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变厚度扁锥壳的非线性固有频率*

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(我刊编委叶开沅来稿)

摘要: 借助于变厚度圆薄板非线性动力学变分方程和协调方程, 给出了变厚度扁薄锥壳的非线性动力学变分方程和协调方程• 假设薄膜张力由两项组成, 将协调方程化为两个独立的方程, 选取变厚度扁锥壳中心最大振幅为摄动参数, 采用摄动变分法, 将变分方程和微分方程线性化• 对周边固定的圆底变厚度扁锥壳的非线性固有频率进行了求解; 一次近似得到了变厚度扁锥壳的线性固有频率, 三次近似得到了变厚度扁锥壳的非线性固有频率, 且绘出了固有频率与静载荷、最大振幅、变厚度参数的特征曲线图• 为动力工程提供了有价值的参考•

关 键 词: 变厚度; 固有频率; 非线性; 摄动变分法

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引 言

变厚度板和壳在工程中被广泛的应用着, 对静力学问题研究较多, 已查出几十篇文章, 可参考文献[1]至[4]• 对非线性动力学问题研究较少[5,6], 本文用文[7]提出的摄动变分法, 取变厚度扁锥壳中心最大振幅为摄动参数, 求出了变厚度扁锥壳的非线性固有频率• 非线性固有频率不但和结构尺寸变化有关, 而且和静载荷、最大振幅都有关• 这对工程设计具有重要意义•

1 变厚度扁薄锥壳混合问题

考虑周边夹紧边界变厚度扁锥壳, 设中心拱高为 f, 厚度为 h, 底边圆半径为 a• 在文[6]中, 基本方程(7)、(10) 的基础上, 对非线性部分 w 加上一个初挠度 w_c 即可• 得到变厚度扁锥壳的能量变分方程

$$\int_{t_1}^{t_2}\int_0^a\left\{D\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial w}{\partial r}-q-\frac{1}{r}\frac{\partial}{\partial r}\left[rN_r\left(\frac{\partial w}{\partial r}+\frac{f}{a}\right)\right]+\rho\frac{\partial^2 w}{\partial t^2}+\right.$$
$$\frac{\partial^2 D}{\partial r^2}\left(\frac{\partial^2 w}{\partial r^2}+\frac{\mu}{r}\frac{\partial w}{\partial r}\right)+\frac{1}{r}\frac{\partial D}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial r}+\mu\frac{\partial^2 w}{\partial r^2}\right)+$$

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$$2\frac{\partial D}{\partial r}\left(\frac{\partial^3 w}{\partial r^3}+\frac{1}{r}\frac{\partial^2 w}{\partial r^2}-\frac{1}{r^2}\frac{\partial w}{\partial r}\right)\Bigg\}\delta w r\,\mathrm{d}r\,\mathrm{d}\tau=0,$$

协调方程为:

$$hr\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}(r^2N_r)=\left[r\frac{\mathrm{d}(rN_r)}{\mathrm{d}r}-\mu rN_r\right]\frac{\partial h}{\partial r}-Eh^2\left[\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2+\frac{f}{a}\frac{\partial w}{\partial r}\right]\cdot$$

这里初挠度为:

$$w_c=-f(1-r/a),\quad h=h_0(1+\varepsilon r/a),$$

$$D=D_0(1+\varepsilon r/a)^3,\quad D_0=Eh_0^3/[12(1-\mu^2)],\quad \rho=\rho_0(1+\varepsilon r/a),$$

其中 ρ_0 为厚度 h_0 板的面密度, $|\varepsilon|<1$•

周边固定夹紧边界条件:

