac{1}{2}\mu_4(E_2\beta_1\beta_2 + 2E_4\beta_3\beta_5) + \frac{1}{6}\kappa_4^2\left(\frac{3}{2}E_7(\beta_1^2\beta_5 + \beta_2^2\beta_5) + 3E_8\beta_1\beta_2\beta_3 + 3E_{10}\beta_3^2\beta_5 + \frac{3}{4}E_{11}(\beta_5^3 + \beta_4^2\beta_5)\right), \tag{28e}$$

其中

$$\mu_1 = \mu_2 = \mu_{11},\quad \mu_3 = \mu_{01},\quad \mu_4 = \mu_5 = \mu_{21}. \tag{29}$$

考虑到 $\sum_k A_{nk}R_k$ 项和 $\sum_n\sum_k R_nR_k(\partial A_{nk}/\partial\beta_i)$ 项需要计算到关于 β_i 的三阶多项式，因此将 A_{nk} 展开到关于 β_i 的二阶多项式

$$A_{11} = \rho\pi(\mu_1E_{14} + H_1\beta_3 + H_2\beta_4 + H_8\beta_1^2 + H_9\beta_2^2 + H_{10}\beta_2^2 + H_{11}\beta_4^2 + H_{11}\beta_5^2 + H_{12}\beta_3\beta_4), \tag{30a}$$

$$A_{12} = A_{21} = \rho\pi(H_2\beta_5 + H_{13}\beta_1\beta_2 + H_{12}\beta_3\beta_5), \tag{30b}$$

$$A_{13} = A_{31} = \rho\pi(H_3\beta_1 + H_{14}\beta_1\beta_3 + H_{15}\beta_1\beta_4 + H_{15}\beta_2\beta_5), \tag{30c}$$

$$A_{14} = A_{41} = \rho\pi(H_4\beta_1 + H_{16}\beta_1\beta_3 + H_{17}\beta_1\beta_4 + H_{18}\beta_2\beta_5), \tag{30d}$$

$$A_{15} = A_{51} = \rho\pi(H_4\beta_2 + H_{17}\beta_1\beta_5 - H_{18}\beta_2\beta_4 + H_{16}\beta_2\beta_3), \tag{30e}$$

$$A_{22} = \rho\pi(\mu_1E_{14} + H_1\beta_3 - H_2\beta_4 + H_9\beta_1^2 + H_8\beta_2^2 + H_{10}\beta_3^2 + H_{11}\beta_4^2 + H_{11}\beta_5^2 - H_{12}\beta_3\beta_4), \tag{30f}$$

$$A_{23} = A_{32} = \rho\pi(H_3\beta_2 + H_{15}\beta_1\beta_5 + H_{14}\beta_2\beta_3 - H_{15}\beta_2\beta_4), \tag{30g}$$

$$A_{24} = A_{42} = \rho\pi(-H_4\beta_2 - H_{16}\beta_2\beta_3 + H_{17}\beta_2\beta_4 - H_{18}\beta_1\beta_5), \tag{30h}$$

$$A_{25} = A_{52} = \rho\pi(H_4\beta_1 + H_{16}\beta_1\beta_3 + H_{17}\beta_2\beta_5 + H_{18}\beta_1\beta_4), \tag{30i}$$

$$A_{33} = \rho\pi(2\mu_3E_{16} + H_5\beta_3 + H_{19}\beta_1^2 + H_{19}\beta_2^2 + H_{20}\beta_3^2 + H_{21}\beta_4^2 + H_{21}\beta_5^2), \tag{30j}$$

$$A_{34} = A_{43} = \rho\pi(H_6\beta_4 + H_{22}\beta_1^2 - H_{22}\beta_2^2 + H_{23}\beta_3\beta_4), \tag{30k}$$

$$A_{35} = A_{53} = \rho\pi(H_6\beta_5 + 2H_{22}\beta_1\beta_2 + H_{23}\beta_3\beta_5), \tag{30l}$$

$$A_{44} = \rho\pi(\mu_4E_{19} + H_7\beta_3 + H_{24}\beta_1^2 + H_{24}\beta_2^2 + H_{25}\beta_3^2 + H_{26}\beta_4^2 + H_{27}\beta_5^2), \tag{30m}$$

$$A_{45} = A_{54} = \rho\pi(H_{28}\beta_4\beta_5), \tag{30n}$$

$$A_{55} = \rho\pi(\mu_4E_{19} + H_7\beta_3 + H_{24}\beta_1^2 + H_{24}\beta_2^2 + H_{25}\beta_3^2 + H_{27}\beta_4^2 + H_{26}\beta_5^2), \tag{30o}$$

