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纤维悬浮剪切湍流中纤维旋转扩散系数的理论研究*

高振宇[1], 林建忠[1,2], 李 俊[1]

(1. 浙江大学 力学系,流体传动及控制国家重点实验室, 杭州 310027;
2. 中国计量学院,杭州 310018)

(我刊编委林建忠来稿)

摘要: 对纤维悬浮剪切湍流中纤维旋转扩散系数进行了理论研究. 首先建立了流场不同脉动速度梯度间的相关矩函数,然后推导出了纤维旋转扩散系数的表达式, 该表达式依赖于特征长度、时间、速度和一个与壁面作用相关的无量纲参数. 得到的纤维旋转扩散系数可以应用于非均匀和非各向同性的湍流场,此外还可以推广到三维湍流场, 因而为纤维悬浮湍流场的研究提供了理论基础.

关 键 词: 纤维悬浮流; 剪切湍流; 旋转扩散系数; 表达式
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引 言

纤维悬浮流在工业部门中已发现越来越多的应用,如复合材料的加工以及环境、化工、纺织工业和造纸工业的应用. 悬浮流的流场特性在纤维产品的质量方面起着重要的作用. 与圆球粒子不同的是,悬浮流中具有非圆球形状的纤维即使是在非常低的浓度下,也会对溶剂流体产生大的作用. 纤维的空间和取向分布依赖于流体的运动,反之亦然. 尽管目前已有大量研究层流情况下纤维悬浮流的结果, 但对工程中非常普遍的纤维悬浮湍流的了解却还不够深入,其原因是湍流场和纤维运动的双重困难. 迄今为止只有为数不多的有关纤维悬浮湍流研究的结果. Asgharian[1] 研究了玻璃纤维在人们呼吸道中的沉积; Kagermann 等[2] 通过假设纤维的尺度小于流场最小涡尺度, 研究了湍流场中纤维的旋转和位移扩散; Krushkal 等[3] 用 Fokker-Planck 方程, 计算了湍流场中的取向分布函数; Bernstein 等[4] 直接测量了玻璃纤维在层流和湍流管道中的轴向和横向取向; Lin 等[5] 用单向耦合方法研究了纤维对湍流场特性的影响; Lin 等分别研究了纤维在具有拟序结构的混合层[6] 和湍流管道流[7] 中的取向分布.

研究纤维在流场中运动最常见的方法之一是欧拉方法, 即用对流-扩散方程(或 Fokker-

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作者简介: 高振宇(1979—) , 男, 沈阳人,博士;
林建忠(1958—) , 男, 福建建阳人, 教授, 博士生导师(联系人. Tel: + 86-571-86836009; E-mail: jzlin@ sfp. zju. edu. cn) .

Plank 方程) 计算纤维取向和位置的概率分布．在这一方法中, 纤维的旋转扩散系数是一个重要的参数．Olson 等[8-9] 在假设纤维和流体相对速度差可忽略的前提下, 获得了纤维的平移和旋转扩散系数．Gao 等[10] 在考虑 Stokes 力和虚假质量力平衡的情况下, 推导得到了纤维的平移和旋转扩散系数, 并讨论了长纤维和短纤维的扩散特性．

然而, 以上提及的研究只是涉及到均匀各向同性湍流, 实际上非均匀和非各向同性的湍流是非常普遍的．因此, 本文的目的是理论推导出纤维悬浮剪切湍流中纤维的旋转扩散系数．

1 纤维取向的对流-扩散方程和旋转扩散系数

纤维在湍流中存在平均运动和脉动, 且其运动是平移和旋转的组合．欧拉方法中, 在时间 t 处于位置且 r 具有取向 $\boldsymbol{p}$ 的纤维可以由概率分布函数 $\psi(r, \boldsymbol{p}, t)$ 表示, ψ 的演变由以下对流-扩散方程控制:

$$\frac{\partial \psi}{\partial t} = D_{\mathrm{r}} \nabla_{\mathrm{r}}^2 \psi - \nabla_{\mathrm{r}} \cdot (\omega \psi) + D_{\mathrm{t}} \nabla^2 \psi - \nabla \cdot (\boldsymbol{V} \psi), \tag{1}$$

