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夹层圆板的非线性弯曲

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摘 要

本文导出了具有软夹心的夹层圆板的非线性轴对称弯曲理论的基本方程和边界条件，并给出了表板很薄情况下的这些方程和边界条件的简化形式．作为算例，研究了在均布横向载荷作用下具有滑动固定边界条件的夹层圆板，使用修正迭代法，得到了相当精确的解析解．

符 号

$r,\ \theta,\ z$	圆柱坐标系
a	夹层圆板的半径
$t,\ h$	表板和夹心厚度
h_0	上、下表板中面间的距离
$E,\ \nu$	表板材料的弹性模量和泊松比
G_2	夹心剪切模量
D_f	表板的抗弯刚度
$D,\ C$	夹层圆板的抗弯和抗剪刚度
q	均布横向载荷
$u_i, v_i, w_{i(i=1,2,3)}$	上表板、夹心和下表板的径向、环向和法向位移
$u,\ w$	夹层圆板中面上点的径向位移和挠度
ψ	上、下表板中面上任意点在变形后的连线与夹层圆板变形前法线的夹角
$e_{ri}, e_{\theta i}, e_{zi}, \gamma_{r\theta i}, \gamma_{\theta zi}, \gamma_{rzi(i=1,2,3)}$	上表板、夹心和下表板点的伸长和剪切应变分量
$\sigma_{ri}, \sigma_{\theta i}, \sigma_{zi}, \tau_{r\theta i}, \tau_{\theta zi}, \tau_{rzi(i=1,2,3)}$	上表板、夹心和下表板点的正应力和剪应力分量
$\sigma_{r0}, \sigma_{\theta 0}$	夹层圆板中面内的径向和环向应力
$U_{i(i=1,2,3)}$	上表板、夹心和下表板的应变能
V	外力功
U	夹层圆板的总势能
M_r, Q_r	夹层圆板的径向弯矩和横向力
m_r	表板的径向弯矩
φ	应力函数
ρ	无量纲径向坐标
k	夹层圆板的无量纲特征参数
$W,\ W_0$	夹层圆板中面上点的无量纲挠度及其中心值
$S_r,\ S_r(0)$	夹层圆板中面内的无量纲径向应力及其中心值
$S_\theta, S_\theta(0), S_\theta(1)$	夹层圆板中面内的无量纲环向应力及其中心和边缘值
P	无量纲横向载荷
$A_2, A_3, B_2, B_3, \alpha_1, \cdots, \alpha_9, \lambda_1, \lambda_2, l_{1,1}, \cdots, l_{11,3}, m_1, \cdots, m_{33}, n_{0,2}, \cdots, n_{22,6}, R_1, \cdots, R_{33}$	辅助量
L	微分算子

一、前 言

夹层板是航空、宇航和船舶制造等工业部门中的重要结构元件．它由三层，即一块厚夹心和两块薄圆板所构成．因为这种板具有高的刚度和轻的重量的特性，所以在工业中应用十分广泛．因此，夹层圆板已经成为一些研究者的对象[1-4]．但是对于其非线性弯曲方面的问题，则由于非线性数学的困难而仅有少数人探讨过．作者[5]使用修正迭代法[6-11]曾对边缘力矩作用下的夹层圆板的非线性弯曲进行过研究，获得了相当精确的三次近似解析解．中国科学院北京力学研究所固体力学研究室板壳组[12]使用摄动法对均布横向载荷作用下的夹层圆板的非线性弯曲求解，得到了简支和固定边界条件下的二次近似解．

本文所研究的夹层板，是上、下表板厚度相等、材料各向同性，夹心材料横观各向同性的薄园板．我们使用变分法导出了这种夹层圆板的非线性轴对称弯曲理论的平衡方程和边界条件．当表板很薄时，还导出了简化方程．最后，使用修正迭代法对承受均布横向载荷作用的具有滑动固定边界条件的夹层圆板进行了研究，获得了相当精确的三次近似解析解．所得结果可供工程设计时参考应用．

二、基 本 方 程

在推导基本方程和边界条件时，引进下列假设：

1．材料服从于虎克定律．
2．夹心横向不可压缩．
3．夹心沿板面方向不能承受载荷．
4．表板为直法线假设，夹心中面法线在变形后保持直线．

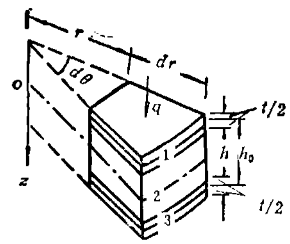

图 1 夹层圆板的坐标和尺寸

现在、考虑在均布横向载荷作用下半径为 a 的夹层圆板，其坐标、尺寸如图 1 所示．在轴对称和上述假设下，夹层圆板中任意一点的位移为：

上表板 $\left[\frac{h}{2}\leqslant z\leqslant\frac{h}{2}+t\right]$

$$u_1=u+\frac{h_0}{2}\psi-\left(z-\frac{h_0}{2}\right)\frac{dw_1}{dr},\quad v_1=0,\quad w_1=w \tag{2.1}$$

下表板 $\left[-\left(\frac{h}{2}+t\right)\leqslant z\leqslant-\frac{h}{2}\right]$

$$u_3=u-\frac{h_0}{2}\psi-\left(z+\frac{h_0}{2}\right)\frac{dw_3}{dr},\quad v_3=0,\quad w_3=w \tag{2.2}$$

夹心 $\left[-\frac{h}{2}\leqslant z\leqslant\frac{h}{2}\right]$

$$u_2=u+\frac{z}{h}\left(h_0\psi+t\frac{dw_2}{dr}\right),\quad v_2=0,\quad w_2=w \tag{2.3}$$

将式（2.1）、（2.2）和（2.3）分别代入下述夹层圆板的几何方程：

$$\varepsilon_{ri}=\frac{\partial u_i}{\partial r}+\frac{1}{2}\left(\frac{dw_i}{dr}\right)^2,\ \varepsilon_{\theta i}=\frac{u_i}{r},\quad \varepsilon_{zi}=\gamma_{r\theta i}=\gamma_{\theta zi}=\gamma_{rzi}=0 \qquad i=1,3 \tag{2.4}$$

$$\gamma_{rz2}=\frac{\partial u_2}{\partial z}+\frac{dw_2}{dr},\quad \varepsilon_{r2}=\varepsilon_{\theta 2}=\varepsilon_{z2}=r_{r\theta 2}=r_{\theta z2}=0 \tag{2.5}$$

便得：

上表板

$$\left.\begin{aligned}\varepsilon_{r1}&=\frac{du}{dr}+\frac{h_0}{2}\frac{d\psi}{dr}-\left(z-\frac{h_0}{2}\right)\frac{d^2w}{dr^2}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2\\ \varepsilon_{\theta 1}&=\frac{u}{r}+\frac{h_0}{2r}\psi-\left(z-\frac{h_0}{2}\right)\frac{1}{r}\frac{dw}{dr}\end{aligned}\right\} \tag{2.6}$$

下表板

$$\left.\begin{aligned}\varepsilon_{r3}&=\frac{du}{dr}-\frac{h_0}{2}\frac{d\psi}{dr}-\left(z+\frac{h_0}{2}\right)\frac{d^2w}{dr^2}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2\\ \varepsilon_{\theta 3}&=\frac{u}{r}-\frac{h_0}{2r}\psi-\left(z+\frac{h_0}{2}\right)\frac{1}{r}\frac{dw}{dr}\end{aligned}\right\} \tag{2.7}$$

夹心

$$\gamma_{rz2}=\frac{h_0}{h}\left(\psi+\frac{dw}{dr}\right) \tag{2.8}$$

将式（2.6)、(2.7）和（2.8）分别代入下述虎克定律：

$$\sigma_{ri}=\frac{E}{1-\nu^2}(\varepsilon_{ri}+\nu\varepsilon_{\theta i}),\quad \sigma_{\theta i}=\frac{E}{1-\nu^2}(\varepsilon_{\theta i}+\nu\varepsilon_{ri}),\quad \sigma_{zi}=\tau_{r\theta i}=\tau_{\theta zi}=\tau_{rzi}=0 \tag{2.9}$$

$$i=1,3$$

$$\tau_{rz2}=G_2\gamma_{rz2},\qquad \sigma_{r2}=\sigma_{\theta 2}=\sigma_{z2}=\tau_{r\theta 2}=\tau_{\theta z2}=0 \tag{2.10}$$

便得：

上表板

$$\left.\begin{aligned}\sigma_{r1}&=\sigma_{r0}+\frac{Eh_0}{2(1-\nu^2)}\left(\frac{d\psi}{dr}+\frac{\nu}{r}\psi\right)-\frac{E}{1-\nu^2}\left(z-\frac{h_0}{2}\right)\left(\frac{d^2w}{dr^2}+\frac{\nu}{r}\frac{dw}{dr}\right)\\ \sigma_{\theta 1}&=\sigma_{\theta 0}+\frac{Eh_0}{2(1-\nu^2)}\left(\frac{\psi}{r}+\nu\frac{d\psi}{dr}\right)-\frac{E}{1-\nu^2}\left(z-\frac{h_0}{2}\right)\left(\frac{1}{r}\frac{dw}{dr}+\nu\frac{d^2w}{dr^2}\right)\end{aligned}\right\} \tag{2.11}$$