其中系数 H 列于附录•

为得到 R_n, 我们首先将其表达为如下形式:

$$R_n = \sum_i c_i\dot\beta_i + \sum_{ij} c_{ij}\dot\beta_i\beta_j + \sum_{ijk} c_{ijk}\dot\beta_i\beta_j\beta_k• \tag{31}$$

然后利用渐近方法, 将式(31) 代入方程(22), 通过合并相同项, 便可求出系数 c_i、c_{ij} 和 c_{ijk}• 考虑到最终的模态系统要保留到 $O(\varepsilon)$ 阶, 因此保留 R_n 到 $O(\varepsilon)$ 阶:

$$R_1 = D_1\dot\beta_1 + D_4\dot\beta_1\beta_3 + D_5\beta_1\dot\beta_3 + D_6\dot\beta_1\beta_4 + D_7\beta_1\dot\beta_4 + D_6\dot\beta_2\beta_5 + D_7\beta_2\dot\beta_5 + D_{14}\beta_1^2\dot\beta_1 + D_{15}\dot\beta_1\beta_2^2 + D_{16}\beta_1\beta_2\dot\beta_2, \tag{32a}$$

$$R_2 = D_1\dot\beta_2 + D_4\dot\beta_2\beta_3 + D_5\beta_2\dot\beta_3 + D_6\dot\beta_1\beta_5 + D_7\beta_1\dot\beta_5 - D_6\dot\beta_2\beta_4 - D_7\beta_2\dot\beta_4 + D_{14}\beta_2^2\dot\beta_2 + D_{15}\dot\beta_2\beta_1^2 + D_{16}\beta_1\beta_2\dot\beta_1, \tag{32b}$$

$$R_3 = D_2\dot\beta_3 + D_8\beta_1\dot\beta_1 + D_8\beta_2\dot\beta_2, \tag{32c}$$

$$R_4 = D_3\dot\beta_4 + D_{11}\beta_1\dot\beta_1 - D_{11}\beta_2\dot\beta_2, \tag{32d}$$

$$R_5 = D_3\dot\beta_5 + D_{11}\dot\beta_1\beta_2 + D_{11}\beta_1\dot\beta_2, \tag{32e}$$

其中系数 D 列于附录•

将式(27) 至式(32) 代入方程(23) 并保留项数至 $O(\varepsilon)$ 阶, 我们最终得到描述圆柱贮箱液体自由晃动的二阶非线性常微分方程组, 即模态系统:

$$\ddot\beta_1 + \sigma_1^2\beta_1 + K_1(\beta_1\dot\beta_1^2 + \ddot\beta_1\beta_1^2 + \beta_1\dot\beta_2^2 + \beta_1\beta_2\ddot\beta_2) + K_2(\ddot\beta_1\beta_2^2 + 2\dot\beta_1\dot\beta_2\beta_2 - 2\beta_1\dot\beta_2^2 - \beta_1\beta_2\ddot\beta_2) + K_3(\ddot\beta_1\beta_4 + \dot\beta_1\dot\beta_4 + \ddot\beta_2\beta_5 + \dot\beta_2\dot\beta_5) - K_4(\beta_1\ddot\beta_4 + \beta_2\ddot\beta_5) + K_5(\ddot\beta_1\beta_3 + \dot\beta_1\dot\beta_3) + K_6\beta_1\ddot\beta_3 = 0, \tag{33a}$$

$$\ddot\beta_2 + \sigma_1^2\beta_2 + K_1(\beta_2\dot\beta_2^2 + \ddot\beta_2\beta_2^2 + \beta_2\dot\beta_1^2 + \beta_1\beta_2\ddot\beta_1) + K_2(\ddot\beta_2\beta_1^2 + 2\dot\beta_1\dot\beta_2\beta_1 - 2\beta_2\dot\beta_1^2 - \beta_1\beta_2\ddot\beta_1) + K_3(\ddot\beta_1\beta_5 + \dot\beta_1\dot\beta_5 - \ddot\beta_2\beta_4 - \dot\beta_2\dot\beta_4) + K_4(\beta_2\ddot\beta_4 - \beta_1\ddot\beta_5) + K_5(\ddot\beta_2\beta_3 + \dot\beta_2\dot\beta_3) + K_6\beta_2\ddot\beta_3 = 0, \tag{33b}$$