式中 $\boldsymbol{V}$ 是纤维的平均位移速度, ω 是纤维角速度, 它与纤维旋转矢量有关, D_{t} 和 D_{r} 分别是位移和旋转扩散系数． 方程右边的项分别是旋转扩散、平均旋转、位移扩散和平均位移[11]． 方程(1) 中的 D_{t} 和 D_{r} 是未知参数且还没有现成的表达式． 湍流场中存在的困难是要建立扩散系数和湍流场特性之间的关系． 在 Olson 的研究中, 旋转扩散系数 D_{r} 表示为[8]:

$$D_{\mathrm{r}} = \frac{1}{2} \frac{\mathrm{d}\, \overline{\boldsymbol{p}'^2}}{\mathrm{d}t}, \tag{2}$$

式中 $\boldsymbol{p}$ 是平行于纤维主轴的单位矢量, $\boldsymbol{p}'$ 是其脉动部分.

2 剪切湍流中纤维的脉动角速度

在方程(2) 中, 旋转扩散系数 D_{r} 依赖于 $\boldsymbol{p}$, Jeffery[12] 给出了纤维取向的变化率 $\dot{\boldsymbol{p}}$ 公式:

$$\dot{\boldsymbol{p}} = -\omega \cdot \boldsymbol{p} + \lambda(\boldsymbol{D} \cdot \boldsymbol{p} - \boldsymbol{D} : \boldsymbol{ppp}), \tag{3}$$

式中 $\omega = (\nabla \boldsymbol{V} - \nabla \boldsymbol{V}^{\mathrm{T}})/2$ 是涡度矢量, $\boldsymbol{D} = (\nabla \boldsymbol{V} + \nabla \boldsymbol{V}^{\mathrm{T}})/2$ 是应变率张量, λ 是一常数, 定义为 $\lambda = (a^2 - 1)/(a^2 + 1)$, 其中 a 是纤维的长径比.

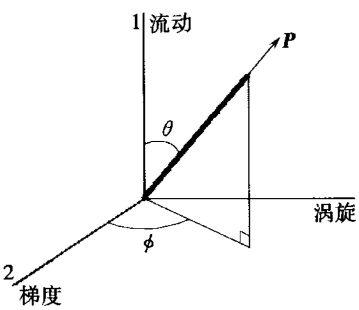

图 1 纤维和坐标系

如图 1 所示, 可以根据纤维的取向角 θ 和 ϕ, 将纤维的取向 $\boldsymbol{p}$ 表示成分量形式:

$$\begin{cases} p_1 = \cos\theta, \\ p_2 = \sin\theta\cos\phi, \\ p_3 = \sin\theta\sin\phi. \end{cases} \tag{4}$$

如果纤维的长径比趋向于无穷大, 则方程(3) 可以写成:

$$\dot{\boldsymbol{p}} = \boldsymbol{k} \cdot \boldsymbol{p} - \boldsymbol{k} : \boldsymbol{ppp}, \tag{5}$$

式中 $\boldsymbol{k} = \nabla \boldsymbol{V}^{\mathrm{T}}$ 是流体速度梯度张量.

对于图 1 所示的有限长径比的纤维, 方程(3) 可以写为:

$$\dot{\boldsymbol{p}} = (\lambda \boldsymbol{D} - \omega) \cdot \delta_r - (\lambda \boldsymbol{D} - \omega) : \delta_r \delta_r \delta_r = (\lambda \boldsymbol{D} - \omega) \cdot (\delta_r \delta_r + \delta_\theta \delta_\theta + \delta_\phi \delta_\phi) - (\lambda \boldsymbol{D} - \omega) : \delta_r \delta_r \delta_r = (\lambda \boldsymbol{D} - \omega) : \delta_r \delta_\theta \delta_\theta + (\lambda \boldsymbol{D} - \omega) : \delta_r \delta_\phi \delta_\phi, \tag{6}$$

式中单位矢量 $\delta = \delta_r \delta_r + \delta_\theta \delta_\theta + \delta_\phi \delta_\phi$, δ_θ 和 δ_ϕ 是 θ 和 ϕ 方向的单位矢量．以上方程的脉动部分为:

$$\dot{\boldsymbol p}' = (\lambda \boldsymbol D' - \boldsymbol\omega') : \delta_r \delta_\theta \delta_\theta + (\lambda \boldsymbol D' - \boldsymbol\omega') : \delta_r \delta_\phi \delta_\phi . \tag{7}$$

