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高阶非完整系统的适应调节*

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摘要: 通过综合利用“加幂积分器”的方法、“不连续投影”技巧和 state scaling 技巧构造出了一个不连续的动态适应控制律, 解决了一类高阶参数非完整系统的适应调节问题. 且所求控制律能确保未知参数的估值在一个预先给定范围内.

关 键 词: 非完整系统; 适应控制; 稳定性

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引 言

近年来, 非完整动态系统的控制和稳定一直是人们关注的问题. 对非完整系统控制问题的研究, 由于几个实际模型及技术方面的原因, 正如 Brockett 指出, 这类非线性系统虽然是可控的, 但不能用已知的线性和非线性控制方法渐近镇定[1], 研究者设法寻找新的控制策略. 然而大多结果是关于无漂移项的一类特殊的非完整系统的研究[2,3].

本文研究如下形式的高阶含参数非完整系统:

$$\begin{cases} \dot{x}_0 = u_0^{p_0}, \\ \dot{x}_i = \left(x_{i+1}^{p_i} + \boldsymbol{f}_i(x_0, x_1, \cdots, x_i)\,\theta\right) u_0^{q_i}, \\ \dot{x}_n = u_1^{p_n} + \boldsymbol{f}_n(x_0, x_1, \cdots, x_n)\,\theta u_0^{q_n}, \end{cases} \tag{1}$$

其中 $\boldsymbol{x} = (x_0, x_1, \cdots, x_n)^{\mathrm{T}} \in \mathrm{R} \times R^n$ 代表状态变量, $\boldsymbol{u} = (u_0, u_1)^{\mathrm{T}} \in R^2$ 控制输入, $\theta \in R^p$ 是未知常参数向量. 函数 $\boldsymbol{f}_i: R^{i+1} \to R^p$ 是 C^1 的. $p_i \geqslant 1$, $i = 0, 1, \cdots, n$ 是正奇数, 而 $q_k \geqslant 1$, $k = 1, \cdots, n-1$ 是正整数.

当 $\theta = \boldsymbol{0}$ 时, 系统(1) 成为文献[2] 中所讨论的形式; 当 $p_i = 1$, $i = 0, \cdots, n$, 和 $q_k = 1$, $k = 1, \cdots, n-1$ 时, 系统(1) 就是文献[3] 所讨论的形式; 而当 $\theta = \boldsymbol{0}$, $p_i = 1$, $i = 0, \cdots, n$, 和 $q_k = 1$, $k = 1, \cdots, n-1$ 时, 系统(1) 就成为一般的标准一次非完整系统(如文献[3]). 就我们所知, 在 $p_i \geqslant 1$, $q_k \geqslant 1$ 和 θ 未知时, 并不曾有人讨论过, 以前的结果可以说是本文结果的特殊情况. 本文通过综合利用“加幂积分器”的办法[4], “不连续投影”技巧和 state scaling 技巧[5] 构造出了一个不连续的动态适应控制律, 解决了高阶参数非完整系统(1) 的适应调节问题, 所求控制律

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能确保未知参数的估计值在一个预先给定范围内.

在本文中, 对系统(1)作如下假设:

(A) $f_i(x_0, x_1, \cdots, x_i) = (x_0^{p_i r_{i+1}}) g_i(x_1, \cdots, x_i)$,

其中 $g_i(\cdot)$ 是 C^1 的, 而且 $g_i(\mathbf{0}) = \mathbf{0}$. 及

$$r_1 = q_1 + p_1 r_2, \cdots, r_{n-1} = q_{n-1} + p_{n-1} r_n, r_n = 0.$$

1 问题描述

定义 1 带不确定参数 θ 的系统(1)的适应调节问题, 是寻找一个如下形式的适应动态反馈律

$$\dot{\hat\theta} = \phi(x_0, \boldsymbol{x}, \hat\theta), \quad \boldsymbol{u} = \boldsymbol{u}(x_0, \boldsymbol{x}, \hat\theta), \tag{2}$$

使得闭环系统(1)~(2)稳定且满足:

i) $\lim_{t\to\infty}(x_0, \boldsymbol{x}) = 0$.

ii) θ 的估计值 $\hat\theta$ 不超出给定的范围.

