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非线性多层渗流系统的数值方法及其应用*

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摘要: 对多层非线性渗流系统提出适合并行计算的迎风分数步差分格式, 利用变分形式、能量方法、差分算子乘积交换性、高阶差分算子的分解、先验估计的理论和技巧, 得到收敛性的最佳阶的误差估计. 该方法已成功的应用到油资源渗流力学运移聚集数值模拟的生产实际中.

关 键 词: 渗流系统; 非线性; 迎风分数步; 收敛性; 油资源数值模拟
中图分类号: O241.82 文献标识码: A

引 言

本文研究非线性多层渗流系统的数值方法. 为简便仅研究三层问题, 置于它们中间的层(弱渗透层)仅有垂直流动时, 需要求解下述一类三维多层对流扩散耦合系统的初边值问题[1~5]

$$\Phi_1(x,y,z)\frac{\partial u}{\partial t}+\boldsymbol{a}(x,y,z,t)\cdot\nabla u-\nabla\cdot(K_1(x,y,z,u,t)\nabla u)+K_2(x,y,z,w,t)\frac{\partial w}{\partial z}\Big|_{z=H_2}=Q_1(x,y,z,t,u),\quad (x,y,z)^{\mathrm T}\in\Omega_1,\ t\in J=(0,T], \tag{1a}$$

$$\Phi_2(x,y,z)\frac{\partial w}{\partial t}=\frac{\partial}{\partial z}\left(K_2(x,y,z,w,t)\frac{\partial w}{\partial z}\right),\quad (x,y,z)^{\mathrm T}\in\Omega_2,\ t\in J, \tag{1b}$$

$$\Phi_3(x,y,z)\frac{\partial v}{\partial t}+\boldsymbol{b}(x,y,z,t)\cdot\nabla v-\nabla\cdot(K_3(x,y,z,v,t)\nabla v)-K_2(x,y,z,w,t)\frac{\partial w}{\partial z}\Big|_{z=H_1}=Q_3(x,y,z,t,v),\quad (x,y,z)^{\mathrm T}\in\Omega_3,\ t\in J, \tag{1c}$$

此处 $\Omega=\bigcup_{i=1}^{3}\Omega_i$, $\Omega_1=\left\{(x,y)\in\Omega_0, H_2<z<H_3\right\}$, $\Omega_2=\left\{(x,y)\in\Omega_0, H_1<z<H_2\right\}$,

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$\Omega_3 = \left\{(x, y) \in \Omega_0, 0 < z < H_1\right\}$, Ω_0 为图 1所示的平面有界区域, $\partial\Omega$, $\partial\Omega_i (i = 1, 2, 3)$ 分别为 Ω 和 Ω_i 的外边界•

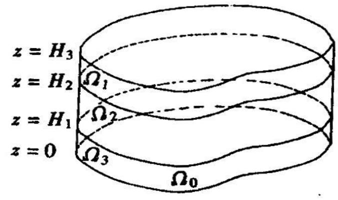

图 1　区域 Ω_1、Ω_2、Ω_3 示意图

初始条件

$$\begin{cases} u(x, y, z, 0) = \Psi_1(x, y, z), & (x, y, z)^T \in \Omega_1, \\ w(x, y, z, 0) = \Psi_2(x, y, z), & (x, y, z)^T \in \Omega_2, \\ v(x, y, z, 0) = \Psi_3(x, y, z), & (x, y, z)^T \in \Omega_3 \bullet \end{cases} \tag{2}$$

边界条件是第一型的

$$\begin{cases} u(x, y, z, t)\mid_{z = H_3, \partial\Omega_1} = 0, \\ w(x, y, z, t)\mid_{\partial\Omega_2} = 0, \\ v(x, y, z, t)\mid_{z = 0, \partial\Omega_3} = 0, \end{cases} \tag{3a}$$

$$\begin{cases} w(x, y, z, t)\mid_{z = H_2} = u(x, y, H_2, t), \\ w(x, y, z, t)\mid_{z = H_1} = v(x, y, H_1, t), \end{cases} \quad (x, y)^T \in \Omega_0 \bullet \ (\text{内边界条件}) \bullet \tag{3b}$$

在渗流力学上, 待求函数 u、w、v 为位势函数, ∇u、∇v、$\partial w/\partial z$ 为达西速度, $\Phi_\alpha (\alpha = 1, 2, 3)$ 为孔隙度函数, $K_1(x, y, z, u, t)$, $K_2(x, y, z, w, t)$ 和 $K_3(x, y, z, v, t)$ 为渗透率函数, $\boldsymbol{a}(x, y, z, t) = (a_1(x, y, z, t), a_2(x, y, z, t), a_3(x, y, z, t))^T$, $\boldsymbol{b}(x, y, z, t) = (b_1(x, y, z, t), b_2(x, y, z, t), b_3(x, y, z, t))^T$ 为相应的对流系数, $Q_1(x, y, z, t, u)$, $Q_3(x, y, z, t, v)$ 为产量项•

对于对流扩散问题已有 Douglas 和 Russell 的著名特征差分方法工作[6, 7], 它克服经典方法可能出现数值解的振荡和失真[1], [8, 9], 解决了用差分方法处理以对流为主的对流扩散问题• 但特征差分方法有着处理边界条件带来的计算复杂性[1], [7], Ewing, Lazarov 等提出用迎风差分格式来解决这类问题[10, 11]• 为解决大规模科学与工程计算(节点个数可多达数万乃至数十万个) 需要采用分数步技术, 将高维问题化为连续解几个一维问题的计算[8, 9], [12]• 文从油气资源勘探、开发和地下水渗流计算的实际问题出发, 研究多层地下渗流系统的数值方法, 提出适合并行计算的二阶耦合迎风分数步差分格式, 利用变分形式、能量方法、差分算子乘积交换性、高阶差分算子的分解、先验估计的理论和技巧, 对二阶格式得到收敛性的最佳阶 l^2 误差估计• 本文的方法已成功应用到三维油资源运移聚集数值模拟计算, 海水入侵预测和防治的工程实践中[13~ 15] ①②