下表板

$$\left.\begin{aligned}\sigma_{r3}&=\sigma_{r0}-\frac{Eh_0}{2(1-\nu^2)}\left(\frac{d\psi}{dr}+\frac{\nu}{r}\psi\right)-\frac{E}{1-\nu^2}\left(z+\frac{h_0}{2}\right)\left(\frac{d^2w}{dr^2}+\frac{\nu}{r}\frac{dw}{dr}\right)\\ \sigma_{\theta 3}&=\sigma_{\theta 0}-\frac{Eh_0}{2(1-\nu^2)}\left(\frac{\psi}{r}+\nu\frac{d\psi}{dr}\right)-\frac{E}{1-\nu^2}\left(z+\frac{h_0}{2}\right)\left(\frac{1}{r}\frac{dw}{dr}+\nu\frac{d^2w}{dr^2}\right)\end{aligned}\right\} \tag{2.12}$$

夹心

$$\tau_{rz2}=\frac{G_2h_0}{h}\left(\psi+\frac{dw}{dr}\right) \tag{2.13}$$

其中

$$\left.\begin{aligned}\sigma_{r0}&=\frac{E}{1-\nu^2}\left[\frac{du}{dr}+\frac{\nu}{r}u+\frac{1}{2}\left(\frac{dw}{dr}\right)^2\right]\\ \sigma_{\theta0}&=\frac{E}{1-\nu^2}\left[\frac{u}{r}+\nu\frac{du}{dr}+\frac{\nu}{2}\left(\frac{dw}{dr}\right)^2\right]\end{aligned}\right\} \tag{2.14}$$

由弹性体应变能的公式:

$$U_i=\frac{1}{2}\iiint\limits_V(\sigma_r\varepsilon_r+\sigma_\theta\varepsilon_\theta+\sigma_z\varepsilon_z+\tau_{r\theta}\gamma_{r\theta}+\tau_{\theta z}\gamma_{\theta z}+\tau_{rz}\gamma_{rz})rdrd\theta dz \quad i=1,\ 2,\ 3 \tag{2.15}$$

得到表板和夹心的应变能公式

$$\left.\begin{aligned}U_i&=\frac{1}{2E}\iiint\limits_{V_i}[(\sigma_{ri}+\sigma_{\theta i})^2-2(1+\nu)\sigma_{ri}\sigma_{\theta i}]rdrd\theta dz \quad i=1,\ 3\\ U_2&=\frac{1}{2G_2}\iiint\limits_{V_2}\tau_{rz2}^2\,rdrd\theta dz\end{aligned}\right\} \tag{2.16}$$

将式(2.11)、(2.12)和(2.13)代入式(2.16),并对z进行积分,便得

$$\left.\begin{aligned}U_1=&\frac{t}{2E}\iint\limits_{S_1}[(\sigma_{r0}+\sigma_{\theta0})^2-2(1+\nu)\sigma_{r0}\sigma_{\theta0}]rdrd\theta+\frac{th_0}{2}\iint\limits_{S_1}\left(\sigma_{r0}\frac{d\psi}{dr}\right.\\ &\left.+\frac{1}{r}\sigma_{\theta0}\psi\right)rdrd\theta+\frac{D_f}{2}\iint\limits_{S_1}\left[\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)^2\right.\\ &\left.-2(1-\nu)\frac{1}{r}\frac{dw}{dr}\frac{d^2w}{dr^2}\right]rdrd\theta+\frac{D}{4}\iint\limits_{S_1}\left[\left(\frac{d\psi}{dr}+\frac{\psi}{r}\right)^2\right.\\ &\left.-2(1-\nu)\frac{\psi}{r}\frac{d\psi}{dr}\right]rdrd\theta\\ U_3=&\frac{t}{2E}\iint\limits_{S_3}\left[(\sigma_{r0}+\sigma_{\theta0})^2-2(1+\nu)\sigma_{r0}\sigma_{\theta0}\right]rdrd\theta-\frac{th_0}{2}\iint\limits_{S_3}\left(\sigma_{r0}\frac{d\psi}{dr}\right.\\ &\left.+\frac{1}{r}\sigma_{\theta0}\psi\right)rdrd\theta+\frac{D_f}{2}\iint\limits_{S_3}\left[\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)^2\right.\\ &\left.-2(1-\nu)\frac{1}{r}\frac{dw}{dr}\frac{d^2w}{dr^2}\right]rdrd\theta+\frac{D}{4}\iint\limits_{S_3}\left[\left(\frac{d\psi}{dr}+\frac{\psi}{r}\right)^2\right.\\ &\left.-2(1-\nu)\frac{\psi}{r}\frac{d\psi}{dr}\right]rdrd\theta\\ U_2=&\frac{C}{2}\iint\limits_{S_2}\left(\psi+\frac{dw}{dr}\right)^2rdrd\theta\end{aligned}\right\} \tag{2.17}$$

其中 $$D_f=\frac{Et^3}{12(1-\nu^2)},\qquad D=\frac{Eth_0^2}{2(1-\nu^2)},\qquad C=\frac{G_2h_0^2}{h}\tag{2.18}$$

均布横向载荷 q 的外力功为

$$V=\iint_S qwrdrd\theta\tag{2.19}$$

这样，夹层圆板的总势能为

$$\begin{aligned}U&=U_1+U_2+U_3-V\\&=\frac{t}{E}\iint_S[(\sigma_{r0}+\sigma_{\theta0})^2-2(1+\nu)\sigma_{r0}\sigma_{\theta0}]rdrd\theta+D_f\iint_S\left[\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)^2-2(1-\nu)\frac{1}{r}\right.\\&\left.\cdot\frac{dw}{dr}\frac{d^2w}{dr^2}\right]rdrd\theta+\frac{D}{2}\iint_S\left[\left(\frac{d\psi}{dr}+\frac{\psi}{r}\right)^2-2(1-\nu)\frac{\psi}{r}\frac{d\psi}{dr}\right]rdrd\theta+\frac{C}{2}\iint_S\left(\psi+\frac{dw}{dr}\right)^2\\&rdrd\theta-\iint_S qwrdrd\theta\end{aligned}\tag{2.20}$$

根据势能原理，以 u、w、ψ 作自变量，对总势能变分为零，有

$$\delta U=0\tag{2.21}$$

将式（2.20）代入，经部分积分后，可得夹层圆板在均布横向载荷 q 作用下的大挠度理论的平衡方程和边界条件：

$$\left.\begin{aligned}&\sigma_{\theta0}-\frac{d}{dr}(r\sigma_{r0})=0\\&2D_f\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-2t\frac{d}{dr}\left(r\sigma_{r0}\frac{dw}{dr}\right)\\&\quad-C\frac{d}{dr}\left[r\left(\psi+\frac{dw}{dr}\right)\right]-qr=0\\&D\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(r\psi)-C\left(\psi+\frac{dw}{dr}\right)=0\end{aligned}\right\}\tag{2.22,a、b、c}$$

当 $r=0$ 时或当 $r=a$ 时，

$$\left.\begin{aligned}&r\sigma_{r0}=0,\ \text{或}\ \delta u=0\\&2rD_f\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-2rt\sigma_{r0}\frac{dw}{dr}-rC\left(\psi+\frac{dw}{dr}\right)=0,\\&\text{或}\quad\delta w=0\\&rM=Dr\left(\frac{d\psi}{dr}+\frac{\nu}{r}\psi\right)=0,\qquad\text{或}\quad\delta\psi=0\\&rm=-D_fr\left(\frac{d^2w}{dr^2}+\frac{\nu}{r}\frac{dw}{dr}\right)=0,\qquad\text{或}\frac{d}{dr}(\delta w)=0\end{aligned}\right\}\tag{2.23}$$

关于应变协调方程，可由方程（2.14）消去位移 u 求得

$$\sigma_{r0}-\frac{d}{dr}(r\sigma_{\theta0})-\frac{E}{2}\left(\frac{dw}{dr}\right)^2=0\tag{2.24}$$

方程（2.22）和（2.24）就是夹层圆板大挠度理论的基本方程，而边界条件（如简支、

完全固定、滑动固定和悬空等）由式（2.23）决定.

引入应力函数 φ：

$$\sigma_{r0}=\frac{1}{r}\frac{d\varphi}{dr},\qquad \sigma_{\theta 0}=\frac{d^2\varphi}{dr^2} \tag{2.25}$$

则方程（2.22,a）自动满足，而方程（2.22,b、c）和（2.24）成为

$$\left.\begin{aligned}&2D_f\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-2t\frac{d}{dr}\left(\frac{d\varphi}{dr}\frac{dw}{dr}\right)\\&\quad -C\frac{d}{dr}\left[r\left(\psi+\frac{dw}{dr}\right)\right]-qr=0\\&D\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(r\psi)-C\left(\psi+\frac{dw}{dr}\right)=0\\&\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{d\varphi}{dr}+\frac{E}{2r}\left(\frac{dw}{dr}\right)^2=0\end{aligned}\right\}\tag{2.26,a、b、c}$$

将方程（2.26,a）乘以 dr，积分一次，可解得

$$\psi=\frac{2D_f}{C}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-\left(\frac{2t}{Cr}\frac{d\varphi}{dr}+1\right)\frac{dw}{dr}-\frac{1}{2C}qr \tag{2.27}$$

将此式代入方程（2.26,b），得

$$\begin{aligned}&\frac{2DD_f}{C}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-(D+2D_f)\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}\\&\quad-\frac{2tD}{C}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\left(\frac{d\varphi}{dr}\frac{dw}{dr}\right)+\frac{2t}{r}\frac{d\varphi}{dr}\frac{dw}{dr}+\frac{1}{2}qr=0\end{aligned}\tag{2.28}$$

于是，基本方程组（2.22）和（2.24）被简化为（2.28）和（2.26,c）两个方程.