当 $r=a$ 时: $w=\partial w/\partial r=0,\ \partial(rN_r)/\partial r-\mu N_r=0$•

当 $r=0$ 时: $w,\partial w/\partial r/r,N_r$ 有限•

初始条件:

$$t=0,\ w=w(0,r),\ \partial w(0,r)/\partial t=0\cdot$$

2　问题的求解

为了便于计算, 引入下列无量纲量(取 $t_2-t_1=2\pi/\omega$, 设 $\tau=\omega t$):

$$x=\frac{r}{a},\ y=\sqrt{12(1-\mu^2)}\frac{w}{h_0},\ N=\frac{12(1-\mu^2)a}{Eh_0^3}rN_r,\ K=\sqrt{12(1-\mu^2)}\frac{f}{h_0},$$

$$Q=\sqrt{12(1-\mu^2)}\frac{a^4}{D_0h_0}q,\quad \Omega^2=\rho_0a^4\omega^2/D_0\cdot$$

代入变分方程和协调方程中得:

$$\int_0^{2\pi}\int_0^1\Bigg\{(1+x\varepsilon)^3L(y)-Q-\frac{1}{x}\frac{\partial}{\partial x}\left[N\left(\frac{\partial y}{\partial x}+K\right)\right]+(1+x\varepsilon)\Omega^2\frac{\partial^2 y}{\partial\tau^2}+$$
$$6(1+x\varepsilon)\varepsilon^2\left(\frac{\partial^2 y}{\partial x^2}+\frac{\mu}{x}\frac{\partial y}{\partial x}\right)+6(1+x\varepsilon)^2\varepsilon\left(\frac{\partial^3 y}{\partial x^3}+\frac{1}{x}\frac{\partial^2 y}{\partial x^2}-\frac{1}{x^2}\frac{\partial y}{\partial x}\right)+$$
$$3(1+x\varepsilon)^2\varepsilon\left(\frac{1}{x^2}\frac{\partial y}{\partial x}+\frac{\mu}{x}\frac{\partial^2 y}{\partial x^2}\right)\Bigg\}\delta y x\,\mathrm{d}x\,\mathrm{d}\tau=0, \tag{1}$$

$$x\frac{\partial}{\partial x}\frac{1}{x}\frac{\partial}{\partial x}(xN)=\varepsilon\left(x\frac{\partial N}{\partial x}-\mu N\right)-$$
$$x^2\varepsilon\frac{\partial}{\partial x}\frac{1}{x}\frac{\partial}{\partial x}(xN)-(1+x\varepsilon)^2\left[\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2+K\frac{\partial y}{\partial x}\right], \tag{2}$$

取　$y=\eta_0+\eta\cos\tau,\ N=N_1\cos\tau+N_2\cos^2\tau,$

其中　$L=\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}x\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}x\frac{\mathrm{d}}{\mathrm{d}x},$

η_0 是静载荷作用下的小挠度解, 且 $\eta_0=Q(x^2-1)^2/64,\ \eta_0'=\varphi=Q(x^3-x)/16$•

取　$\eta=\sum_{i=1}^{\infty}\eta_i\eta_0^i,\ N_1=\sum_{i=1}^{\infty}s_{1i}\eta_0^i,\ N_2=\sum_{i=1}^{\infty}s_{2i}\eta_0^i,\ \Omega^2=\sum_{i=0}^{\infty}\Omega_i^2\eta_0^i,$

这里 η_0 为中心最大振幅, 将这些式子代入(1)、(2) 中, 按 η_0 的同次幂项可得一系列边值问题•

一次近似边值问题

$$\int_0^1\Bigg\{(1+x\varepsilon)^3L(\eta_1)-\frac{1}{x}\frac{\partial}{\partial x}[s_{11}(K+\varphi)]-(1+x\varepsilon)\Omega_0^2\eta_1+$$
$$6(1+x\varepsilon)\varepsilon^2\left(\frac{\partial^2\eta_1}{\partial x^2}+\frac{\mu}{x}\frac{\partial\eta_1}{\partial x}\right)+6(1+x\varepsilon)^2\varepsilon\left(\frac{\partial^3\eta_1}{\partial x^3}+\frac{1}{x}\frac{\partial^2\eta_1}{\partial x^2}-\frac{1}{x^2}\frac{\partial\eta_1}{\partial x}\right)+$$

$$3(1+x\varepsilon)^2\varepsilon\left(\frac{1}{x^2}\frac{\partial\eta_1}{\partial x}+\frac{\mu}{x}\frac{\partial^2\eta_1}{\partial x^2}\right)\Big\}\eta_1 x\,dx=0, \tag{3}$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xs_{11})=\left(x\frac{ds_{11}}{dx}-\mu s_{11}\right)\varepsilon-$$
$$x^2\varepsilon\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xs_{11})-(1+x\varepsilon)^2(K+\varphi)\frac{d\eta_1}{dx}, \tag{4}$$

