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二阶非线性阻尼方程的振动准则*

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(我刊编委郭兴明来稿)

摘要: 利用 Riccati 变换和积分平均方法, 研究了一类具有阻尼项的二阶非线性方程的解为振动的若干充分条件, 建立了一些判定上述方程为振动的充分准则, 结果推广并加强了已有的一些振动准则•

关 键 词: 二阶非线性阻尼方程; 振动性; Riccati 变换; 积分平均技巧
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1 引言及问题的提出

考虑如下二阶非线性阻尼方程

$$[p(t)\phi(y)u(y')]' + r(t)y'(t) + q(t)f(y)g(y') = 0, \qquad t \geqslant t_0, \tag{1}$$

其中

$$p(t) \in C'([t_0, \infty), (0, \infty)), \quad r(t) \in C([t_0, \infty), (-\infty, \infty)),$$
$$q(t) \in C([t_0, \infty), [0, \infty)),$$

且存在

$$T \geqslant t_0, q(t) \neq 0, \qquad t \in [T, \infty),$$
$$f(y), g(y), \phi(y), u(y) \in C((-\infty, \infty), (-\infty, \infty)),$$
$$yf(y) > 0, \quad yu(y) > 0, y \neq 0•$$

方程(1)的解 $y(t)$ 称为是正常解, 如果 $y(t)$ 是方程(1)的非常数解且 $\sup_{t \geqslant t_0} |y(t)| > 0$ (参见文献[1]); 一个正常解称为是振动的, 如果它有任意大零点, 否则称为是非振动的; 如果方程(1)的所有正常解都是振动的, 则称方程(1)是振动的•

不同类型的线性和非线性微分方程的振动与非振动性, 已被很多学者所研究, 文献[2]~[6]分别研究了如下线性方程

$$y''(t) + q(t)y = 0, \tag{2}$$

$$(p(t)y')' + q(t)y = 0, \tag{3}$$

及非线性情形

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$$(p(t)y')' + q(t)f(y) = 0 \tag{4}$$

的振动性• 有阻尼项的方程的振动准则还可从文献[7]、[8]及其所列参考文献中找到•

受文献[2]的启发，本文去掉文献[1]的准则中关于 $\partial H/\partial s \leqslant 0$ 的限制，考虑 $H(t,s)k(s)$ 型函数(它对 s 的偏导数不一定非正)，得到一些新的振动准则• 这些准则改进并推广了文献[1]及其它一些已知结果•

2 主要结果

首先定义函数类 Φ• 记 $D_0 = \{(t,s): t > s \geqslant t_0\}$，$D = \{(t,s): t \geqslant s \geqslant t_0\}$，称 $H = H(t,s) \in \Phi$，如果 $H \in C(D,(-\infty,\infty))$，满足

(i) $H(t,t) = 0$，$t \geqslant t_0$，$H(t,s) > 0$ 在 D_0 上；

(ii) $H(t,s)$ 在 D_0 上对 s 有连续且非正的偏导数•

下面总假设下列条件成立：

(A_1) $f(y)/y \geqslant \mu > 0$，$y \neq 0$，μ 为常数；

(A_2) $0 < C_1 \leqslant \psi(y) \leqslant C_2$，$y \neq 0$，$C_1$、$C_2$ 为常数；

(A_3) $0 < M_1 \leqslant y/u(y) \leqslant M_2$，$y \neq 0$，$M_1$、$M_2$ 为常数；

(A_4) $g(y) \geqslant K > 0$，$y \neq 0$，K 为常数•

定理1 设条件(A_1)～(A_4)成立，且存在 $H \in \Phi$，$h \in (D,(-\infty,\infty))$ 及 $k(s)$，$a(s) \in C'([t_0,\infty),(0,\infty))$ 满足

$$\frac{\partial}{\partial s}(H(t,s)k(s)) - H(t,s)k(s)\left[\frac{M_1 r(s)}{C_2 p(s)} - \frac{a'(s)}{a(s)}\right] = h(t,s)\sqrt{H(t,s)}, \quad (t,s) \in D, \tag{5}$$

$$\limsup_{t\to\infty} \frac{1}{H(t,t_0)}\int_{t_0}^{t}\left\{H(t,s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t,s)\right\}\mathrm{d}s = \infty, \tag{6}$$

$$Q(t) = K\mu a(t)q(t) - \left(\frac{M_2}{C_1} - \frac{M_1}{C_2}\right)\frac{a(t)r^2(t)}{4p(t)}, \tag{7}$$