由方程（2.28）、（2.26,c）和边界条件（2.23）不难看出，当夹心剪切模量 G_2 无限增强（即 C→∞）时，它们将转化为单层圆板（厚度为 h＋2t）的基本方程和边界条件.

三、方 程 的 简 化

若表板很薄，即 $t\ll h$，那么上述方程可被简化. 此时式（2.1）—（2.3）成为：

上表板

$$u_1=u+\frac{1}{2}h_0\psi,\qquad v_1=0,\qquad w_1=w \tag{3.1}$$

下表板

$$u_3=u-\frac{1}{2}h_0\psi,\qquad v_3=0,\qquad w_3=w \tag{3.2}$$

夹心

$$u_2=u+z\psi,\qquad v_2=0,\qquad w_2=w \tag{3.3}$$

这时，板总厚度可用两表板中面间的距离 h_0 来量度.

几何方程和虎克定律仍为式（2.4）、（2.5）和（2.9）、（2.10）. 经过类似的运算，

得表板和夹心的应变和应力公式

$$\varepsilon_{r1}=\frac{du}{dr}+\frac{h_0}{2}\frac{d\psi}{dr}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2,\qquad \varepsilon_{\theta1}=\frac{u}{r}+\frac{h_0}{2r}\psi \tag{3.4}$$

$$\varepsilon_{r3}=\frac{du}{dr}-\frac{h_0}{2}\frac{d\psi}{dr}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2,\qquad \varepsilon_{\theta3}=\frac{u}{r}-\frac{h_0}{2r}\psi \tag{3.5}$$

$$\gamma_{rz2}=\psi+\frac{dw}{dr} \tag{3.6}$$

$$\sigma_{r1}=\sigma_{r0}+\frac{Eh_0}{2(1-\nu^2)}\left(\frac{d\psi}{dr}+\frac{\nu}{r}\psi\right),\quad \sigma_{\theta1}=\sigma_{\theta0}+\frac{Eh_0}{2(1-\nu^2)}\left(\frac{\psi}{r}+\nu\frac{d\psi}{dr}\right) \tag{3.7}$$

$$\sigma_{r3}=\sigma_{r0}-\frac{Eh_0}{2(1-\nu^2)}\left(\frac{d\psi}{dr}+\frac{\nu}{r}\psi\right),\quad \sigma_{\theta3}=\sigma_{\theta0}-\frac{Eh_0}{2(1-\nu^2)}\left(\frac{\psi}{r}+\nu\frac{d\psi}{dr}\right) \tag{3.8}$$

$$\tau_{rz2}=G_2\left(\psi+\frac{dw}{dr}\right) \tag{3.9}$$

这里，σ_{r0} 和 $\sigma_{\theta0}$ 仍与式（2.14）相同.

将式（3.7）—（3.9）代入应变能公式（2.16），并应用式（2.19），得总势能

$$U=\frac{t}{E}\iint_S[(\sigma_{r0}+\sigma_{\theta0})^2-2(1+\nu)\sigma_{r0}\sigma_{\theta0}]rdrd\theta+\frac{D}{2}\iint_S\left[\left(\frac{d\psi}{dr}+\frac{\psi}{r}\right)^2-2(1-\nu)\cdot\frac{\psi}{r}\frac{d\psi}{dr}\right]rdrd\theta+\frac{G_2h_0}{2}\iint_S\left(\psi+\frac{dw}{dr}\right)^2rdrd\theta-\iint_S qwrdrd\theta \tag{3.10}$$

其中 D 仍由（2.18）的第二式表示.

对总势能变分为零，即

$$\delta U=0 \tag{3.11}$$

推得简化情况下的平衡方程和边界条件：

$$\left.\begin{aligned}&\sigma_{\theta0}-\frac{d}{dr}(r\sigma_{r0})=0\\&2t\frac{d}{dr}\left(r\sigma_{r0}\frac{dw}{dr}\right)+G_2h_0\frac{d}{dr}\left[r\left(\psi+\frac{dw}{dr}\right)\right]+qr=0\\&Dr\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(r\psi)-G_2h_0r\left(\psi+\frac{dw}{dr}\right)=0\end{aligned}\right\} \tag{3.12,a、b、c}$$

当 $r=0$ 或当 $r=a$ 时，

$$\left.\begin{aligned}&r\sigma_{r0}=0,\quad\text{或}\quad\delta u=0\\&rQ_r=G_2h_0r\left(\psi+\frac{dw}{dr}\right)=-2tr\sigma_{r0}\frac{dw}{dr},\quad\text{或}\quad\delta w=0\\&rM_r=Dr\left(\frac{d\psi}{dr}+\frac{\nu}{r}\psi\right)=0,\quad\text{或}\quad\delta\psi=0\end{aligned}\right\} \tag{3.13}$$

而应变协调方程仍为方程（2.24）.

引入如式（2.25）的应力函数φ，则方程（3.12,a）自动满足.

将方程（3.12,b）乘以 dr，积分一次，可解得

$$\psi=-\frac{1}{G_2h_0}\left(\frac{2t}{r}\frac{d\psi}{dr}\frac{dw}{dr}+\frac{1}{2}qr\right)-\frac{dw}{dr} \tag{3.14}$$

将式（3.14）代入方程（3.12c），得

$$Dr\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-2t\left(1-\frac{D}{G_2h_0}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\right)\left(\frac{d\varphi}{dr}\frac{dw}{dr}\right)-\frac{1}{2}qr^2=0 \tag{3.15}$$

则方程（3.15）和（2.26,c）就是表板很薄情况下的基本方程组．显而易见，此简化方程较前大为简单．由于实际情况中夹层板大多属于这种情况，所以这组方程非常实用．若将Reissner[13]所得的夹层矩形板的大挠度方程进行极坐标变换，所得方程与此简化方程完全相同

四、在滑动固定边界条件下夹层圆板的非线性轴对称弯曲问题的求解

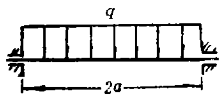

图 2　具有滑动固定边界条件的夹层圆板

现在，应用简化方程，研究如图 2 所示的夹层圆板的非线性轴对称弯曲问题．若以 σ_{r0} 和 w 为未知数，由方程(3.15)、(2.26,c) 和（3.13）得边值问题：

$$\left.\begin{aligned}&D\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}=2t\left(\frac{1}{r}-\frac{D}{G_2h_0}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\right)\left(r\sigma_{r0}\frac{dw}{dr}\right)+\frac{1}{2}qr\\&\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(r^2\sigma_{r0})=-\frac{E}{2r}\left(\frac{dw}{dr}\right)^2\end{aligned}\right\} \tag{4.1}$$

在 $r=a$ 时，

$$w=0,\qquad \psi=-\frac{1}{G_2h_0}\left(2t\sigma_{r0}\frac{dw}{dr}+\frac{1}{2}qr\right)-\frac{dw}{dr}=0,\qquad \sigma_{r0}=0 \tag{4.2}$$

在 $r=0$ 时，

$$\psi=-\frac{1}{G_2h_0}\left(2t\sigma_{r0}\frac{dw}{dr}+\frac{1}{2}qr\right)-\frac{dw}{dr}=0,\qquad \sigma_{r0}\text{ 有限} \tag{4.3}$$

为计算简单起见，引入无量纲量

$$\left.\begin{aligned}&\rho=\frac{r}{a},\qquad W=\sqrt{2(1-\nu^2)}\frac{w}{h_0},\qquad S_r=\frac{2ta^2}{D}\sigma_{r0}\\&k=\frac{D}{G_2h_0a^2},\qquad P=\frac{\sqrt{2(1-\nu^2)}a^4}{2h_0D}q\end{aligned}\right\} \tag{4.4}$$

则边值问题（4.1)—(4.3）成为

$$\left.\begin{aligned}&L\left(\rho\frac{dW}{d\rho}\right)=\left(\frac{1}{\rho}-kL\right)\left(\rho S_r\frac{dW}{d\rho}\right)+P\rho\\&L(\rho^2S_r)=-\frac{1}{\rho}\left(\frac{dW}{d\rho}\right)^2\end{aligned}\right\} \tag{4.5,a,b}$$

在 $\rho=1$ 时,

$$W=0,\qquad \frac{dW}{d\rho}=-k\left(S_r\frac{dW}{d\rho}+P\rho\right),\qquad S_r=0 \tag{4.6,a、b、c}$$

在 $\rho=0$ 时,

$$\frac{dW}{d\rho}=-kS_r\frac{dW}{d\rho},\qquad S_r\text{ 有限} \tag{4.7,a、b}$$

其中
$$L(\cdots)=\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}(\cdots) \tag{4.8}$$

我们使用修正迭代法求解此组非线性边值问题. 在一次近似中, 先略去方程 (4.5,a) 和边界条件 (4.6,b)、(4.7,a) 中的非线性项, 然后将由方程 (4.5,a) 所得的解 W_1 代入方程(4.5,b) 的右端, 便得下列线性边值问题

$$\left.\begin{aligned}&L\left(\rho\frac{dW_1}{d\rho}\right)=P\rho\\&L(\rho^2S_{r1})=-\frac{1}{\rho}\left(\frac{dW_1}{d\rho}\right)^2\end{aligned}\right\} \tag{4.9,a、b}$$

在 $\rho=1$ 时

$$W_1=0,\quad \frac{dW_1}{d\rho}=-kP\rho,\quad S_{r1}=0 \tag{4.10,a、b、c}$$

在 $\rho=0$ 时,

$$\frac{dW_1}{d\rho}=0,\quad S_{r1}\text{ 有限} \tag{4.11,a、b}$$

应用边界条件 (4.10,a、b) 和 (4.11,a), 方程 (4.9,a) 的解是

$$W_1=\frac{1}{8}\left(\frac{1}{\lambda_1}-\frac{1}{2}\lambda_2\rho^2+\frac{1}{4}\rho^4\right)P \tag{4.12}$$

其中
$$\lambda_1=\frac{4}{1+16k},\quad \lambda_2=1+8k \tag{4.13}$$

我们以无量纲中心挠度作为迭代参数:

$$W_0=W|_{\rho=0} \tag{4.14}$$

应用式 (4.12), 得一次近似中的载荷和中心挠度的关系式

$$P=\alpha_1W_0 \tag{4.15}$$

其中
$$\alpha_1=8\lambda_1 \tag{4.16}$$

显而易见, 式 (4.15) 就是夹层圆板小挠度理论的解.