另一方面, $\dot{\boldsymbol p}$可以表示为:

$$\dot{\boldsymbol p} = \dot p_1 \boldsymbol e_1 + \dot p_2 \boldsymbol e_2 + \dot p_3 \boldsymbol e_3 =$$

$$-\sin\theta\cdot\dot\theta\boldsymbol e_1 + (\cos\theta\cos\phi\cdot\dot\theta - \sin\theta\sin\phi\cdot\dot\phi)\boldsymbol e_2 + (\cos\theta\sin\phi\cdot\dot\theta + \sin\theta\cos\phi\cdot\dot\phi)\boldsymbol e_3 =$$

$$\dot\theta\cdot(-\sin\theta\boldsymbol e_1 + \cos\theta\cos\phi\boldsymbol e_2 + \cos\theta\sin\phi\boldsymbol e_3) +$$

$$\sin\theta\cdot\dot\phi\cdot(-\sin\phi\boldsymbol e_2 + \cos\phi\boldsymbol e_3) = \dot\theta\delta_\theta + \sin\theta\cdot\dot\phi\,\delta_\phi .$$

相应的脉动部分为:

$$\dot{\boldsymbol p}' = \dot\theta'\delta_\theta + \sin\theta\cdot\dot\phi'\delta_\phi . \tag{8}$$

结合方程(7)和方程(8),可以得到脉动角速度:

$$\begin{cases} \dot\theta' = (\lambda\boldsymbol D' - \boldsymbol\omega') : \delta_r\delta_\theta, \\ \dot\phi' = \dfrac{1}{\sin\theta}(\lambda\boldsymbol D' - \boldsymbol\omega') : \delta_r\delta_\phi . \end{cases} \tag{9}$$

联立方程(8)和方程(9),可以得到:

$$\dot{\boldsymbol p}'^2 = \dot\theta'^2 + \sin^2\theta\cdot\dot\phi'^2 = [(\lambda\boldsymbol D' + \boldsymbol\omega') : \delta_r\delta_\theta]^2 + [(\lambda\boldsymbol D' + \boldsymbol\omega') : \delta_r\delta_\phi]^2 . \tag{10}$$

由笛卡儿坐标和球坐标的变换公式,可知:

$$\begin{cases} \delta_r = \cos\theta\boldsymbol e_1 + \sin\theta\cos\phi\boldsymbol e_2 + \sin\theta\sin\phi\boldsymbol e_3, \\ \delta_\theta = \dfrac{\partial\delta_r}{\partial\theta} = -\sin\theta\boldsymbol e_1 + \cos\theta\cos\phi\boldsymbol e_2 + \cos\theta\sin\phi\boldsymbol e_3, \\ \delta_\phi = \dfrac{1}{\sin\theta}\cdot\dfrac{\partial\delta_r}{\partial\phi} = 0\cdot\boldsymbol e_1 - \sin\phi\boldsymbol e_2 + \cos\phi\boldsymbol e_3, \end{cases} \tag{11}$$

于是可以得到:

$$\delta_r\delta_\theta = \begin{pmatrix} -\sin\theta\cos\theta & \cos^2\theta\cos\phi & \cos^2\theta\sin\phi \\ -\sin^2\theta\cos\phi & \sin\theta\cos\theta\cos^2\phi & \sin\theta\cos\theta\sin\phi\cos\phi \\ -\sin^2\theta\cos\phi & \sin\theta\cos\theta\sin\phi\cos\phi & \sin\theta\cos\theta\sin^2\phi \end{pmatrix}, \tag{12}$$

$$\delta_r\delta_\phi = \begin{pmatrix} 0 & -\cos\theta\sin\phi & \cos\theta\cos\phi \\ 0 & -\sin\theta\sin\phi\cos\phi & \sin\theta\cos^2\phi \\ 0 & -\sin\theta\sin^2\phi & \sin\theta\sin\phi\cos\phi \end{pmatrix}. \tag{13}$$

