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某些四阶时滞微分方程解的稳定性*

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(郭兴明推荐)

摘要: 利用 Liapunov 函数法,得到了一个新的、证明某些四阶非线性时滞微分方程零解渐近稳定的结果• 建立结果的限制性条件弱于其他文献给出的的方法•

关 键 词: 四阶非线性时滞微分方程; 稳定性; Liapunov 泛函

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引 言

在微分方程理论和应用领域中,稳定性是非常重要的问题• 至今,确定线性和非线性微分方程解稳定性的有效方法,仍是 Liapunov 直接法(即 Liapunov 第二种方法)• 其优点在于,一般不需要预先知道解的情况,就可知道解的稳定性• 今天,它不仅是研究微分方程的极好工具,而且广泛用于控制系统、动力系统、时间滞后系统、电力系统分析、时变非线性反馈系统等的理论研究中• 它的主要特征为构造一类标量函数或泛函,也就是 Liapunov 函数或泛函• 但是寻找一类具有时滞(或非时滞) 的高阶微分方程的 Liapunov 函数或泛函,一般来说是困难的• 最近几年来,利用 Liapunov 直接法,得到了多种二阶、三阶、四阶、五阶和六阶非线性时滞(或非时滞) 微分方程解稳定性和有界性的极好的结果(见文献[1~ 31] 及其参考文献)• 值得注意的是,对于四阶非线性时滞微分方程解的稳定性研究结果却不多(见文献[1]、[14]、[18]、[21]、[26])•

1973 年, Sinha[21] 研究了如下四阶时滞微分方程:

$$x^{(4)}(t) + f(x''(t))x'''(t) + f_2(x'(t), x''(t))x''(t) + g(x'(t-r)) + h(x(t-r)) = 0,$$

证明了该方程是渐近稳定的• 之后在 1989 年, Okoronkwo[14] 研究了一种四阶标量时滞微分方程

$$x^{(4)}(t) + f(x''(t))x'''(t) + \alpha_2 x''(t) + \beta_2 x''(t-h) + g(x'(t-h)) + \alpha_4 x(t) + \beta_4 x(t-h) = P(t)$$

的解具有一致渐近稳定性和有界性的充分条件• 1998 年, Bereketoglu[1] 给出四阶时滞微分方程

$$x^{(4)}(t) + e(t, x(t), x'(t) + x''(t) + x'''(t))x'''(t) + f(t, x''(t-\tau)) + g(t, x'(t-\tau)) + h(x(t-\tau)) = 0$$

零解的一致渐近稳定性的充分条件• Sadek[18] 于 2004 年在对下面形式的四阶非线性时滞微分

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方程

$$x^{(4)} + \alpha_1 \dddot{x} + \alpha_2 \ddot{x} + \alpha_3 \dot{x} + f(x(t-r)) = 0$$

和

$$x^{(4)} + \alpha_1 \dddot{x} + \alpha_2 \ddot{x} + \phi(\dot{x}(t-r)) + f(x) = 0$$

的研究中, 通过构造两个新的 Liapunov 泛函, 得出方程组零解具有渐近稳定性的充分条件· 最近, 在文献[26] 中, 作者对四阶非线性微分方程

$$x^{(4)} + \varphi(\ddot{x})\dddot{x} + h(\dot{x})\ddot{x} + \phi(\dot{x}(t)) + f(x(t-r)) = 0$$

和

$$x^{(4)} + \varphi(\ddot{x})\dddot{x} + h(\dot{x})\ddot{x} + \phi(\dot{x}(t-r)) + f(x(t)) = 0$$

的零解的渐近稳定性作了研究·

在本文中, 我们将研究怎样通过构造一个新的 Liapunov 泛函, 讨论下面方程的渐近稳定性问题:

$$x^{(4)} + \varphi(\ddot{x})\dddot{x} + h(\dot{x})\ddot{x} + \phi(\dot{x}(t-r)) + f(x(t-r)) = 0, \tag{1}$$

其中 r 为正常数, $\varphi(\ddot{x})$、$h(\dot{x})$、$\phi(\dot{x})$ 和 $f(x)$ 为连续函数, $\phi(0) = f(0) = 0$· 导函数 $\mathrm{d}\phi/\mathrm{d}\dot{x} \equiv \phi'(\dot{x})$ 和 $\mathrm{d}f/\mathrm{d}x \equiv f'(x)$ 存在且连续·