则方程(1)是振动的•

证明 若方程(1)存在一非振动解 $y(t)$，则存在充分大的 $T_0 \geqslant t_0$，使得对所有 $t \geqslant T_0$，$y(t) \neq 0$，无妨设 $y(t) > 0$，令

$$W(t) = a(t)\frac{p(t)\psi(y)u(y')}{y}, \tag{8}$$

对 $W(t)$ 求导并结合方程(1)，条件(A_1)～(A_4)得

$$\begin{aligned}
W'(t) &= a'(t)\frac{p(t)\psi(y)u(y')}{y} + a(t)\frac{[p(t)\psi(y)u(y')]'}{y} - \\
&\quad a(t)\frac{p(t)\psi(y)u(y')y'}{y^2} \leqslant \\
&\quad \frac{a'(t)}{a(t)}W(t) - \frac{r(t)y'}{p(t)\psi(y)u(y')}W(t) - \\
&\quad \frac{y'}{a(t)p(t)\psi(y)u(y')}W^2(t) - K\mu a(t)q(t) = \\
&\quad \frac{a'(t)}{a(t)}W(t) - \frac{y'}{p(t)\psi(y)u(y')}\left[r(t)W(t) + \frac{M^2(t)}{a(t)}\right] - K\mu a(t)q(t) = \\
&\quad \frac{a'(t)}{a(t)}W(t) - \frac{y'}{p(t)\psi(y)u(y')}\left[\frac{W(t)}{\sqrt{a(t)}} + \frac{\sqrt{a(t)}}{2}r(t)\right]^2 +
\end{aligned}$$

$$\frac{a(t)r^2(t)y'}{4p(t)\psi(y)u(y')} - K\mu a(t)q(t) \leqslant$$

$$\frac{a'(t)}{a(t)}W(t) - \frac{M_1}{C_2 p(t)}\left[\frac{W(t)}{\sqrt{a(t)}} + \frac{\sqrt{a(t)}}{2}r(t)\right]^2 +$$

$$\frac{M_2 a(t)r^2(t)}{4C_1 p(t)} - K\mu a(t)q(t) =$$

$$-\left[\frac{M_1 r(t)}{C_2 p(t)} - \frac{a'(t)}{a(t)}\right]W(t) - \frac{M_1}{C_2 a(t)p(t)}W^2(t) -$$

$$K\mu a(t)q(t) + \left(\frac{M_2}{C_1} - \frac{M_1}{C_2}\right)\frac{a(t)r^2(t)}{4p(t)},$$

即

$$W'(t) \leqslant -\left[\frac{M_1 r(t)}{C_2 p(t)} - \frac{a'(t)}{a(t)}\right]W(t) - \frac{M_1}{C_2 a(t)p(t)}W^2(t) - Q(t). \tag{9}$$

两边同乘 $H(t, s)k(s)$ 并从 T 到 t 积分, $t \geqslant T \geqslant t_0$, 并结合(5)式,得

$$\int_T^t H(t, s)k(s)Q(s)\mathrm{d}s \leqslant -\int_T^t H(t, s)k(s)W'(s)\mathrm{d}s -$$

$$\int_T^t\left[\frac{M_1 r(s)}{C_2 p(s)} - \frac{a'(s)}{a(s)}\right]H(t, s)k(s)W(s)\mathrm{d}s - \int_T^t \frac{M_1 H(t, s)k(s)}{C_2 a(s)p(s)}W^2(s)\mathrm{d}s =$$

$$H(t, T)k(T)W(T) + \int_T^t\left\{\frac{\partial}{\partial s}[H(t, s)k(s)] -\right.$$

$$\left.H(t, s)k(s)\left[\frac{M_1 r(s)}{C_2 p(s)} - \frac{a'(s)}{a(s)}\right]\right\}W(s)\mathrm{d}s - \int_T^t \frac{M_1 H(t, s)k(s)}{C_2 a(s)p(s)}W^2(s)\mathrm{d}s =$$