引入速度梯度张量 $\boldsymbol K_{ij} = \partial u_i/\partial x_j$ 可得:

$$\lambda\boldsymbol D' + \boldsymbol\omega' =$$

$$\frac{1}{2}\begin{pmatrix} 2\lambda K'_{11} & (\lambda+1)K'_{12} + (\lambda-1)K'_{21} & (\lambda+1)K'_{13} + (\lambda-1)K'_{31} \\ (\lambda-1)K'_{12} + (\lambda+1)K'_{21} & 2\lambda K'_{22} & (\lambda+1)K'_{23} + (\lambda-1)K'_{32} \\ (\lambda-1)K'_{13} + (\lambda+1)K'_{31} & (\lambda-1)K'_{23} + (\lambda+1)K'_{32} & 2\lambda K'_{33} \end{pmatrix},$$

二阶张量间的双点积为:

$$(\lambda\boldsymbol D + \boldsymbol\omega) : \delta\delta = (\lambda\boldsymbol D + \boldsymbol\omega)_{\cdot j}^{i}\cdot(\delta\delta)_{\cdot l}^{k}\boldsymbol g_i \boldsymbol g^j \boldsymbol g_k \boldsymbol g^l =$$

$$(\lambda\boldsymbol D + \boldsymbol\omega)_{\cdot j}^{i}\cdot(\delta\delta)_{\cdot l}^{k}\delta_{\cdot i}^{l}\delta_k^j = (\lambda\boldsymbol D + \boldsymbol\omega)_{\cdot j}^{i}\cdot(\delta\delta)_{\cdot i}^{j} .$$

对二维流场可以建立纤维脉动角速度与流场脉动速度的关系:

$$\dot\theta' = -\sin\theta\cos\theta\lambda K'_{11} - \frac{1}{2}\sin^2\theta\cos\phi[(\lambda+1)K'_{12} + (\lambda-1)K'_{21}] +$$

$$\frac{1}{2}\cos^2\theta\cos\phi[(\lambda-1)K'_{12} + (\lambda+1)K'_{21}] + \sin\theta\cos\theta\cos^2\phi\lambda K'_{22}, \tag{14}$$

$$\sin\theta\cdot\dot{\phi}' = -\frac{1}{2}\cos\theta\sin\phi[(\lambda-1)K'_{12}+(\lambda+1)K'_{21}] - \sin\theta\sin\phi\cos\phi\lambda K'_{22}. \tag{15}$$

将方程(14)、方程(15)代入方程(10)，可以得到 $\langle p'^2\rangle$ 的表达式，式中相关矩和系数列于表 1 中.

为了得到纤维脉动取向的均方值，需要推导以上脉动速度梯度的相关矩，即对脉动速度梯度取统计平均:

$$\bar{f} = \int_{-\infty}^{\infty}\int\cdots\int_{-\infty}^{\infty} f\cdot p(t,T)\,d\omega_1 t_1\,d\omega_2 t_2\,d\omega_3 t_3\,d\omega_1 T_1\,d\omega_2 T_2\,d\omega_3 T_3. \tag{16}$$