假设 1 未知参数满足

$$\theta \in \Omega_\theta = \left\{\theta: \theta_{\min} < \theta < \theta_{\max}\right\}, \tag{3}$$

其中 $\theta_{\min} = (\theta_{1\min}, \cdots, \theta_{p\min})^{\mathrm T}$, $\theta_{\max} = (\theta_{1max}, \cdots, \theta_{pmax})^T$ 是已知的, $\theta_{\min} > \mathbf{0}$. 令 $\hat\theta$ 表示θ 的估计值, 而 $\tilde\theta$ 是估计误差. (即 $\tilde\theta = \hat\theta - \theta$), 一个不连续投影函数[5]可定义为

$$\mathrm{Proj}_{\hat\theta}(*) = \begin{cases} \mathbf{0}, & \hat\theta_i = \theta_{i\max},\ * > \mathbf{0}, \\ \mathbf{0}, & \hat\theta_i = \theta_{i\min},\ * < \mathbf{0}, \\ *, & \text{其它}. \end{cases}$$

设 Γ > 0 是对角矩阵, 可证明对任何函数 τ, 投影函数有下面性质[5]:

$$\begin{aligned} &(\mathrm{P1}) \quad \hat\theta \in \left\{\hat\theta: \theta_{\min} \leqslant \hat\theta \leqslant \theta_{\max}\right\}; \\ &(\mathrm{P2}) \quad \tilde\theta^{\mathrm T}(\Gamma^{-1}\mathrm{Proj}_{\hat\theta}(\Gamma\tau) - \tau) \leqslant \mathbf{0}, \quad \forall \tau. \end{aligned} \tag{4}$$

2 全局适应调节律的构造

定理 1 存在如下形式的不连续适应动态控制律

$$u_0 = \alpha_0(u_0), \quad u_1 = \alpha_1(x_0, x_1, \cdots, \hat\theta), \quad \dot{\hat\theta} = \phi(x_0, x_1, \cdots, x_n, \hat\theta)$$

满足定义 1, 即使得系统(1)的适应调节问题可解.

令

$$u_0 = - x_0. \tag{5}$$

引理 2 对 $t_0 \geqslant 0$ 和任何初始条件 $x_0(t_0) \in \mathrm R$, 解 $x_0(t)$ 存在且

$$x_0^{p_0-1}(t) = \begin{cases} \dfrac{1}{x_0^{1-p_0} + (p_0 - 1)(t - t_0)}, & p_0 > 1, \\ x_0 e^{-(t-t_0)}, & p_0 = 1. \end{cases}$$

注 1 从引理 2 可知, 集合 $\left\{(x_0, \boldsymbol{x}) \in \mathrm R \times R^n, x_0 \neq 0\right\}$ 是闭环系统(1)和(5)的正不变集.

定理 1 的证明

1) 当 $x_0(t_0) \neq 0$ 时

选取下面的坐标变换[5]：

$$z_0 = x_0,\quad z_1 = \frac{x_1}{x_0^{r_1}},\quad \cdots,\ z_n = \frac{x_n}{x_0^{r_n}}. \tag{6}$$

由(6)式及假设(A)，系统(1)变为下面的系统

$$\begin{cases} \dot z_0 = -z_0^{p_0}, \\ \dot z_i = (-1)^{q_i} z_{i+1}^{p_i} + (-1)^{q_i} \boldsymbol{g}_i(x_1, \cdots, x_i)\theta - r_i z_i z_0^{p_0 - 1}, \\ \dot z_n = u_1^{p_n} + (-1)^{q_n} \boldsymbol{g}_n(x_1, \cdots, x_n)\theta z_0^{q_n}. \end{cases} \tag{7}$$

