应用数学和力学, 第 26 卷 第 1 期(2005 年 1 月)
Applied Mathematics and Mechanics
应用数学和力学编委会编
重 庆 出 版 社 出 版

文章编号: 1000_0887(2005) 01_0016_09

压电压磁复合材料中界面裂纹对弹性波的散射*

周振功， 王 彪

(哈尔滨工业大学 复合材料研究所, 哈尔滨 150001)

(我刊编委王彪来稿)

摘要： 利用 Schmidt 方法分析了压电压磁复合材料中可导通界面裂纹对反平面简谐波的散射问题• 经过富里叶变换得到了以裂纹面上的间断位移为未知变量的对偶积分方程• 在求解对偶积分方程的过程中, 裂纹面上的间断位移被展开成雅可比多项式的形式• 数值模拟分析了裂纹长度、波速和入射波频率对应力强度因子、电位移强度因子、磁通量强度因子的影响• 从结果中可以看出, 压电压磁复合材料中可导通界面裂纹的反平面问题的应力奇异性形式与一般弹性材料中的反平面问题应力奇异性形式相同•

关 键 词： 界面裂纹； 弹性波； 压电压磁复合材料； 对偶积分方程
中图分类号： O346.58 文献标识码： A

引 言

近几年, 由于含有压电相和压磁相的复合材料在先进材料系统中迅速发展, 已引起了人们的广泛注意• 压电压磁复合材料具有大的磁电系数、静电和磁场的耦合系数, 而压电或压磁材料不具有这种新型耦合特性, 在某些情况下压电压磁复合材料的耦合特性甚至比单相磁电材料高出上百倍, 因此压电压磁复合材料所独有的磁_电_机械能的相互转换功能[1]已广泛用于电子封装、传感器和作动器中, 如: 磁场探测器、声学与超声装置、声纳等设备中• 但由于在压电压磁复合材料制备过程中会存在一些缺陷, 如裂纹、空洞, 因此会在使用过程中失效• 所以对压电压磁复合材料的磁_电_弹性之间的相互作用及其断裂特性研究是重要的[2~3]• Van Suchtelen1972 年提出了压电和压磁材料的结合可以导致出现新的材料特性即磁电耦合效应[4]• 此后许多研究者分析了 $BaTiO_3$_$CoFe_2O_4$ 复合材料的磁电耦合特性, 而研究压电压磁复合材料磁电耦合特性的理论工作也只是近期进行的[1~3, 5~13]• 据了解, 对于压电压磁复合材料中界面裂纹对弹性波的散射问题的研究还没有进行过•

本文将利用 Schmidt[14~15] 方法分析压电压磁复合材料中可导通界面裂纹对弹性波的散射

* 收稿日期： 2003_10_30； 修订日期： 2004_09_08
基金项目： 国家自然科学基金资助项目(50232030, 10172030)；黑龙江省自然科研基金资助项目(A0301)
作者简介： 周振功(1963—), 河南镇平县人, 教授, 博士, 博导(联系人. Tel: + 86_451_86402396; Fax: + 86_451_86418251; E_mail: zhouzhg@hit.edu.cn)

问题,经过富里叶变换,问题的求解可以转化为一对对偶积分方程的求解• 为了求解对偶积分方程,裂纹面上的间断位移被展开成雅可比多项式的形式• 这一求解过程与文献[2~ 3]中的求解过程不同• 最终给出了动应力强度因子的数值解•

1 基本方程

设有一位于两不同压电压磁复合材料界面上的裂纹,长度为 $2l$,如图 1 所示• 设其受与裂纹面垂直的反平面简谐波入射,ω 是入射波频率,$-\tau_0$ 是入射波幅值• 如公认采取的技术一样,在以下的有关变量中舍去简谐时间项 $e^{-i\omega t}$• 这里仅考虑反平面机械位移、平面电位移和平面磁场的情况,从而本问题的边界条件可表示为(本文只考虑扰动场):

$$\begin{cases} \tau_{yz}^{(1)}(x,0^+) = \tau_{yz}^{(2)}(x,0^-) = -\tau_0 & (|x| \leqslant l), \\ w^{(1)}(x,0^+) = w^{(2)}(x,0^-) & (|x| > l), \end{cases} \tag{1}$$