$$H(t, T)k(T)W(T) - \int_T^t\left\{\frac{M_1 H(t, s)k(s)}{C_2 a(s)p(s)}W^2(s) -\right.$$

$$\left.h(t, s)\sqrt{H(t, s)k(s)}W(s)\right\}\mathrm{d}s =$$

$$H(t, T)k(T)W(T) - \int_T^t\left\{\sqrt{\frac{M_1 H(t, s)k(s)}{C_2 a(s)p(s)}}W(s) -\right.$$

$$\left.\frac{h(t, s)}{2}\sqrt{\frac{C_2 a(s)p(s)}{M_1}}\right\}^2\mathrm{d}s + \frac{C_2}{4M_1}\int_T^t a(s)p(s)h^2(t, s)\mathrm{d}s \leqslant$$

$$H(t, T)k(T)W(T) + \frac{C_2}{4M_1}\int_T^t a(s)p(s)h^2(t, s)\mathrm{d}s,$$

从而

$$\int_T^t\left\{H(t, s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t, s)\right\}\mathrm{d}s \leqslant H(t, T)k(T)W(t) \leqslant$$

$$H(t, T)k(T)|W(t)| \leqslant H(t, t_0)k(T)|W(t)|,$$

于是对所有 $t \geqslant T$, 有

$$\int_{t_0}^t\left\{H(t, s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t, s)\right\}\mathrm{d}s =$$

$$\int_{t_0}^T\left\{H(t, s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t, s)\right\}\mathrm{d}s +$$

$$\int_T^t\left\{H(t, s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t, s)\right\}\mathrm{d}s \leqslant$$

$$\int_{t_0}^{T} H(t,s)k(s)Q(s)ds + H(t,t_0)k(T)|W(T)| \leqslant$$

$$H(t,t_0)\left[\int_{t_0}^{T} k(s)|Q(s)|ds + K(T)|W(T)|\right].$$

$$\lim_{t\to\infty}\sup\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left\{H(t,s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t,s)\right\}ds \leqslant$$

$$\int_{t_0}^{T} k(s)|Q(s)|ds + k(T)|W(T)| < \infty.$$

这与(6)式矛盾, 定理1得证.

推论1 若定理1中的条件(6)被替换为

$$\lim_{t\to\infty}\sup\frac{1}{H(t,t_0)}\int_{t_0}^{t} H(t,s)k(s)Q(s)ds = \infty, \tag{10}$$

$$\lim_{t\to\infty}\sup\frac{1}{H(t,t_0)}\int_{t_0}^{t} a(s)p(s)h^2(t,s)ds < \infty, \tag{11}$$

则方程(1)是振动的.

通过适当选取$H(t,s)$和$k(s)$, 可获得方程(1)的一些具体的振动准则.

令$H(t,s) = (t-s)^{n-1}, (t,s)\in D, k(s) = s$, 其中$n > 2$为整数, 则

$$h(t,s) = -(t-s)^{(n-3)/2}\left\{ns - t + (t-s)s\left[\frac{M_1 r(s)}{C_2 p(s)} - \frac{a'(s)}{a(s)}\right]\right\}.$$

推论2 设条件$(A_1)\sim(A_4)$成立, 若存在$a(t)\in C'([t_0,\infty),(0,\infty))$, $n > 2$为整数,

$$\lim_{t\to\infty}\sup t^{1-n}\int_{t_0}^{t}\left\{(t-s)^{n-1}sQ(s) - \frac{C_2 a(s)p(s)(t-s)^{n-3}}{4M_1}\times\right.$$

$$\left.\left\{ns - t + (t-s)s\left[\frac{M_1 r(s)}{C_2 p(s)} - \frac{a'(s)}{a(s)}\right]\right\}^2\right\}ds = \infty, \tag{12}$$

则方程(1)是振动的.

注1 若$u(y)\equiv y, g(y)\equiv 1$, 取$M_1 = M_2 = 1, K = 1, k(s)\equiv 1, \partial H/\partial s = -h(t,s)\sqrt{H(t,s)}$, 即得文献[1]的定理1、推论2、推论3.