通常问题是正定的, 即满足

$$0 < \Phi_* \leqslant \Phi_a \leqslant \Phi^*, \quad 0 < K_* \leqslant K_a \leqslant K^*, \qquad a = 1, 2, 3, \tag{4a}$$

$$\left|\frac{\partial K_1(u)}{\partial u}\right| + \left|\frac{\partial K_3(v)}{\partial v}\right| \left|\frac{\partial K_2(w)}{\partial w}\right| \leqslant K^*, \tag{4b}$$

此处 Φ_*、Φ^*、K_*、K^* 均为正常数•

假定问题(1) ~ (4) 的精确解是正则的, 即

$$\frac{\partial^2 u}{\partial t^2} \in L^\infty(L^\infty(\Omega_1)), \quad u \in L^\infty(W^{4,\infty}(\Omega_1)) \cap W^{1,\infty}(W^{1,\infty}(\Omega_1)),$$

① 山东大学数学研究所, 胜利油田计算中心: 多层油资源运移聚集定量数值模拟技术研究, 1999. 6.

② 山东大学数学研究所, 胜利石油管理局物探研究院: 油资源运移聚集精细数值模拟研究, 2003. 10.

$$\frac{\partial^2 v}{\partial t^2} \in L^\infty(L^\infty(\Omega_3)),\ v \in L^\infty(W^{4,\infty}(\Omega_3)) \cap W^{1,\infty}(W^{1,\infty}(\Omega_3)),$$
$$\frac{\partial^2 w}{\partial t^2} \in L^\infty(L^\infty(\Omega_2)),\ w \in L^\infty(W^{4,\infty}(\Omega_2)) \cap W^{1,\infty}(W^{1,\infty}(\Omega_2)),$$

且 $Q_1(x, y, z, t, u)$, $Q_3(x, y, z, t, v)$ 在解的 ε_0_邻域满足 Lipschitz 连续条件, 即存在常数 M, 当 $|\varepsilon_i| \leqslant \varepsilon_0(1 \leqslant i \leqslant 4)$ 时有

$$|Q_1(u(x,y,z,t)+\varepsilon_1) - Q_1(u(x,y,z,t)+\varepsilon_2)| \leqslant M|\varepsilon_1-\varepsilon_2|, \quad (x,y,z,t) \in \Omega_1 \times J,$$
$$|Q_3(v(x,y,z,t)+\varepsilon_3) - Q_3(v(x,y,z,t)+\varepsilon_4)| \leqslant M|\varepsilon_3-\varepsilon_4|, \quad (x,y,z,t) \in \Omega_3 \times J.$$

本文中记号 M 和 ε 分别表示普通的正常数和普通小的正数, 在不同处有不同的含义.

1 迎风分数步差分格式

为了用差分方法求解, 我们用网格区域 $\Omega_{0,h}$ 代替 Ω_0, 在平面 (x, y) 上步长为 h_1, $x_i = ih_1$, $y_j = jh_1$. 在 z 方向步长为 h_2, $z_k = H_2 + kh_2$, $h_2 = (H_3 - H_2)/N_1$, 此处 N_1 为某一正整数. $t^n = n\Delta t$, 用 $\Omega_{1,h}$ 代替 Ω_1. 用 $\partial\Omega_{1,h}$, $\partial\Omega_{0,h}$ 分别表示 $\Omega_{1,h}$ 和 $\Omega_{0,h}$ 的边界. 类似的, 对 Ω_2 在 z 方向的步长取为 h_3, $z_k = H_1 + kh_3$, $h_3 = (H_2 - H_1)/N_2$, 用 $\Omega_{2,h}$ 代替 Ω_2. 对 Ω_3 在 z 方向的步长取为 h_4, $z_k = kh_4$, $h_4 = H_1/N_3$, 同样用 $\Omega_{3,h}$ 代替 Ω_3. 此处 N_2、N_3 为正整数, $\partial\Omega_{2,h}$, $\partial\Omega_{3,h}$ 表示 $\Omega_{2,h}$, $\Omega_{3,h}$ 的外侧边界. 记 $U(x_i, y_j, z_k, t^n) = U^n_{ijk}$, $V(x_i, y_j, z_k, t^n) = V^n_{ijk}$, $W(x_i, y_j, z_k, t^n) = W^n_{ijk}$. 记 δ_x、δ_y、δ_z、$\delta_{\bar{x}}$、$\delta_{\bar{y}}$、$\delta_{\bar{z}}$ 分别为 x、y 和 z 方向的向前、向后差商算子, d_tU^n 为网格函数 U^n_{ijk} 在 $t = t_n$ 的向前差商.

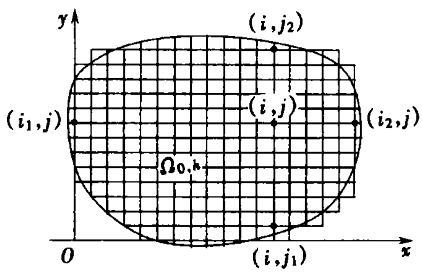

图 2 网域 $\Omega_{0,h}$ 示意图

为了得到高精度计算格式, 在 Ω_2 上对方程(1a) 在 $(x, y, z_{N_2-1/2}, t)$ 点展开

$$\left[K_2(x,y,z,w,t)\frac{\partial u}{\partial z}\right]_{N_2-1/2} = \left[K_2(x,y,z,u,t)\frac{\partial w}{\partial z}\right]_{N_2} - \frac{h_3}{2}\left[\frac{\partial}{\partial z}\left(K_2(x,y,z,u,t)\frac{\partial w}{\partial z}\right)\right]_{N_2} + O(h_3^2).$$

于是在 Ω_1 上在点 (x, y, H_2, t) 有

$$\Phi_1(x,y,z,H_2)\frac{\partial u}{\partial t} + \boldsymbol{a}(x,y,z,t)\cdot\nabla u - \nabla\cdot(K_1(x,y,z,u,t)\nabla u) = - \left[K_2(x,y,z,w,t)\frac{\partial w}{\partial z}\right]_{N_2-1/2} + Q_1(u) + O(h_3^2), \tag{5a}$$

此处 $\Phi_1(x,y,z,H_2) = \Phi_1(x,y,z) + (h_3/2)\Phi_2(x,y,H_2)$.