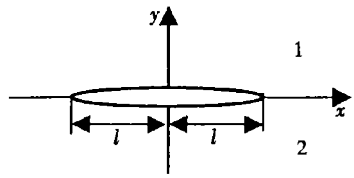

图 1 压电压磁复合材料中的界面裂纹

$$\begin{cases} \phi^{(1)}(x,0^+) = \phi^{(2)}(x,0^-) \\ D_y^{(1)}(x,0^+) = D_y^{(2)}(x,0^-) \end{cases} \quad (|x| \leqslant \infty), \tag{2}$$

$$\begin{cases} \psi^{(1)}(x,0^+) = \psi^{(2)}(x,0^-) \\ B_y^{(1)}(x,0^+) = B_y^{(2)}(x,0^-) \end{cases} \quad (|x| \leqslant \infty), \tag{3}$$

$$w^{(1)}(x,y) = w^{(2)}(x,y) = 0 \quad ((x^2+y^2)^{1/2} \to \infty), \tag{4}$$

这里 $\tau_{zk}^{(i)}$、$D_k^{(i)}$ 和 $B_k^{(i)}$ $(k = x, y, i = 1, 2)$ 分别是反平面剪切应力、平面电位移和平面磁通量• $w^{(i)}$、$\phi^{(i)}$ 和 $\psi^{(i)}$ 分别是机械位移、电势和磁势• 上标 i $(i = 1, 2)$ 分别对应上半平面 1 和下半平面 2,如图 1 所示• 这里我们仅考虑 τ_0 为正值的情况•

本构方程可表示为:

$$\tau_{zk}^{(i)} = c_{44}^{(i)} w_{,k}^{(i)} + e_{15}^{(i)} \phi_{,k}^{(i)} + q_{15}^{(i)} \psi_{,k}^{(i)} \quad (k = x, y;\ i = 1, 2), \tag{5}$$

$$D_k^{(i)} = e_{15}^{(i)} w_{,k}^{(i)} - \varepsilon_{11}^{(i)} \phi_{,k}^{(i)} - d_{11}^{(i)} \psi_{,k}^{(i)} \quad (k = x, y;\ i = 1, 2), \tag{6}$$

$$B_k^{(i)} = q_{15}^{(i)} w_{,k}^{(i)} - d_{11}^{(i)} \phi_{,k}^{(i)} - \mu_{11}^{(i)} \psi_{,k}^{(i)} \quad (k = x, y;\ i = 1, 2), \tag{7}$$

这里 $c_{44}^{(i)}$ 是剪切模量,$e_{15}^{(i)}$ 是压电系数,$\varepsilon_{11}^{(i)}$ 是介电参数,$q_{15}^{(i)}$ 是压磁系数,$d_{11}^{(i)}$ 是电磁系数,$\mu_{11}^{(i)}$ 是磁通率•

控制方程可表示为:

$$c_{44}^{(i)} \nabla^2 w^{(i)} + e_{15}^{(i)} \nabla^2 \phi^{(i)} + q_{15}^{(i)} \nabla^2 \psi^{(i)} = \rho^{(i)} \frac{\partial^2 w^{(i)}}{\partial t^2} \quad (i = 1, 2), \tag{8}$$

$$e_{15}^{(i)} \nabla^2 w^{(i)} - \varepsilon_{11}^{(i)} \nabla^2 \phi^{(i)} - d_{11}^{(i)} \nabla^2 \psi^{(i)} = 0 \quad (i = 1, 2), \tag{9}$$

$$q_{15}^{(i)} \nabla^2 w^{(i)} - d_{11}^{(i)} \nabla^2 \phi^{(i)} - \mu_{11}^{(i)} \nabla^2 \psi^{(i)} = 0 \quad (i = 1, 2), \tag{10}$$

这里 $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ 是二维拉普拉斯算子• $\rho^{(i)}$ 是压电压磁复合材料密度•

由于本问题几何和载荷的对称性,只需考虑右半平面 $0 \leqslant x < \infty$,$-\infty \leqslant y < \infty$ 就可以•