注 2　因为 $\boldsymbol{g}_i(\cdot)$ 是 C^1 的且 $\boldsymbol{g}_i(\boldsymbol{0}) = \boldsymbol{0}$. 由 Taylor 展式有

$$\begin{aligned} |\boldsymbol{g}_i(x_1, \cdots, x_i)\theta| &\leqslant (|x_1| + \cdots + |x_i|)\gamma_i(x_1, \cdots, x_i)\theta \leqslant \\ &(|z_1 z_0^{r_1}| + \cdots + |z_i z_0^{r_i}|)\gamma_i(z_1 z_0^{r_1}, \cdots, z_i z_0^{r_i})\theta \leqslant \\ &(|z_1| + \cdots + |z_i|)\boldsymbol{h}_i(z_0, z_1, \cdots, z_i)\theta, \end{aligned}$$

其中 $\boldsymbol{h}_i(\cdot)$ 是 C^∞ 函数.

第一步　令 $V_1(z_0, z_1) = (nz_0^2 + z_1^2)/2$，则 V_1 沿(7)式的导数为

$$\begin{aligned} \dot V_1(z_0, z_1) = &-nz_0^{p_0+1} + z_1((-1)^{q_1} z_2^{p_1} + (-1)^{q_1}\boldsymbol{g}_1(x_1)\theta + r_1 z_1 z_0^{p_0 - 1}) \leqslant \\ &-nz_0^{p_0+1} + (-1)^{q_1} z_1(z_2^{p_1} - z_2^{*\,p_1}) + (-1)^{q_1} z_1 z_2^{*\,p_1} + \\ &z_1^2 \boldsymbol{h}_1(z_0, z_1)\hat\theta + r_1 z_1^2 z_0^{p_0 - 1} + (-1)^{q_1+1} z_1 \boldsymbol{g}_1(x_1)\theta. \end{aligned} \tag{8}$$

令虚拟控制律为

$$\begin{aligned} z_2^{*\,p_1} = &(-1)^{q_1+1}(n + c_1 + \boldsymbol{h}_1(z_0, z_1)\hat\theta + r_1 z_0^{p_0 - 1}) = \\ &(-1)^{q_1+1} z_1 \beta_1(z_0, z_1, \hat\theta), \end{aligned} \tag{9}$$

其中 $c_1 > 0$ 是一待定常数. 则

$$\begin{aligned} \dot V_1(z_0, z_1) \leqslant &-nz_0^{p_0+1} - nz_1^2 - c_1 z_1^2 + (-1)^{q_1} z_1(z_2^{p_1} - z_2^{*\,p_1}) + \\ &(-1)^{q_1+1} z_1 \boldsymbol{g}_1(x_1)\theta. \end{aligned} \tag{10}$$

递归步　设在第 $k-1$ 步，存在正定的 C^1 Liapunov 函数 $V_{k-1}(z_0, z_1, \cdots, z_{k-1}, \hat\theta)$ 及一组 C^0 的虚拟控制律 $z_1^*, \cdots, z_k^*$ 定义为

$$\begin{cases} z_1^* = 0, & \xi_1 = z_1 - z_1^*, \\ z_2^{*\,p_1} = (-1)^{q_1+1}\xi_1\beta_1(z_0, z_1, \hat\theta), & \xi_2 = z_2^{p_2} - z_2^{*\,p_1}, \\ \quad\vdots & \quad\vdots \\ z^{*\,p_1\cdots p_{k-1}} = (-1)^{q_{k-1}+1}\xi_{k-1}\beta_{k-1}(z_0, z_1, \cdots, z_{k-1}, \hat\theta), & \xi_k = z_k^{p_1\cdots p_{k-1}} - z_k^{*\,p_1\cdots p_{k-1}} \end{cases} \tag{11}$$