类似的在 Ω_3 上在点 (x, y, H_1, t) 有

$$\Phi_3(x,y,z,H_1)\frac{\partial v}{\partial t} + \boldsymbol{b}(x,y,z,t)\cdot\nabla v - \nabla\cdot(K_3(x,y,z,v,t)\nabla v) = \left[K_2(x,y,z,w,t)\frac{\partial w}{\partial z}\right]_{1/2} + Q_3(v) + O(h_3^2), \tag{5b}$$

此处 $\Phi_3(x,y,z,H_1) = \Phi_3(x,y,z) + (h_3/2)\Phi_2(x,y,H_1)$.

方程(5a)可近似分裂为

$$\left(1-\frac{\Delta t}{\Phi_1}\frac{\partial}{\partial x}\left(K_1\frac{\partial}{\partial x}\right)+\frac{\Delta t}{\Phi_1}a_1\frac{\partial}{\partial x}\right)\left(1-\frac{\Delta t}{\Phi_1}\frac{\partial}{\partial y}\left(K_1\frac{\partial}{\partial y}\right)+\frac{\Delta t}{\Phi_1}a_2\frac{\partial}{\partial y}\right)\left(1-\frac{\Delta t}{\Phi_1}\frac{\partial}{\partial z}\left(K_1\frac{\partial}{\partial z}\right)+\frac{\Delta t}{\Phi_1}a_3\frac{\partial}{\partial z}\right)u^{n+1}=$$
$$u^n-\frac{\Delta t}{\Phi_1}\left\{\left(K_2\frac{\partial w^{n+1}}{\partial z}\right)_{N_2-1/2}-Q_1(x,y,z,t^{n+1},u^{n+1})\right\}. \tag{6}$$

其对应的二阶迎风分数步差分格式为

$$\left(\Phi_1-\Delta t\left(1+\frac{h_1}{2}\frac{|a_1^n|}{K_1(U^n)}\right)^{-1}\delta_{\bar{x}}(K_1(U^n)\delta_x)+\Delta t\,\delta_{a_1^n,U^n,x}\right)U_{ijk}^{n+1/3}=$$
$$\Phi_{1,ijk}U_{ijk}^n+\Delta t\left\{-K_2(x_i,y_j,z_{N_2-1/2},U_{ij,N_2}^n,t^n)\delta_{\bar{z}}W_{ij,N_2}^{n+1}+Q_1(x_i,y_j,z_k,U_{ijk}^{n+1})\right\},\quad i_1(j)<i<i_2(j), \tag{7a}$$

$$U_{ijk}^{n+1/3}=0,\qquad (x_i,y_j,z_k)^{\mathrm T}\in\partial\Omega_{1,h}, \tag{7b}$$

$$\left(\Phi_1-\Delta t\left(1+\frac{h_1}{2}\frac{|a_2^n|}{K_1(U^n)}\right)^{-1}\delta_{\bar{y}}(K_1(U^n)\delta_y)+\Delta t\,\delta_{a_2^n,U^n,y}\right)U_{ijk}^{n+2/3}=$$
$$\Phi_{1,ijk}U_{ijk}^{n+1/3},\qquad j_1(i)<j<j_2(i), \tag{7c}$$

$$U_{ijk}^{n+2/3}=0,\qquad (x_i,y_j,z_k)^{\mathrm T}\in\partial\Omega_{1,h}, \tag{7d}$$

$$\left(\Phi_1-\Delta t\left(1+\frac{h_1}{2}\frac{|a_3^n|}{K_1(U^n)}\right)^{-1}\delta_{\bar{z}}(K_1(U^n)\delta_z)+\Delta t\,\delta_{a_3^n,U^n,z}\right)U_{ijk}^{n+1}=$$
$$\Phi_{1,ijk}U_{ijk}^{n+2/3},\qquad 0<k<N_1, \tag{7e}$$

$$U_{ijk}^{n+1}=0,\qquad (x_i,y_j,z_k)^{\mathrm T}\in\partial\Omega_{1,h}, \tag{7f}$$

此处

$$\delta_{a_1^n,U^n,x}u_{ijk}=a_{1,ijk}^n[H(a_{1,ijk}^n)K_1(U^n)_{ijk}^{-1}\cdot K_1(U^n)_{i-1/2,jk}\delta_{\bar{x}}+(1-H(a_{1,ijk}^n))K_1(U^n)_{ijk}^{-1}\cdot K_1(U^n)_{i+1/2,jk}\delta_x]u_{ijk},$$
$$\delta_{a_2^n,U^n,y}u_{ijk}=a_{2,ijk}^n[H(a_{2,ijk}^n)K_1(U^n)_{ijk}^{-1}\cdot K_1(U^n)_{i,j-1/2,k}\delta_{\bar{y}}+(1-H(a_{2,ijk}^n))K_1(U^n)_{ijk}^{-1}\cdot K_1(U^n)_{i,j+1/2,k}\delta_y]u_{ijk},$$
$$\delta_{a_3^n,U^n,z}u_{ijk}=a_{3,ijk}^n[H(a_{3,ijk}^n)K_1(U^n)_{ijk}^{-1}\cdot K_1(U^n)_{ij,k-1/2}\delta_{\bar{z}}+(1-H(a_{3,ijk}^n))K_1(U^n)_{ijk}^{-1}\cdot K_1(U^n)_{ij,k+1/2}\delta_z]u_{ijk},$$
$$H(z)=\begin{cases}1, & z\geqslant 0,\\ 0, & z<0.\end{cases}$$

在实际计算时式(7a)中 $\delta_{\bar{z}}W_{ij,N_2}^{n+1}$ 近似的取为 $\delta_{\bar{z}}W_{ij,N_2}^n$, U_{ijk}^{n+1} 近似的取为 U_{ijk}^n.