本文研究动机主要来自 Sadek[18] 和 Tunç[26] 的论文, 但条件和 Bereketoglu[1]、Okoronkwo[14] 和 Sinha[21] 的完全不同·

显然, 式(1) 等价于系统

$$\begin{cases} \dot{x} = y,\ \dot{y} = z,\ \dot{z} = u, \\ \dot{u} = -\varphi(z)u - h(y)z - \phi(y) - f(x) + \int_{t-r}^{t} \phi'(y(s))z(s)\mathrm{d}s + \\ \qquad \int_{t-r}^{t} f'(x(s))y(s)\mathrm{d}s \cdot \end{cases} \tag{2}$$

1 预 备 知 识

为便于得出本文的主要结果, 首先给出一些重要的一般自治时滞系统的稳定性准则· 考虑

$$\dot{x} = f(x_t),\ x_t = x(t+\theta),\qquad -r \leqslant \theta \leqslant 0,\ t \geqslant 0, \tag{3}$$

其中 $f: C_H \to \mathscr{R}^n$ 为 C_H 到 $\mathscr{R}^n$ 的连续映射, $f(0) = 0$, $C_H := \left\{ \phi \in C([-r, 0], \mathscr{R}^n): \|\phi\| \leqslant H \right\}$; 且当 $H_1 < H$ 时(H_1、H 为正常数), $L(H_1) > 0$, 且当 $\|\phi\| \leqslant H_1$, 有 $|f(\phi)| \leqslant L(H_1)$·

定义 1 $x(t, 0, \phi)$ 定义在$[0, \infty)$ 上, 存在序列$\left\{ t_n \right\}$: 当 $n \to \infty$ 时 $t_n \to \infty$· 若元素 $\psi \in C_H$, 满足 $\|x_{t_n}(\phi) - \psi\| \to 0 (n \to \infty)$、$x_{t_n}(\phi) = x(t_n + \theta, 0, \phi)(-r \leqslant \theta \leqslant 0)$, 则称元素 ψ 为属于 ϕ 的 ω 极限集, 记为 $\Omega(\phi)$·

定义 2(见文献[31]) 若集合 $Q \subset C_H$ 满足: 对任意 $\phi \in Q$, 式(3) 的解 $x(t, 0, \phi)$ 定义在 $[0, \infty)$ 上, 并且当 $t \in [0, \infty)$ 有 $x_t(\phi) \in Q$, 则称集合 Q 为不变集合·

引理 1(见文献[3]、[8]、[31]) 若 $\phi \in C_H$ 是定义在$[0, \infty)$ 上的式(3) 的解 $x_t(\phi)$ 且满足 $x_0(\phi) = \phi$, 又 $\|x_t(\phi)\| \leqslant H_1 < H$, $t \in [0, \infty)$, 那么 $\Omega(\phi)$ 称为非空紧不变集且有

$$\operatorname{dist}(x_t(\phi), \Omega(\phi)) \to 0, \qquad 当\ t \to \infty·$$

引理 2(见文献[3]、[31]) 令 $V(\phi): C_H \to \mathrm{R}$ 为满足局部 Lipschitz 条件的连续泛函, $V(0) = 0$ 且满足下列条件:

(i) $W_1(|\phi(0)|) \leqslant V(\phi) \leqslant W_2(\|\phi\|)$, $W_1(r)$、$W_2(r)$ 为权函数·

(ii) $\dot{V}_{(3)}(\phi) \leqslant 0$, 当 $\phi \in C_H$·

则式(3) 零解是一致稳定的· 若定义 $Z = \left\{ \phi \in C_H: \dot{V}_{(3)}(\phi) = 0 \right\}$, 则式(3) 零解是渐近稳定

的, 并证明 Z 中最大不变集是 $Q=\{0\}$.