方程(8)~(10)经富里叶变换后,其解可假设为:

$$\begin{cases} w^{(1)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_1(s) e^{-\gamma_1 y} \cos(sx)\,ds, \\ \phi^{(1)}(x,y) = \dfrac{\mu_{11}^{(1)} e_{15}^{(1)} - d_{11}^{(1)} q_{15}^{(1)}}{\varepsilon_{11}^{(1)} \mu_{11}^{(1)} - \mu_{11}^{(1)2}} w^{(1)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_1(s) e^{-sy} \cos(sx)\,ds, \\ \psi^{(1)}(x,y) = \dfrac{q_{15}^{(1)} \varepsilon_{11}^{(1)} - d_{11}^{(1)} e_{15}^{(1)}}{\varepsilon_{11}^{(1)} \mu_{11}^{(1)} - \mu_{11}^{(1)2}} w^{(1)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_1(s) e^{-sy} \cos(sx)\,ds, \end{cases}$$

$$(y \geqslant 0) \tag{11}$$

$$\begin{cases} w^{(2)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_2(s)\mathrm{e}^{\gamma_2 y}\cos(sx)\,\mathrm{d}s, \\ \phi^{(2)}(x,y) = \dfrac{\mu_{11}^{(2)}e_{15}^{(2)} - d_{11}^{(2)}q_{15}^{(2)}}{\varepsilon_{11}^{(2)}\mu_{11}^{(2)} - \mu_{11}^{(2)2}}w^{(2)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_2(s)\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s, \\ \psi^{(2)}(x,y) = \dfrac{q_{15}^{(2)}\varepsilon_{11}^{(2)} - d_{11}^{(2)}e_{15}^{(2)}}{\varepsilon_{11}^{(2)}\mu_{11}^{(2)} - \mu_{11}^{(2)2}}w^{(2)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_2(s)\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s, \end{cases}$$

$$(y \leqslant 0), \tag{12}$$

这里 $A_1(s)$、$B_1(s)$、$C_1(s)$、$A_2(s)$、$B_2(s)$ 和 $C_2(s)$ 是未知函数,

$$\gamma_1^2 = s^2 - \omega^2/c_1^2,\quad c_1^2 = \mu^{(1)}/\rho^{(1)},\quad \mu^{(1)} = c_{44}^{(1)} + \frac{a_1 e_{15}^{(1)}}{a_0} + \frac{a_2 q_{15}^{(1)}}{a_0},$$

$$a_0 = \varepsilon_{11}^{(1)}\mu_{11}^{(1)} - d_{11}^{(1)2},\quad a_1 = \mu_{11}^{(1)}e_{15}^{(1)} - d_{11}^{(1)}q_{15}^{(1)},\quad a_2 = q_{15}^{(1)}\varepsilon_{11}^{(1)} - d_{11}^{(1)}e_{15}^{(1)},$$

$$\gamma_2^2 = s^2 - \omega^2/c_2^2,\quad c_2^2 = \mu^{(2)}/\rho^{(2)},\quad \mu^{(2)} = c_{44}^{(2)} + \frac{a_4 e_{15}^{(2)}}{a_3} + \frac{a_5 q_{15}^{(2)}}{a_3},$$

$$a_3 = \varepsilon_{11}^{(2)}\mu_{11}^{(2)} - d_{11}^{(2)^2},\quad a_4 = \mu_{11}^{(2)}e_{15}^{(2)} - d_{11}^{(2)}q_{15}^{(2)},\quad a_5 = q_{15}^{(2)}\varepsilon_{11}^{(2)} - d_{11}^{(2)}e_{15}^{(2)}.$$

从而由方程(5)~(7)可得:

$$\tau_{yz}^{(1)}(x,y) = -\frac{2}{\pi}\int_0^\infty \left\{ \gamma_1\left(c_{44}^{(1)} + \frac{a_1 e_{15}^{(1)}}{a_0} + \frac{a_2 q_{15}^{(1)}}{a_0}\right)A_1(s)\mathrm{e}^{-\gamma_1 y} + s[e_{15}^{(1)}B_1(s) + q_{15}^{(1)}C_1(s)]\mathrm{e}^{-sy}\right\}\cos(sx)\,\mathrm{d}s, \tag{13}$$

$$D_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty s[\varepsilon_{11}^{(1)}B_1(s) + d_{11}^{(1)}C_1(s)]\mathrm{e}^{-sy}\cos(sx)\,\mathrm{d}s, \tag{14}$$

$$B_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty s[d_{11}^{(1)}B_1(s) + \mu_{11}^{(1)}C_1(s)]\mathrm{e}^{-sy}\cos(sx)\,\mathrm{d}s, \tag{15}$$