将式(4.15)代入 式 (4.12), 有

$$W_1=\left(1-\frac{1}{2}\lambda_1\lambda_2\rho^2+\frac{1}{4}\lambda_1\rho^4\right)W_0 \tag{4.17}$$

再将这个 W_1 值代入方程(4.9,b), 应用边界条件(4.10,c)和(4.11,b), 方程(4.9,b)的解是

$$S_{r1}=\frac{\lambda_1^2}{4}\left[\left(\frac{1}{2}\lambda_2^2-\frac{1}{3}\lambda_2+\frac{1}{12}\right)-\frac{1}{2}\lambda_2^2\rho^2+\frac{1}{3}\lambda_2\rho^4-\frac{1}{12}\rho^6\right]W_0^2 \tag{4.18}$$

此即一次近似中的径向应力公式.

在二次近似中，有下面的边值问题

$$\left.\begin{aligned}&L\left(\rho\frac{dW_2}{d\rho}\right)=\left(\frac{1}{\rho}-kL\right)\left(\rho S_{r1}\frac{dW_1}{d\rho}\right)+P\rho\\&L\left(\rho^2S_{r2}\right)=-\frac{1}{\rho}\left(\frac{dW_2}{d\rho}\right)^2\end{aligned}\right\}\qquad(4.19,a、b)$$

在 $\rho=1$ 时，

$$W_2=0,\quad \frac{dW_2}{d\rho}=-k\left(S_{r1}\frac{dW_1}{d\rho}+P\rho\right),\quad S_{r2}=0\qquad(4.20,a、b、c)$$

在 $\rho=0$ 时，

$$\frac{dW_2}{d\rho}=-kS_{r1}\frac{dW_1}{d\rho},\quad S_{r2}\text{有限}\qquad(4.21,a、b)$$

应用式(4.17)和(4.18)，在边界条件(4.20,a、b) 和(4.21,a)情况下，方程(4.19,a)的解是

$$\begin{aligned}W_2=&B_2+\frac{1}{2}A_2\rho^2+\frac{1}{32}P\rho^4-\frac{\lambda_1^3}{64}\left[\frac{1}{4}\left(\lambda_2^4+\lambda_2^3-\frac{7}{3}\lambda_2^2+\lambda_2-\frac{1}{6}\right)\rho^4\right.\\&-\frac{1}{9}\left(3\lambda_2^3-2\lambda_2^2-\frac{1}{3}\lambda_2+\frac{1}{12}\right)\rho^6+\frac{5\lambda_2}{48}\left(\frac{4}{3}\lambda_2-1\right)\rho^8-\frac{1}{60}\left(\frac{3}{2}\lambda_2-1\right)\rho^{10}\\&\left.+\frac{1}{1080}\rho^{12}\right]W_0^3\end{aligned}\qquad(4.22)$$

其中

$$A_2=-\frac{\lambda_2}{8}P+\frac{\lambda_1^3}{64}\left(\lambda_2^4-\lambda_2^3+\frac{1}{9}\lambda_2^2+\frac{5}{36}\lambda_2-\frac{2}{45}\right)W_0^3$$

$$B_2=\frac{1}{8\lambda_1}P-\frac{\lambda_1^3}{128}\left(\frac{1}{2}\lambda_2^4-\frac{5}{6}\lambda_2^3+\frac{5}{9}\lambda_2^2-\frac{191}{1080}\lambda_2+\frac{1}{45}\right)W_0^3$$

利用定义(4.14)，由式(4.22)得二次近似中的载荷和中心挠度的关系 式

$$P=\alpha_1W_0+\alpha_3W_0^3\qquad(4.23)$$

其中

$$\alpha_3=\frac{\lambda_1^4}{16}\left(\frac{1}{2}\lambda_2^4-\frac{5}{6}\lambda_2^3+\frac{5}{9}\lambda_2^2-\frac{191}{1080}\lambda_2+\frac{1}{45}\right)\qquad(4.24)$$

将式(4.23)代入式(4.22)，得二次近似挠度公 式

$$\begin{aligned}W=&W_0-\frac{\lambda_1}{2}\left[\lambda_2W_0-\frac{\lambda_1^3}{128}\left(\frac{1}{2}\lambda_2^5-\frac{2}{3}\lambda_2^4+\frac{1}{18}\lambda_2^3+\frac{281}{1080}\lambda_2^2-\frac{49}{360}\lambda_2\right.\right.\\&\left.\left.+\frac{1}{45}\right)W_0^3\right]\rho^2+\frac{\lambda_1}{4}\left[W_0-\frac{\lambda_1^3}{128}\left(\lambda_2^5-2\lambda_2^3+\frac{29}{18}\lambda_2^2-\frac{529}{1080}\lambda_2+\frac{11}{180}\right)W_0^3\right]\rho^4\\&+\frac{\lambda_1^3}{192}\left[\left(\lambda_2^3-\frac{2}{3}\lambda_2^2-\frac{1}{9}\lambda_2+\frac{1}{36}\right)\rho^6-\frac{5\lambda_2}{16}\left(\frac{4}{3}\lambda_2-1\right)\rho^8+\frac{1}{20}\left(\frac{3}{2}\lambda_2-1\right)\rho^{10}\right.\\&\left.-\frac{1}{360}\rho^{12}\right]W_0^3\end{aligned}\qquad(4.25)$$

将此式代入方程(4.19,b)，应用边界条件(4.20,c) 和(4.21,b)，得方程(4.19,b)的解，

即二次近似径向应力公式

$$
\begin{aligned}
S_{r2}=&(n_{0,2}W_0^2+n_{0,4}W_0^4+n_{0,6}W_0^6)-(n_{2,2}W_0^2+n_{2,4}W_0^4+n_{2,6}W_0^6)\rho^2\\
&-(n_{4,2}W_0^2+n_{4,4}W_0^4+n_{4,6}W_0^6)\rho^4-(n_{6,2}W_0^2+n_{6,4}W_0^4+n_{6,6}W_0^6)\rho^6\\
&-(n_{8,4}W_0^4+n_{8,6}W_0^6)\rho^8-(n_{10,4}W_0^4+n_{10,6}W_0^6)\rho^{10}-(n_{12,4}W_0^4+n_{12,6}W_0^6)\rho^{12}\\
&-(n_{14,4}W_0^4+n_{14,6}W_0^6)\rho^{14}-(n_{16,6}\rho^{16}+n_{18,6}\rho^{18}+n_{20,6}\rho^{20}+n_{22,6}\rho^{22})W_0^6
\end{aligned}
\tag{4.26}
$$

其中

$$n_{0,2}=-\frac{\lambda_1^2}{4}\left(\frac{1}{2}\lambda_2^2-\frac{1}{3}\lambda_2+\frac{1}{12}\right)$$

$$n_{0,4}=+\frac{\lambda_1^5}{1536}\left(\frac{1}{2}\lambda_2^6-\lambda_2^5+\frac{11}{15}\lambda_2^4-\frac{11}{40}\lambda_2^3+\frac{593}{7560}\lambda_2^2-\frac{71}{3024}\lambda_2+\frac{1}{240}\right)$$

$$
\begin{aligned}
n_{0,6}=-\frac{\lambda_1^8}{786432}\Big(&\frac{1}{2}\lambda_2^{10}-\frac{26}{15}\lambda_2^9+\frac{12}{5}\lambda_2^8-\frac{6229}{3780}\lambda_2^7+\frac{2153}{3780}\lambda_2^6-\frac{2164}{14175}\lambda_2^5\\
&+\frac{64741}{453600}\lambda_2^4-\frac{186223}{1603800}\lambda_2^3+\frac{4317389}{89812800}\lambda_2^2-\frac{5363}{534600}\lambda_2+\frac{6431}{7484400}\Big)
\end{aligned}
$$

$$n_{2,2}=\frac{1}{8}\lambda_1^2\lambda_2^2$$

$$n_{2,4}=-\frac{\lambda_1^5\lambda_2}{512}\left(\frac{1}{2}\lambda_2^5-\frac{2}{3}\lambda_2^4+\frac{1}{18}\lambda_2^3+\frac{281}{1080}\lambda_2^2-\frac{49}{360}\lambda_2+\frac{1}{45}\right)$$