2 主要结论

在讨论主要定理前, 首先引入记号:

$$\Omega:=\{(x,y,z,u)\in\mathscr{R}^4: |x|<H_1, |y|<H_1, |z|<H_1, |u|<H_1, H_1\leqslant H\},$$

$$\varphi_1(z)=\begin{cases}\dfrac{1}{z}\displaystyle\int_0^z\varphi(z)\,\mathrm{d}z, & z\neq 0,\\ \varphi(0), & z=0,\end{cases}$$

和 $$\phi_1(y)=\begin{cases}\phi(y)/y, & y\neq 0,\\ \phi'(0), & y=0.\end{cases}$$

本文的主要结果如下:

定理 假设存在正常数 α_1、α_2、α_3、α_4、Δ 和 ε, 使得对 Ω 中的每个 x、y、z、u, 下列条件成立:

(i) $\alpha_1\alpha_2\alpha_3-\alpha_3\phi'(y)-\alpha_1\alpha_4\varphi(z)\geqslant\Delta>0$,

其中 $\varepsilon\leqslant\Delta/(2\alpha_1\alpha_3 D)$, $D=\alpha_1\alpha_2+\alpha_2\alpha_3/\alpha_4$;

(ii) $0<\alpha_4-\alpha_1\Delta/(4\alpha_3)<f'(x)\leqslant\alpha_4$;

(iii) $\phi'(y)\geqslant\alpha_3$ 和 $0\leqslant\phi_1(y)-\alpha_3<\dfrac{\Delta}{8\alpha_3}\sqrt{\dfrac{\alpha_4}{2\alpha_1\alpha_3}}$;

(iv) $0\leqslant h(y)-\alpha_2\leqslant\dfrac{\alpha_1}{8}\sqrt{\dfrac{\varepsilon\Delta}{\alpha_3}}$;

(v) $\varphi(z)\geqslant\alpha_1$, $\varphi_1(z)-\varphi(z)<\Delta/(2\alpha_1^2\alpha_3)$.

那么系统(2)的零解是渐进稳定的, 并得到

$$r<2\min\left\{\frac{\varepsilon\alpha_3}{d_2\alpha_4+2\lambda+d_2\alpha_1\alpha_2},\ \frac{3\Delta}{16\alpha_1\alpha_3(\alpha_4+\alpha_1\alpha_2+2\mu)},\ \frac{3\varepsilon\alpha_1}{4d_1(\alpha_4+\alpha_1\alpha_2)}\right\}. \tag{4}$$

和

$$d_1=\varepsilon+1/\alpha_1,\quad d_2=\varepsilon+\alpha_4/\alpha_3,$$
$$\lambda=(\alpha_4/2)(d_1+d_2+1)>0 \text{ 和 } \mu=(\alpha_1\alpha_2/2)(d_1+d_2+1)>0.$$

附注 1 根据定理中条件(i)、(iii)、(v)可得:

$$\varphi(z)<\alpha_2\alpha_3/\alpha_4,\quad \phi'(y)<\alpha_1\alpha_2.$$

附注 2 该定理需满足的条件比文献[18]中定理 1、定理 2 的充分条件要弱, 因此, 该定理包含了上述文献中得出的结果.

证明 先定义一个 Liapunov 函数 $V=V(x_t,y_t,z_t,u_t)$:

$$\begin{aligned}2V(x_t,y_t,z_t,u_t)={}&2d_2\int_0^x f(\xi)\,\mathrm{d}\xi+2d_2\int_0^y h(\eta)\eta\,\mathrm{d}\eta-d_1\alpha_4y^2+2\int_0^y\phi(\eta)\,\mathrm{d}\eta+d_1\alpha_2z^2+\\&2\int_0^z\varphi(\tau)\tau\,\mathrm{d}\tau-d_2z^2+d_1u^2+2f(x)y+2d_1f(x)z+2d_1\phi(y)z+2d_2y\int_0^z\varphi(\tau)\,\mathrm{d}\tau+\\&2d_2yu+2zu+2\lambda\int_{-r}^0\int_{t+s}^t y^2(\theta)\,\mathrm{d}\theta\mathrm{d}s+2\mu\int_{-r}^0\int_{t+s}^t z^2(\theta)\,\mathrm{d}\theta\mathrm{d}s.\end{aligned} \tag{5}$$

明显地, $V(0,0,0,0)=0$.

