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一类不确定非线性系统的适应 H_infinity 控制*

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(郭兴明推荐)

摘要: 主要讨论了一类具有不确定参数的非线性系统的通过适应输出反馈达到干扰衰减的问题• 通过构造降维观测器, 利用 Backstepping 方法设计输出反馈控制器, 使闭环系统具有不确定参数的标准的增益问题可解, 并使系统达到内稳定•

关 键 词: 非线性系统; 适应控制; 输出反馈
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引 言

近年来非线性系统 H_∞ 问题得到了广泛的研究, 取得了很多突破性的进展[1~4], 文献[5]中基于降维观测器研究了 H_∞ 控制问题• 本文中, 主要考虑了对一类广泛具有不确定参数、时变非线性系统的 H_∞ 控制问题, 利用降维观测器及 Backstepping 方法及 Sontag[6] 中提出的 ISS 稳定性概念和改变能量函数的技巧, 放宽了不确定系统的条件, 构造了输出反馈补偿器使得标准 L_2_增益的干扰衰减问题的可解, 推广了文献[5]和文献[7]的结果•

1 问题描述

在本文中主要讨论下述不确定的非线性系统

$$\begin{cases} \dot{z} = \boldsymbol{q}_0(t, z, y) + \boldsymbol{q}_1(t, z, y)\boldsymbol{w}, \\ \dot{x}_1 = x_2 + f_1(t, z, y) + g_1(t, z, y)\boldsymbol{w}, \\ \vdots \\ \dot{x}_n = u + f_n(t, z, y) + g_n(t, z, y)\boldsymbol{w}, \\ y = x_1, \end{cases} \tag{1}$$

其中, $(z, \boldsymbol{x}) \in R^{n_0+n}$ 是状态变量, $u \in \mathrm{R}$ 是控制输入, $y \in \mathrm{R}$ 是输出, $\boldsymbol{w} \in R^m$ 是系统(1) 的扰动输入• 假设 y 是可测量的, 其余状态变量是不可量测的• $\boldsymbol{q}_0, \boldsymbol{q}_1, f_i, g_i (1 \leqslant i \leqslant n)$ 均是不确定的, 但是 Lipschitz 连续的•

对系统(1) 均作如下假设

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假设 1　对每一个 $i(1 \leqslant i \leqslant n)$, 存在一个未知的正常数 p_i^*, 使得

$$\begin{cases} |f_i(t,z,y)| \leqslant p_i^* \varphi_{i1}(|y|) + p_i^* \varphi_{i2}(|z|), \\ |g_i(t,z,y)| \leqslant \psi_{i1}(y) + \psi_{i2}(z) + \psi_{i0}, \end{cases} \quad \forall (t,z,y) \in \mathrm{R}_+ \times R^{n_0} \times \mathrm{R}, \tag{2}$$

其中 $\varphi_{i1}, \varphi_{i2}, \psi_{i1}, \psi_{i2}$ 是已知的光滑非负函数, ψ_{i0} 是正常数, 不失一般性假设

$$\varphi_{i1}(0) = 0, \quad \varphi_{i2}(0) = 0.$$

假设 2　对系统(1) 中的 z_子系统存在 ISS Liapunov 函数 V_0, 即存在 K_∞ 函数 $\underline{\alpha}, \bar{\alpha}, \alpha_0, \gamma_0$ 和 γ_1, 满足下列不等式

$$\begin{cases} \underline{\alpha}(|z|) \leqslant V_0(t,z) \leqslant \bar{\alpha}(z), \\ \dfrac{\partial V_0}{\partial t}(t,z) + \dfrac{\partial V_0}{\partial z}(t,z)[\boldsymbol{q}_0(t,z,y) + \boldsymbol{q}_1(t,z,y)\boldsymbol{w}] \leqslant \\ \qquad - \alpha_0(|z|) + \gamma_0(|y|) + \gamma_1(|\boldsymbol{w}|). \end{cases} \tag{3}$$

对任意的 $t \in \mathrm{R}_+$, $z \in R^{n_0}$, $y \in \mathrm{R}$, 且 $\boldsymbol{w} \in R^m$. 由文献[10] 从假设 2 可以得到 z_子系统对于输入 y, $\boldsymbol{w}$ 是 ISS 稳定的, 特别的, 当 $y = 0$, $\boldsymbol{w} = \boldsymbol{0}$ 时, z_子系统在 $z = \boldsymbol{0}$ 是全局一致渐近稳定的.