边界条件:

当 $x=1$ 时: $\eta_1=0,\ d\eta_1/dx=0,\ ds_{11}/dx-\mu s_{11}=0$•

当 $x=0$ 时: $\eta_1=0,\ d\eta_1/dx,\ s_{11}$ 有限•

二次近似边值问题

$$\int_0^1\Big\{\eta_1\Big\{(1+x\varepsilon)^3L(\eta_2)-\frac{1}{x}\frac{\partial}{\partial x}[s_{12}(K+\varphi)]-(1+x\varepsilon)(\Omega_0^2\eta_2+\Omega_1^2\eta_1)+$$
$$6(1+x\varepsilon)\varepsilon^2\left(\frac{\partial^2\eta_2}{\partial x^2}+\frac{\mu}{x}\frac{\partial\eta_2}{\partial x}\right)+6(1+x\varepsilon)^2\varepsilon\left(\frac{\partial^3\eta_2}{\partial x^3}+\frac{1}{x}\frac{\partial^2\eta_2}{\partial x^2}-\frac{1}{x^2}\frac{\partial\eta_2}{\partial x}\right)+$$
$$3(1+x\varepsilon)^2\varepsilon\left(\frac{1}{x^2}\frac{\partial\eta_2}{\partial x}+\frac{\mu}{x}\frac{\partial^2\eta_2}{\partial x^2}\right)\Big\}+$$
$$2\eta_2\Big\{(1+x\varepsilon)^3L(\eta_1)-\frac{1}{x}\frac{\partial}{\partial x}[s_{11}(K+\varphi)]-(1+x\varepsilon)\Omega_0^2\eta_1+$$
$$6(1+x\varepsilon)\varepsilon^2\left(\frac{\partial^2\eta_1}{\partial x^2}+\frac{\mu}{x}\frac{\partial\eta_1}{\partial x}\right)+6(1+x\varepsilon)^2\varepsilon\left(\frac{\partial^3\eta_1}{\partial x^3}+\frac{1}{x}\frac{\partial^2\eta_1}{\partial x^2}-\frac{1}{x^2}\frac{\partial\eta_1}{\partial x}\right)+$$
$$3(1+x\varepsilon)^2\varepsilon\left(\frac{1}{x^2}\frac{\partial\eta_1}{\partial x}+\frac{\mu}{x}\frac{\partial^2\eta_1}{\partial x^2}\right)\Big\}\Big\}x\,dx=0, \tag{5}$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xs_{12})=\left(x\frac{ds_{12}}{dx}-\mu s_{12}\right)\varepsilon-$$
$$x^2\varepsilon\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xs_{12})-(1+x\varepsilon)^2(K+\varphi)\frac{d\eta_2}{dx}, \tag{6}$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xs_{22})=-\frac{1}{2}\left(\frac{d\eta_1}{dx}\right)^2, \tag{7}$$

边界条件:

当 $x=1$ 时: $\eta_2=0,\ d\eta_2/dx=0,\ ds_{12}/dx-\mu s_{12}=0,\ ds_{22}/dx-\mu s_{22}=0$•