及 $\beta_1(z_0, z_1, \hat\theta) > 0, \cdots, \beta_{k-1}(z_0, z_1, \cdots, z_{k-1}, \hat\theta) > 0$ 是光滑函数，使得

$$\begin{aligned} \dot V_{k-1} \leqslant &-(n - k + 2)(z_0^{p_0+1} + \xi_1^2 + \cdots + \xi_{k-1}^2) + \xi_{k-1}^{2-1/(p_1\cdots p_{k-2})}(z_k^{p_{k-1}} - z_k^{*\,p_{k-1}}) - \\ &(c_1\xi_1^2 + \cdots + c_{k-1}(z_0, z_1, \cdots, z_{k-2}, \hat\theta)\xi_{k-1}^2) + \left(\frac{\partial W_{k-1}}{\partial\hat\theta} + \cdots + \frac{\partial W_2}{\partial\hat\theta}\right)\dot{\hat\theta} - \\ &\Bigg[(-1)^{q_1}\xi_1\boldsymbol{g}_1(x_1) + \left((-1)^{q_2}\frac{\partial W_2}{\partial z_2}\boldsymbol{g}_2(x_1, x_2) + (-1)^{q_1}\frac{\partial W_2}{\partial z_1}\boldsymbol{g}_1(x_1)\right) + \\ &\left((-1)^{q_{k-1}}\frac{\partial W_{k-1}}{\partial z_{k-1}}\boldsymbol{g}_{k-1}(x_1, \cdots, x_{k-1}) + \cdots + (-1)^{q_1}\frac{\partial W_{k-1}}{\partial z_1}\boldsymbol{g}_1(x_1)\right)\Bigg]\theta. \end{aligned} \tag{12}$$

我们将证明(12)式在第 k 步同样成立. 考虑

$$\begin{cases} V_k(z_0, z_1, \cdots, z_k, \theta) = V_{k-1}(z_0, z_1, \cdots, z_{k-1}, \theta) + W_k(z_0, z_1, \cdots, z_k, \theta), \\ W_k(z_0, z_1, \cdots, z_k, \theta) = \int_{z_k^*}^{z_k} (s^{p_1\cdots p_{k-1}} - z_k^{*\,p_1\cdots p_{k-1}})^{2-1/(p_1\cdots p_{k-1})}\mathrm{d}s. \end{cases} \tag{13}$$

类似[4]中的证明, 有 $V_k(z_0, z_1, \cdots, z_k, \theta)$ 是 C^1、正定和真的, 同时 W_k 满足

$$\begin{cases} \dfrac{\partial W_k}{\partial z_k} = \xi_k^{2-1/(p_1\cdots p_{k-1})}, \\ \dfrac{\partial W_k}{\partial z_l} = -\left(2 - \dfrac{1}{p_1\cdots p_{k-1}}\right)\dfrac{\partial z_k^{*\,p_1\cdots p_{k-1}}}{\partial z_l}\int_{z_k^*}^{z_k}(s^{p_1\cdots p_{k-1}} - z_k^{*\,p_1\cdots p_{k-1}})^{1-1/(p_1\cdots p_{k-1})}\mathrm{d}s, \end{cases} \tag{14}$$