系统(1) 的干扰衰减稳定性, 就是存在一个 K 函数 γ, 使得对任意的正实数 ε, 找到输出反馈控制律具有如下形式

$$\dot{\chi} = v(\chi, y), \quad u = \mu(\chi, y), \tag{4}$$

满足下列性质(*)

• 当 $\boldsymbol{w} = \boldsymbol{0}$ 时, 闭环系统(1)、(4) 在原点是全局一致渐近稳定的.

• 当 $\boldsymbol{w} \in L_{\infty e}^m$ 时, 从原点出发的闭环系统的解满足.

$$\int_{t_0}^{t} |y(\tau)|^2 \mathrm{d}\tau \leqslant \varepsilon \int_{t_0}^{t} \gamma(|\boldsymbol{w}(\tau)|) \mathrm{d}\tau, \qquad \forall t \geqslant t_0 \geqslant 0. \tag{5}$$

注 1　若 $\gamma(r) = kr^2$,

$$\int_{t_0}^{t} |y(\tau)|^2 \mathrm{d}\tau \leqslant k\varepsilon \int_{t_0}^{t} |\boldsymbol{w}(\tau)|^2 \mathrm{d}\tau, \qquad \forall t \geqslant t_0 \geqslant 0.$$

$$\|y\|_{2[t_0,t]} \leqslant \sqrt{k\varepsilon} \|\boldsymbol{w}\|_{2[t_0,t]}, \qquad \forall t \geqslant t_0 \geqslant 0.$$

称系统(1) 的 L_2_增益的干扰衰减问题用输出反馈是可解的.

注 2　(5) 式等价于存在一个正定的 Liapunov 函数 V_c, 使其沿闭环系统(1)、(4) 的轨线满足下列不等式

$$\dot{V}_c \leqslant - W_c(z, \boldsymbol{x}) - \eta(y^2) + \gamma(\|\boldsymbol{w}\|),$$

其中 η 是一个 K_∞ 函数.

2　鲁棒控制设计

2.1　观测器的设计

引入如下降维观测器

$$\begin{cases} \dot{\xi}_i = \xi_{i+1} + L_{i+1}y - L_i(\xi_1 + L_1 y), \qquad 1 \leqslant i \leqslant n-2, \\ \dot{\xi}_{n-1} = u - L_{n-1}(\xi_1 + L_1 y), \end{cases} \tag{6}$$

选择实数 $L_i(1 \leqslant i \leqslant n-1)$, 使得矩阵

$$A = \begin{pmatrix} -L_1 & 1 & 0 & \cdots & 0 \\ -L_2 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \\ -L_{n-2} & 0 & 0 & \cdots & 1 \\ -L_{n-1} & 0 & 0 & \cdots & 0 \end{pmatrix}$$

是稳定矩阵, 对 $\forall 1 \leqslant i \leqslant n-1$, 令

$$\zeta_i = \frac{x_{i+1} - \xi_i - L_i y}{p_i^*}, \tag{7}$$

$$\boldsymbol{f}^*(t, \boldsymbol{z}, y) = (f_2(t, \boldsymbol{z}, y) - L_1 f_1(t, \boldsymbol{z}, y), \cdots, f_n(t, \boldsymbol{z}, y) - L_{n-1} f_1(t, \boldsymbol{z}, y))^{\mathrm{T}}, \tag{8}$$

$$\boldsymbol{g}^*(t, \boldsymbol{z}, y) = (g_2(t, \boldsymbol{z}, y) - L_1 g_1(t, \boldsymbol{z}, y), \cdots, g_n(t, \boldsymbol{z}, y) - L_{n-1} g_1(t, \boldsymbol{z}, y))^{\mathrm{T}}, \tag{9}$$

记 $p^* := \max\left\{1, p_{i+1}^*, p_1^*, L_i p_1^*; 1 \leqslant i \leqslant n-1\right\}$ 则有

$$\dot{\zeta} = \boldsymbol{A}\zeta + \frac{1}{p^*}\boldsymbol{f}^*(t, \boldsymbol{z}, y) + \frac{1}{p^*}\boldsymbol{g}^*(t, \boldsymbol{z}, y)\boldsymbol{w}, \tag{10}$$

显然若 $\boldsymbol{w} = \boldsymbol{0}$ 时, f_i 关于 $\boldsymbol{z}, y$ 一致趋于 0, 则当 $t \to +\infty$ 时, ζ 就趋于 0. 这时, 不可量测的状态变量 $(x_2, \cdots, x_n)$ 就可以由 $(\xi_1 + L_1 y, \cdots, \xi_{n-1} + L_{n-1} y)$ 通过观测器(6) 来估计.

设 $\boldsymbol{P}$ 是正定矩阵, 满足方程 $\boldsymbol{PA} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} = -2\boldsymbol{I}$.