当 $x=0$ 时: $\eta_2=0,\ d\eta_2/dx,\ s_{12},\ s_{22}$ 有限•

三次近似边值问题

$$\int_0^1\Big\{\eta_1\Big\{(1+x\varepsilon)^3L(\eta_3)-\frac{1}{x}\frac{\partial}{\partial x}\left[s_{13}(K+\varphi)+\frac{3}{4}s_{22}\frac{\partial\eta_1}{\partial x}\right]-$$
$$(1+x\varepsilon)(\Omega_0^2\eta_3+\Omega_1^2\eta_2+\Omega_2^2\eta_1)+6(1+x\varepsilon)\varepsilon^2\left(\frac{\partial^2\eta_3}{\partial x^2}+\frac{\mu}{x}\frac{\partial\eta_3}{\partial x}\right)+$$
$$6(1+x\varepsilon)^2\varepsilon\left(\frac{\partial^3\eta_3}{\partial x^3}+\frac{1}{x}\frac{\partial^2\eta_3}{\partial x^2}-\frac{1}{x^2}\frac{\partial\eta_3}{\partial x}\right)+3(1+x\varepsilon)^2\varepsilon\left(\frac{1}{x^2}\frac{\partial\eta_3}{\partial x}+\frac{\mu}{x}\frac{\partial^2\eta_3}{\partial x^2}\right)\Big\}+$$
$$2\eta_2\Big\{(1+x\varepsilon)^3L(\eta_2)-\frac{1}{x}\frac{\partial}{\partial x}[s_{12}(K+\varphi)]-$$
$$(1+x\varepsilon)(\Omega_0^2\eta_2+\Omega_1^2\eta_1)+6(1+x\varepsilon)\varepsilon^2\left(\frac{\partial^2\eta_2}{\partial x^2}+\frac{\mu}{x}\frac{\partial\eta_2}{\partial x}\right)+$$
$$6(1+x\varepsilon)^2\varepsilon\left(\frac{\partial^3\eta_2}{\partial x^3}+\frac{1}{x}\frac{\partial^2\eta_2}{\partial x^2}-\frac{1}{x^2}\frac{\partial\eta_2}{\partial x}\right)+3(1+x\varepsilon)^2\varepsilon\left(\frac{1}{x^2}\frac{\partial\eta_2}{\partial x}+\frac{\mu}{x}\frac{\partial^2\eta_2}{\partial x^2}\right)\Big\}+$$

$$3\eta_3\left\{(1+x\varepsilon)^3L(\eta_1)-\frac{1}{x}\frac{\partial}{\partial x}[s_{11}(K+\varphi)]-(1+x\varepsilon)\Omega_0^2\eta_1+\right.$$
$$6(1+x\varepsilon)\varepsilon^2\left(\frac{\partial^2\eta_1}{\partial x^2}+\frac{\mu}{x}\frac{\partial\eta_1}{\partial x}\right)+6(1+x\varepsilon)^2\varepsilon\left(\frac{\partial^3\eta_1}{\partial x^3}+\frac{1}{x}\frac{\partial^2\eta_1}{\partial x^2}-\frac{1}{x^2}\frac{\partial\eta_1}{\partial x}\right)+$$
$$\left.3(1+x\varepsilon)^2\varepsilon\left(\frac{1}{x^2}\frac{\partial\eta_1}{\partial x}+\frac{\mu}{x}\frac{\partial^2\eta_1}{\partial x^2}\right)\right\}\bigg\}x\,\mathrm{d}x=0, \tag{8}$$

$$x\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}(xs_{13})=\left(x\frac{\mathrm{d}s_{13}}{\mathrm{d}x}-\mu s_{13}\right)\varepsilon-$$
$$x^2\varepsilon\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}(xs_{13})-(1+x\varepsilon)^2(K+\varphi)\frac{\mathrm{d}\eta_3}{\mathrm{d}x}, \tag{9}$$

$$x\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}x(s_{23})=\left(x\frac{\mathrm{d}s_{23}}{\mathrm{d}x}-\mu s_{23}\right)\varepsilon-x^2\varepsilon\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}x(s_{23})-(1+x\varepsilon)^2\frac{\mathrm{d}\eta_1}{\mathrm{d}x}\frac{\mathrm{d}\eta_2}{\mathrm{d}x}, \tag{10}$$

边界条件:

当 $x=1$ 时: $\eta_3=0$, $\mathrm{d}\eta_3/\mathrm{d}x=0$, $\mathrm{d}s_{13}/\mathrm{d}x-\mu s_{13}=0$, $\mathrm{d}s_{23}/\mathrm{d}x-\mu s_{23}=0$•

当 $x=0$ 时: $\eta_3=0$, $\mathrm{d}\eta_3/\mathrm{d}x$, s_{13}, s_{23} 有限•

下面就上述 3 个边值问题进行求解

根据边界条件, 取 $\eta_1=(1-x^2)^2$, $\eta_2=(1-x^2)^2x^2$, $\eta_3=0$•

解一次近似边值问题可得

$$s_{11}=\frac{Q}{192}\left(\frac{5-3\mu}{1-\mu}x-6x^3+4x^5-x^7\right)-\frac{4K}{15}\left(\frac{6-4\mu}{1-\mu}x-5x^2+x^4\right)-$$
$$(Q/20\,160)[21(3\mu-7)\varepsilon+256(\mu-2)]x\varepsilon/(1-\mu)-$$
$$Q(672x^4\varepsilon+210x^5\varepsilon^2-2\,304x^6\varepsilon-210x^7\varepsilon^2+1\,280x^8\varepsilon+63x^9\varepsilon^2)/20\,160+$$
$$K\{[8(2\mu-5)\varepsilon+70(\mu-2)]x\varepsilon/(1-\mu)+$$
$$105x^3\varepsilon+28x^4\varepsilon^2-35x^5\varepsilon-12x^6\varepsilon^2\}/105, \tag{11}$$

$$\Omega_0^2=[(73\,920+27\,720\varepsilon^2\mu^2+8\,448\varepsilon^2\mu^2+164\,736\varepsilon-49\,280\varepsilon^3\mu+$$
$$138\,600\varepsilon^2-190\,080\varepsilon\mu-73\,920\mu+40\,832\varepsilon^3-166\,320\varepsilon^2\mu+25\,344\varepsilon\mu^2)+$$
$$4K^2(11\,781+4\,422\varepsilon^2+13\,640\varepsilon-4\,389\mu-1\,254\varepsilon^2\mu-4\,400\varepsilon\mu)/15+$$
$$KQ(112\,776\varepsilon^2\mu+1\,744\,798\varepsilon\mu-10\,085\,920-11\,655\,930\varepsilon-$$
$$3\,713\,504\varepsilon^2+3\,839\,680\mu)/81\,120+Q^2(113\,664\varepsilon+35\,607\varepsilon^2+731\,170-$$
$$38\,610\mu-11\,583\varepsilon^2\mu-40\,448\varepsilon\mu)/79\,872]/[(1-\mu)(256\varepsilon+693)]• \tag{12}$$

解二次近似边值问题可得

$$s_{12}=(2K/105)\{((12-16\mu)/(1-\mu))x-35x^2+28x^4-9x^6\}-$$
$$(Q/1\,920)\{((4-6\mu)/(1-\mu))x-30x^3+50x^5-35x^7+9x^9\}-$$
$$\frac{K\varepsilon}{420}\left(\frac{35(x+\mu)+32x\varepsilon}{1-\mu}+210x^3+56x^4-280x^5-96x^6+105x^7+40x^8\varepsilon\right)-$$
$$(Q\varepsilon/6\,652\,800)(7\,680x+110\,880x^4+36\,450x^5\varepsilon-237\,600x^6-86\,625x^7\varepsilon+$$
$$184\,800x^8+72\,796x^9\varepsilon-50\,400x^{10}+20\,790x^{11}\varepsilon)+6\,360x\varepsilon/(1-\mu), \tag{13}$$