表 1 $\langle p'^2\rangle$ 表达式中的相关矩和系数

相关矩	系数
$(\partial u'/\partial x)^2$	$\lambda^2\sin^2\theta\cos^2\theta$
$(\partial u'/\partial y)^2$	$[(1/2)(\lambda-1)\cos^2\theta\cos\phi-(1/2)(\lambda+1)\sin^2\theta\cos\phi]^2+(1/4)(\lambda-1)^2\cos^2\theta\sin^2\phi$
$(\partial v'/\partial x)^2$	$[(1/2)(\lambda+1)\cos^2\theta\cos\phi-(1/2)(\lambda-1)\sin^2\theta\cos\phi]^2+(1/4)(\lambda+1)^2\cos^2\theta\sin^2\phi$
$(\partial v'/\partial y)^2$	$\lambda^2\sin^2\theta\cos^2\theta\cos^4\phi+\lambda^2\sin^2\theta\sin^2\phi\cos^2\phi$
$(\partial u'/\partial x)\cdot(\partial u'/\partial y)$	$\lambda\sin\theta\cos\theta\cos\phi(1-\lambda\cos2\theta)$
$(\partial u'/\partial x)\cdot(\partial v'/\partial x)$	$-\lambda\sin\theta\cos\theta\cos\phi(1+\lambda\cos2\theta)$
$(\partial u'/\partial x)\cdot(\partial v'/\partial y)$	$-2\lambda^2\sin^2\theta\cos^2\theta\cos^2\phi$
$(\partial u'/\partial y)\cdot(\partial v'/\partial y)$	$\lambda\sin\theta\cos\theta\cos\phi(-2\lambda\sin^2\theta\cos^2\phi+\lambda-1)$
$(\partial v'/\partial x)\cdot(\partial v'/\partial y)$	$\lambda\sin\theta\cos\theta\cos\phi(-2\lambda\sin^2\theta\cos^2\phi+\lambda+1)$
$(\partial u'/\partial y)\cdot(\partial v'/\partial x)$	$-2\lambda^2\sin^2\theta\cos^2\theta\cos^2\phi+(1/2)(\lambda^2-1)(\cos^2\theta-\cos^2\theta\cos^2\phi+\cos^2\phi)$

3 剪切湍流的随机理论

为了研究剪切湍流的时均和脉动流动结构，需要以物理现象为基础，引入反映湍流随机特性的假定，即根据脉动速度的概率分布，将流场脉动速度表示为[13]:

$$v_i' = v_i'' - \left(l'_{(i)m} + \frac{1}{2}r'_m\right)\frac{\partial v_i}{\partial x_m} - \frac{1}{2}r'_m\frac{\partial v_m}{\partial x_i}, \tag{17}$$

式中 v_i'' 是无任何时均速度梯度湍流场中的基本脉动流速，具有各向同性的性质，可以表示为与时均流速无关的脉动流场的函数; r' 表示讨论点至涡旋轴心的距离，与涡旋半径有关; $l'_{(i)}$ 表示轴心瞬时流速分量 v_i 在相对运动中保持不变的长度，类似于普朗特混合长度. 方程(17)不仅与基本脉动流速有关，而且也与涡旋的尺度有关. 与普朗特动量传递理论公式相比，方程(17)增加了基本脉动流速，这弥补了普朗特理论中时均流速梯度为零时脉动流速也为零的缺陷.

假定上述脉动速度的概率分布为:

$$v_i'' = U\omega_i t_i, \tag{18}$$

$$r_i' = \left[\frac{1}{2}(1+i)\,\omega_i T_i + m(-\omega_j^2 T_j^2 + \omega_k^2 T_k^2)\right]L, \tag{19}$$

$$l'_{(i)j} = 2m\omega_i T_i L + (1+i)\,\omega_j t_j U T, \tag{20}$$

式中 U、L 和 T 分别是特征速度、长度和时间; m 是与壁面作用有关的无量纲参数，所有以上 4 个参数依赖于空间坐标; ωt 和 ωT 分别表示基本脉动速度和涡半径在统计空间的矢量半径. 概率分布密度可以表示为:

$$p(t,T)=\left(\frac{1}{\sqrt{\pi}}\right)^{6}\exp[-\omega_l^2(t_l^2+T_l^2)],\tag{21}$$

$$\omega_l^2(t_l^2+T_l^2)=\omega_1^2(t_1^2+T_1^2)+\omega_2^2(t_2^2+T_2^2)+\omega_3^2(t_3^2+T_3^2),\tag{22}$$

式中 ω 是统计频率． 将以上方程用于二维平行剪切湍流, 假设湍流场沿 x 和 z 方向均匀, 即 $u(y)=\gamma y$ 以及 $v=w=0$, 此时脉动速度可以简化为:

$$u'=u''-\left(l'_{(1)2}+\frac{1}{2}r'_2\right),\quad v'=v''-\frac{1}{2}\gamma r'_1.\tag{23}$$