取 $V_\zeta = \zeta^{\mathrm{T}}\boldsymbol{P}\zeta$, 沿(10) 求导, 由假设 1, 有

$$\begin{aligned} \dot{V}_\zeta = & -2\zeta^{\mathrm{T}}\zeta + 2\zeta^{\mathrm{T}}\boldsymbol{P}\frac{1}{p^*}(\boldsymbol{f}^* + \boldsymbol{g}^*\boldsymbol{w}) \leqslant \\ & -|\zeta|^2 + 2|\boldsymbol{P}|^2\left(\frac{1}{p^{*2}}|\boldsymbol{f}^*|^2 + |\boldsymbol{g}^*|^2\boldsymbol{w}^2\right) \leqslant \\ & -|\zeta|^2 + 4|\boldsymbol{P}|^2\left[\sum_{i=1}^{n-1}(\varphi_{i+1,1}(|y|) + \varphi_{11}(|y|))\right]^2 + \\ & 4|\boldsymbol{P}|^2\left[\sum_{i=1}^{n-1}(\varphi_{i+1,2}(|\boldsymbol{z}|) + \varphi_{11}(|\boldsymbol{z}|))\right]^2 + \\ & 6|\boldsymbol{P}|^2\left[\sum_{i=1}^{n-1}(\psi_{i+1,0} + \psi_{10}\right]^2\boldsymbol{w}^2 + 3|\boldsymbol{P}|^2\left[\sum_{i=1}^{n-1}(\psi_{i+1,1}(|y|) + \psi_{11}(|y|))\right]^4 + \\ & 3|\boldsymbol{P}|^2\left[\sum_{i=1}^{n-1}(\psi_{i+1,2}(|\boldsymbol{z}|) + \psi_{11}(|\boldsymbol{z}|))\right]^4 + 6|\boldsymbol{P}|^2\boldsymbol{w}^4, \end{aligned}$$

由 $\varphi_{i+1,1}$, φ_{11}, $\varphi_{i+1,2}$, φ_{12}, $\psi_{i+1,1}$, ψ_{11} 的光滑性, 存在非负光滑函数 $\phi_1(y)$, $\phi_2(\boldsymbol{z})$ 及非负常数 ψ_0, 使得

$$\dot{V}_\zeta \leqslant -|\zeta|^2 + \phi_1(y) + \phi_2(\boldsymbol{z}) + \psi_0\boldsymbol{w}^2 + 6|\boldsymbol{P}|^2\boldsymbol{w}^4. \tag{11}$$

原系统(1) 的干扰衰减问题的可解性就转化为下列系统从 $\boldsymbol{w}$ 到 y 用新状态变量 $(y, \xi_1, \cdots, \xi_{n-1})$ 的干扰衰减问题

$$\begin{cases} \dot{\boldsymbol{z}} = \boldsymbol{q}_0(t, \boldsymbol{z}, y) + \boldsymbol{q}_1(t, \boldsymbol{z}, y)\boldsymbol{w}, \\ \dot{\zeta} = \boldsymbol{A}\zeta + \dfrac{1}{p^*}\boldsymbol{f}^*(t, \boldsymbol{z}, y) + \dfrac{1}{p^*}\boldsymbol{g}^*(t, \boldsymbol{z}, y)\boldsymbol{w}, \\ \dot{y} = \xi_1 + L_1 y + p^*\zeta_1 + f_1(t, \boldsymbol{z}, y) + g_1(t, \boldsymbol{z}, y)\boldsymbol{w}, \\ \dot{\xi}_i = \xi_{i+1} + L_{i+1} - L_i(\xi_1 + L_1 y), \qquad 1 \leqslant i \leqslant n-2, \\ \dot{\xi}_{n-1} = u - L_{n-1}(\xi_1 + L_1 y). \end{cases} \tag{12}$$

2.2 递归设计

考虑(12)的 (ζ, y)_子系统

$$\begin{cases} \dot{\zeta} = A\zeta + \dfrac{1}{p^*}f^*(t, z, y) + \dfrac{1}{p^*}g^*(t, z, y)w, \\ \dot{y} = \xi_1 + L_1 y + p^*\zeta_1 + f_1(t, z, y) + g_1(t, z, y)w, \end{cases} \tag{13}$$

将 ξ_1 认为是虚拟控制输入•

取 Liapunov 函数

$$V_1 = V_\zeta(\zeta) + \frac{1}{2}\eta(y^2) + \frac{1}{2}(\hat{p} - p)^2, \tag{14}$$