$$s_{22}=(1/6)(((5-3\mu)/(1-\mu))-6x^3+4x^5-x^7), \tag{14}$$

$$\Omega_1^2=[760\,381\,440(4\,587\,520\varepsilon^4\mu+27\,684\,888\varepsilon^3\mu+1\,363\,824\varepsilon^3\mu^2+$$
$$4\,514\,103\varepsilon^2\mu^2+172\,032\varepsilon^4\mu^2-4\,759\,552\varepsilon^4-29\,048\,712\varepsilon^3-56\,326\,842\varepsilon^2-$$
$$41\,023\,752\varepsilon+6\,936\,930\mu+36\,267\,000\varepsilon\mu+51\,812\,739\varepsilon^2\mu+4\,756\,752\varepsilon\mu^2-$$
$$6\,936\,930)+8\,192K^2(34\,033\,965\,966-291\,228\,169\,070\mu+39\,395\,648\,292\varepsilon-$$
$$35\,770\,414\,680\varepsilon\mu-15\,737\,941\,947\varepsilon^2\mu-2\,550\,955\,264\varepsilon^3\mu+3\,347\,545\,344\varepsilon^3+$$

$17\,200\,042\,587\varepsilon^2) - 64KQ(189\,788\,014\,416 - 143\,182\,943\,904\mu +$
$243\,182\,238\,299\varepsilon - 13\,943\,250\,944\varepsilon^2\mu + 22\,237\,745\,152\varepsilon^3 - 184\,043\,024\,165\varepsilon\mu -$
$84\,136\,277\,056\varepsilon^3\mu + 115\,822\,139\,680\varepsilon^2) + 7Q^2(18\,944\,260\,335 - 12\,812\,014\,215\mu +$
$25\,964\,260\,608\varepsilon + 12\,960\,271\,472\varepsilon^2 - 8\,059\,943\,884\varepsilon^2\mu + 2\,456\,360\,192\varepsilon^3 -$
$1\,358\,559\,488\varepsilon^3\mu - 17\,103\,440\,640\varepsilon\mu)]/[411\,873\,280(\mu - 1)(256\varepsilon + 693)]$• (15)

解三次近似边值问题可得

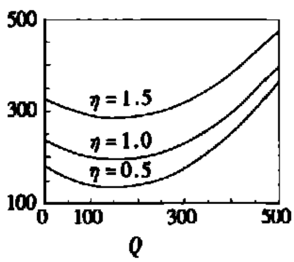
$s_{13} = 0;$

$$\begin{aligned}\Omega_2^2 = {} & 12(289\,856\,845\,278 - 289\,856\,845\,278\mu + 723\,573\,594\,744\varepsilon - 781\,268\,239\,752\varepsilon\mu + \\ & 743\,550\,486\,492\varepsilon^2 - 833\,032\,032\,789\varepsilon^2\mu + 375\,393\,820\,152\varepsilon^3 - \\ & 427\,731\,537\,768\varepsilon^3\mu + 89\,025\,925\,120\varepsilon^4 - 102\,820\,630\,528\varepsilon^4\mu + \\ & 7\,948\,206\,080\varepsilon^5 - 9\,395\,240\,960\varepsilon^5\mu + 1\,447\,034\,880\varepsilon^5\mu^2 + \\ & 13\,794\,705\,408\varepsilon^4\mu^2 + 52\,337\,717\,616\varepsilon^3\mu^2 + 89\,481\,546\,297\varepsilon^2\mu^2 + \\ & 57\,694\,645\,008\varepsilon\mu^2)/[169(1-\mu)(693+256\varepsilon)^3] - \\ & K^2(6\,244\,931\,386\,624\varepsilon^3\mu + 21\,139\,071\,967\,717\varepsilon^2\mu + 33\,458\,639\,718\,504\varepsilon\mu - \\ & 18\,557\,385\,172\,326 - 16\,848\,038\,721\,349\varepsilon^2 - 28\,462\,435\,069\,068\varepsilon + \\ & 20\,547\,695\,830\,662\mu - 4\,958\,256\,921\,856\varepsilon^3 - 693\,699\,477\,504\varepsilon^4 + \\ & 742\,789\,611\,520\varepsilon^4\mu)/[112\,385(1-\mu)(693+256\varepsilon)^3] + \\ & KQ(189\,710\,369\,771\,520\varepsilon^3\mu + 620\,228\,846\,144\,544\varepsilon^2\mu + \\ & 944\,929\,235\,596\,346\varepsilon\mu - 930\,234\,483\,116\,939\varepsilon - 605\,026\,567\,068\,560\varepsilon^2 + \\ & 558\,625\,545\,445\,968\mu - 562\,308\,572\,465\,640 - 193\,827\,208\,407\,040\varepsilon^3 - \\ & 27\,424\,935\,903\,232\varepsilon^4 + 23\,160\,311\,250\,944\varepsilon^4\mu)/[83\,661\,760(1-\mu)(639+256\varepsilon)^3] - \\ & Q^2(679\,005\,992\,665\,856\varepsilon^3\mu + 2\,157\,665\,858\,197\,564\varepsilon^2\mu - 3\,672\,462\,438\,078\,720\varepsilon + \\ & 1\,820\,307\,918\,041\,613\mu - 2\,080\,224\,396\,287\,037 + 3\,182\,940\,531\,733\,248\varepsilon\mu - \\ & 859\,575\,743\,651\,584\varepsilon^3 + 84\,630\,048\,735\,232\varepsilon^4\mu - 121\,245\,152\,444\,416\varepsilon^4 - \\ & 2\,556\,959\,391\,180\,480\varepsilon^2)/[29\,066\,485\,760(1-\mu)(693+256\varepsilon)^3],\end{aligned} \tag{16}$$