$$\ddot\beta_3 + \sigma_0^2\beta_3 + K_8(\dot\beta_1^2 + \dot\beta_2^2) + K_{10}(\beta_1\ddot\beta_1 + \beta_2\ddot\beta_2) = 0, \tag{33c}$$

$$\ddot\beta_4 + \sigma_2^2\beta_4 + K_7(\dot\beta_2^2 - \dot\beta_1^2) + K_9(\beta_2\ddot\beta_2 - \beta_1\ddot\beta_1) = 0, \tag{33d}$$

$$\ddot\beta_5 + \sigma_2^2\beta_5 - 2K_7\dot\beta_1\dot\beta_2 - K_9(\beta_2\ddot\beta_1 + \beta_1\ddot\beta_2) = 0, \tag{33e}$$

其中系数 K 列于附录•

3　数值仿真及讨论

对于线性晃动, 自由液面波高响应为

$$\eta = AJ_1(\kappa_{11}r)\cos\theta\cos\sigma_1 t, \tag{34}$$

其中, A 为波高幅值, 由初始条件确定•

对于方程组(33), 采用四阶 Runge-Kutta 方法进行数值积分, 贮箱半径 $a = 1$ m, 平均充液深度 $h = 1$ m, 取初始线性波高作为初始条件:

$$\beta_1(0) = A,\quad \beta_i(0) = 0,\quad i \neq 1,\quad \dot\beta_i(0) = 0,\quad i = 1, \cdots, 5; \tag{35}$$

A 分别取为 0.02 m, 0.1 m, 0.4 m•

图 1~ 3 为不同初始波高条件下 $r = 1, \theta = 0$ 处的自由液面线性波高响应和非线性波高响应时间曲线• 从图中可见, 当初始波高幅值很小时 $(A/h \leqslant 1/10)$, 根据多维模态方法计算出的波高响应和线性理论求解结果几乎相同; 而当初始波高幅值逐渐增大时, 表现出典型的非线性现象: a) 波峰大于波谷; b) 晃动频率小于线性晃动固有频率, 相同的现象存在于二维矩形贮箱[2]•

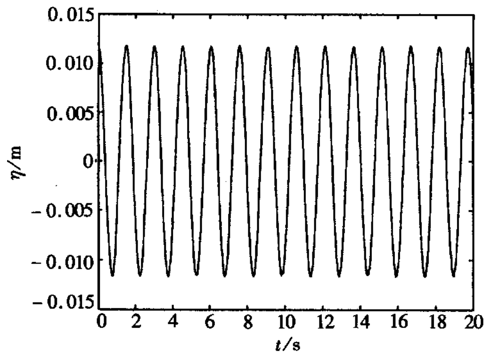

图 1　$A = 0.02$ m 时 $r = 1, \theta = 0$ 处的自由液面波高响应时间曲线

图 4 和图 5 表示 $A = 0.02$ m 和 $A = 0.4$ m 时, 半个晃动周期内, $\theta = 0$ 截面在不同时刻的波形图• 从图中可见, 当初始波高幅值较大时, 由于发生耗散现象, 线性意义下的波节$(\theta = \pi/2)$ 在非线性晃动波中将不复存在• 此外, 不同于小幅线性晃动, 在这种情况下, 液面永远不会达到平面状态• Admire[18] 采用逐步近似法求解了任意轴对称形状贮箱液体有限幅自由晃动问题, 得出了同样的结论•