将方程(18) 至方程(20) 代入方程(23), 可以得到脉动速度的随机变量表达式:

$$u'=U\omega_1t_1-\gamma\Big[2mL\,\omega_1T_1+(1+i)UT\omega_2t_2+\frac{1}{4}(1+i)L\,\omega_2T_2+\frac{1}{2}mL(-\omega_3^2T_3^2+\omega_1^2T_1^2)\Big],\tag{24}$$

$$v'=U\omega_2t_2-\gamma\Big[\frac{1}{4}(1+i)L\,\omega_1T_1+\frac{1}{2}mL(-\omega_2^2T_2^2+\omega_3^2T_3^2)\Big].\tag{25}$$

4 纤维的旋转扩散系数

将方程(24)、方程(25) 对 x 和 y 求导, 可以得到脉动速度的梯度:

$$\frac{\partial u'}{\partial x}=\frac{\partial U}{\partial x}\omega_1t_1-\gamma\Big[2\frac{\partial(mL)}{\partial x}\omega_1T_1+(1+i)\frac{\partial(UT)}{\partial x}\omega_2t_2\Big]-\gamma\Big[\frac{1}{4}(1+i)\frac{\partial L}{\partial x}\omega_2T_2+\frac{1}{2}\frac{\partial(mL)}{\partial x}(-\omega_3^2T_3^2+\omega_1^2T_1^2)\Big],\tag{26}$$

$$\frac{\partial u'}{\partial y}=\frac{\partial U}{\partial y}\omega_1t_1-\gamma\Big[2\frac{\partial(mL)}{\partial y}\omega_1T_1+(1+i)\frac{\partial(UT)}{\partial y}\omega_2t_2\Big]-\gamma\Big[\frac{1}{4}(1+i)\frac{\partial L}{\partial y}\omega_2T_2+\frac{1}{2}\frac{\partial(mL)}{\partial y}(-\omega_3^2T_3^2+\omega_1^2T_1^2)\Big],\tag{27}$$

$$\frac{\partial v'}{\partial x}=\frac{\partial U}{\partial x}\omega_2t_2-\gamma\Big[\frac{1}{4}(1+i)\frac{\partial L}{\partial x}\omega_1T_1+\frac{1}{2}\frac{\partial(mL)}{\partial x}(-\omega_2^2T_2^2+\omega_3^2T_3^2)\Big],\tag{28}$$

$$\frac{\partial v'}{\partial y}=\frac{\partial U}{\partial y}\omega_2t_2-\gamma\Big[\frac{1}{4}(1+i)\frac{\partial L}{\partial y}\omega_1T_1+\frac{1}{2}\frac{\partial(mL)}{\partial y}(-\omega_2^2T_2^2+\omega_3^2T_3^2)\Big].\tag{29}$$

将方程(26) 至方程(29) 代入方程(16) 并积分, 可以得到脉动速度梯度的统计平均值:

$$\overline{\left(\frac{\partial u'}{\partial x}\right)^2}=\frac{9}{4}\gamma^2\left(\frac{\partial mL}{\partial x}\right)^2+\frac{1}{16}\gamma^2\left(\frac{\partial L}{\partial x}\right)^2 i+\gamma^2\frac{\partial UT}{\partial x}i+\frac{1}{2}\left(\frac{\partial U}{\partial x}\right)^2,$$

$$\overline{\left(\frac{\partial u'}{\partial y}\right)^2}=\frac{9}{4}\gamma^2\left(\frac{\partial mL}{\partial y}\right)^2+\frac{1}{16}\gamma^2\left(\frac{\partial L}{\partial y}\right)^2 i+\gamma^2\frac{\partial UT}{\partial y}i+\frac{1}{2}\left(\frac{\partial U}{\partial y}\right)^2,$$

$$\overline{\left(\frac{\partial v'}{\partial x}\right)^2}=\frac{1}{4}\gamma^2\left(\frac{\partial mL}{\partial x}\right)^2+\frac{1}{16}\gamma^2\left(\frac{\partial L}{\partial x}\right)^2 i+\frac{1}{2}\left(\frac{\partial U}{\partial x}\right)^2,$$

$$\overline{\left(\frac{\partial v'}{\partial y}\right)^2}=\frac{1}{4}\gamma^2\left(\frac{\partial mL}{\partial y}\right)^2+\frac{1}{16}\gamma^2\left(\frac{\partial L}{\partial y}\right)^2 i+\frac{1}{2}\left(\frac{\partial U}{\partial y}\right)^2,$$