$$
\begin{aligned}
n_{2,6}=\frac{\lambda_1^8}{131072}\Big(&\frac{1}{4}\lambda_2^{10}-\frac{2}{3}\lambda_2^9+\frac{1}{2}\lambda_2^8+\frac{67}{360}\lambda_2^7-\frac{311}{648}\lambda_2^6+\frac{2261}{9720}\lambda_2^5\\
&+\frac{26761}{1166400}\lambda_2^4-\frac{13289}{194400}\lambda_2^3+\frac{11699}{388800}\lambda_2^2-\frac{49}{8100}\lambda_2+\frac{1}{2025}\Big)
\end{aligned}
$$

$$n_{4,2}=-\frac{1}{12}\lambda_1^2\lambda_2$$

$$n_{4,4}=\frac{\lambda_1^5}{1536}\left(\lambda_2^6+\frac{1}{2}\lambda_2^5-\frac{8}{3}\lambda_2^4+\frac{5}{3}\lambda_2^3-\frac{31}{135}\lambda_2^2-\frac{3}{40}\lambda_2+\frac{1}{45}\right)$$

$$
\begin{aligned}
n_{4,6}=-\frac{\lambda_1^8}{393216}\Big(&\lambda_2^{10}-\frac{4}{3}\lambda_2^9-\frac{17}{9}\lambda_2^8+\frac{2591}{540}\lambda_2^7-\frac{3383}{1080}\lambda_2^6-\frac{167}{1620}\lambda_2^5+\frac{101}{81}\lambda_2^4\\
&-\frac{452309}{583200}\lambda_2^3+\frac{15341}{64800}\lambda_2^2-\frac{3733}{97200}\lambda_2+\frac{11}{4050}\Big)
\end{aligned}
$$

$$n_{6,2}=\frac{1}{48}\lambda_1^2$$

$$n_{6,4}=-\frac{\lambda_1^5}{9216}\left(9\lambda_2^5-7\lambda_2^4-\frac{14}{3}\lambda_2^3+\frac{16}{3}\lambda_2^2-\frac{559}{360}\lambda_2+\frac{11}{60}\right)$$

$$
\begin{aligned}
n_{6,6}=\frac{\lambda_1^8}{2359296}\Big(&3\lambda_2^{10}+6\lambda_2^9-27\lambda_2^8+21\lambda_2^7+\frac{1823}{180}\lambda_2^6-\frac{1118}{45}\lambda_2^5+\frac{2705}{162}\lambda_2^4\\
&-\frac{2386}{405}\lambda_2^3+\frac{462781}{388800}\lambda_2^2-\frac{1091}{8100}\lambda_2+\frac{3}{400}\Big)
\end{aligned}
$$

$$n_{8,4}=\frac{\lambda_1^4}{3840}\left(\frac{14}{3}\lambda_2^3-\frac{13}{4}\lambda_2^2-\frac{1}{3}\lambda_2+\frac{1}{12}\right)$$

$$n_{8,6}=-\frac{\lambda_1^7}{491520}\Big(3\lambda_2^8-\frac{7}{6}\lambda_2^7-\frac{581}{72}\lambda_2^6+\frac{1063}{108}\lambda_2^5-\frac{11861}{3240}\lambda_2^4-\frac{401}{4320}\lambda_2^3$$

$$+\frac{4957}{12960}\lambda_2^2-\frac{1153}{12960}\lambda_2+\frac{11}{2160}\Big)$$

$$n_{10,4}=-\frac{\lambda_1^4\lambda_2}{23040}\Big(\frac{49}{6}\lambda_2-6\Big)$$

$$n_{10,6}=\frac{\lambda_1^7}{368640}\Big(\frac{7}{3}\lambda_2^7-\frac{105}{32}\lambda_2^6-\frac{25}{48}\lambda_2^5+\frac{2441}{864}\lambda_2^4-\frac{81031}{51840}\lambda_2^3+\frac{5327}{17280}\lambda_2^2$$

$$-\frac{97}{8640}\lambda_2-\frac{29}{8640}\Big)$$

$$n_{12,4}=\frac{\lambda_1^4}{32256}\Big(\frac{47}{30}\lambda_2-1\Big)$$

$$n_{12,6}=-\frac{\lambda_1^7}{24772608}\Big(89\lambda_2^6-\frac{2387}{15}\lambda_2^5+\frac{3128}{45}\lambda_2^4+\frac{1787}{90}\lambda_2^3-\frac{108443}{5400}\lambda_2^2+\frac{3841}{900}\lambda_2-\frac{161}{450}\Big)$$

$$n_{14,4}=-\frac{1}{645120}\lambda_1^4$$

$$n_{14,6}=\frac{\lambda_1^7}{49545216}\Big(\frac{914}{15}\lambda_2^5-\frac{697}{6}\lambda_2^4+\frac{1391}{20}\lambda_2^3-\frac{1219}{120}\lambda_2^2-\frac{488}{225}\lambda_2+\frac{43}{150}\Big)$$

$$n_{16,6}=-\frac{\lambda_1^6}{53084160}\Big(27\lambda_2^3-\frac{147}{4}\lambda_2^2+\frac{221}{18}\lambda_2+\frac{1}{18}\Big)$$

$$n_{18,6}=\frac{\lambda_1^6}{53084160}\Big(\frac{113}{36}\lambda_2^2-\frac{11}{3}\lambda_2+1\Big)$$

$$n_{20,6}=-\frac{\lambda_1^6}{486604800}\Big(\frac{3}{2}\lambda_2-1\Big)$$

$$n_{22,6}=\frac{1}{17517772800}\lambda_1^6$$

在三次近似中，关于挠度 W 的边值问题为

$$L\Big(\rho\frac{dW_3}{d\rho}\Big)=\Big(\frac{1}{\rho}-kL\Big)\Big(\rho S_{r2}\frac{dW_2}{d\rho}\Big)+P\rho \tag{4.27}$$

在 $\rho=1$ 时，

$$W_3=0,\qquad \frac{dW_3}{d\rho}=-k\Big(S_{r2}\frac{dW_2}{d\rho}+P\rho\Big) \tag{4.28}$$

在 $\rho=0$ 时，

$$\frac{dW_3}{d\rho}=-kS_{r2}\frac{dW_2}{d\rho} \tag{4.29}$$

应用式(4.25)和(4.26)，在边界条件(4.28)和(4.29)情况下，方程(4.27)的解是

$$W_3=B_3+\frac{1}{2}A_3\rho^2+\frac{1}{4}\Big(\frac{1}{8}P+\frac{m_1}{8}-km_3\Big)\rho^4+\frac{1}{6}\Big(\frac{m_3}{24}-km_5\Big)\rho^6+\frac{1}{8}\Big(\frac{m_5}{48}$$

$$-km_7\Big)\rho^8+\frac{1}{10}\Big(\frac{m_7}{80}-km_9\Big)\rho^{10}+\frac{1}{12}\Big(\frac{m_9}{120}-km_{11}\Big)\rho^{12}+\frac{1}{14}\Big(\frac{m_{11}}{168}-km_{13}\Big)\rho^{14}$$

$$+\frac{1}{16}\left(\frac{m_{13}}{224}-km_{15}\right)\rho^{16}+\frac{1}{18}\left(\frac{m_{15}}{288}-km_{17}\right)\rho^{18}+\frac{1}{20}\left(\frac{m_{17}}{360}-km_{19}\right)\rho^{20}$$

$$+\frac{1}{22}\left(\frac{m_{19}}{440}-km_{21}\right)\rho^{22}+\frac{1}{24}\left(\frac{m_{21}}{528}-km_{23}\right)\rho^{24}+\frac{1}{26}\left(\frac{m_{23}}{624}-km_{25}\right)\rho^{26}$$

$$+\frac{1}{28}\left(\frac{m_{25}}{728}-km_{27}\right)\rho^{28}+\frac{1}{30}\left(\frac{m_{27}}{840}-km_{29}\right)\rho^{30}+\frac{1}{32}\left(\frac{m_{29}}{960}-km_{31}\right)\rho^{32}$$

$$+\frac{1}{34}\left(\frac{m_{31}}{1088}-km_{33}\right)\rho^{34}+\frac{m_{33}}{44064}\rho^{36} \tag{4.30}$$