其中 η 是待定的函数, $p \geqslant \max\left\{p^*, p^{*2}, p_1^{*2}, p_1^*\right\}$ 未知常量•

由假设 1 和不等式(11), V_1 沿(12)的导数满足

$$\dot{V}_1 \leqslant -|\zeta|^2 + \phi_1(y) + \phi_2(z) + \psi_0 w^2 + 6|P|^2 w^4 + (\hat{p} - p)\dot{\hat{p}} + \eta'(y^2)y[\xi_1 + L_1 y + p^*\zeta_1 + f_1(t, z, y) + g_1(t, z, y)w],$$

我们有

$$\eta'(y^2)y[p^*\zeta_1 + f_1(t, z, y)] \leqslant \frac{1}{2}|\zeta|^2 + p\eta' y\left[y\varphi_{11}(|y|) + \frac{3}{4}\eta' y\right] + \varphi_{12}(|z|)^2,$$

其中 $\varphi_{11}(y)$ 是光滑非负函数

$$\eta'(y^2)yg_1(t, z, y)w \leqslant \eta' y\left[\frac{\eta' y}{2}\psi_{11}(|y|)^2 + \frac{\eta'^3 y^3}{16} + \frac{\eta' y}{4}\right] + \psi_{12}(|z|)^4 + (1 + \psi_{10}^2)w^2,$$

$$\dot{V}_1 \leqslant \frac{1}{2}|\zeta|^2 + \eta' y\left\{\left[\xi_1 + L_1 y + \frac{1}{\eta'(y^2)}y\hat{\phi}_1(y) + y\psi_{11}(y)\right] + p\left[y\varphi_{11}(|y|) + \frac{3}{4}\eta' y\right]\right\} + \phi_2(z) + \varphi_{12}(|z|)^2 + \psi_{12}(|z|)^4 + (\hat{p} - p)\dot{\hat{p}} + (1 + \psi_0 + \psi_{10}^2)w^2 + 6|P|^2 w^4, \tag{15}$$

存在光滑非负函数 $\hat{\phi}_1(y)$ 满足: $\phi_1(y) \leqslant y^2\hat{\phi}_1(y)$, $\forall y \in \mathrm{R}$, 令

$$\begin{cases} \omega_1 = -\sigma\hat{p} + \eta' y\left[y\varphi_{11}(|y|) + \dfrac{3}{4}\eta' y\right], \\ \vartheta_1 = -\dfrac{\varepsilon^{-1}y}{\eta'(y^2)} - yv(y^2) - L_1 y - \dfrac{1}{\eta'(y^2)}y\hat{\phi}_1(y) - \\ \qquad y\psi_{11}(y) - \hat{p}\left[y\varphi_{11}(|y|) + \dfrac{3}{4}\eta' y\right], \\ \xi_2 = \xi_1 - \vartheta_1(y, \hat{p}), \end{cases} \tag{16}$$

其中 $\sigma > 0$, ν 是光滑非减函数, $\nu(0) > n - 1$• 则有

$$\dot{V}_1 \leqslant \frac{1}{2}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta' y^2\nu(y^2) + \eta' y\xi_2 - \sigma(\hat{p} - p)\hat{p} + (\hat{p} - p)(\dot{\hat{p}} - \omega_1) + \phi_2(z) + \varphi_{12}(|z|)^2 + \psi_{12}(|z|)^4 + (1 + \psi_0 + \psi_{10}^2)w^2 + 6|P|^2 w^4• \tag{17}$$

假设已经设计了光滑函数 $\vartheta_j(1 \leqslant j \leqslant k)$ 及 ω_k 使得下面的不等式成立

$$\dot{V}_k \leqslant \frac{1}{2^k}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta' y^2(\nu(y^2) - k + 1) -$$

$$\sum_{j=2}^{k} c_j \xi_j^2 + \xi_k \xi_{k+1} - \sigma(\hat{p} - p)\hat{p} +$$
$$\left(\frac{1}{\lambda}(\hat{p} - p) - \sum_{j=2}^{k} \xi_j^2 \frac{\partial \vartheta_{j-1}}{\partial \hat{p}}\right)(\dot{\hat{p}} - \omega_k) + \phi_2(z) + k\varphi_{12}(|z|)^2 +$$
$$k\psi_{12}(|z|)^4 + (k + \psi_0 + k\psi_{10}^2)\boldsymbol{w}^2 + 6|\boldsymbol{P}|^2\boldsymbol{w}^4, \tag{18}$$

其中 $c_j > 0$, 且 $\xi_1 = \eta'(y^2)y$, $\xi_j = \xi_{j-1} - \vartheta_{j-1}(y, \dots, \xi_{j-2}, \hat{p})$• $2 \leqslant j \leqslant k + 1$, $\xi_n = u - L_n y$, 记

$$\Delta_k = \phi_2(z) + k\varphi_{12}(|z|)^2 + k\psi_{12}(|z|)^4 +$$
$$(k + \psi_0 + k\psi_{10}^2)\boldsymbol{w}^2 + 6|\boldsymbol{P}|^2\boldsymbol{w}^4, \tag{19}$$