$$\overline{\frac{\partial u'}{\partial x}\cdot\frac{\partial v'}{\partial x}}=-\frac{1}{8}\gamma^2\left(\frac{\partial mL}{\partial x}\right)^2+\frac{1}{4}\gamma^2\frac{\partial L}{\partial x}\cdot\frac{\partial mL}{\partial x}(1+i)-\frac{1}{2}\gamma\frac{\partial U}{\partial x}\cdot\frac{\partial UT}{\partial x}(1+i),$$

$$\overline{\frac{\partial u'}{\partial y}\cdot\frac{\partial v'}{\partial y}}=-\frac{1}{8}\gamma^2\left(\frac{\partial mL}{\partial y}\right)^2+\frac{1}{4}\gamma^2\frac{\partial L}{\partial y}\cdot\frac{\partial mL}{\partial y}(1+i)-\frac{1}{2}\gamma\frac{\partial U}{\partial y}\cdot\frac{\partial UT}{\partial y}(1+i),$$

$$\overline{\frac{\partial u'}{\partial x}\cdot\frac{\partial v'}{\partial y}}=-\frac{1}{8}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial mL}{\partial y}+\frac{1}{4}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial L}{\partial y}(1+i)-\frac{1}{2}\gamma\frac{\partial UT}{\partial x}\cdot\frac{\partial U}{\partial y}(1+i),$$

$$\overline{\frac{\partial u'}{\partial y}\cdot\frac{\partial v'}{\partial x}}=-\frac{1}{8}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial mL}{\partial y}+\frac{1}{4}\gamma^2\frac{\partial mL}{\partial y}\cdot\frac{\partial L}{\partial x}(1+i)-\frac{1}{2}\gamma\frac{\partial UT}{\partial y}\cdot\frac{\partial U}{\partial x}(1+i),$$

$$\overline{\frac{\partial u'}{\partial x}\cdot\frac{\partial u'}{\partial y}} = \frac{9}{4}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial mL}{\partial y} + \frac{1}{16}\gamma^2\frac{\partial L}{\partial x}\cdot\frac{\partial L}{\partial y}\mathrm{i} + \gamma^2\frac{\partial UT}{\partial x}\cdot\frac{\partial UT}{\partial y}\mathrm{i} + \frac{1}{2}\frac{\partial U}{\partial x}\cdot\frac{\partial U}{\partial y},$$

$$\overline{\frac{\partial v'}{\partial x}\cdot\frac{\partial v'}{\partial y}} = \frac{1}{4}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial mL}{\partial y} + \frac{1}{16}\gamma^2\frac{\partial L}{\partial x}\cdot\frac{\partial L}{\partial y}\mathrm{i} + \frac{1}{2}\frac{\partial U}{\partial x}\cdot\frac{\partial U}{\partial y}.$$

取上式的实值部分, 即得到各脉动速度梯度间的相关矩:

$$\overline{\left(\frac{\partial u'}{\partial x}\right)^2} = \frac{9}{4}\gamma^2\left(\frac{\partial mL}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial U}{\partial x}\right)^2, \tag{30}$$

$$\overline{\left(\frac{\partial u'}{\partial y}\right)^2} = \frac{9}{4}\gamma^2\left(\frac{\partial mL}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial U}{\partial y}\right)^2, \tag{31}$$

$$\overline{\left(\frac{\partial v'}{\partial x}\right)^2} = \frac{1}{4}\gamma^2\left(\frac{\partial mL}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial U}{\partial x}\right)^2, \tag{32}$$

$$\overline{\left(\frac{\partial v'}{\partial y}\right)^2} = \frac{1}{4}\gamma^2\left(\frac{\partial mL}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial U}{\partial y}\right)^2, \tag{33}$$

$$\overline{\frac{\partial u'}{\partial x}\cdot\frac{\partial v'}{\partial x}} = -\frac{1}{8}\gamma^2\left(\frac{\partial mL}{\partial x}\right)^2 + \frac{1}{4}\gamma^2\frac{\partial L}{\partial x}\cdot\frac{\partial mL}{\partial x} - \frac{1}{2}\gamma\frac{\partial U}{\partial x}\cdot\frac{\partial UT}{\partial x}, \tag{34}$$