其中

$$A_3=-\left(\frac{\lambda_2}{8}P+\frac{\lambda_2 m_1}{8}+\frac{m_3}{24}+\frac{m_5}{48}+\frac{m_7}{80}+\frac{m_9}{120}+\frac{m_{11}}{168}+\frac{m_{13}}{224}+\frac{m_{15}}{288}+\frac{m_{17}}{360}\right.$$

$$\left.+\frac{m_{19}}{440}+\frac{m_{21}}{528}+\frac{m_{23}}{624}+\frac{m_{25}}{728}+\frac{m_{27}}{840}+\frac{m_{29}}{960}+\frac{m_{31}}{1088}+\frac{m_{33}}{1224}\right)$$

$$B_3=-\left[\frac{1}{2}A_3+\frac{1}{32}P+\frac{m_1}{32}+\frac{m_3}{4}\left(\frac{1}{36}-k\right)+\frac{m_5}{6}\left(\frac{1}{64}-k\right)+\frac{m_7}{8}\left(\frac{1}{100}-k\right)\right.$$

$$+\frac{m_9}{10}\left(\frac{1}{144}-k\right)+\frac{m_{11}}{12}\left(\frac{1}{196}-k\right)+\frac{m_{13}}{14}\left(\frac{1}{256}-k\right)+\frac{m_{15}}{16}\left(\frac{1}{324}-k\right)$$

$$+\frac{m_{17}}{18}\left(\frac{1}{400}-k\right)+\frac{m_{19}}{20}\left(\frac{1}{484}-k\right)+\frac{m_{21}}{22}\left(\frac{1}{576}-k\right)+\frac{m_{23}}{24}\left(\frac{1}{676}-k\right)$$

$$+\frac{m_{25}}{26}\left(\frac{1}{784}-k\right)+\frac{m_{27}}{28}\left(\frac{1}{900}-k\right)+\frac{m_{29}}{30}\left(\frac{1}{1024}-k\right)+\frac{m_{31}}{32}\left(\frac{1}{1156}-k\right)$$

$$+\frac{m_{33}}{34}\left(\frac{1}{1296}-k\right)$$

$$m_1=l_{1,1}n_{0,2}W_0^3+(l_{1,1}n_{0,4}+l_{1,3}n_{0,2})W_0^5+(l_{1,1}n_{0,6}+l_{1,3}n_{0,4})W_0^7+l_{1,3}n_{0,6}W_0^9$$

$$m_3=(l_{3,1}n_{0,2}-l_{1,1}n_{2,2})W_0^3+(l_{3,1}n_{0,4}+l_{3,3}n_{0,2}-l_{1,1}n_{2,4}-l_{1,3}n_{2,2})W_0^5+(l_{3,1}n_{0,6}$$
$$+l_{3,3}n_{0,4}-l_{1,1}n_{2,6}-l_{1,3}n_{2,4})W_0^7+(l_{3,3}n_{0,6}-l_{1,3}n_{2,6})W_0^9$$

$$m_5=-(l_{3,1}n_{2,2}+l_{1,1}n_{4,2})W_0^3+(l_{5,3}n_{0,2}-l_{3,1}n_{2,4}-l_{3,3}n_{2,2}-l_{1,1}n_{4,4}-l_{1,3}n_{4,2})W_0^5$$
$$+(l_{5,3}n_{0,4}-l_{3,1}n_{2,6}-l_{3,3}n_{2,4}-l_{1,1}n_{4,6}-l_{1,3}n_{4,4})W_0^7+(l_{5,3}n_{0,6}-l_{3,3}n_{2,6}$$
$$-l_{1,3}n_{4,6})W_0^9$$

$$m_7=-(l_{3,1}n_{4,2}+l_{1,1}n_{6,2})W_0^3+(l_{7,3}n_{0,2}-l_{5,3}n_{2,2}-l_{3,1}n_{4,4}-l_{3,3}n_{4,2}-l_{1,1}n_{6,4}$$
$$-l_{1,3}n_{6,2})W_0^5+(l_{7,3}n_{0,4}-l_{5,3}n_{2,4}-l_{3,1}n_{4,6}-l_{3,3}n_{4,4}-l_{1,1}n_{6,6}-l_{1,3}n_{6,4})W_0^7$$
$$+(l_{7,3}n_{0,6}-l_{5,3}n_{2,6}-l_{3,3}n_{4,6}-l_{1,3}n_{6,6})W_0^9$$

$$m_9=-l_{3,1}n_{6,2}W_0^3+(l_{9,3}n_{0,2}-l_{7,3}n_{2,2}-l_{5,3}n_{4,2}-l_{3,1}n_{8,4}-l_{3,3}n_{6,2}-l_{1,1}n_{8,4})W_0^5$$
$$+(l_{9,3}n_{0,4}-l_{7,3}n_{2,4}-l_{5,3}n_{4,4}-l_{3,1}n_{6,6}-l_{3,3}n_{6,4}-l_{1,1}n_{8,6}-l_{1,3}n_{8,4})W_0^7+(l_{9,3}n_{0,6}$$
$$-l_{7,3}n_{2,6}-l_{5,3}n_{4,6}-l_{3,3}n_{6,6}-l_{1,3}n_{8,6})W_0^9$$

$$m_{11}=(l_{11,3}n_{0,2}-l_{9,3}n_{2,2}-l_{7,3}n_{4,2}-l_{5,3}n_{6,2}-l_{3,1}n_{8,4}-l_{1,1}n_{10,4})W_0^5+(l_{11,3}n_{0,4}$$
$$-l_{9,3}n_{2,4}-l_{7,3}n_{4,4}-l_{5,3}n_{6,4}-l_{3,1}n_{8,6}-l_{3,3}n_{8,4}-l_{1,1}n_{10,6}-l_{1,3}n_{10,4})W_0^7$$
$$+(l_{11,3}n_{0,6}-l_{9,3}n_{2,6}-l_{7,3}n_{4,6}-l_{5,3}n_{6,6}-l_{3,3}n_{8,6}-l_{1,3}n_{10,6})W_0^9$$

$$m_{13}=-[(l_{11,3}n_{2,2}+l_{9,3}n_{4,2}+l_{7,3}n_{6,2}+l_{3,1}n_{10,4}+l_{1,1}n_{12,4})W_0^5+(l_{11,3}n_{2,4}+l_{9,3}n_{4,4}$$
$$+l_{7,3}n_{6,4}+l_{5,3}n_{8,4}+l_{3,1}n_{10,6}+l_{3,3}n_{10,4}+l_{1,1}n_{12,6}+l_{1,3}n_{12,4})W_0^7+(l_{11,3}n_{2,6}$$

$$+l_{9,3}n_{4,6}+l_{7,3}n_{6,6}+l_{5,3}n_{8,6}+l_{3,3}n_{10,6}+l_{1,3}n_{12,6})W_0^9]$$

$$m_{15}=-[(l_{11,3}n_{4,2}+l_{9,3}n_{6,2}+l_{3,1}n_{12,4}+l_{1,1}n_{14,4})W_0^5+(l_{11,3}n_{4,4}+l_{9,3}n_{6,4}+l_{7,3}n_{8,4}$$
$$+l_{5,3}n_{10,4}+l_{3,1}n_{12,6}+l_{3,3}n_{12,4}+l_{1,1}n_{14,6}+l_{1,3}n_{14,4})W_0^7+(l_{11,3}n_{4,6}+l_{9,3}n_{6,6}$$
$$+l_{7,3}n_{8,6}+l_{5,3}n_{10,6}+l_{3,3}n_{12,6}+l_{1,3}n_{14,6})W_0^9]$$

$$m_{17}=-[(l_{11,3}n_{6,2}+l_{3,1}n_{14,4})W_0^5+(l_{11,3}n_{6,4}+l_{9,3}n_{8,4}+l_{7,3}n_{10,4}+l_{5,3}n_{12,4}$$
$$+l_{3,1}n_{14,6}+l_{3,3}n_{14,4}+l_{1,1}n_{16,6})W_0^7+(l_{11,3}n_{6,6}+l_{9,3}n_{8,6}+l_{7,3}n_{10,6}+l_{5,3}n_{12,6}$$
$$+l_{3,3}n_{14,6}+l_{1,3}n_{16,6})W_0^9]$$

$$m_{19}=-[(l_{11,3}n_{8,4}+l_{9,3}n_{10,4}+l_{7,3}n_{12,4}+l_{5,3}n_{14,4}+l_{3,1}n_{16,6}+l_{1,1}n_{18,6})W_0^7$$
$$+(l_{11,3}n_{8,6}+l_{9,3}n_{10,6}+l_{7,3}n_{12,6}+l_{5,3}n_{14,6}+l_{3,3}n_{16,6}+l_{1,3}n_{18,6})W_0^9]$$

$$m_{21}=-[(l_{11,3}n_{10,4}+l_{9,3}n_{12,4}+l_{7,3}n_{14,4}+l_{3,1}n_{18,6}+l_{1,1}n_{20,6})W_0^7+(l_{11,3}n_{10,6}$$
$$+l_{9,3}n_{12,6}+l_{7,3}n_{14,6}+l_{5,3}n_{16,6}+l_{3,3}n_{18,6}+l_{1,3}n_{20,6})W_0^9]$$

$$m_{23}=-[l_{11,3}n_{12,4}+l_{9,3}n_{14,4}+l_{3,1}n_{20,6}+l_{1,1}n_{22,6})W_0^7+(l_{11,3}n_{12,6}+l_{9,3}n_{14,6}$$
$$+l_{7,3}n_{16,6}+l_{5,3}n_{18,6}+l_{3,3}n_{20,6}+l_{1,3}n_{22,6})W_0^9]$$

$$m_{25}=-[(l_{11,3}n_{14,4}+l_{3,1}n_{22,6})W_0^7+(l_{11,3}n_{14,6}+l_{9,3}n_{16,6}+l_{7,3}n_{18,6}+l_{5,3}n_{20,6}$$
$$+l_{3,3}n_{22,6})W_0^9]$$

$$m_{27}=-(l_{11,3}n_{16,6}+l_{9,3}n_{18,6}+l_{7,3}n_{20,6}+l_{5,3}n_{22,6})W_0^9$$