取

$$V_{k+1} = V_k(\zeta, y, \dots, \xi_{k+1}) + \frac{1}{2}\xi_{k+1}^2 =$$
$$V_k(\zeta, y, \dots, \xi_{k+1}) + \frac{1}{2}(\xi_k - \vartheta_k(y, \dots, \xi_{k-1}, \hat{p}))^2,$$

V_{k+1} 沿(12) 式对 t 求导,

$$\dot{V}_{k+1} = \dot{V}_k(\zeta, y, \dots, \xi_{k+1}) + \xi_{k+1}\dot{\xi}_{k+1} \leqslant$$
$$-\frac{1}{2^k}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta' y^2(\nu(y^2) - k + 1) -$$
$$\sum_{j=2}^{k} c_j \xi_j^2 + \xi_k \xi_{k+1} - \sigma(\hat{p} - p)\hat{p} +$$
$$\left((\hat{p} - p) - \sum_{j=2}^{k} \xi_j^2 \frac{\partial \vartheta_{j-1}}{\partial \hat{p}}\right)(\dot{\hat{p}} - \omega_k) + \Delta_k +$$
$$\xi_{k+1}\Big[\xi_{k+1} + L_{k+1}y - L_k(\xi_1 + L_1 y) -$$
$$\frac{\partial \vartheta_k}{\partial y}(\xi_1 + L_1 y + p^* \zeta_1 + f_1 + g_1\boldsymbol{w}) -$$
$$\sum_{j=1}^{k-1} \frac{\partial \vartheta_k}{\partial \xi_j}(\xi_{j+1} + L_{j+1}y - L_j(\xi_1 + L_1 y) - \frac{\partial \vartheta_k}{\partial \hat{p}}\dot{\hat{p}}\Big]$$

与第一步相同的技巧和简单运算, 存在光滑非负函数 ϕ_{21}, 使得

$$-\xi_{k+1}\frac{\partial \vartheta_k}{\partial y}(p^* \zeta_1 + f_1 + g_1\boldsymbol{w}) \leqslant$$
$$\frac{1}{2^{k+1}}|\zeta|^2 + \eta' y^2 + p\xi_{k+1}^2\phi_{21}(y, \xi_1, \dots, \xi_k) + \varphi_{12}(|z|)^2 +$$
$$\psi_{12}(|z|)^4 + (1 + \psi_{10}^2)\boldsymbol{w}^2,$$

其中 $p \geqslant \max\left\{p^*, p^{*2}, p_1^*, P_1^{*2}\right\}$• 则

$$\dot{V}_{k+1} \leqslant \frac{-1}{2^{k+1}}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta' y^2(\nu(y^2) - k) - \sum_{j=2}^{k} c_j \xi_j^2 - \sigma(\hat{p} - p)\hat{p} +$$
$$\left((\hat{p} - p) - \sum_{j=2}^{k} \xi_j^2 \frac{\partial \vartheta_{j-1}}{\partial \hat{p}}\right)(\dot{\hat{p}} - \omega_k) + \Delta_{k+1} +$$
$$\xi_{k+1}\Big[\xi_{k+1} + L_{k+1}y - L_k(\xi_1 + L_1 y) + \xi_k - \frac{\partial \vartheta_k}{\partial y}(\xi_1 + L_1 y) -$$
$$\sum_{j=1}^{k-1} \frac{\partial \vartheta_k}{\partial \xi_j}(\xi_{j+1} + L_{j+1}y - L_j(\xi_1 + L_1 y)) + p\xi_{k+1}^2\phi_{21} - \frac{\partial \vartheta_k}{\partial \hat{p}}\dot{\hat{p}}\Big], \tag{20}$$

定义

$$\begin{cases} \omega_{k+1} = \omega_k + \xi_{k+1}^2 \phi_{21}, \\ \vartheta_{k+1} = -c_{k+1}\xi_{k+1} - \xi_k - L_{k+1}y + L_k(\xi_1 + L_1 y) + \dfrac{\partial \vartheta_k}{\partial y}(\xi_1 + L_1 y) + \dfrac{\partial \vartheta_k}{\partial \hat{p}}\omega_{k+1} + \\ \qquad \displaystyle\sum_{j=1}^{k-1}\frac{\partial \vartheta_k}{\partial \xi_j}(\xi_{j+1} + L_{j+1}y - L_j(\xi_1 + L_1 y)) - \left(\hat{p} - \sum_{j=2}^{k}\xi_j^2\frac{\partial \vartheta_{j-1}}{\partial \hat{p}}\right)\xi_{k+1}\phi_{21}, \\ \xi_{k+2} = \xi_{k+1} - \vartheta_{k+1}, \end{cases} \tag{21}$$