其中 $l = 0, 1, \cdots, k-1$. 则 V_k 沿(7)式的子系统$(z_0, z_1, \cdots, z_k)$ 导数为

$$\begin{aligned} &\dot V_k(z_0, z_1, \cdots, z_k, \theta) = \dot V_{k-1}(z_0, z_1, \cdots, z_{k-1}, \theta) + \frac{\partial W_k}{\partial z_k}\dot z_k + \sum_{l=0}^{k-1}\frac{\partial W_k}{\partial z_l}\dot z_l + \frac{\partial W_k}{\partial \theta}\dot\theta \leqslant \\ &-(n-k+2)(z_0^{p_0-1} + \xi_1^2 + \cdots + \xi_{k-1}^2) + \xi_{k-1}^{2-1/(p_1\cdots p_{k-2})}(z_k^{p_{k-1}} - z_k^{*\,p_{k-1}}) - \\ &-(c_1\xi_1^2 + \cdots c_{k-1}(z_0, z_1, \cdots, z_{k-2}, \theta)\xi_{k-1}^2) + \left(\frac{\partial W_k}{\partial \theta} + \cdots + \frac{\partial W_2}{\partial \theta}\right)\dot\theta - \\ &\left[(-1)^{q_1}\xi_1 \boldsymbol{g}_1(x_1) + \cdots + \left(\frac{\partial W_k}{\partial z_k}\boldsymbol{g}_k(z_1, \cdots, z_k) + \cdots + \frac{\partial W_k}{\partial z_1}\boldsymbol{g}_k(x_1)\right)\right]\theta + \\ &\xi_k^{2-1/(p_1\cdots p_{k-1})}((-1)^{q_k}z_{k+1}^{p_k} + \boldsymbol{g}_k(x_1, \cdots, x_k)\theta + r_k z_k z_0^{p_0-1}) + \frac{\partial W_k}{\partial z_0}(-z_0^{p_0}) + \\ &\sum_{l=1}^{k-1}\frac{\partial W_k}{\partial z_l}((-1)^{q_l}z_{l+1}^{p_l} + \boldsymbol{g}_l(x_1, \cdots, x_l)\theta + r_l z_l z_0^{p_0-1}). \end{aligned} \tag{15}$$

下面我们估计(15)式右边各项的范围:

$$\begin{aligned} &|\xi_{k-1}^{2-1/(p_1\cdots p_{k-2})}(z_k^{p_{k-1}})| \leqslant \\ &\quad |\xi_{k-1}|^{2-1/(p_1\cdots p_{k-1})}2^{1-1/(p_1\cdots p_{k-1})}|z_k^{p_1\cdots p_{k-1}} - z_k^{*\,p_1\cdots p_{k-1}}|^{1-1/(p_1\cdots p_{k-1})} \leqslant \\ &\quad \frac{\xi_{k-1}^2}{3} + \xi_k^2\rho_{k1}, \qquad \rho_{k1} > 0; \end{aligned} \tag{16}$$

$$\begin{cases} |(-1)^{q_k}\boldsymbol{g}_k\theta + r_k z_k z_0^{p_0-1}| \leqslant \\ \quad (|\xi_1|^{1/(p_1\cdots p_{k-1})} + \cdots + |\xi_k|^{1/(p_1\cdots p_{k-1})})\gamma_k(z_0, \cdots, z_k, \theta), \\ |\xi_k^{2-1/(p_1\cdots p_{k-1})}((-1)^{q_k}\boldsymbol{g}_k\theta + r_k z_k z_0^{p_0-1})| \leqslant \\ \quad \dfrac{\xi_1^2 + \cdots + \xi_{k-1}^2}{3} + \xi_k^2\rho_{k2}(z_0, \cdots, z_k, \theta), \end{cases} \tag{17}$$

$\rho_{k2}(z_0, \cdots, z_k, \theta) > 0$ 是光滑函数. 从(14)式, 有

$$\begin{aligned} &\left|\frac{\partial W_k}{\partial z_l}\right| = \left(2 - \frac{1}{p_1\cdots p_{k-1}}\right)\left|\frac{\partial z_k^{*\,p_1\cdots p_{k-1}}}{\partial z_l}\int_{z_k^*}^{z_k}(s^{p_1\cdots p_{k-1}} - z^{*\,p_1\cdots p_{k-1}})^{1-1/(p_1\cdots p_{k-1})}\mathrm{d}s\right| \leqslant \\ &\quad \left(2 - \frac{1}{p_1\cdots p_{k-1}}\right)|z_k^* - z_k||\xi_k|^{1-1/(p_1\cdots p_{k-1})}\left|\frac{\partial z_k^{*\,p_1\cdots p_{k-1}}}{\partial z_l}\right| \leqslant \\ &\quad a_k|\xi_k|\left|\frac{\partial z_k^{*\,p_1\cdots p_{k-1}}}{\partial z_l}\right|, \qquad a_k > 0;\ l = 0, \cdots, k-1. \end{aligned} \tag{18}$$