对方程(1b)的差分格式是

$$\Phi_{2,ijk}\frac{W_{ijk}^{n+1}-W_{ijk}^n}{\Delta t}=\delta_{\bar{z}}(K_2(W^n)\delta_z W^{n+1})_{ijk},\qquad 0<k<N_2;\ (i,j)\in\Omega_{0,h}. \tag{8}$$

方程(1c)可近似分裂为

$$\left(1-\frac{\Delta t}{\Phi_3}\frac{\partial}{\partial x}\left(K_3\frac{\partial}{\partial x}\right)+\frac{\Delta t}{\Phi_3}b_1\frac{\partial}{\partial x}\right)\left(1-\frac{\Delta t}{\Phi_3}\frac{\partial}{\partial y}\left(K_3\frac{\partial}{\partial y}\right)+\frac{\Delta t}{\Phi_3}b_2\frac{\partial}{\partial y}\right)\left(1-\frac{\Delta t}{\Phi_3}\frac{\partial}{\partial z}\left(K_3\frac{\partial}{\partial z}\right)+\frac{\Delta t}{\Phi_3}b_3\frac{\partial}{\partial z}\right)v_{ijk}^{n+1}=$$

$$v_{ijk}^{n+1} + \frac{\Delta t}{\Phi_3}\left\{\left(K_2\frac{\partial w^{n+1}}{\partial z}\right)_{1/2} + Q_3(x_i, y_j, z_k, t^{n+1}, v_{ijk}^{n+1})\right\}, \tag{9}$$

其对应的迎风分数步差分格式为

$$\left(\Phi_3 - \Delta t\left(1 + \frac{h_1}{2}\frac{|b_1^n|}{K_3(V^n)}\right)^{-1}\delta_x(K_3(V^n)\delta_{\bar{x}}) + \Delta t\,\delta_{b_1^n, V^n, x}\right)V_{ijk}^{n+1/3} =$$
$$\Phi_{3,ijk}V_{ijk}^n + \Delta t\Big\{K_2(x_i, y_j, z_{1/2}, V_{ij,0}^n, t^n)\,\delta_{\bar{z}}W_{ij,0}^{n+1} +$$
$$Q_3(x_i, y_j, z_k, t^n, V_{ijk}^{n+1})\Big\}, \qquad i_1(j) < i < i_2(j), \tag{10a}$$

$$V_{ijk}^{n+1/3} = 0, \qquad (x_i, y_j, z_k)^{\mathrm{T}} \in \partial\Omega_{3,h}, \tag{10b}$$

$$\left(\Phi_3 - \Delta t\left(1 + \frac{h_1}{2}\frac{|b_2^n|}{K_3(V^n)}\right)^{-1}\delta_y(K_3(V^n)\delta_{\bar{y}}) + \Delta t\,\delta_{b_2^n, V^n, y}\right)V_{ijk}^{n+2/3} =$$
$$\Phi_{3,ijk}V_{ijk}^{n+1/3}, \qquad j_1(i) < j < j_2(i), \tag{10c}$$

$$V_{ijk}^{n+2/3} = 0, \qquad (x_i, y_j, z_k)^{\mathrm{T}} \in \partial\Omega_{3,h}, \tag{10d}$$

$$\left(\Phi_3 - \Delta t\left(1 + \frac{h_1}{2}\frac{b_3^n}{K_3(V^n)}\right)^{-1}\delta_z(K_3(V^n)\delta_{\bar{z}}) + \Delta t\,\delta_{b_3^n, V^n, z}\right)V_{ijk}^{n+1} =$$
$$\Phi_{3,ijk}V_{ijk}^{n+2/3}, \qquad 0 < k < N_3, \tag{10e}$$

$$V_{ijk}^{n+1} = 0, \qquad (x_i, y_j, z_k)^{\mathrm{T}} \in \partial\Omega_{3,h}, \tag{10f}$$

此处算子 $\delta_{b_1^n, V^n, x}$, $\delta_{b_2^n, V^n, y}$, $\delta_{b_3^n, V^n, z}$, 的定义和 $\delta_{a_1^n, u^n, x}$, $\delta_{a_2^n, u^n, y}$, $\delta_{a_3^n, u^n, z}$ 是一样的• 在实际计算时(10a) 中 $\delta_{\bar{z}}w_{ij,0}^{n+1}$ 近似取为 $\delta_{\bar{z}}w_{ij,0}^n$, V_{ijk}^{n+1} 近似的取为 V_{ijk}^n•

差分格式(7), 差分格式(8)和差分格式(10)的计算程序是: 若已知时刻 $t = t^n$ 的差分解 $\{U_{ijk}^n, W_{ijk}^n, V_{ijk}^n\}$ 时, 寻求下一时刻 $t = t^{n+1}$ 的 $\{U_{ijk}^{n+1}, W_{ijk}^{n+1}, V_{ijk}^{n+1}\}$• 首先由式(7a), 式(7b)用追赶法求出过渡层的解 $\{U_{ijk}^{n+1/3}\}$, 再由式(7c), 式(7d) 求出 $\{U_{ijk}^{n+2/3}\}$, 以后再由式(7e), 式(7f) 求出 $\{U_{ijk}^{n+1}\}$• 与此同时可并行的由式(10a), 式(10b)用追赶法求出过渡层的解 $\{V_{ijk}^{n+1/3}\}$, 再由式(10c), 式(10d) 求出 $\{V_{ijk}^{n+2/3}\}$, 以后再由式(10e), 式(10f) 求出 $\{V_{ijk}^{n+1}\}$• 最后由式(8) 利用内边界条件(3b) 求出 $\{W_{ijk}^{n+1}\}$• 由正定性条件(4), 此差分解存在且唯一•