$$\tau_{yz}^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty \left\{ \gamma_2\left(c_{44}^{(2)} + \frac{a_4 e_{15}^{(2)}}{a_3} + \frac{a_5 q_{15}^{(2)}}{a_3}\right)A_2(s)\mathrm{e}^{\gamma_2 y} + s[e_{15}^{(2)}B_2(s) + q_{15}^{(2)}C_2(s)]\mathrm{e}^{sy}\right\}\cos(sx)\,\mathrm{d}s, \tag{16}$$

$$D_y^{(2)}(x,y) = -\frac{2}{\pi}\int_0^\infty s[\varepsilon_{11}^{(2)}B_2(s) + d_{11}^{(2)}C_2(s)]\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s, \tag{17}$$

$$B_y^{(2)}(x,y) = -\frac{2}{\pi}\int_0^\infty s[d_{11}^{(2)}B_2(s) + \mu_{11}^{(2)}C_2(s)]\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s. \tag{18}$$

为了求解问题, 裂纹面上的位移之差定义为:

$$f(x) = w^{(1)}(x,0^+) - w^{(2)}(x,0^-). \tag{19}$$

把方程(11)~(12)代入到方程(19), 并利用富里叶变换和边界条件(2)~(3)可得:

$$f(s) = A_1(s) - A_2(s), \tag{20}$$

$$\frac{a_1}{a_0}A_1(s) - \frac{a_4}{a_3}A_2(s) + B_1(s) - B_2(s) = 0, \tag{21}$$

$$\frac{a_2}{a_0}A_1(s) - \frac{a_5}{a_3}A_2(s) + C_1(s) - C_2(s) = 0. \tag{22}$$

把方程(13)~(18)代入到方程(1)~(3)中可得:

$$-\left(c_{44}^{(1)} + \frac{a_1 e_{15}^{(1)}}{a_0} + \frac{a_2 q_{15}^{(1)}}{a_0}\right)\gamma_1 A_1(s) - se_{15}^{(1)}B_1(s) - sq_{15}^{(1)}C_1(s) -$$

$$\left(c_{44}^{(2)}+\frac{a_4e_{15}^{(2)}}{a_3}+\frac{a_5q_{15}^{(2)}}{a_3}\right)\gamma_2A_2(s)-se_{15}^{(2)}B_2(s)-sq_{15}^{(2)}C_2(s)=0, \tag{23}$$

$$\varepsilon_{11}^{(1)}B_1(s)+d_{11}^{(1)}C_1(s)+\varepsilon_{11}^{(2)}B_2(s)+d_{11}^{(2)}C_2(s)=0, \tag{24}$$

$$d_{11}^{(1)}B_1(s)+\mu_{11}^{(1)}C_1(s)+d_{11}^{(2)}B_2(s)+\mu_{11}^{(2)}C_2(s)=0. \tag{25}$$

利用 6 个方程(20)~(25)求解六个未知数 $A_1(s)$、$B_1(s)$、$C_1(s)$、$A_2(s)$、$B_2(s)$、$C_2(s)$，并利用边界条件(1)可得:

$$\frac{2}{\pi}\int_0^\infty g_1(s)f(s)\cos(sx)\,\mathrm{d}s=-\tau_0\qquad(0\leqslant x\leqslant l), \tag{26}$$

$$\int_0^\infty f(s)\cos(sx)\,\mathrm{d}s=0\qquad(x>l), \tag{27}$$

这里 $g_1(s)$ 是一已知函数, 具体可见附录, 且 $\lim\limits_{s\to\infty}g_1(s)/s=\beta_1$, β_1 是一与上下半平面材料性质有关的常数(见附录). 当上下两边平面材料相同时可得 $\beta_1=-c_{44}^{(1)}/2$. 为了确定未知函数 $f(s)$, 必须求解上述对偶积分方程(26)、(27).

2 对偶积分方程的求解

这里可以利用 Schmidt[14~15] 方法来求解对偶积分方程(26)~(27), 未知函数 $f(x)$ 可以展开成如下级数形式:

$$f(x)=\sum_{n=1}^{\infty}b_nP_{2n-2}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1-\frac{x^2}{l^2}\right)^{1/2}\qquad(0\leqslant x\leqslant l), \tag{28}$$

这里 b_n 是未知系数, $P_n^{(1/2,1/2)}(x)$ 是雅可比多项式[16].