$$\overline{\frac{\partial u'}{\partial y}\cdot\frac{\partial v'}{\partial y}} = -\frac{1}{8}\gamma^2\left(\frac{\partial mL}{\partial y}\right)^2 + \frac{1}{4}\gamma^2\frac{\partial L}{\partial y}\cdot\frac{\partial mL}{\partial y} - \frac{1}{2}\gamma\frac{\partial U}{\partial y}\cdot\frac{\partial UT}{\partial y}, \tag{35}$$

$$\overline{\frac{\partial u'}{\partial x}\cdot\frac{\partial v'}{\partial y}} = -\frac{1}{8}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial mL}{\partial y} + \frac{1}{4}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial L}{\partial y} - \frac{1}{2}\gamma\frac{\partial UT}{\partial x}\cdot\frac{\partial U}{\partial y}, \tag{36}$$

$$\overline{\frac{\partial u'}{\partial y}\cdot\frac{\partial v'}{\partial x}} = -\frac{1}{8}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial mL}{\partial y} + \frac{1}{4}\gamma^2\frac{\partial mL}{\partial y}\cdot\frac{\partial L}{\partial x} - \frac{1}{2}\gamma\frac{\partial UT}{\partial y}\cdot\frac{\partial U}{\partial x}, \tag{37}$$

$$\overline{\frac{\partial u'}{\partial x}\cdot\frac{\partial u'}{\partial y}} = \frac{9}{4}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial mL}{\partial y} + \frac{1}{2}\frac{\partial U}{\partial x}\cdot\frac{\partial U}{\partial y}, \tag{38}$$

$$\overline{\frac{\partial v'}{\partial x}\cdot\frac{\partial v'}{\partial y}} = \frac{1}{4}\gamma^2\frac{\partial mL}{\partial x}\cdot\frac{\partial mL}{\partial y} + \frac{1}{2}\frac{\partial U}{\partial x}\cdot\frac{\partial U}{\partial y}. \tag{39}$$

根据表 1 中的相关矩和系数可计算 $\overline{\dot{p}'^2}$. 对于非常小的旋转扩散($\overline{p'^2} \ll 1$), 旋转扩散的均方值为:

$$\overline{p'^2} = \int_0^t (t - \tau)\,\overline{\dot{p}'^2}\,\mathrm{d}\tau. \tag{40}$$

将方程(40)代入方程(2), 可以得到纤维的旋转扩散系数:

$$D_{\mathrm{r}} = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t (t - \tau)\,\overline{\dot{p}'^2}\,\mathrm{d}\tau. \tag{41}$$

5 结 论

建立了二维平行剪切湍流场中各脉动速度梯度的相关矩函数, 在此基础上, 推导出了旋转扩散系数表达式, 该式依赖于 4 个参数, 即特征长度、时间、速度和与壁面作用相关的无量纲参数. 这 4 个参数由出现在剪切湍流场运动方程中的具体参数确定. 推导出的旋转扩散系数表达式可应用于非均匀和非各向同性的湍流场, 并能扩展到三维湍流场, 因而为求解纤维悬浮湍流场提供了理论基础.

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Theoretical Research on the Rotational Dispersion Coefficient of Fiber in the Turbulent Shear Flow of Fiber Suspension

GAO Zhen-yu[1], LIN Jian-zhong[1,2], LI Jun[1]
(1. Department of Mechanics, State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou 310027, P.R.China;
2. China Metrology University, Hangzhou 310018, P.R.China)

Abstract: The rotational dispersion coefficient of the fiber in the turbulent shear flow of fiber suspension was studied theoretically. The function of correlation moment between the different fluctuating velocity gradient of the flow was built. Then the expression of rotational dispersion coefficient is derived. Which is dependent on the characteristic length, time, velocity and a dimensionless parameter related to the effect of wall. The derived expression of rotational dispersion coefficient can be employed to the inhomogeneous and non-isotropic turbulent flows. Furthermore it can be expanded to three-dimensional turbulent flows and serves as the theoretical basis for solving the turbulent flow of fiber suspension.

Key words: fiber suspension; turbulent shear flow; rotational dispersion coefficient; expression