2 收敛性分析

为理论分析简便, 设 $\Omega = \{(x, y, z) \mid 0 < x < 1, 0 < y < 1, 0 < z < 3\}$, $\Omega_0 = \{(x, y) \mid 0 < x < 1, 0 < y < 1)\}$, $\Omega_1 = \{(x, y, z) \mid 0 < x < 1, 0 < y < 1, 2 < z < 3\}$, $\Omega_2 = \{(x, y, z) \mid 0 < x < 1, 0 < y < 1, 1 < z < 2\}$, $\Omega_3 = \{(x, y, z) \mid 0 < x < 1, 0 < y < 1, 0 < z < 1\}$, $h = 1/N$, $t^n = n\Delta t$• 定义网络函数空间 H_h 的内积[16~19]•

首先对格式(7)、格式(8)、格式(10)进行收敛性分析• 设 u、v、w 为问题(1) ~ (4) 的精确解, U、V、W 为格式(7)、格式(8)、格式(10) 的差分解, 记误差函数为 $\xi = u - U$, $\zeta = v - V$, $\omega = w - W$• 方程(7a) ~ 方程(7f) 消去 $U^{n+1/3}$, $U^{n+2/3}$, 若问题(1) ~ (4) 的精确解 u、v、w 是正则的, 则有下述误差方程

$$\Phi_{1,ijk}\frac{\xi_{ijk}^{n+1} - \xi_{ijk}^n}{\Delta t} - \left\{\left(1 + \frac{h}{2}\frac{|a_{1,ijk}^n|}{K_1(U^n)_{ijk}}\right)^{-1}\delta_x(K_1(U^n)\delta_{\bar{x}}\xi^{n+1})_{ijk} +\right.$$
$$\left(1 + \frac{h}{2}\frac{|a_{1,ijk}^n|}{K_1(U^n)_{ijk}}\right)^{-1}\delta_x([K_1(u^{n+1}) - K_1(U^n)]\delta_{\bar{x}}u^{n+1})_{ijk} +$$

$$\left[\left(1+\frac{h}{2}\frac{|a_{1,ijk}^{n+1}|}{K_1(u^{n+1})_{ijk}}\right)^{-1}-\left(1+\frac{h}{2}\frac{|a_{1,ijk}^{n}|}{K_1(U^{n})_{ijk}}\right)^{-1}\right]\delta_x(K_1(u^{n+1})\delta_x u^{n+1})_{ijk}\Bigg\}-$$

$$\left\{\left(1+\frac{h}{2}\frac{|a_{2,ijk}^{n}|}{K_1(U^{n})_{ijk}}\right)^{-1}\delta_y(K_1(U^n)\delta_y\xi^{n+1})_{ijk}+\cdots\right\}-$$

$$\left\{\left(1+\frac{h}{2}\frac{|a_{3,ijk}^{n}|}{K_1(U^{n})_{ijk}}\right)^{-1}\delta_z(K_1(U^n)\delta_z\xi^{n+1})_{ijk}+\cdots\right\}+$$

$$\left\{\delta_{a_1^n,U^n,x}\xi_{ijk}^{n+1}+\delta_{a_1^{n+1},u^{n+1},x}u_{ijk}^{n+1}-\delta_{a_1^n,U^n,x}u_{ijk}^{n+1}\right\}+$$

$$\left\{\delta_{a_2^n,U^n,y}\xi_{ijk}^{n+1}+\cdots\right\}+\left\{\delta_{a_3^n,U^n,z}\xi_{ijk}^{n+1}+\cdots\right\}+$$

$$\Delta t\left\{\left(1+\frac{h}{2}\frac{|a_{1,ijk}^{n+1}|}{K_1(U^{n})_{ijk}}\right)^{-1}\delta_x(K_1(U^n)\delta_x)\times\right.$$

$$\left(\Phi_1^{-1}\left(1+\frac{h}{2}\frac{|a_2^{n}|}{K_1(U^{n})}\right)^{-1}\delta_y(K_1(U^n)\delta_y\xi^{n+1})\cdot\right)_{ijk}+$$

$$\left(1+\frac{h}{2}\frac{|a_{1,ijk}^{n}|}{K_1(U^{n})_{ijk}}\right)^{-1}\delta_x(K_1(U^n)\delta_x)\times$$

$$\left.\left(\Phi_1^{-1}\left(1+\frac{h}{2}\frac{|a_3^{n}|}{K_1(U^{n})}\right)^{-1}\delta_z(K_1(U^n)\delta_z\xi_{ijk}^{n+1})\cdot\right)_{ijk}+\cdots\right\}-$$

$$\Delta t\left\{\left(1+\frac{h}{2}\frac{|a_{1,ijk}^{n}|}{K_1(U^{n})_{ijk}}\right)^{-1}\delta_x(K_1(U^n)\delta_x)\left(\Phi_1^{-1}\delta_{a_2^n,U^n,y}\xi^{n+1})\cdot\right)_{ijk}+\cdots\right\}-$$

$$\Delta t\left\{\delta_{a_1^n,U^n,x}\left(\Phi_1^{-1}\left(1+\frac{h}{2}\frac{|a_2^{n}|}{K_1(U^{n})}\right)^{-1}\delta_y(K_1(U^n)\delta_y\xi^{n+1})\right)_{ijk}+\cdots\right\}+$$

$$\Delta t\left\{\delta_{a_1^n,U^n,x}(\Phi_1^{-1}\delta_{a_2^n,U^n,y}\xi^{n+1})_{ijk}+\cdots\right\}-$$

$$(\Delta t)^2\left\{\left(1+\frac{h}{2}\frac{|a_1^{n}|}{K_1(U^{n})}\right)^{-1}\delta_x(K_1(U^n)\delta_x)\times\right.$$