注意到式(5)中 $2V$ 可改写为

$$2V=\frac{1}{\alpha_3}[f(x)+\alpha_3y+d_1\alpha_3z]^2+\frac{1}{\varphi_1(z)}[u+\varphi_1(z)z+d_2\varphi_1(z)y]^2+\left[d_1-\frac{1}{\varphi_1(z)}\right]u^2+$$

$$[d_1\alpha_2 - d_2 - d_1^2\alpha_3]z^2 + 2d_2\int_0^y h(\eta)\eta d\eta - d_1\alpha_4 y^2 - d_2^2\varphi_1(z)y^2 +$$
$$2\int_0^y \phi(\eta)d\eta - \alpha_3 y^2 + 2d_1[\phi_1(y) - \alpha_3]yz + 2d_2\int_0^x f(\xi)d\xi - \left(\frac{1}{\alpha_3}\right)f^2(x) +$$
$$\left[2\int_0^z \varphi(\tau)\tau d\tau - \varphi_1(z)z^2\right] + 2\lambda\int_{-r}^0\int_{t+s}^t y^2(\theta)d\theta ds + 2\mu\int_{-r}^0\int_{t+s}^t z^2(\theta)d\theta ds \cdot \tag{6}$$

根据定理的假设, 常数 d_1、d_2 的定义以及微分中值定理与积分中值定理, 可知:

$$2V \geqslant \varepsilon\left(\alpha_4 - \frac{\alpha_1\Delta}{4\alpha_3}\right)x^2 + \left(\frac{\Delta\alpha_4}{2\alpha_1\alpha_3^2}\right)y^2 + \left(\frac{\Delta}{4\alpha_1^2\alpha_3}\right)z^2 + \varepsilon u^2 +$$
$$2d_1[\phi_1(y) - \alpha_3]yz + 2\lambda\int_{-r}^0\int_{t+s}^t y^2(\theta)d\theta ds + 2\mu\int_{-r}^0\int_{t+s}^t z^2(\theta)d\theta ds \cdot \tag{7}$$

记 W_5 为

$$W_5 \equiv \left(\frac{\Delta\alpha_4}{4\alpha_1\alpha_3^2}\right)y^2 + 2d_1[\phi_1(y) - \alpha_3]yz + \left(\frac{\Delta}{8\alpha_1^2\alpha_3}\right)z^2 \cdot$$

根据不等式

$$d_1^2[\phi_1(y) - \alpha_3]^2 < \frac{4}{\alpha_1^2}[\phi_1(y) - \alpha_3]^2 < \frac{\alpha_4\Delta^2}{32\alpha_1^3\alpha_3^3} \qquad (\text{对所有 } y),$$

得到下面关于 W_5 的估计

$$W_5 \geqslant \left[\frac{1}{2\alpha_3}\sqrt{\frac{\Delta\alpha_4}{\alpha_1}}|y| - \frac{1}{2\alpha_1}\sqrt{\frac{\Delta}{2\alpha_3}}|z|\right]^2 \geqslant 0 \cdot$$

结合该估计式与关于 $2V$ 的估计式(7) 可得

$$2V \geqslant \varepsilon\left(\alpha_4 - \frac{\alpha_1\Delta}{4\alpha_3}\right)x^2 + \left(\frac{\Delta\alpha_4}{4\alpha_1\alpha_3^2}\right)y^2 + \left(\frac{\Delta}{8\alpha_1^2\alpha_3}\right)z^2 + \varepsilon u^2 +$$
$$2\lambda\int_{-r}^0\int_{t+s}^t y^2(\theta)d\theta ds + 2\mu\int_{-r}^0\int_{t+s}^t z^2(\theta)d\theta ds \cdot$$

显然 $V(x_t, y_t, z_t, u_t)$ 满足引理 2 的条件(i)•

记 $dV(x_t, y_t, z_t, u_t)/dt = dV/dt$ 为 $V = V(x_t, y_t, z_t, u_t)$ 的导数, 直接计算式(5) 和式(2) , 可得

$$\frac{d}{dt}V = -[\alpha_4 - f'(x)]\left[y + \frac{d_1 z}{2}\right]^2 - [h(y) - d_1\phi'(y) - d_2\varphi_1(z)]z^2 +$$
$$\frac{d_1^2}{4}[\alpha_4 - f'(x)]z^2 - [d_1\varphi(z) - 1]u^2 - \left[d_2\frac{\phi(y)}{y} - \alpha_4\right]y^2 -$$
$$d_1[h(y) - \alpha_2]zu + (d_1u + z + d_2y)\int_{t-r}^t f'(x(s))y(s)ds +$$
$$(d_1u + z + d_2y)\int_{t-r}^t \phi'(y(s))z(s)ds + \lambda y^2 r -$$
$$\lambda\int_{t-r}^t y^2(s)ds + \mu z^2 r - \mu\int_{t-r}^t z^2(s)ds \cdot$$