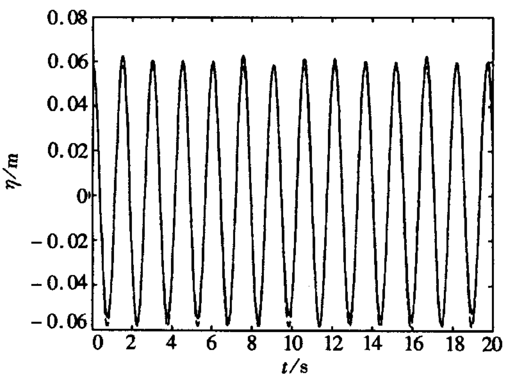

图 2　同图 1 但 $A = 0.1$ m

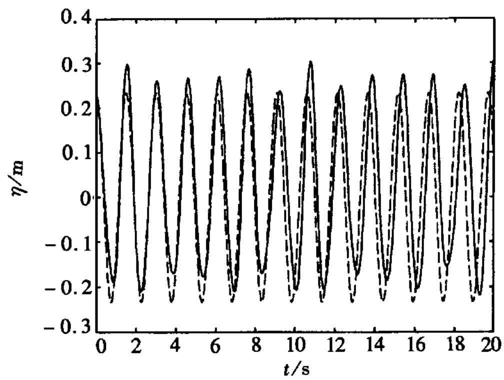

图 3　同图 1 但 $A = 0.4$ m

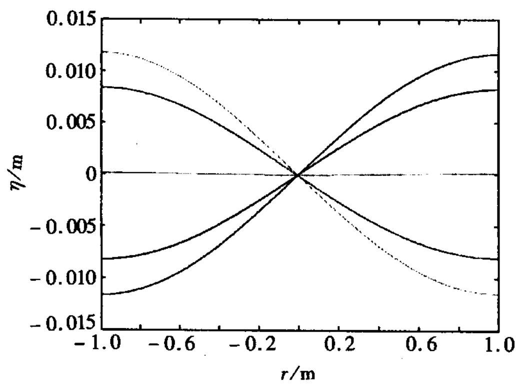

图 4　$A = 0.02$ m 时半个晃动周期内 $\theta = 0$ 截面不同时刻的波形图

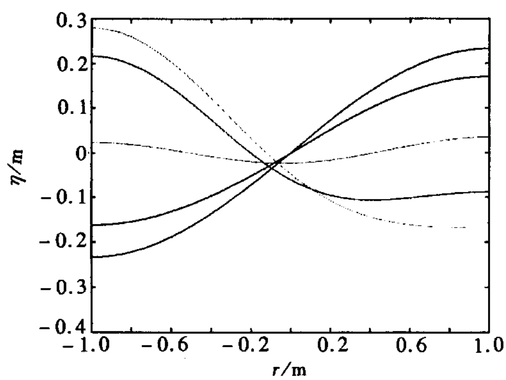

图 5　同图 4 但 $A = 0.4$ m

4　结　　论

利用多维模态理论研究了圆柱贮箱液体非线性自由晃动问题,结论如下:

1. 推导出了描述圆柱贮箱液体自由晃动的五维渐近模态系统(包括两阶主模态和三阶次模态)•

2. 通过对所得模态系统的数值积分,揭示了重要的非线性现象,如波高峰值和波相的变化、耗散现象的发生等•

3. 多维模态方法非常省时并且数值稳定,在我们下面的工作中,将继续发展这种方法,用来求解更为复杂的液体晃动问题•

致谢　感谢挪威理工大学 O. M. Faltinsen 教授在公式推导中提供的讨论和建议•

附　　录

1. 系数 E

$$E_1 = \int_0^a rJ_1^2(\kappa_{11}r)J_0(\kappa_{01}r)\mathrm{d}r,\quad E_2 = \int_0^a rJ_1^2(\kappa_{11}r)J_2(\kappa_{21}r)\mathrm{d}r,$$

$$E_3 = \int_0^a rJ_0^3(\kappa_{01}r)\mathrm{d}r,\quad E_4 = \int_0^a rJ_2^2(\kappa_{21}r)J_0(\kappa_{01}r)\mathrm{d}r,\quad E_5 = \int_0^a rJ_1^4(\kappa_{11}r)\mathrm{d}r,$$

$$E_6 = \int_0^a rJ_1^2(\kappa_{11}r)J_0^2(\kappa_{01}r)\mathrm{d}r,\quad E_7 = \int_0^a rJ_1^2(\kappa_{11}r)J_2^2(\kappa_{21}r)\mathrm{d}r,$$

$$E_8 = \int_0^a rJ_1^2(\kappa_{11}r)J_0(\kappa_{01}r)J_2(\kappa_{21}r)\mathrm{d}r,\quad E_9 = \int_0^a rJ_1^4(\kappa_{01}r)\mathrm{d}r,$$

$$E_{10} = \int_0^a rJ_0^2(\kappa_{01}r)J_2^2(\kappa_{21}r)\mathrm{d}r,\quad E_{11} = \int_0^a rJ_2^4(\kappa_{21}r)\mathrm{d}r,$$

$$E_{12} = \int_0^a r(J_1'(\kappa_{11}r))^2\mathrm{d}r,\quad E_{13} = \int_0^a r^{-1}J_1^2(\kappa_{11}r)\mathrm{d}r,\quad E_{14} = \int_0^a rJ_1^2(\kappa_{11}r)\mathrm{d}r,$$