定理2 设条件$(A_1)\sim(A_4)$成立, $H(t,s)$、$h(t,s)$、$k(s)$、$a(s)$的定义同定理1. 设

$$0 < \inf_{s\geqslant t_0}\left[\lim_{t\to\infty}\inf\frac{H(t,s)}{H(t,t_0)}\right] \leqslant \infty, \tag{13}$$

若存在$\varphi\in C([t_0,\infty),(-\infty,\infty))$, 使$t\geqslant t_0, T\geqslant t_0$时

$$\lim_{t\to\infty}\sup\frac{1}{H(t,t_0)}\int_{t_0}^{t} a(s)p(s)h^2(t,s)ds < \infty, \tag{14}$$

$$\lim_{t\to\infty}\sup\int_{t_0}^{t}\frac{\varphi_+^2(s)}{k(s)a(s)p(s)}ds = \infty, \tag{15}$$

$$\lim_{t\to\infty}\sup\frac{1}{H(t,T)}\int_{T}^{t}\left\{H(t,s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t,s)\right\}ds \geqslant \varphi(T), \tag{16}$$

其中$Q(t)$的定义同定理1, $\varphi_+(t) = \max\{\varphi(t), 0\}$, 则方程(1)是振动的.

证明 设若不然, 方程(1)存在非振动解$y(t)$, 则存在充分大的$T_0\geqslant t_0$, 使得对所有$t\geqslant T_0$, $y\neq 0$, 无妨设$y(t) > 0$, 令

$$W(t) = a(t)\frac{p(t)\psi(y)u(y')}{y},$$

对 $W(t)$ 求导, 类似于定理 1 的证法, 可得

$$\int_T^t\left\{H(t,s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t,s)\right\}\mathrm{d}s \leqslant H(t,T)k(T)W(T) - \int_T^t\left\{\sqrt{\frac{M_1H(t,s)k(s)}{C_2a(s)p(s)}}W(s) - \frac{h(t,s)}{2}\sqrt{\frac{C_2a(s)p(s)}{M_1}}\right\}^2\mathrm{d}s,$$

$$\limsup_{t\to\infty}\frac{1}{H(t,T)}\int_T^t\left\{H(t,s)k(s)Q(s) - \frac{C_2}{4M_1}a(s)p(s)h^2(t,s)\right\}\mathrm{d}s \leqslant k(T)W(T) - \liminf_{t\to\infty}\frac{1}{H(t,T)}\int_T^t\left\{\sqrt{\frac{M_1H(t,s)k(s)}{C_2a(s)p(s)}}W(s) - \frac{h(t,s)}{2}\sqrt{\frac{C_2a(s)p(s)}{M_1}}\right\}^2\mathrm{d}s.$$

由(16)式, 对 $T \geqslant T_0$, 有

$$\begin{cases} k(T)W(t) \geqslant \varphi(T) + \liminf_{t\to\infty}\dfrac{1}{H(t,T)}\displaystyle\int_T^t\left\{\sqrt{\frac{M_1H(t,s)k(s)}{C_2a(s)p(s)}}W(s) - \frac{h(t,s)}{2}\sqrt{\frac{C_2a(s)p(s)}{M_1}}\right\}^2\mathrm{d}s, \quad k(T)W(T) \geqslant \varphi(T),\ T \geqslant T_0, \\ \liminf_{t\to\infty}\dfrac{1}{H(t,T_0)}\displaystyle\int_{T_0}^t\left\{\sqrt{\frac{M_1H(t,s)k(s)}{C_2a(s)p(s)}}W(s) - \frac{h(t,s)}{2}\sqrt{\frac{C_2a(s)p(s)}{M_1}}\right\}^2\mathrm{d}s \leqslant k(T_0)W(T_0) - \varphi(T_0) = M < \infty, \end{cases} \tag{17}$$

因此, 对 $t \geqslant T_0$, 有

$$\infty > \liminf_{t\to\infty}\frac{1}{H(t,T_0)}\int_{T_0}^t\left\{\sqrt{\frac{M_1H(t,s)k(s)}{C_2a(s)p(s)}}W(s) - \frac{h(t,s)}{2}\sqrt{\frac{C_2a(s)p(s)}{M_1}}\right\}^2\mathrm{d}s \leqslant \liminf_{t\to\infty}\frac{1}{H(t,T_0)}\int_{T_0}^t\left\{\frac{M_1H(t,s)k(s)}{C_2a(s)p(s)}W^2(s) - h(t,s)\sqrt{H(t,s)k(s)}W(s)\right\}\mathrm{d}s. \tag{18}$$