$$m_{29}=-(l_{11,3}n_{18,6}+l_{9,3}n_{20,6}+l_{7,3}n_{22,6})W_0^9$$

$$m_{31}=-(l_{11,3}n_{20,6}+l_{9,3}n_{22,6})W_0^9$$

$$m_{33}=-l_{11,3}n_{22,6}W_0^9$$

$$l_{1,1}=-\lambda_1\lambda_2$$

$$l_{1,3}=\frac{\lambda_1^4}{128}\left(\frac{1}{2}\lambda_2^5-\frac{2}{3}\lambda_2^4+\frac{1}{18}\lambda_2^3+\frac{281}{1080}\lambda_2^2-\frac{49}{360}\lambda_2+\frac{1}{45}\right)$$

$$l_{3,1}=\lambda_1$$

$$l_{3,3}=-\frac{\lambda_1^4}{128}\left(\lambda_2^5-2\lambda_2^3+\frac{29}{18}\lambda_2^2-\frac{529}{1080}\lambda_2+\frac{11}{180}\right)$$

$$l_{5,3}=\frac{\lambda_1^3}{96}\left(3\lambda_2^3-2\lambda_2^2-\frac{1}{3}\lambda_2+\frac{1}{12}\right)$$

$$l_{7,3}=-\frac{5\lambda_1^3\lambda_2}{384}\left(\frac{4}{3}\lambda_2-1\right)$$

$$l_{9,3}=\frac{\lambda_1^3}{384}\left(\frac{3}{2}\lambda_2-1\right)$$

$$l_{11,3}=-\frac{\lambda_1^3}{5760}$$

利用定义(4.14)，由式(4.30)得三次近似中的载荷和中心挠度的九次方关系式

$$P=\sum_{i=0}^{4}\alpha_{2i+1}W_0^{2i+1} \tag{4.31}$$

其中 $\alpha_6=-8\lambda_1[R_1(l_{1,1}n_{0,4}+l_{1,3}n_{0,2})+R_3(l_{3,1}n_{0,4}+l_{3,3}n_{0,2}-l_{1,1}n_{2,4}-l_{1,3}n_{2,2})$
$+R_5(l_{5,3}n_{0,2}-l_{3,1}n_{2,4}-l_{3,3}n_{2,2}-l_{1,1}n_{4,4}-l_{1,3}n_{4,2})+R_7(l_{7,3}n_{0,2}-l_{5,3}n_{2,2}$

$$
\begin{aligned}
&-l_{3,1}n_{4,4}-l_{3,3}n_{4,2}-l_{1,1}n_{6,4}-l_{1,3}n_{6,2})+R_9(l_{9,3}n_{0,2}-l_{7,3}n_{2,2}-l_{5,3}n_{4,2}\\
&-l_{3,1}n_{6,4}-l_{3,3}n_{6,2}-l_{1,1}n_{8,4})+R_{11}(l_{11,3}n_{0,2}-l_{9,3}n_{2,2}-l_{7,3}n_{4,2}-l_{5,3}n_{6,2}\\
&-l_{3,1}n_{8,4}-l_{1,1}n_{10,4})-R_{13}(l_{11,3}n_{2,2}+l_{9,3}n_{4,2}+l_{7,3}n_{6,2}+l_{3,1}n_{10,4}+l_{1,1}n_{12,4})\\
&-R_{15}(l_{11,3}n_{4,2}+l_{9,3}n_{6,2}+l_{3,1}n_{12,4}+l_{1,1}n_{14,4})-R_{17}(l_{11,3}n_{6,2}+l_{3,1}n_{14,4})]
\end{aligned}
\tag{4.32,a}
$$

$$
\begin{aligned}
a_7=&-8\lambda_1[R_1(l_{1,1}n_{0,6}+l_{1,3}n_{0,4})+R_3(l_{3,1}n_{0,6}+l_{3,3}n_{0,4}-l_{1,1}n_{2,6}-l_{1,3}n_{2,4})\\
&+R_5(l_{5,3}n_{0,4}-l_{3,1}n_{2,6}-l_{3,3}n_{2,4}-l_{1,1}n_{4,6}-l_{1,3}n_{4,4})+R_7(l_{7,3}n_{0,4}-l_{5,3}n_{2,4}\\
&-l_{3,1}n_{4,6}-l_{3,3}n_{4,4}-l_{1,1}n_{6,6}-l_{1,3}n_{6,4})+R_9(l_{9,3}n_{0,4}-l_{7,3}n_{2,4}-l_{5,3}n_{4,4}\\
&-l_{3,1}n_{6,6}-l_{3,3}n_{6,4}-l_{1,1}n_{8,6}-l_{1,3}n_{8,4})+R_{11}(l_{11,3}n_{0,4}-l_{9,3}n_{2,4}-l_{7,3}n_{4,4}\\
&-l_{5,3}n_{6,4}-l_{3,1}n_{8,6}-l_{3,3}n_{8,4}-l_{1,1}n_{10,6}-l_{1,3}n_{10,4})-R_{13}(l_{11,3}n_{2,4}+l_{9,3}n_{4,4}\\
&+l_{7,3}n_{6,4}+l_{5,3}n_{8,4}+l_{3,1}n_{10,6}+l_{3,3}n_{10,4}+l_{1,1}n_{12,6}+l_{1,3}n_{12,4})-R_{15}(l_{11,3}n_{4,4}\\
&+l_{9,3}n_{6,4}+l_{7,3}n_{8,4}+l_{5,3}n_{10,4}+l_{3,1}n_{12,6}+l_{3,3}n_{12,4}+l_{1,1}n_{14,6}+l_{1,3}n_{14,4})\\
&-R_{17}(l_{11,3}n_{6,4}+l_{9,3}n_{8,4}+l_{7,3}n_{10,4}+l_{5,3}n_{12,4}+l_{3,1}n_{14,6}+l_{3,3}n_{14,4}+l_{1,1}n_{16,6})\\
&-R_{19}(l_{11,3}n_{8,4}+l_{9,3}n_{10,4}+l_{7,3}n_{12,4}+l_{5,3}n_{14,4}+l_{3,1}n_{16,6}+l_{1,1}n_{18,6})\\
&-R_{21}(l_{11,3}n_{10,4}+l_{9,3}n_{12,4}+l_{7,3}n_{14,4}+l_{3,1}n_{18,6}+l_{1,1}n_{20,6})-R_{23}(l_{11,3}n_{12,4}\\
&+l_{9,3}n_{14,4}+l_{3,1}n_{20,6}+l_{1,1}n_{22,6})-R_{25}(l_{11,3}n_{14,4}+l_{3,1}n_{22,6})]
\end{aligned}
\tag{4.32,b}
$$

$$
\begin{aligned}
a_9=&-8\lambda_1[R_1l_{1,3}n_{0,6}+R_3(l_{3,3}n_{0,6}-l_{1,3}n_{2,6})+R_5(l_{5,3}n_{0,6}-l_{3,3}n_{2,6}-l_{1,3}n_{4,6})\\
&+R_7(l_{7,3}n_{0,6}-l_{5,3}n_{2,6}-l_{3,3}n_{4,6}-l_{1,3}n_{6,6})+R_9(l_{9,3}n_{0,6}-l_{7,3}n_{2,6}-l_{5,3}n_{4,6}\\
&-l_{3,3}n_{6,6}-l_{1,3}n_{8,6})+R_{11}(l_{11,3}n_{0,6}-l_{9,3}n_{2,6}-l_{7,3}n_{4,6}-l_{5,3}n_{6,6}-l_{3,3}n_{8,6}\\
&-l_{1,3}n_{10,6})-R_{13}(l_{11,3}n_{2,6}+l_{9,3}n_{4,6}+l_{7,3}n_{6,6}+l_{5,3}n_{8,6}+l_{3,3}n_{10,6}+l_{1,3}n_{12,6})\\
&-R_{15}(l_{11,3}n_{4,6}+l_{9,3}n_{6,6}+l_{7,3}n_{8,6}+l_{5,3}n_{10,6}+l_{3,3}n_{12,6}+l_{1,3}n_{14,6})-R_{17}(l_{11,3}n_{6,6}\\
&+l_{9,3}n_{8,6}+l_{7,3}n_{10,6}+l_{5,3}n_{12,6}+l_{3,3}n_{14,6}+l_{1,3}n_{16,6})-R_{19}(l_{11,3}n_{8,6}+l_{9,3}n_{10,6}\\
&+l_{7,3}n_{12,6}+l_{5,3}n_{14,6}+l_{3,3}n_{16,6}+l_{1,3}n_{18,6})-R_{21}(l_{11,3}n_{10,6}+l_{9,3}n_{12,6}\\
&+l_{7,3}n_{14,6}+l_{5,3}n_{16,6}+l_{3,3}n_{18,6}+l_{1,3}n_{20,6})-R_{23}(l_{11,3}n_{12,6}+l_{9,3}n_{14,6}+l_{7,3}n_{16,6}\\
&+l_{5,3}n_{18,6}+l_{3,3}n_{20,6}+l_{1,3}n_{22,6})-R_{25}(l_{11,3}n_{14,6}+l_{9,3}n_{16,6}+l_{7,3}n_{18,6}+l_{5,3}n_{20,6}\\
&+l_{3,3}n_{22,6})-R_{27}(l_{11,3}n_{16,6}+l_{9,3}n_{18,6}+l_{7,3}n_{20,6}+l_{5,3}n_{22,6})-R_{29}(l_{11,3}n_{18,6}\\
&+l_{9,3}n_{20,6}+l_{7,3}n_{22,6})-R_{31}(l_{11,3}n_{20,6}+l_{9,3}n_{22,6})-R_{33}l_{11,3}n_{22,6}]
\end{aligned}
\tag{4.32,c}
$$