$$E_{15} = \int_0^a r(J_0'(\kappa_{01}r))^2\mathrm{d}r,\quad E_{16} = \int_0^a rJ_0^2(\kappa_{01}r)\mathrm{d}r,\quad E_{17} = \int_0^a r(J_2'(\kappa_{21}r))^2\mathrm{d}r,$$

$$E_{18} = \int_0^a r^{-1}J_2^2(\kappa_{21}r)\mathrm{d}r,\quad E_{19} = \int_0^a rJ_2^2(\kappa_{21}r)\mathrm{d}r,\quad E_{20} = \int_0^a r(J_1'(\kappa_{11}r))^2J_0(\kappa_{01}r)\mathrm{d}r,$$

$$E_{21} = \int_0^a r(J_1'(\kappa_{11}r))^2J_2(\kappa_{21}r)\mathrm{d}r,\quad E_{22} = \int_0^a r^{-1}J_1^2(\kappa_{11}r)J_0(\kappa_{01}r)\mathrm{d}r,$$

$$E_{23} = \int_0^a r^{-1}J_1^2(\kappa_{11}r)J_2(\kappa_{21}r)\mathrm{d}r,\quad E_{24} = \int_0^a rJ_1'(\kappa_{11}r)J_0'(\kappa_{01}r)J_1(\kappa_{11}r)\mathrm{d}r,$$

$$E_{25} = \int_0^a rJ_1'(\kappa_{11}r)J_2'(\kappa_{21}r)J_1(\kappa_{11}r)\mathrm{d}r,\quad E_{26} = \int_0^a r(J_0'(\kappa_{01}r))^2J_0(\kappa_{01}r)\mathrm{d}r,$$

$$E_{27} = \int_0^a r(J_2'(\kappa_{21}r))^2J_0(\kappa_{01}r)\mathrm{d}r,\quad E_{28} = \int_0^a r^{-1}J_2^2(\kappa_{21}r)J_0(\kappa_{01}r)\mathrm{d}r,$$

$$E_{29} = \int_0^a rJ_0'(\kappa_{01}r)J_2'(\kappa_{21}r)J_2(\kappa_{21}r)\mathrm{d}r,\quad E_{30} = \int_0^a r(J_1'(\kappa_{11}r))^2J_1^2(\kappa_{11}r)\mathrm{d}r,$$

$$E_{31} = \int_0^a r(J_1'(\kappa_{11}r))^2J_0^2(\kappa_{01}r)\mathrm{d}r,\quad E_{32} = \int_0^a r(J_1'(\kappa_{11}r))^2J_2^2(\kappa_{21}r)\mathrm{d}r,$$

$$E_{33} = \int_0^a r(J_1'(\kappa_{11}r))^2J_0(\kappa_{01}r)J_2(\kappa_{21}r)\mathrm{d}r,\quad E_{34} = \int_0^a r^{-1}J_1^4(\kappa_{11}r)\mathrm{d}r,$$

$$E_{35} = \int_0^a r^{-1}J_1^2(\kappa_{11}r)J_0^2(\kappa_{01}r)\mathrm{d}r,\quad E_{36} = \int_0^a r^{-1}J_1^2(\kappa_{11}r)J_2^2(\kappa_{21}r)\mathrm{d}r,$$

$$E_{37} = \int_0^a r^{-1}J_1^2(\kappa_{11}r)J_0(\kappa_{01}r)J_2(\kappa_{21}r)\mathrm{d}r,\quad E_{38} = \int_0^a rJ_1'(\kappa_{11}r)J_0'(\kappa_{01}r)J_0(\kappa_{01}r)J_1(\kappa_{11}r)\mathrm{d}r,$$

$$E_{39} = \int_0^a rJ_1'(\kappa_{11}r)J_0'(\kappa_{01}r)J_1(\kappa_{11}r)J_2(\kappa_{21}r)\mathrm{d}r,\quad E_{40} = \int_0^a rJ_1'(\kappa_{11}r)J_2'(\kappa_{21}r)J_0(\kappa_{01}r)J_1(\kappa_{11}r)\mathrm{d}r,$$