记

$$\alpha(t) = \frac{1}{H(t,T_0)}\int_{T_0}^t\frac{M_1H(t,s)k(s)}{C_2a(s)p(s)}W^2(s)\mathrm{d}s, \qquad t > T_0,$$

$$\beta(t) = \frac{1}{H(t,T_0)}\int_{T_0}^t h(t,s)\sqrt{H(t,s)k(s)}W(s)\mathrm{d}s, \qquad t > T_0,$$

由(18)式, 得

$$\liminf_{t\to\infty}[\alpha(t) - \beta(t)] < \infty. \tag{19}$$

下证

$$\int_{T_0}^{\infty}\frac{k(s)W^2(s)}{a(s)p(s)}\mathrm{d}s < \infty. \tag{20}$$

若不然,

$$\int_{T_0}^{\infty}\frac{k(s)W^2(s)}{a(s)p(s)}\mathrm{d}s=\infty. \tag{21}$$

由(13)式知,存在常数 $K_1>0$, 使

$$\inf_{s\geqslant t_0}\left[\liminf_{t\to\infty}\frac{H(t,s)}{H(t,t_0)}\right]>K_1>0. \tag{22}$$

取充分大的正数 K_2, 由(21)式知, 存在 $T_1>T_0$, 使 $t>T_1$ 时, 有

$$\int_{T_0}^{t}\frac{k(s)W^2(s)}{a(s)p(s)}\mathrm{d}s\geqslant\frac{K_2}{K_1}.$$

从而对 $t>T_1$, 有

$$\begin{aligned}\alpha(t)=&\frac{1}{H(t,T_0)}\int_{T_0}^{t}\frac{M_1H(t,s)}{C_2}\mathrm{d}\left[\int_{T_0}^{s}\frac{k(\tau)W^2(\tau)}{a(\tau)p(\tau)}\mathrm{d}\tau\right]=\\&\frac{1}{H(t,T_0)}\int_{T_0}^{t}\frac{M_1}{C_2}\left[-\frac{\partial H(t,s)}{\partial s}\right]\left[\int_{T_0}^{s}\frac{k(\tau)W^2(\tau)}{a(\tau)p(\tau)}\mathrm{d}\tau\right]\mathrm{d}s\geqslant\\&\frac{1}{H(t,T_0)}\int_{T_1}^{t}\frac{M_1}{C_2}\left[-\frac{\partial H(t,s)}{\partial s}\right]\left[\int_{T_0}^{s}\frac{k(\tau)W^2(\tau)}{a(\tau)p(\tau)}\mathrm{d}\tau\right]\mathrm{d}s\geqslant\\&\frac{1}{H(t,T_0)}\int_{T_1}^{t}\frac{M_1K_2}{C_2K_1}\left[-\frac{\partial H(t,s)}{\partial s}\right]\mathrm{d}s=\frac{M_1K_2}{C_2K_1}\frac{H(t,T_1)}{H(t,T_0)}\qquad(t\geqslant T_1).\end{aligned} \tag{23}$$

由(22)式,存在 $T_2>T_1$, 使

$$\frac{H(t,T_1)}{H(t,T_0)}\geqslant K_1,\qquad t\geqslant T_2;$$

由(23)式,得

$$\alpha(t)\geqslant\frac{M_1K_2}{C_2},\qquad t\geqslant T_2.$$

因 K_2 为充分大正数,从而有

$$\lim_{t\to\infty}\alpha(t)=\infty. \tag{24}$$

现取一序列 $\{t_n\}_{n=1}^{\infty}\subset(T_0,\infty)$, 使 $\lim\limits_{n\to\infty}t_n=\infty$ 且满足

$$\lim_{n\to\infty}[\alpha(t_n)-\beta(t_n)]=\liminf_{t\to\infty}[\alpha(t)-\beta(t)].$$