方程(28)经富立叶变换后为[17]:

$$f(s)=\sum_{n=1}^{\infty}b_nG_n\frac{1}{s}\mathrm{J}_{2n-1}(sl),\quad G_n=2\sqrt{\pi}(-1)^{n-1}\frac{\Gamma(2n-1/2)}{(2n-2)!}, \tag{29}$$

这里 $\Gamma(x)$ 和 $\mathrm{J}_n(x)$ 分别是伽玛函数和贝赛尔函数.

把方程(29)代入到方程(26)~(27)中, 方程(27)能够自动满足, 方程(26)经过在区间 $[0, x]$ 上对 x 积分后变为:

$$\sum_{n=1}^{\infty}b_nG_n\int_0^\infty\frac{g_1(s)}{s^2}\mathrm{J}_{2n-1}(sl)\sin(sx)\,\mathrm{d}s=-\frac{\pi\tau_0x}{2}. \tag{30}$$

利用关系式[16]

$$\int_0^\infty\frac{1}{s}\mathrm{J}_n(sa)\sin(bs)\,\mathrm{d}s=\begin{cases}\dfrac{\sin[n\arcsin(b/a)]}{n} & (a>b),\\[2ex] \dfrac{a^n\sin(n\pi/2)}{n[b+\sqrt{b^2-a^2}]^n} & (b>a).\end{cases} \tag{31}$$

方程(30)的半无限积分可变为:

$$\int_0^\infty\frac{1}{s}\left[\beta_1+\left(\frac{g_1(s)}{s}-\beta_1\right)\right]\mathrm{J}_{2n-1}(sl)\sin(sx)\,\mathrm{d}s=$$
$$\frac{\beta_1}{2n-1}\sin\left[(2n-1)\arcsin\left(\frac{x}{l}\right)\right]+\int_0^\infty\frac{1}{s}\frac{g_1(s)-s\beta_1}{s}\mathrm{J}_{2n-1}(sl)\sin(sx)\,\mathrm{d}s.$$

从而方程(30)中半无限积分可容易进行数值求解. 至此方程(30)可以利用 Schmidt 方法求解未知系数 b_n. 这一方法在文中被删去, 具体可参见文献[14].

3 强度因子

若未知系数 b_n 获得，整个扰动应力场、扰动电位移场和磁场就可以获得。但对于断裂力学，重要的是确定裂纹尖端附近的扰动应力场、扰动电位移场和磁场。沿裂纹面的应力 $\tau_{yz}^{(1)}$，电位移 $D_y^{(1)}$ 和磁通量 $B_y^{(1)}$ 可分别表示为：

$$\tau_{yz}^{(1)}(x,0)=\frac{2}{\pi}\sum_{n=1}^{\infty}b_nG_n\int_0^{\infty}\frac{g_1(s)}{s}\mathrm{J}_{2n-1}(sl)\cos(xs)\,\mathrm{d}s, \tag{32}$$

$$D_y^{(1)}(x,0)=\frac{2}{\pi}\sum_{n=1}^{\infty}b_nG_n\int_0^{\infty}\frac{g_2(s)}{s}\mathrm{J}_{2n-1}(sl)\cos(xs)\,\mathrm{d}s, \tag{33}$$

$$B_y^{(1)}(x,0)=\frac{2}{\pi}\sum_{n=1}^{\infty}b_nG_n\int_0^{\infty}\frac{g_3(s)}{s}\mathrm{J}_{2n-1}(sl)\cos(xs)\,\mathrm{d}s, \tag{34}$$

这里 $g_2(s)$ 和 $g_3(s)$ 是已知函数，具体可见附录，且 $\lim_{s\to\infty}g_2(s)/s=\beta_2$，$\lim_{s\to\infty}g_3(s)/s=\beta_3$。$\beta_2$ 和 β_3 是与材料性质有关的常数(见附录)。当上下两边平面材料项同时可得 $\beta_2=-e_{15}^{(1)}/2$ 而 $\beta_3=-q_{15}^{(1)}/2$。

通过观察方程(32)～(34)，利用下列关系式[16]：

$$\int_0^{\infty}\mathrm{J}_n(sa)\sin(bs)\,\mathrm{d}s=\begin{cases}\dfrac{\cos[n\arcsin(b/a)]}{\sqrt{a^2-b^2}} & (a>b),\\[2ex] -\dfrac{a^n\sin(n\pi/2)}{\sqrt{b^2-a^2}\,[b+\sqrt{b^2-a^2}]^n} & (b>a).\end{cases} \tag{35}$$