$$E_{41}=\int_0^a rJ_1'(\kappa_{11}r)J_2'(\kappa_{21}r)J_1(\kappa_{11}r)J_2(\kappa_{21}r)\,\mathrm{d}r,\quad E_{42}=\int_0^a r(J_0'(\kappa_{11}r))^2J_1^2(\kappa_{11}r)\,\mathrm{d}r,$$

$$E_{43}=\int_0^a r(J_0'(\kappa_{01}r))^2J_0^2(\kappa_{01}r)\,\mathrm{d}r,\quad E_{44}=\int_0^a r(J_0'(\kappa_{01}r))^2J_2^2(\kappa_{21}r)\,\mathrm{d}r,$$

$$E_{45}=\int_0^a rJ_0'(\kappa_{01}r)J_2'(\kappa_{21}r)J_1^2(\kappa_{11}r)\,\mathrm{d}r,\quad E_{46}=\int_0^a rJ_0'(\kappa_{01}r)J_2'(\kappa_{21}r)J_0(\kappa_{01}r)J_2(\kappa_{21}r)\,\mathrm{d}r,$$

$$E_{47}=\int_0^a rJ_0'(\kappa_{01}r)J_2'(\kappa_{21}r)J_1(\kappa_{11}r)J_2(\kappa_{21}r)\,\mathrm{d}r,\quad E_{48}=\int_0^a r(J_2'(\kappa_{21}r))^2J_1^2(\kappa_{11}r)\,\mathrm{d}r,$$

$$E_{49}=\int_0^a r(J_2'(\kappa_{21}r))^2J_0^2(\kappa_{01}r)\,\mathrm{d}r,\quad E_{50}=\int_0^a r(J_2'(\kappa_{21}r))^2J_2^2(\kappa_{21}r)\,\mathrm{d}r,$$

$$E_{51}=\int_0^a r^{-1}J_0^2(\kappa_{01}r)J_2^2(\kappa_{21}r)\,\mathrm{d}r,\quad E_{52}=\int_0^a r^{-1}J_2^4(\kappa_{21}r)\,\mathrm{d}r,$$

2. 系数 H

$$H_1=\kappa_1^2E_{20}+E_{22}+\mu_1^2E_1,\quad H_2=\frac{1}{2}(\kappa_1^2E_{21}-E_{23}+\mu_1^2E_2),\quad H_3=\kappa_1\kappa_3E_{24}+\mu_1\mu_3E_1,$$

$$H_4=\frac{1}{2}\kappa_1\kappa_4E_{25}+E_{23}+\frac{1}{2}\mu_1\mu_4E_2,\quad H_5=2(\kappa_3^2E_{26}+\mu_3^2E_3),\quad H_6=\kappa_3\kappa_4E_{29}+\mu_3\mu_4E_4,$$

$$H_7=\kappa_4^2E_{27}+4E_{28}+\mu_4^2E_4,\quad H_8=\mu_1\left(\frac{3}{4}\kappa_1^2E_{30}+\frac{1}{4}E_{34}+\frac{3}{4}\kappa_1^2E_5\right),$$

$$H_9=\mu_1\left(\frac{1}{4}\kappa_1^2E_{30}+\frac{3}{4}E_{34}+\frac{1}{4}\kappa_1^2E_5\right),\quad H_{10}=\mu_1(\kappa_1^2E_{31}+E_{35}+\kappa_1^2E_6),$$

$$H_{11}=\frac{1}{2}\mu_1(\kappa_1^2E_{32}+E_{36}+\kappa_1^2E_7),\quad H_{12}=\mu_1(\kappa_1^2E_{33}-E_{37}+\kappa_1^2E_8),$$

$$H_{13}=\frac{1}{2}\mu_1(\kappa_1^2E_{30}-E_{34}+\kappa_1^2E_5),\quad H_{14}=(\mu_1+\mu_3)\kappa_1\kappa_3E_{38}+(\kappa_1^2\mu_3+\kappa_3^2\mu_1)E_6,$$

$$H_{15}=\frac{1}{2}((\mu_1+\mu_3)\kappa_1\kappa_3E_{39}+(\kappa_1^2\mu_3+\kappa_3^2\mu_1)E_8),$$

$$H_{16}=\frac{1}{2}(\mu_1+\mu_4)\kappa_1\kappa_4E_{40}+(\mu_1+\mu_4)E_{37}+\frac{1}{2}(\kappa_1^2\mu_4+\kappa_4^2\mu_1)E_8,$$