注意到定理的假设、式(4) 、附注 1, 并运用微分中值定理与积分中值定理, 可得到:

$$0 \leqslant \alpha_4 - f'(x);$$
$$[h(y) - d_1\phi'(y) - d_2\varphi_1(z)]z^2 \geqslant$$
$$\left[\alpha_2 - \left(\varepsilon + \frac{1}{\alpha_1}\right)\phi'(y) - \left(\varepsilon + \frac{\alpha_4}{\alpha_3}\right)\varphi_1(z)\right]z^2 =$$
$$\left[\alpha_2 - \frac{1}{\alpha_1}\phi'(y) - \frac{\alpha_4}{\alpha_3}\varphi_1(z)\right]z^2 - \varepsilon[\phi'(y) + \varphi_1(z)]z^2 \geqslant$$

$$\left(\frac{1}{\alpha_1\alpha_3}\right)[\alpha_1\alpha_2\alpha_3-\alpha_3\phi'(y)-\alpha_1\alpha_4\varphi_1(z)]z^2-\varepsilon[\alpha_1\alpha_2+\varphi_1(z)]z^2=$$
$$\left(\frac{1}{\alpha_1\alpha_3}\right)[\alpha_1\alpha_2\alpha_2-\alpha_3\phi'(y)-\alpha_1\alpha_4\varphi(\theta z)]z^2-\varepsilon[\alpha_1\alpha_2+\varphi(\theta z)]z^2\geqslant$$
$$\left(\frac{\Delta}{\alpha_1\alpha_3}\right)z^2-\varepsilon\left[\alpha_1\alpha_2+\frac{\alpha_2\alpha_3}{\alpha_4}\right]z^2\geqslant$$
$$\left(\frac{\Delta}{\alpha_1\alpha_3}\right)z^2-\varepsilon Dz^2\geqslant\left(\frac{\Delta}{2\alpha_1\alpha_3}\right)z^2,$$

其中 $0\leqslant\theta\leqslant1$,

$$[d_1\varphi(z)-1]u^2\geqslant\left[\left(\varepsilon+\frac{1}{\alpha_1}\right)\varphi(z)-1\right]u^2\geqslant$$
$$\left[\left(\varepsilon+\frac{1}{\alpha_1}\right)\alpha_1-1\right]u^2=\varepsilon\alpha_1u^2$$

和

$$\left[d_2\frac{\phi(y)}{y}-\alpha_4\right]y^2\geqslant\left[\left(\varepsilon+\frac{\alpha_4}{\alpha_3}\right)\alpha_3-\alpha_4\right]y^2\geqslant\varepsilon\alpha_3y^2.$$

综上所述, 可推导出

$$\frac{\mathrm{d}}{\mathrm{d}t}V\leqslant-\varepsilon\alpha_3y^2-\left(\frac{\Delta}{2\alpha_1\alpha_3}\right)z^2-\varepsilon\alpha_1u^2+\frac{d_1^2}{4}[\alpha_4-f'(x)]z^2-$$
$$d_1[h(y)-\alpha_2]zu+(d_1u+z+d_2y)\int_{t-r}^{t}f'(x(s))y(s)\mathrm{d}s+$$
$$(d_1u+z+d_2y)\int_{t-r}^{t}\phi'(y(s))z(s)\mathrm{d}s+$$
$$\lambda y^2r-\lambda\int_{t-r}^{t}y^2(s)\mathrm{d}s+\mu z^2r-\mu\int_{t-r}^{t}z^2(s)\mathrm{d}s.\qquad(8)$$

现在考虑项

$$W_6\equiv\frac{d_1^2}{4}[\alpha_4-f'(x)]z^2-\left(\frac{\varepsilon\alpha_1}{4}\right)u^2-d_1[h(y)-\alpha_2]zu-\left(\frac{\Delta}{16\alpha_1\alpha_3}\right)z^2,$$

该式包含于式(8)中, 根据不等式

$$\frac{d_1^2}{4}[\alpha_4-f'(x)]<\frac{1}{\alpha_1^2}[\alpha_4-f'(x)]<\frac{\Delta}{4\alpha_1\alpha_3}\text{ 和 }\frac{d_1^2}{4}[h(y)-\alpha_2]^2\leqslant\frac{\varepsilon\Delta}{64\alpha_3},$$