由(19)式,知存在自然数 N, 使 $n>N$ 时,有

$$\alpha(t_n)-\beta(t_n)\leqslant M. \tag{25}$$

由(24)式,得

$$\lim_{n\to\infty}\beta(t_n)=\infty. \tag{26}$$

由(25)、(26)式,对充分大的 n, 有

$$1-\frac{\beta(t_n)}{\alpha(t_n)}\leqslant\frac{M}{\alpha(t_n)}<\frac{1}{2},\ \frac{\beta(t_n)}{\alpha(t_n)}>\frac{1}{2}. \tag{27}$$

由(26)、(27)式,得

$$\lim_{n\to\infty}\frac{\beta(t_n)}{\alpha(t_n)}\beta(t_n)=\infty. \tag{28}$$

另一方面,由 Schwarz 不等式,对任意的自然数 n, 有

$$\beta^2(t_n)=\frac{1}{H^2(t_n,T_0)}\left[\int_{T_0}^{t_n}\sqrt{\frac{M_1H(t_n,s)k(s)}{C_2a(s)p(s)}}W(s)\sqrt{\frac{C_2a(s)p(s)}{M_1}}h(t_n,s)\mathrm{d}s\right]^2\leqslant$$

$$\left[\frac{1}{H(t_n,T_0)}\int_{T_0}^{t_n}\frac{M_1H(t_n,s)k(s)}{C_2a(s)p(s)}W^2(s)\mathrm{d}s\right]\times$$

$$\left[\frac{1}{H(t_n,T_0)}\int_{T_0}^{t_n}\frac{C_2a(s)p(s)}{M_1}h^2(t_n,s)\mathrm{d}s\right]\leqslant$$

$$\alpha(t_n)\frac{1}{H(t,T_0)}\int_{T_0}^{t_n}\frac{C_2a(s)p(s)}{M_1}h^2(t_n,s)\mathrm{d}s;$$

$$\frac{\beta^2(t_n)}{\alpha(t_n)}\leqslant\frac{C_2}{M_1H(t_n,T_0)}\int_{T_0}^{t_n}a(s)p(s)h^2(t_n,s)\mathrm{d}s\cdot$$

由(22)式得,

$$\lim_{t\to\infty}\inf\frac{H(t,T_0)}{H(t,t_0)}>K,$$

从而有 $T_3>T_0$, 使

$$\frac{H(t,T_0)}{H(t,t_0)}>K_1,\qquad t>T_3,$$

因而对充分大的 n, 有

$$\frac{H(t_n,T_0)}{H(t_n,t_0)}>K_1,$$

$$\frac{\beta^2(t_n)}{\alpha(t_n)}\leqslant\frac{C_2}{M_1K_1}\frac{1}{H(t_n,t_0)}\int_{T_0}^{t_n}a(s)p(s)h^2(t_n,s)\mathrm{d}s\cdot$$

由(28)式,得

$$\begin{cases}\lim\limits_{n\to\infty}\dfrac{1}{H(t_n,t_0)}\displaystyle\int_{t_0}^{t_n}a(s)p(s)h^2(t_n,s)\mathrm{d}s=\infty,\\ \limsup\limits_{t\to\infty}\dfrac{1}{H(t,t_0)}\displaystyle\int_{t_0}^{t}a(s)p(s)h^2(t,s)\mathrm{d}s=\infty\cdot\end{cases}\tag{29}$$

这与(14)式矛盾,从而(20)式成立,于是由(17)式有

$$\int_{T_0}^{\infty}\frac{\varphi_+^2(s)}{k(s)a(s)p(s)}\mathrm{d}s\leqslant\int_{T_0}^{\infty}\frac{k(s)W^2(s)}{a(s)p(s)}\mathrm{d}s<\infty\cdot$$

这与(15)式矛盾,从而方程(1)是振动的• 定理2证毕•

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Oscillation Criteria for Second Order Nonlinear Differential Equation With Damping

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Abstract: By the generalized Riccati transformation and the integral averaging technique. Some sufficient conditions of oscillation of the solutions for second order nonlinear differential equations with damping were discussed. Some sufficient oscillation criteria for previous equations were built up. Some oscillation criteria have been expanded and strengthened in some other known results.

Key words: second order nonlinear differential equation with damping; oscillation; Riccati transformation; integral averaging technique