由此可得三次近似非线性固有频率

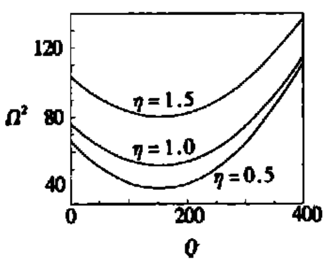
$$\Omega^2 = \Omega_0^2 + \Omega_1^2\eta_0 + \Omega_2^2\eta_0^2 \tag{17}$$

下面我们绘出特征曲线图(参见图 1~ 图 3)•



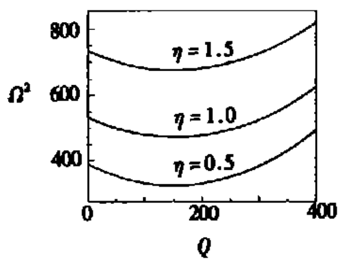
$\varepsilon = 0,\ K = 3,\ \mu = 0.3$

图 1 特征曲线图



$\varepsilon = -0.5,\ K = 3,\ \mu = 0.3$

图 2 特征曲线图



$\varepsilon = 0.5,\ K = 3,\ \mu = 0.3$

图 3 特征曲线图

3 讨 论

从文中式(12)、(15)、(16)和特征关系式(17)可看出, 变厚度扁锥壳的非线性振动时的固有频率, 不但和材料性质、结构形状有关, 而且和厚度变化、静载荷、振幅的大小都有关系• 这

给工程技术人员设计提供了参考依据• 本文采用的方法,也可推广到结构不同的非线性振动模式,得到不同的非线性固有频率•

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Nonlinear Natural Frequency of Shallow Conical Shiells With Variable Thickness

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Abstract: The nonlinear dynamical variation equation and compatible equation of the shallow conical shell with variable thickness are obtained by the theory of nonlinear dynamical variation equation and compatible equation of the circular thin plate with variable thickness. Assuming the thin film tension is composed of two items. The compatible equation is transformed into two independent equations. Selecting the maximum amplitude in the center of the shallow conical shells with variable thickness as the perturbation parameter, the variation equation and the differential equation are transformed into linear expression by theory of perturbation variation method. The nonlinear natural frequency of shallow conical shells with circular bottom and variable thickness under the fixed boundary conditions is solved; in the first approximate equation, the linear natural frequency of shallow conical shells with variable thickness is obtained, in the third approximate equation, the nonlinear natural frequency of it is obtained. The figures of the characteristic curves of the natural frequency varying with stationary loads, large amplitude, and variable thickness coefficient are plotted. A valuable reference is given for dynamic engineering.

Key words: variable thickness; natural frequency; nonlinear; perturbation variation method