与文献[4]中类似有

$$\left|\frac{\partial z_k^{*\,p_1\cdots p_{k-1}}}{\partial z_l}((-1)^{q_l}z_{l+1}^{p_l} + (-1)^{q_l}\boldsymbol{g}_l\theta + r_l z_l z_0^{p_0-1})\right| \leqslant$$

$$(|\xi_1|+\cdots+|\xi_k|)C_{kl}(z_0,\cdots,z_k,\hat{\theta}).\tag{19}$$

对于 C^{∞} 函数 $C_{kl}\geqslant 0,\ l=1,\cdots,k-1$，则合并(18)和(19)式有

$$\sum_{l=1}^{k-1}\frac{\partial W_k}{\partial z_l}\left((-1)^{q_l}z_{l+1}^{p_l}+(-1)^{q_l}\boldsymbol{g}_l\theta+r_l z_l z_0^{p_0-1}\right)\leqslant$$
$$a_k|\xi_k|\sum_{l=1}^{k-1}\left|\frac{\partial z_k^{*\,p_1\cdots p_{k-1}}}{\partial z_l}\left((-1)^{q_l}z_{l+1}^{p_l}+(-1)^{q_l}\boldsymbol{g}_l\theta+r_l z_l z_0^{p_0-1}\right)\right|\leqslant$$
$$\frac{\xi_1^2+\cdots+\xi_{k-1}^2}{3}+\xi_k^2\rho_{k3}(z_0,\cdots,z_k,\hat{\theta}),\tag{20}$$

其中 $\rho_{k3}\geqslant 0$ 是光滑函数．由 Young 不等式有

$$\left|\frac{\partial W_k}{\partial z_0}\dot{z}_0\right|\leqslant a_k|\xi_k|\left|\frac{\partial z_k^{*\,p_1\cdots p_{k-1}}}{\partial z_0}\right||z_0^{p_0}|\leqslant z_0^{p_0+1}+\xi_k^2\rho_{k4}(\cdot),\tag{21}$$

其中 $\rho_{k4}(\cdot)=\xi_k^{p_0-1}\left(a_k(\partial z_k^{*\,p_1\cdots p_{k-1}}/\partial z_0)\right)^{p_0+1}\geqslant 0$ 是光滑函数，把式(16)～(21)代入式(15)

$$\dot{V}_k\leqslant-(n-k+1)(z_0^{p_0+1}+\xi_1^2+\cdots+\xi_{k-1}^2)+(-1)^{q_k}\xi_k^{2-1/(p_1\cdots p_{k-1})}z_{k+1}^{p_k}-$$
$$-(c_1\xi_1^2+\cdots+c_{k-1}(z_0,z_1,\cdots,z_{k-2},\hat{\theta})\xi_{k-1}^2)+\left(\frac{\partial W_k}{\partial\hat{\theta}}+\cdots+\frac{\partial W_2}{\partial\hat{\theta}}\right)\dot{\hat{\theta}}-$$
$$\left[(-1)^{q_1}\xi_1\boldsymbol{g}_1(x_1)+\left((-1)^{q_k}\frac{\partial W_k}{\partial z_k}\boldsymbol{g}_k(x_1,\cdots,x_k)+\cdots+(-1)^{q_1}\frac{\partial W_k}{\partial z_1}\boldsymbol{g}_1(x_1)\right)\right]\theta+$$
$$\xi_k^2(\rho_{k1}+\rho_{k2}(\cdot)+\rho_{k3}(\cdot)+\rho_{k4}(\cdot)),$$