$$\left(\Phi_1^{-1}\left(1+\frac{h}{2}\frac{|a_2^{n}|}{K_1(U^{n})}\right)^{-1}\delta_y(K_1(U^n)\delta_y)\times\right.$$

$$\left.\left(\Phi_1^{-1}\left(1+\frac{h}{2}\frac{|a_3^{n}|}{K_1(U^{n})}\right)^{-1}\delta_z(K_1(U^n)\delta_z)\xi^{n+1}\right)\cdot\right)_{ijk}+\cdots\Bigg\}=$$

$$-K_2(x_i,y_j,z_{N-1/2},U_{ij,N}^n,t^n)\delta_z\omega_{ij,N}^{n+1}+Q_1(u_{ijk}^{n+1})-$$

$$Q_1(U^n)+\varepsilon_{1,ijk}^{n+1},\qquad 1\leqslant i,j,k\leqslant N-1, \tag{11a}$$

$$\xi_{ijk}^{n+1}=0,\qquad (x_i,y_j,z_k)^{\mathrm T}\in\partial\Omega_1, \tag{11b}$$

此处 $|\varepsilon_{1,ijk}^{n+1}|\leqslant M\left\{\|\partial^2u/\partial t^2\|_{L^\infty(L^\infty)},\ \|u\|_{L^\infty(W^4,\infty)}\right\}\left\{\Delta t+h^2\right\}$.

方程(10a)~方程(10f)消去 $V^{n+1/3}$, $V^{n+2/3}$, 可得类似的误差方程

$$\Phi_{3,ijk}\frac{\zeta_{ijk}^{n+1}-\zeta_{ijk}^{n}}{\Delta t}-\left\{\left(1+\frac{h}{2}\frac{|b_{1,ijk}^{n}|}{K_3(V^{n})_{ijk}}\right)^{-1}\delta_x(K_3(V^n)\delta_x\zeta^{n+1})_{ijk}+\cdots\right\}-$$

$$\left\{\left(1+\frac{h}{2}\frac{|b_{2,ijk}^{n}|}{K_3(V^{n})_{ijk}}\right)^{-1}\delta_y(K_3(V^n)\delta_y\zeta^{n+1})_{ijk}+\cdots\right\}-$$

$$\left\{\left(1+\frac{h}{2}\frac{|b_{3,ijk}^{n}|}{K_3(V^{n})_{ijk}}\right)^{-1}\delta_z(K_3(V^n)\delta_z\zeta^{n+1})_{ijk}+\cdots\right\}+$$

$$\left\{\delta_{b_1^n, V^n, x}\zeta_{ijk}^{n+1} + \delta_{b_1^{n+1}, v^{n+1}, x}v_{ijk}^{n+1} - \delta_{b_1^n, V^n, x}v_{ijk}^{n+1}\right\} +$$
$$\left\{\delta_{b_2^n, V^n, y}\zeta_{ijk}^{n+1} + \cdots\right\} + \left\{\delta_{b_3^n, V^n, z}\zeta_{ijk}^{n+1} + \cdots\right\} +$$
$$\Delta t\left\{\left(1 + \frac{h}{2}\frac{|b_{1,ijk}^n|}{K_3(V^n)_{ijk}}\right)^{-1}\delta_x(K_3(V^n)\delta_x)\times\right.$$
$$\left.\left(\Phi_3^{-1}\left(1+\frac{h}{2}\frac{|b_2^n|}{K_3(V^n)}\right)^{-1}\delta_y(K_3(V^n)\delta_y\zeta^{n+1})\right)_{ijk} + \cdots\right\} -$$
$$\Delta t\left\{\left(1 + \frac{h}{2}\frac{|b_{1,ijk}^n|}{K_3(V^n)_{ijk}}\right)^{-1}\delta_x(K_3(V^n)\delta_x)\left(\Phi_3^{-1}\delta_{b_2^n, V^n, y}\zeta^{n+1}\right)_{ijk} + \cdots\right\} -$$
$$\Delta t\left\{\delta_{b_1^n, V^n, x}\left(\Phi_3^{-1}\left(1+\frac{h}{2}\frac{|b_2^n|}{K_3(V^n)}\right)^{-1}\delta_y(K_3(V^n)\delta_y\zeta^{n+1})\right)_{ijk} + \cdots\right\} +$$
$$\Delta t\left\{\delta_{b_1^n, V^n, x}(\Phi_3^{-1}\delta_{b_2^n, V^n, y}\zeta^{n+1})_{ijk} + \cdots\right\} =$$
$$K_2(x_i, y_j, z_{1/2}, V_{ij,0}^n, t^n)\delta_z\omega_{ij,0}^{n+1} + Q_3(v_{ijk}^{n+1}) -$$
$$Q_3(V_{ijk}^{n+1}) + \varepsilon_{3,ijk}^{n+1}, \qquad 1 \leqslant i, j, k \leqslant N-1, \tag{12a}$$
$$\xi_{ijk}^{n+1} = 0, \qquad (x_i, y_j, z_k)^{\mathrm{T}} \in \partial\Omega_{3,h}, \tag{12b}$$

此处 $|\varepsilon_{3,ijk}^{n+1}| \leqslant M\left\{\|\partial^2 v/\partial t^2\|_{L^\infty(L^\infty)}, \|v\|_{L^\infty(W^{4,\infty})}\right\}\left\{\Delta t + h^2\right\}$•

方程(8)的误差方程

$$\Phi_{2,ijk}\frac{\omega_{ijk}^{n+1} - \omega_{ijk}^n}{\Delta t} = \delta_z(K_2(W^n)\delta_z\omega^{n+1})_{ijk} +$$
$$\delta_z([K_2(w^{n+1}) - K_2(W^n)]\delta_z\omega^{n+1})_{ijk} + \varepsilon_{2,ijk}^{n+1}, \qquad 1 \leqslant i, j, k \leqslant n-1, \tag{13}$$