其中 $c_{k+1} > 0$. 将(21)代入(20),就有

$$\dot{V}_{k+1} \leqslant -\frac{1}{2^{k+1}}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta' y^2(\nu(y^2) - k) - \sum_{j=2}^{k+1}c_j\xi_j^2 + \xi_{k+1}\xi_{k+2} - \sigma(\hat{p} - p)\hat{p} + \left((\hat{p} - p) - \sum_{j=2}^{k+1}\xi_j^2\frac{\partial \vartheta_{j-1}}{\partial \hat{p}}\right)(\dot{\hat{p}} - \omega_{k+1}) + \Delta_{k+1}, \tag{22}$$

在第 n 步,我们令 $u = \xi_n y + L_n y = \vartheta_n + L_n y$, $\dot{\hat{p}} = \omega_n$, 则

$$V_n = V_{n-1} + \frac{1}{2}\xi_n^2 = V_\zeta + \frac{1}{2}\eta(y^2) + \frac{1}{2\lambda}(\hat{p} - p)^2 + \frac{1}{2}\sum_{i=1}^{n-1}(\xi_i - \vartheta_i(y, \ldots, \xi_{i-1}, \hat{p}))^2,$$

$$\begin{aligned} \dot{V}_n = {} & \dot{V}_{n-1} + \xi_n\dot{\xi}_n \leqslant \\ & -\frac{1}{2^n}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta' y^2(\nu(y^2) - n + 1) - \sum_{j=2}^{n-1}c_j\xi_j^2 - \sigma(\hat{p} - p)\hat{p} + \\ & \left((\hat{p} - p) - \sum_{j=2}^{n-1}\xi_j^2\frac{\partial \vartheta_{j-1}}{\partial \hat{p}}\right)(\dot{\hat{p}} - \omega_{n-1}) + \Delta_n + \\ & \xi_n\Big[u + \xi_{n-1} - L_{n-1}(\xi_1 + L_1 y) - \frac{\partial \vartheta_{n-1}}{\partial y}(\xi_1 + L_1 y) - \\ & \sum_{j=1}^{n-2}\frac{\partial \vartheta_{n-1}}{\partial \xi_j}(\xi_{k+1} + L_{j+1}y - L_j(\xi_1 + L_1 y)) + p\xi_n\phi_{21} - \frac{\partial \vartheta_{n-1}}{\partial \hat{p}}\dot{\hat{p}}\Big], \end{aligned}$$

同样,令

$$\begin{cases} \omega_n = \omega_{n-1} + \xi_n^2\phi_{21}, \\ \vartheta_n = -c_n\xi_n - \xi_{n-1} - L_n y + L_{n-1}(\xi_1 + L_1 y) + \\ \qquad \dfrac{\partial \vartheta_{n-1}}{\partial y}(\xi_1 + L_1 y) + \dfrac{\partial \vartheta_{n-1}}{\partial \hat{p}}\omega_n + \displaystyle\sum_{j=1}^{n-2}\frac{\partial \vartheta_{n-1}}{\partial \xi_j}(\xi_{j+1} + L_{j+1}y - \\ \qquad L_j(\xi_1 + L_1 y)) - \left(\hat{p} - \displaystyle\sum_{j=2}^{n-1}\xi_j^2\frac{\partial \vartheta_{j-1}}{\partial \hat{p}}\right)\xi_n\phi_{21}, \end{cases} \tag{23}$$

$$\dot{V}_n \leqslant -\frac{1}{2^n}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta' y^2(\nu(y^2) - n + 1) - \sum_{j=2}^{n}c_j\xi_j^2 - \sigma(\hat{p} - p)\hat{p} + \left((\hat{p} - p) - \sum_{j=2}^{n}\xi_j^2\frac{\partial \vartheta_{j-1}}{\partial \hat{p}}\right)(\dot{\hat{p}} - \omega_n) + \xi_n[u - L_n y + \vartheta_n] + \Delta_n, \tag{24}$$

取 $u = L_n y - \vartheta_n$, $\dot{\hat{p}} = \omega_n$, 即有

$$\dot{V}_n \leqslant -\frac{1}{2^n}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta' y^2(\nu(y^2) - n + 1) -$$

$$\sum_{j=2}^{n} c_j \xi_j^2 - \sigma(\hat{p}-p)\hat{p} + \phi_2(z) + n\varphi_{12}(|z|)^2 + n\psi_{12}(|z|)^4 + (n+\psi_0+n\psi_{10}^2)\boldsymbol{w}^2 + 6|\boldsymbol{P}|^2\boldsymbol{w}^4 \cdot \tag{25}$$