显然 W_6 满足

$$W_6\leqslant\left(\frac{\Delta}{4\alpha_1\alpha_3}\right)z^2-\left[\frac{\sqrt{\varepsilon\alpha_1}}{2}|u|-\frac{1}{4}\sqrt{\frac{\Delta}{\alpha_1\alpha_3}}|z|\right]^2\leqslant\left(\frac{\Delta}{4\alpha_1\alpha_3}\right)z^2.$$

将上式代入前面 $\mathrm{d}V/\mathrm{d}t$ 的不等式, 有

$$\frac{\mathrm{d}}{\mathrm{d}t}V\leqslant-\varepsilon\alpha_3y^2-\left(\frac{3\Delta}{16\alpha_1\alpha_3}\right)z^2-\left(\frac{3\varepsilon\alpha_1}{4}\right)u^2+(d_1u+z+$$
$$d_2y)\int_{t-r}^{t}f'(x(s))y(s)\mathrm{d}s+(d_1u+z+d_2y)\int_{t-r}^{t}\phi'(y(s))z(s)\mathrm{d}s+$$
$$\lambda y^2r-\lambda\int_{t-r}^{t}y^2(s)\mathrm{d}s+\mu z^2r-\mu\int_{t-r}^{t}z^2(s)\mathrm{d}s.$$

注意到 $f'(x)\leqslant\alpha_4$, $|\phi'(y)|\leqslant\alpha_1\alpha_2$ 和 $2uv\leqslant u^2+v^2$, 则有

$$\frac{\mathrm{d}}{\mathrm{d}t}V\leqslant-\left[\varepsilon\alpha_3-\frac{1}{2}(d_2\alpha_4+2\lambda+d_2\alpha_1\alpha_2)r\right]y^2-$$
$$\left(\frac{3\Delta}{16\alpha_1\alpha_3}-\frac{1}{2}(\alpha_4+\alpha_1\alpha_2+2\mu)r\right)z^2-\left(\frac{3}{4}\varepsilon\alpha_1-\frac{1}{2}d_1(\alpha_4+\alpha_1\alpha_2)r\right)u^2+$$

$$\left[\frac{\alpha_4}{2}(d_1+d_2+1)-\lambda\right]\int_{t-r}^{t}y^2(s)\mathrm{d}s+\left[\frac{\alpha_1\alpha_2}{2}(d_1+d_2+1)-\mu\right]\int_{t-r}^{t}z^2(s)\mathrm{d}s. \tag{9}$$

若取 $\lambda=(\alpha_4/2)(d_1+d_2+1)>0$ 和 $\mu=(\alpha_1\alpha_2/2)(d_1+d_2+1)>0$, 由不等式(9)可知

$$\frac{\mathrm{d}}{\mathrm{d}t}V\leqslant\left[\varepsilon\alpha_3-\frac{1}{2}(d_2\alpha_4+2\lambda+d_2\alpha_1\alpha_2)r\right]y^2-\left(\frac{3\Delta}{16\alpha_1\alpha_3}-\frac{1}{2}(\alpha_4+\alpha_1\alpha_2+2\mu)r\right)z^2-\left(\frac{3}{4}\varepsilon\alpha_1-\frac{1}{2}d_1(\alpha_4+\alpha_1\alpha_2)r\right)u^2.$$

若取

$$r<2\min\left\{\frac{\varepsilon\alpha_3}{d_2\alpha_4+2\lambda+d_2\alpha_1\alpha_2},\frac{3\Delta}{16\alpha_1\alpha_3(\alpha_4+\alpha_1\alpha_2+2\mu)},\frac{3\varepsilon\alpha_1}{4d_1(\alpha_4+\alpha_1\alpha_2)}\right\},$$

实际上, 可得到

$$\mathrm{d}V(x_t,y_t,z_t,u_t)/\mathrm{d}t\leqslant-\rho(y^2+z^2+u^2)\qquad(\text{对一些常数 }\rho>0).$$

根据 $\mathrm{d}V(x_t,y_t,z_t,u_t)/\mathrm{d}t=0$ 和系统(2), 易知 $x=y=z=u=0$. 因此, 引理 2 的所有条件得到满足, 所以式(1) 的零解是渐进稳定的.

定理得证.

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On the Stability of Solutions of Certain Fourth_Order Delay Differential Equations

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Abstract: By the use of the Liapunov functional approach, a new result is obtained to ascertain the asymptotic stability of zero solution of a certain fourth_order non_linear differential equation with delay. The established result is less restrictive than those reported in the literature.

Key words: non_linear delay differential equation of fourth order; stability; Liapunov functional