此处 $|\varepsilon_{2,ijk}^{n+1}| \leqslant M\left\{\|\partial^2 w/\partial t^2\|_{L^\infty(L^\infty)}, \|w\|_{L^\infty(W^{4,\infty})}\right\}\left\{\Delta t + h^2\right\}$•

对方程(11a), 方程(12a), 方程(13)分别乘以 $2\Delta t\xi_{ijk}^{n+1}$, $2\Delta t\zeta_{ijk}^{n+1}$, 和 $2\Delta t\omega_{ijk}^{n+1}$, 在 $\Omega_{1,h}$, $\Omega_{3,h}$, $\Omega_{2,h}$ 上作内积, 分部求和并经估算和对时间 t 求和 $0 \leqslant n \leqslant L$, 注意到 $\xi^0 = \zeta^0 = \omega^0 = 0$, 故有

$$\left\{\|\Phi_1^{1/2}\xi^{L+1}\|^2 + \|\Phi_3^{1/2}\zeta^{L+1}\|^2 + \|\Phi_2^{1/2}\omega^{L+1}\|^2\right\} +$$
$$\Delta t\sum_{n=0}^{L}\left\{\|\Phi_1^{1/2}d_t\xi^n\|^2 + \|\Phi_3^{1/2}d_t\zeta^n\|^2 + \|\Phi_2^{1/2}d_t\omega^n\|^2\right\}\Delta t +$$
$$\sum_{n=0}^{L}\left\{\|K_1^{n,1/2}\delta_x\xi^{n+1}\|^2 + \|K_1^{n,1/2}\delta_y\xi^{n+1}\|^2 + \|K_1^{n,1/2}\delta_z\xi^{n+1}\|^2 +\right.$$
$$\|K_3^{n,1/2}\delta_x\zeta^{n+1}\|^2 + \|K_3^{n,1/2}\delta_y\zeta^{n+1}\| +$$
$$\left.\|K_3^{n,1/2}\delta_z\zeta^{n+1}\|^2 + \|\Phi_2^{1/2}\delta_z\omega^{n+1}\|^2\right\}\Delta t \leqslant$$
$$M\left\{\sum_{n=0}^{L}[\|\xi^{n+1}\|^2 + \|\zeta^{n+1}\|^2 + \|\omega^{n+1}\|^2]\Delta t + (\Delta t)^2 + h^4\right\}• \tag{14}$$

应用 Gronwall 引理可得

$$\left\{\|\Phi_1^{1/2}\xi^{L+1}\|^2 + \|\Phi_3^{1/2}\zeta^{L+1}\|^2 + \|\Phi_2^{1/2}\omega^{L+1}\|^2\right\} +$$
$$\Delta t\sum_{n=0}^{L}\left\{\|\Phi_1^{1/2}d_t\xi^n\|^2 + \|\Phi_3^{1/2}d_t\zeta^n\|^2 + \|\Phi_2^{1/2}d_t\omega^n\|^2\right\}\Delta t +$$
$$\sum_{n=0}^{L}\left\{\|K_1^{n,1/2}\delta_x\xi^{n+1}\|^2 + \|K_1^{n,1/2}\delta_y\xi^{n+1}\|^2 + \|K_1^{n,1/2}\delta_z\xi^{n+1}\|^2 +\right.$$
$$\|K_3^{n,1/2}\delta_x\zeta^{n+1}\|^2 + \|K_3^{n,1/2}\delta_y\zeta^{n+1}\|^2 + \|K_3^{n,1/2}\delta_z\zeta^{n+1}\|^2 +$$

$$\| K_2^{n,\gamma_2} \delta_{\bar{z}} \omega^{n+1} \|^2 \Big\} \Delta t \leqslant M\Big\{ (\Delta t)^2 + h^4 \Big\}. \tag{15}$$

定理 假定问题(1)~(4)的精确解满足光滑性条件

$$(\partial^2 u/\partial t^2) \in L^\infty(L^\infty(\Omega_1)),\ u \in L^\infty(W^{4,\infty}(\Omega_1)) \cap W^{1,\infty}(W^{1,\infty}(\Omega_1)),$$

$$(\partial^2 v/\partial t^2) \in L^\infty(L^\infty(\Omega_3)),\ v \in L^\infty(W^{4,\infty}(\Omega_3)) \cap W^{1,\infty}(W^{1,\infty}(\Omega_3)),$$

$$(\partial^2 w/\partial t^2) \in L^\infty(L^\infty(\Omega_2)),\ w \in L^\infty(W^{4,\infty}(\Omega_2)).$$

采用迎风分数步差分格式(7)、(8)、(10)逐层计算,则下述误差估计式成立

$$\| u - U \|_{L^\infty(J;l^2)} + \| v - V \|_{L^\infty(J;l^2)} + \| w - W \|_{L^\infty(J;l^2)} + \| u - U \|_{L^2(J;h^1)} + \| v - V \|_{L^2(J;h^1)} + \| w - W \|_{L^2(J;h^1)} \leqslant M^*\Big\{ \Delta t + h^2 \Big\}, \tag{16}$$

此处 $\| g \|_{L^\infty(J;x)} = \sup_{n\Delta t \leqslant T} \| f^n \|_x$, $\| g \|_{L^2(J;x)} = \sup_{L\Delta t \leqslant T} \Big\{ \sum_{n=0}^{L} \| g^n \|_x^2 \Delta t \Big\}^{1/2}$, M^* 依赖函数 u, v, w 及其导函数.