所以选取如下虚拟控制律

$$z_{k+1}^{*\,p_1\cdots p_k}(z_0,z_1,\cdots,z_k,\hat{\theta})=$$
$$-(-1)^{q_k}\xi_k(n-k+1+c_k(z_0,z_1,\cdots,z_{k-1},\hat{\theta})+\rho_{k1}+\rho_{k2}+\rho_{k3}+\rho_{k4})=$$
$$(-1)^{q_{k+1}}(\xi_k\beta_k(z_0,z_1,\cdots,z_k,\hat{\theta})),\tag{22}$$

$$\dot{V}_k(z_0,z_1,\cdots,z_k,\hat{\theta})\leqslant$$
$$-(n-k+1)(z_0^{p_0+1}+\xi_1^2+\cdots+\xi_k^2)+\xi_k^{2-1/(p_1\cdots p_{k-1})}(z_{k+1}^{p_k}-z_{k+1}^{*\,p_k})-$$
$$-(c_1\xi_1^2+\cdots+c_k(z_0,z_1,\cdots,z_{k-1},\hat{\theta})\xi_k^2)+\left(\frac{\partial W_k}{\partial\hat{\theta}}+\cdots+\frac{\partial W_2}{\partial\hat{\theta}}\right)\dot{\hat{\theta}}-$$
$$\left[(-1)^{q_1}\xi_1\boldsymbol{g}_1(x_1)+\left((-1)^{q_k}\frac{\partial W_k}{\partial z_k}\boldsymbol{g}_k(x_1,\cdots,x_k)+\cdots+(-1)^{q_1}\frac{\partial W_k}{\partial z_1}\boldsymbol{g}_1(x_1)\right)\right]\theta.\tag{23}$$

由以上归纳法可知，(12)式在第 $k=n+1$ 步也成立．选择

$$u_1=z_{n+1}=z_{n+1}^{*}=(-1)^{q_n+1}(\xi_n\beta(z_0,\cdots,z_n,\hat{\theta}))^{1/(p_1\cdots p_n)},\tag{24}$$

从而有

$$\dot{V}_n(z_0,z_1,\cdots,z_n,\hat{\theta})\leqslant$$
$$-(z_0^{p_0+1}+\xi_1^2+\cdots+\xi_n^2)-(c_1\xi_1^2+\cdots+c_n(z_0,z_1,\cdots,z_{n-1},\hat{\theta})\xi_n^2)-$$
$$\left[(-1)^{q_1}\xi_1\boldsymbol{g}_1(x_1)+\left((-1)^{q_n}\frac{\partial W_n}{\partial z_n}\boldsymbol{g}_n(x_1,\cdots,x_n)+\cdots+\right.\right.$$
$$\left.\left.(-1)^{q_1}\frac{\partial W_n}{\partial z_1}\boldsymbol{g}_1(x_1)\right)\right]\theta+\left(\frac{\partial W_n}{\partial\hat{\theta}}+\cdots+\frac{\partial W_2}{\partial\hat{\theta}}\right)\dot{\hat{\theta}}.\tag{25}$$

最后，我们确定光滑函数 $c_1,\cdots,c_n(z_0,z_1,\cdots,z_{n-1},\hat{\theta})$．令

$$\tau=\left[(-1)^{q_1}\xi_1 \boldsymbol{g}_1(x_1)+\left((-1)^{q_n}\frac{\partial W_n}{\partial z_n}\boldsymbol{g}_n(x_1,\cdots,x_n)+\cdots+(-1)^{q_1}\frac{\partial W_n}{\partial z_1}\boldsymbol{g}_1(x_1)\right)\right]^{\mathrm{T}},$$