3 储能函数的选取

首先选择光滑函数 v，满足

$$\eta'(y^2)y^2(v(y^2)-n+1) \geqslant \eta(y^2),$$

由 $\eta'(y^2) \neq 0$, $\forall y$, 则这样的光滑函数总是存在的• 又由 $\phi_2(z)$, $\varphi_{12}(|z|)$, $\psi_{12}(|z|)$ 均是光滑函数且在原点为 0, 所以存在光滑的 K_∞ 函数 α 及 γ 使得

$$\phi_2(z) + n\varphi_{12}(|z|)^2 + n\psi_{12}(|z|)^4 \leqslant \alpha(|z|^2), \tag{26}$$

$$(n+\psi_0+n\psi_{10}^2)\boldsymbol{w}^2 + 6|\boldsymbol{P}|^2\boldsymbol{w}^4 \leqslant \gamma(|\boldsymbol{w}|), \tag{27}$$

由 $-\sigma(\hat{p}-p)\hat{p} \leqslant -(\sigma/2)(\hat{p}-p)^2 + (\sigma/2)p^2$, 并记 $d = (\sigma/2)p^2$, 则

$$\dot{V}_n \leqslant -\frac{1}{2^n}|\zeta|^2 - \varepsilon^{-1}y^2 - \eta(y^2) - \sum_{j=2}^{n} c_j\xi_j^2 - \frac{\sigma}{2}(\hat{p}-p)^2 + \alpha(|z|^2) + \gamma(|\boldsymbol{w}|) + d, \tag{28}$$

类似于文献[7], 令

$$U_0(t,z) = \int_0^{V_0(t,z)} \rho(\tau)\,\mathrm{d}\tau, \tag{29}$$

$V_0(t,z)$ 是假设 2 中定义的, $\rho: \mathrm{R}_+ \to \mathrm{R}_+$ 是光滑非减函数, $\rho(\tau) > 0$, $\forall \tau \geqslant 0$• 显然 U_0 是正定的且为径向无界的•

引理 设假设 1 与假设 2 中的函数 γ_0, α_0 和 φ_{i2} 满足下列局部性质

$$\lim_{s\to 0^+}\sup \frac{\gamma_0(s)}{s^2} < +\infty, \quad \lim_{s\to 0^+}\sup \frac{\varphi_{i2}(s)^2}{s^2} < +\infty, \qquad \forall 1 \leqslant i \leqslant n, \tag{30}$$

取 $V_c = V_n + U_0$, 则有不等式

$$\dot{V}_c \leqslant \alpha(|z|) - \frac{1}{2^n}|\zeta|^2 - \eta(y^2) - \sum_{j=2}^{n} c_j\xi_j^2 - \frac{\sigma}{2}(\hat{p}-p)^2 + \gamma(|\boldsymbol{w}|) + d, \tag{31}$$

其中

$$\begin{cases} \alpha(r) = \dfrac{1}{4}\rho \circ \underline{\alpha}(r)\alpha_0(r), \\ \gamma(r) = \rho \circ \underline{\alpha} \circ \alpha_0^{-1} \circ 4\gamma_1(r)\gamma_1(r) + (n+\psi_0+n\psi_{10}^2)r^2 + 6|\boldsymbol{P}|^2 r^4 \cdot \end{cases} \tag{32}$$

证明

$$\dot{U}_0 \leqslant \rho(V_0(t,z))(-\alpha_0(|z|) + \gamma_0(|y|) + \gamma_1(|\boldsymbol{w}|)) = \rho(V_0(t,z))\left(-\frac{1}{2}\alpha_0(|z|) - \frac{1}{2}\alpha_0(|z|) + \gamma_0(|y|) + \gamma_1(|\boldsymbol{w}|)\right) \cdot$$

1) 若 $-(1/2)\alpha_0(|z|) + \gamma_0(|y|) + \gamma_1(|\boldsymbol{w}|) < 0$, 则

$$\dot{U}_0 \leqslant -\frac{1}{2}\rho(V_0(t,z))\alpha_0(|z|) \leqslant -\frac{1}{2}\rho \circ \underline{\alpha}(|z|)\alpha_0(|z|) \cdot$$

2) 若 $-(1/2)\alpha_0(|z|) + \gamma_0(|y|) + \gamma_1(|\boldsymbol{w}|) \geqslant 0$, 则当