$$
R_1=\frac{1}{8\lambda_1},\quad R_3=\frac{1}{4}\left(\frac{1}{18}+k\right),\quad R_5=\frac{1}{6}\left(\frac{3}{64}+k\right),\quad R_7=\frac{1}{8}\left(\frac{1}{25}+k\right),
$$

$$
R_9=\frac{1}{10}\left(\frac{5}{144}+k\right),\quad R_{11}=\frac{1}{12}\left(\frac{3}{98}+k\right),\quad R_{13}=\frac{1}{14}\left(\frac{7}{256}+k\right),\quad R_{15}=\frac{1}{16}\left(\frac{2}{81}+k\right),
$$

$$
R_{17}=\frac{1}{18}\left(\frac{9}{400}+k\right),\quad R_{19}=\frac{1}{20}\left(\frac{5}{242}+k\right),\quad R_{21}=\frac{1}{22}\left(\frac{11}{576}+k\right),
$$

$$
R_{23}=\frac{1}{24}\left(\frac{3}{169}+k\right),\quad R_{25}=\frac{1}{26}\left(\frac{13}{784}+k\right),\quad R_{27}=\frac{1}{28}\left(\frac{7}{450}+k\right),
$$

$$
R_{29}=\frac{1}{30}\left(\frac{15}{1024}+k\right),\quad R_{31}=\frac{1}{32}\left(\frac{4}{289}+k\right),\quad R_{33}=\frac{1}{34}\left(\frac{17}{1296}+k\right)
$$

最后，引入环向应力 $\sigma_{\theta\theta}$ 的无量纲量：

$$
S_\theta=\frac{2ta^2}{D}\sigma_{\theta\theta}
\tag{4.33}
$$

再应用式(4.26)，可由方程(2.12,a)得二次近似无量纲环向应力公式

$$S_{\theta 2}=(n_{0,2}W_0^2+n_{0,4}W_0^4+n_{0,6}W_0^6)-3(n_{2,2}W_0^2+n_{2,4}W_0^4+n_{2,6}W_0^6)\rho^2 -5(n_{4,2}W_0^2+n_{4,4}W_0^4+n_{4,6}W_0^6)\rho^4-7(n_{6,2}W_0^2+n_{6,4}W_0^4+n_{6,6}W_0^6)\rho^6 -9(n_{8,4}W_0^4+n_{8,6}W_0^6)\rho^8-11(n_{10,4}W_0^4+n_{10,6}W_0^6)\rho^{10}-13(n_{12,4}W_0^4 +n_{12,6}W_0^6)\rho^{12}-15(n_{14,4}W_0^4+n_{14,6}W_0^6)\rho^{14}-(17n_{16,6}\rho^{16}+19n_{18,6}\rho^{18} +21n_{20,6}\rho^{20}+23n_{22,6}\rho^{22})W_0^6 \tag{4.34}$$

由式(4.26)和(4.34)，我们还可写出夹层圆板中心和边缘的应力公式

$$S_r(0)=S_\theta(0)=n_{0,2}W_0^2+n_{0,4}W_0^4+n_{0,6}W_0^6$$

$$S_\theta(1)=-\frac{\lambda_1^2}{2}\Big[\Big(\frac{1}{2}\lambda_2^2-\frac{2}{3}\lambda_2+\frac{1}{4}\Big)W_0^2+\frac{\lambda_1^3}{192}\Big(\frac{1}{4}\lambda_2^6-\frac{3}{4}\lambda_2^5+\frac{13}{15}\lambda_2^4-\frac{113}{240}\lambda_2^3 +\frac{104}{945}\lambda_2^2-\frac{17}{10080}\lambda_2-\frac{5}{2016}\Big)W_0^4+\frac{\lambda_1^6}{49152}\Big(\frac{1}{8}\lambda_2^{10}-\frac{17}{30}\lambda_2^9+\frac{17}{15}\lambda_2^8-\frac{4021}{3024}\lambda_2^7 +\frac{577}{560}\lambda_2^6-\frac{32051}{56700}\lambda_2^5+\frac{1252103}{5443200}\lambda_2^4-\frac{3178631}{44906400}\lambda_2^3+\frac{382043}{23950080}\lambda_2^2-\frac{59}{25200}\lambda_2 +\frac{353}{2138400}\Big)W_0^6\Big]$$

应用公式(4.23)，(4.31)和(4.35)，我们得到了下面一些有用的结果．图3内示出了几个不同k值下的载荷P与中心挠度W_0的关系曲线．较精确的三次近似结果用实线表示，二次近似结果用虚线表示．显而易见，随着中心挠度的增大，板的刚度也增大，并且，对于相同的P值，具有较大k值的板将产生较大的中心挠度．由图看出，二次和三次近似曲线异常接近，这说明解的精确度很高．为了便于应用，我们用图4表示本文二次与三次近似解的相对误差．由此图可知，随着特征参数k的增大，它们的相对误差在增加．一般说来，由于二次近似解非常简单，对于工程设计说来，采用它是比较方便的．

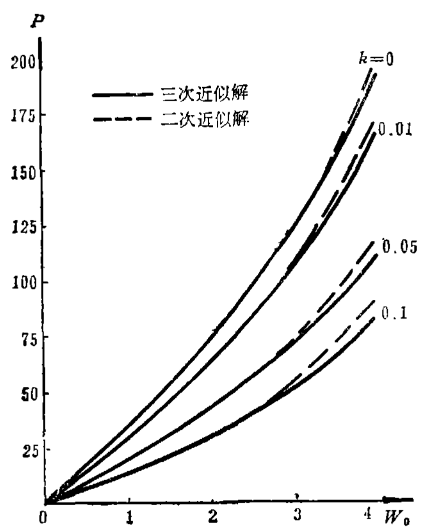

图3 各种k值下的载荷与中心挠度关系曲线

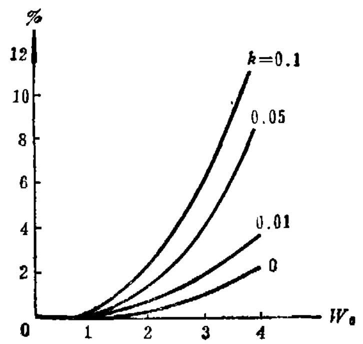

图4 二次与三次近似解的相对误差

在图5、6内，给出了各种k值下的夹层圆板中心和边缘的径向和环向应力曲线．由图看出，中心的径向和环向应力始终是正值，边缘的环向应力始终是负值，亦即夹层圆板的边缘区域是承受压应力的．而且，对于相同的中心挠度值，具有较大k值的夹层圆板将有较大

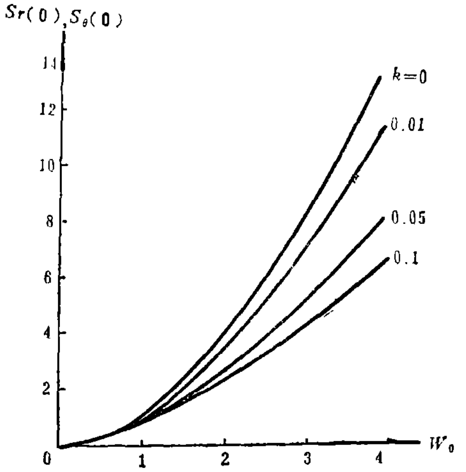

图 5　各种k值下的夹层圆板中心的径向和环向应力

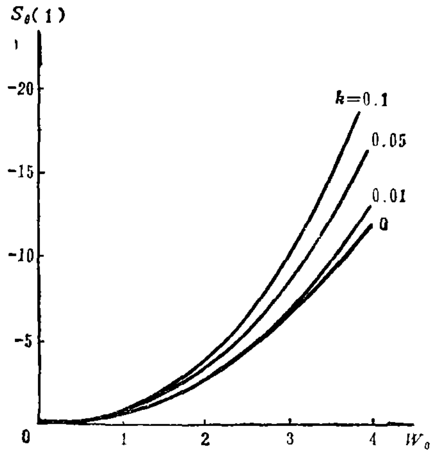

图 6　各种 k 值下的夹层圆板边缘的环向应力

的边缘环向应力以及较小的中心径向和环向应力.

五、结　　论

1．本文给出了具有软夹心的夹层园板在均布横向载荷作用下的非线性轴对称弯曲理论的基本方程和边界条件.

2．本文给出了这种夹层圆板在表板很薄时的上述方程和边界条件的简化形式.

3．应用简化方程，使用修正迭代法求解了滑动固定边界条件下的夹层圆板．获得了挠度的二次和三次近似解和应力的二次近似解．由二次和三次近似数值结果非常一致的事实，说明本文所得到的解的精确度是相当高的．对于工程设计说来，一般使用二次近似解就能满足精确度要求.

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Nonlinear Bending of Circular Sandwich Plates

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Abstract

In this paper, foundamental equations and boundary conditions of nonlinear, axsymmetrical bending theory for the circular sandwich plates (with a soft core) are derived by means of the method of calculus of variations. In most cases, the sandwich plates are composed of very thin faces. Then the preceding foundamental equations and boundary conditions are simplified considerably. For an illustrative example, a circular sandwich plate with edge clamped but free to slip under the action of uniformly transverse load is considered A more accurate solution of the problem of nonlinear bending of circular plate sandwich has been obtained by the aid of the modified iteration method. All results are presented in figures which are ready for direct application.