$$H_{17}=\frac{1}{2}((\mu_1+\mu_4)\kappa_1\kappa_4E_{41}+(\kappa_1^2\mu_4+\kappa_4^2\mu_1)E_7),\quad H_{18}=(\mu_1+\mu_4)E_{36},$$

$$H_{19}=\mu_3\kappa_3^2(E_{42}+E_6),\quad H_{20}=2\mu_3\kappa_3^2(E_{43}+E_9),\quad H_{21}=\mu_3\kappa_3^2(E_{44}+E_{10}),$$

$$H_{22}=\frac{1}{4}((\mu_3+\mu_4)\kappa_3\kappa_4E_{45}+(\kappa_3^2\mu_4+\kappa_4^2\mu_3)E_8),$$

$$H_{23}=(\mu_3+\mu_4)\kappa_3\kappa_4E_{46}+(\kappa_3^2\mu_4+\kappa_4^2\mu_3)E_{10},$$

$$H_{24}=\mu_4\left(\frac{1}{2}\kappa_4^2E_{48}+2E_{36}+\frac{1}{2}\kappa_4^2E_7\right),\quad H_{25}=\mu_4(\kappa_4^2E_{49}+4E_{51}+\kappa_4^2E_{10}),$$

$$H_{26}=\mu_4\left(\frac{3}{4}\kappa_4^2E_{50}+E_{52}+\frac{3}{4}\kappa_4^2E_{11}\right),\quad H_{27}=\mu_4\left(\frac{1}{4}\kappa_4^2E_{50}+3E_{52}+\frac{1}{4}\kappa_4^2E_{11}\right),$$

$$H_{28}=\mu_4\left(\frac{1}{2}\kappa_4^2E_{50}-2E_{52}+\frac{1}{2}\kappa_4^2E_{11}\right).$$

3. 系数 D

$$D_1=\frac{1}{\mu_1},\quad D_2=\frac{1}{\mu_3},\quad D_3=\frac{1}{\mu_4},\quad D_4=\frac{\mu_1E_1-H_1D_1}{\mu_1E_{14}},\quad D_5=\frac{\mu_1E_1-H_3D_2}{\mu_1E_{14}},$$

$$D_6=\frac{0.5\mu_1E_2-H_2D_1}{\mu_1E_{14}},\quad D_7=\frac{0.5\mu_1E_2-H_4D_3}{\mu_1E_{14}},\quad D_8=\frac{\mu_3E_1-H_3D_1}{2\mu_3E_{16}},$$

$$D_{11}=\frac{0.5\mu_4E_2-H_4D_1}{\mu_4E_{19}},\quad D_{14}=\frac{(3/8)\kappa_1^2E_5-H_8D_1-H_3D_8-H_4D_{11}}{\mu_1E_{14}},$$

$$D_{15}=\frac{(1/8)\kappa_1^2E_5-H_9D_1-H_4D_{11}}{\mu_1E_{14}},\quad D_{16}=\frac{0.25\kappa_1^2E_5-H_{13}D_1-H_3D_8}{\mu_1E_{14}}.$$

4. 系数 K

$$K_1=\frac{\mu_1\mu_3E_1D_8}{E_{14}}+\frac{\mu_1\mu_4E_2D_{11}}{2E_{14}}+\frac{3\kappa_1^2E_5}{8E_{14}}+\mu_1D_{14},\quad K_2=\frac{\mu_1\mu_4E_2D_{11}}{2E_{14}}+\frac{\kappa_1^2E_5}{8E_{14}}+\mu_1D_{15},$$

$$K_3 = \frac{\mu_1 E_2}{2E_{14}} + \mu_1 D_6,\ K_4 = -\frac{\mu_1 \mu_4 E_2 D_3}{2E_{14}} - \mu_1 D_7,\ K_5 = \frac{\mu_1 E_1}{E_{14}} + \mu_1 D_4,$$

$$K_6 = \frac{\mu_1 E_1}{E_{14}} + \mu_1 D_5,\ K_7 = -\frac{\mu_4 H_2 D_1 D_1}{2E_{19}} - \mu_4 D_{11},\ K_8 = \frac{\mu_3 H_1 D_1 D_1}{4E_{16}} + \mu_3 D_8,$$

$$K_9 = -\frac{\mu_4 E_2}{2E_{19}} - \mu_4 D_{11},\ K_{10} = \frac{\mu_3 E_1}{2E_{16}} + \mu_3 D_8.$$

[参 考 文 献]

[1] Abramson H N. The dynamic behavior of liquids in moving containers[R]. NASA SP-106, 1966.