3 油资源运移聚集数值模拟的实际应用

3.1 模型问题的三维数值模拟

本文所提出的方法已成功的应用于胜利油田三维数值模拟, 问题的数学模型是下述一组非线性偏微分方程初边值问题

$$\nabla\cdot\left(K \frac{kr_o}{\mu_o} \nabla \phi_o\right) + B_o q = -\Phi_3'\left(\frac{\partial \phi_o}{\partial t} - \frac{\partial \phi_w}{\partial t}\right),\ \boldsymbol{X} = (x, y, z)^T \in \Omega,\ t \in J = (0, T], \tag{17a}$$

$$\nabla\cdot\left(k \frac{kr_w}{\mu_w} \nabla \phi_w\right) + B_w q = \Phi_3'\left(\frac{\partial \phi_o}{\partial t} - \frac{\partial \phi_w}{\partial t}\right),\qquad \boldsymbol{X} \in \Omega,\ t \in J, \tag{17b}$$

此处 ϕ_o、ϕ_w 是油相, 水相流动位势.

Hubbert, Dembicki, Calalan 等学者做过的油水二次运移聚集的著名石油地质水动力学实验, 在实验室理想条件下显示了油水运移、聚集、分离的过程[20~22], 我们以胜利油田提供的地质参数, 模仿Hubbert 等到的著名实验进行数值模拟, 模拟结果和实验结果基本吻合, 且有很强的物理力学特性, 十分清晰地看到油水运移、分离、聚集的全过程. 同时得知计算格式具有很强的稳定性和收敛性.

3.2 惠民凹陷多层数值模拟

对于多层问题的油资源运移聚集数值模拟, 应用本方法已成功对惠民凹陷进行数值模拟, 问题的数学模型是

$$\nabla\cdot\left(K_1 \frac{kr_o}{\mu_o} \nabla \phi_o\right) + B_o q - \left(K_2 \frac{kr_o}{\mu_o} \frac{\partial \phi_o}{\partial z}\right)_{z=H_2} = -\Phi_3'\left(\frac{\partial \phi_o}{\partial t} - \frac{\partial \phi_w}{\partial t}\right),\qquad \boldsymbol{X} \in \Omega_1,\ t \in J, \tag{18a}$$

$$\nabla\cdot\left(K_1 \frac{kr_w}{\mu_w} \nabla \phi_w\right) + B_w q - \left(K_2 \frac{kr_w}{\mu_w} \frac{\partial \phi_w}{\partial z}\right)_{z=H_2} = \Phi_3'\left(\frac{\partial \phi_o}{\partial t} - \frac{\partial \phi_w}{\partial t}\right),\qquad \boldsymbol{X} \in \Omega_1,\ t \in J, \tag{18b}$$

$$\frac{\partial}{\partial z}\left(K_2 \frac{kr_o}{\mu_o} \frac{\partial \phi_o}{\partial z}\right) = -\Phi_3'\left(\frac{\partial \phi_o}{\partial t} - \frac{\partial \phi_w}{\partial t}\right),\qquad \boldsymbol{X} \in \Omega_2,\ t \in J, \tag{19a}$$

$$\frac{\partial}{\partial z}\left(K_2\frac{kr_{\mathrm{w}}}{\mu_{\mathrm{w}}}\frac{\partial\psi_{\mathrm{w}}}{\partial z}\right)=\Phi_3'\left(\frac{\partial\psi_{\mathrm{o}}}{\partial t}-\frac{\partial\psi_{\mathrm{w}}}{\partial t}\right),\qquad \boldsymbol{X}\in\Omega_2,\ t\in J, \tag{19b}$$

$$\nabla\cdot\left(K_3\frac{kr_{\mathrm{o}}}{\mu_{\mathrm{o}}}\nabla\psi_{\mathrm{o}}\right)+B_{\mathrm{w}}q+\left(K_2\frac{kr_{\mathrm{o}}}{\mu_{\mathrm{o}}}\frac{\partial\psi_{\mathrm{o}}}{\partial z}\right)_{z=H_1}=$$
$$-\Phi_3'\left(\frac{\partial\psi_{\mathrm{o}}}{\partial t}-\frac{\partial\psi_{\mathrm{w}}}{\partial t}\right),\qquad \boldsymbol{X}\in\Omega_3,\ t\in J, \tag{20a}$$

$$\nabla\cdot\left(K_3\frac{kr_{\mathrm{w}}}{\mu_{\mathrm{w}}}\nabla\psi_{\mathrm{w}}\right)+B_{\mathrm{w}}q+\left(K_3\frac{kr_{\mathrm{w}}}{\mu_{\mathrm{w}}}\frac{\partial\psi_{\mathrm{w}}}{\partial z}\right)_{z=H_1}=$$
$$\Phi_3'\left(\frac{\partial\psi_{\mathrm{o}}}{\partial t}-\frac{\partial\psi_{\mathrm{w}}}{\partial t}\right),\qquad \boldsymbol{X}\in\Omega_3,\ t\in J, \tag{20b}$$

此处 ψ_{o}, ψ_{w} 是油相, 水相流动位势•

4 结　　论

本文从油气资源勘探、开发和地下水渗流计算的实际问题出发, 研究非线性多层渗流耦合系统的数值方法及其应用, 提出适合并行计算的二阶耦合迎风分数步差分格式, 应用微分方程先验估计的理论和技巧, 得到收敛性的最佳阶 l^2 误差估计, 并已成功的应用到油资源运移聚集数值模拟计算, 海水入侵预测和防治的工程实践中, 是一种高效的渗流力学工程计算方法•

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Numerical Method and Application for the Three_Dimensional Nonlinear System of Dynamics of Fluids in Porous Media

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Abstract: For the system of multilayer dynamics of fluids in porous media, the second order upwind finite difference fractional steps schemes applicable to parallel arithmetic are put forward. Some techniques, such as calculus of variations, energy method, multiplicative commutation rule of difference operators, decomposition of high order difference operators and prior estimates were adopted. Optimal order estimates were derived to determine the error in the second order approximate solution. These methods have already been applied to the numerical simulation of migration_accumulation of oil resources.

Key words: coupled system; nonlinear; upwind fractional step; convergence; numerical simulation of oil resource