$$\dot{\hat\theta}=\mathrm{Proj}_{\hat\theta}(\tau),$$

代入不等式(25),就得

$$\dot V_n(z_0,z_1,\cdots,z_n,\hat\theta)\leqslant -(z_0^{p_0+1}+\xi_1^2+\cdots+\xi_n^2)-(c_1\xi_1^2+\cdots+c_n(z_0,z_1,\cdots,z_{n-1},\hat\theta)\xi_n^2)+\left|\left(\frac{\partial W_n}{\partial\hat\theta}+\cdots+\frac{\partial W_2}{\partial\hat\theta}\right)\tau\right|-\tau^{\mathrm{T}}\theta, \tag{26}$$

$$\left|\frac{\partial W_k}{\partial\hat\theta}\right|\leqslant a_k\,|\,\xi_k\,|\,\left|\frac{\partial z_k^{*\,p_1\cdots p_{k-1}}}{\partial\hat\theta}\right|,\qquad a_k>0. \tag{27}$$

由(18)和(27)式,存在光滑函数 $d_1,\cdots,d_n(z_0,z_1,\cdots,z_{n-1},\hat\theta)$ 使得

$$\left|\left(\frac{\partial W_n}{\partial\hat\theta}+\cdots+\frac{\partial W_2}{\partial\hat\theta}\right)\tau\right|\leqslant d_1\xi_1^2+\cdots+d_n(z_0,z_1,\cdots,z_{n-1},\hat\theta)\xi_n^2,$$

则,选取 $c_i(z_0,z_1,\cdots,z_{i-1},\hat\theta)\geqslant d_i(z_0,z_1,\cdots,z_{i-1},\hat\theta)$,由(26)式有

$$\dot V_n(z_0,z_1,\cdots,z_n,\hat\theta)\leqslant -(z_0^{p_0+1}+\xi_1^2+\cdots+\xi_n^2)-\tau^{\mathrm{T}}\theta. \tag{28}$$

定义一新的正定函数 V_θ 为

$$V_\theta=V_n+\frac{1}{2}\theta^{\mathrm{T}}\theta.$$

因(28)式及投影函数的性质,V_θ 的导数为

$$\dot V_\theta\leqslant -(z_0^{p_0+1}+\xi_1^2+\cdots+\xi_n^2)-\tau^{\mathrm{T}}\theta+\theta\dot{\hat\theta}\leqslant -(z_0^{p_0+1}+\xi_1^2+\cdots+\xi_n^2),$$

所以,$(z_0,z_1,\cdots,z_n)^{\mathrm{T}}\in L_2^{n+1}$,容易验证 $\dot z$ 是有界的. 由稳定性定理及 Barbalat 引理[6],有 $z\to\mathbf{0}$, $t\to+\infty$,故系统(7)~(24)的平凡解 $(z_0,z_1,\cdots,z_n)$ 是全局稳定且 $\hat\theta$ 不超出预先给定的值范围.

2) 设 $x_0(t_0)=0$

不失一般性,不妨设 $t_0=0$,下面我们讨论如下的开关策略:对控制输入 u_0 和 u_1,选

$$u_0=u_0^*,\quad u_0^*\neq 0, \tag{29}$$

则状态 z_0 并不会无界. 给定任何有限的 $t'>0$,在区间 $(0,t']$,在 $\boldsymbol{u}=\boldsymbol{u}^*(z_0,\boldsymbol{z})$ 时,应用类似不等式(28),可知系统(1)的子系统 $\boldsymbol{z}$ 仍在区间 $[0,t']$ 有界,因在 t',$x_0(t')\neq 0$,把输入 u_0 和 u_1 换成(5)和(24)式.

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Adaptive Regulation of High Order Nonholonomic Systems

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Abstract: The problem of adaptive regulation for a class of high_order parametric nonholonomic systems chained_form is discussed. Using adding a power integrator technique and state scaling with discontinuous projection technique, a discontinuous adaptive dynamic controller was constructed. The controller guarantees the estimated value of unknown parameter in the prescribed extent.

Key words: nonholonomic system; adaptive control; triangular system; stabilization