[2] Frandsen J B. Sloshing motions in excited tanks[J]. Journal of Computational Physics, 2004, **196**(1): 53-87.

[3] Wu G X, Ma Q W, Taylor R E. Numerical simulation of sloshing waves in a 3D tank based on a finite element method[J]. Applied Ocean Research, 1998, **20**(6): 337-355.

[4] Guillot M J. Application of a discontinuous Galerkin finite element method to liquid sloshing[J]. Journal of Offshore Mechanics and Arctic Engineering, 2006, **128**(1): 1-10.

[5] Kita E, Katsuragawa J, Kamiya N. Application of Trefftz-type boundary element method to simulation of two-dimensional sloshing problem[J]. Engineering Analysis With Boundary Elements, 2004, **28**(6): 677-683.

[6] Ushijima S. Three-dimensional arbitrary Lagrangian-Eulerian numerical prediction method for non-linear free surface oscillation[J]. International Journal for Numerical Methods in Fluids, 1998, **26**(5): 605-623.

[7] 岳宝增, 李俊峰. 三维液体非线性晃动及其复杂现象[J]. 力学学报, 2002, **34**(6): 949-955.

[8] 郑磊, 李俊峰, 王天舒, 等. 计算液体晃动 ALE 网格速度的高精度方法[J]. 力学与实践, 2007, **29**(1): 14-16.

[9] Narimanov G S. Movement of a tank partly filled by a fluid: the taking into account of non-smallness of amplitude[J]. Prikl Math Mech, 1957, **21**: 513-524.

[10] Moiseiev N N. On the theory of nonlinear vibrations of a liquid of finite volume[J]. Prikl Math Mech, 1958, **22**: 612-621.

[11] Hutton R E. An investigation of resonant, nonlinear, nonplanar free surface oscillations of a fluid [R]. NASA TN D-1870, 1963.

[12] Faltinsen O M. A nonlinear theory of sloshing in rectangular tanks[J]. Journal of Ship Research, 1974, **18**(4): 224-241.

[13] Ikeda T, Murakami S. Autoparametric resonances in a structure/fluid interaction system carrying a cylindrical liquid tank[J]. Journal of Sound and Vibration, 2005, **285**(3): 517-546.

[14] Miles J W. Internally resonant surface waves in a circular cylinder[J]. Journal of Fluid Mechanics, 1984, **149**: 1-14.

[15] Miles J W. Resonantly forced surface waves in a circular cylinder[J]. Journal of Fluid Mechanics, 1984, **149**: 15-31.

[16] Faltinsen O M, Rognebakke O F, Lukovsky I A, et al. Multidimensional modal analysis of nonlinear sloshing in a rectangular tank with finite water depth[J]. Journal of Fluid Mechanics, 2000, **407**: 201-234.

[17] Faltinsen O M, Rognebakke O F, Timolha A N. Resonant three dimensional nonlinear sloshing in a square-base basin[J]. Journal of Fluid Mechanics, 2003, **487**: 1-42.

[18] Admire J R. Nonlinear fluid oscillations in a partially filled axisymmetric container of general shape [R]. NASA TN D-5808, 1970.

Multidimensional Modal Analysis of Liquid Nonlinear Sloshing in Right Circular Cylindrical Tank

YU Yan-sheng[1], MA Xing-rui[2], WANG Ben-li[1]

(1. Research Center of Satellite Technology, Harbin Institute of Technology, Harbin 150001, P.R.China;

2. China Aerospace Science and Technology Corporation, Beijing 100830, P.R.China)

Abstract: The multidimensional modal theory proposed by Faltinsen, et al (2000) was applied to solve liquid nonlinear free sloshing in right circular cylindrical tank. After selecting the leading modes and fixing the order of magnitudes based on the Narimanov-Moiseiev third order asymptotic hypothesis, the general infinite dimensional modal system was reduced to a five dimensional asymptotic modal system (the system of second order nonlinear ordinary differential equations coupling the generalized time dependent coordinates of free surface wave elevation). The numerical integrations of this modal system discover most important nonlinear phenomena, which agree well with both pervious analytic theories and experimental observations. The results indicate that the multidimensional modal method is a very good tool for solving liquid nonlinear sloshing dynamics and will be developed to investigate more complex sloshing problem in our following work.

Key words: circular cylindrical tank; nonlinear free sloshing; multidimensional modal method; asymptotic modal system; dispersion effect