$\gamma_1(|\boldsymbol{w}|) > \gamma_0(|y|)$ 且 $-(1/2)\alpha_0(|z|) + \gamma_0(|y|) \leqslant 0$

时,就有

$$\dot{U}_0 \leqslant \frac{1}{2}\rho(V_0(t,z))\alpha_0(|z|) + \rho(V_0(t,z))\left[-\frac{1}{2}\alpha_0(|z|) + \gamma_0(|y|)\right] + \rho(V_0(t,z))\gamma_1(|w|) \leqslant -\frac{1}{2}\rho\circ\underline{\alpha}(|z|)\alpha_0(|z|) + \rho\circ\alpha\circ\alpha_0^{-1}\circ 4\gamma_1(|w|)\gamma_1(|w|).$$

当 $\gamma_1(|w|) > \gamma_0(|y|)$ 且 $-(1/2)\alpha_0(|z|) + \gamma_0(|y|) > 0$ 时, 有 $(1/2)\alpha_0(|z|) \leqslant \gamma_0(|y|) + \gamma_1(|w|)$, 且

$$\dot{U}_0 \leqslant \frac{1}{2}\rho\circ\underline{\alpha}(|z|)\alpha_0(|z|) + \rho\circ\alpha\circ\alpha_0^{-1}\circ 4\gamma_1(|w|)\gamma_1(|w|),$$

同理可得 $\gamma_0(|y|) \geqslant \gamma_1(|w|)$ 时,有

$$\dot{U}_0 \leqslant \frac{1}{2}\rho\circ\underline{\alpha}(|z|)\alpha_0(|z|) + \rho\circ\alpha\circ\alpha_0^{-1}\circ 4\gamma_0(|y|)\gamma_0(|y|).$$

综合 1)、2),就有

$$\dot{U}_0 \leqslant \frac{1}{2}\rho\circ\underline{\alpha}(|z|)\alpha_0(|z|) + \rho\circ\alpha\circ\alpha_0^{-1}\circ 4\gamma_0(|y|)\gamma_0(|y|) + \rho\circ\alpha\circ\alpha_0^{-1}\circ 4\gamma_1(|w|)\gamma_1(|w|).$$

由文献[7]中的引理 2 及条件(31),存在正定函数 ρ, 使得

$$\beta(z) = \Phi_2(z) + n\varphi_{11}(|z|)^2 + n\psi_{12}(|z|)^4 \leqslant \frac{1}{4}\rho\circ\underline{\alpha}(|z|)\alpha_0(|z|).$$

同样,我们选择合适的光滑的 K_∞ 函数 η, $\eta'(y^2) \neq 0$, 满足

$$\eta(y^2) \geqslant \rho\circ\alpha\circ\alpha_0^{-1}\circ 4\gamma_0(|y|)\gamma_0(|y|),$$

$$\dot{V}_c = \dot{V}_n + \dot{U}_0 \leqslant \alpha(|z|) - \frac{1}{2^n}|\zeta|^2 - \varepsilon^{-1}y^2 - \sum_{j=2}^{n}c_j\xi_j^2 - \frac{\sigma}{2}(\hat{p}-p)^2 + \gamma(|w|) + d. \tag{33}$$

4 主 要 结 果

定理　在假设 1,假设 2 及引理的条件下,不确定系统(1)的 H_∞ 控制问题在动态输出反馈(4)下是可解的.

证明　有

$$\dot{V}_c \leqslant \alpha(|z|) - \frac{1}{2^n}|\zeta|^2 - \varepsilon^{-1}y^2 - \sum_{j=2}^{n}c_j\xi_j^2 - \frac{\sigma}{2}(\hat{p}-p)^2 + \gamma(|w|) + d, \tag{34}$$

当 $w = 0$, $d = 0$ 时, $\dot{V}_c \leqslant W(z,\zeta,y)$ 是负定的, 而 $V_c = V_n + U_0$ 关于 (z,ζ,y,ξ) 或 (z,x,ξ) 是正定的径向无界函数, 则由 LaSalle_Yoshizawa 定理[8], 闭环系统(1)、(4)在原点的全局一致渐近稳定的. 性质(*)中的(5)可以从(34)直接得到.

注 3　由(34)式,当 $d = 0$ 时,闭环系统是 ISS 稳定的.

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Adaptive H_infinity Control of a Class of Uncertain Nonlinear Systems

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Abstract: It is concerned with the problem of disturbance attenuation with stability for uncertain nonlinear systems by adaptive output feedback. By a partial_state observer and Backstepping technique, an adaptive output feedback controller is constructed, which can solve the standard gain disturbance attenuation problem with internal stability.

Key words: nonlinear systems; adaptive control; output feedback