应力场、电位移场和磁通量的奇异部分可分别表示为 $(x>l)$：

$$\tau=-\frac{2\beta_1}{\pi}\sum_{n=1}^{\infty}b_nG_nH_n(x), \tag{36}$$

$$D=-\frac{2\beta_2}{\pi}\sum_{n=1}^{\infty}b_nG_nH_n(x), \tag{37}$$

$$B=-\frac{2\beta_3}{\pi}\sum_{n=1}^{\infty}b_nG_nH_n(x), \tag{38}$$

这里 $H_n(x)=\dfrac{(-1)^{n-1}l^{2n-1}}{\sqrt{x^2-l^2}\,[x+\sqrt{x^2-l^2}]^{2n-1}}$。

可得应力强度因子 K 为：

$$K=\lim_{x\to l^+}\sqrt{2(x-l)}\cdot\tau=-\frac{4\beta_1}{\sqrt{\pi l}}\sum_{n=1}^{\infty}b_n\frac{\Gamma(2n-1/2)}{(2n-2)!}. \tag{39}$$

可得电位移强度因子 K^D 为：

$$K^D=\lim_{x\to l^+}\sqrt{2(x-l)}\cdot D=-\frac{4\beta_2}{\sqrt{\pi l}}\sum_{n=1}^{\infty}b_n\frac{\Gamma(2n-1/2)}{(2n-2)!}=\frac{\beta_2}{\beta_1}K. \tag{40}$$

可得磁通量强度因子 K^B 为：

$$K^B=\lim_{x\to l^+}\sqrt{2(x-l)}\cdot B=-\frac{4\beta_3}{\sqrt{\pi l}}\sum_{n=1}^{\infty}b_n\frac{\Gamma(2n-1/2)}{(2n-2)!}=\frac{\beta_3}{\beta_1}K. \tag{41}$$

4 结论

从文献[12～15]中的研究结果可知，可以利用 Schmidt 方法来求解方程(30)，且选取级数

的前十项就可以满足有关精度, 在 $-l \leqslant x \leqslant l,\ y=0$ 上, $\tau_{yz}^{(1)}/\tau_0$ 非常接近于 -1, 这也证明了本文的解满足边界条件• 有关材料性质[3, 9, 10]假设为

$$c_{44}^{(1)}=44.0(\mathrm{GPa}),\quad e_{15}^{(1)}=5.8(\mathrm{C/m^2}),\quad \varepsilon_{11}^{(1)}=5.64\times 10^{-9}(\mathrm{C^2/Nm^2}),$$
$$q_{15}^{(1)}=275.0(\mathrm{N/Am}),\quad d_{11}^{(1)}=0.005\times 10^{-1}(\mathrm{Ns/VC}),$$
$$\mu_{11}^{(1)}=-297.0\times 10^{-6}(\mathrm{Ns^2/C^2}),$$
$$c_{44}^{(2)}=54.0(\mathrm{GPa}),\quad e_{15}^{(2)}=7.8(\mathrm{C/m^2}),\quad \varepsilon_{11}^{(2)}=3.64\times 10^{-9}(\mathrm{C^2/Nm^2}),$$
$$q_{15}^{(2)}=175.0(\mathrm{N/Am}),\quad d_{11}^{(2)}=0.008\times 10^{-9}(\mathrm{Ns/VC}),$$
$$\mu_{11}^{(2)}=-197.0\times 10^{-6}(\mathrm{Ns^2/C^2}),$$

有关数值结果由图 2~ 图 4 表示出来•

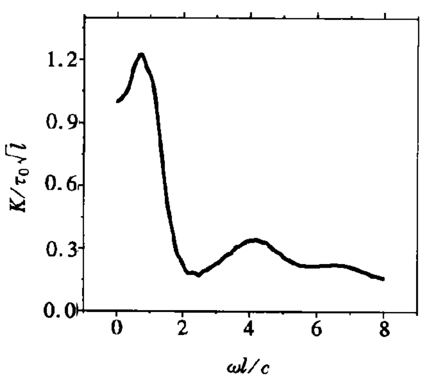

图 2　界面裂纹应力强度因子随 $\omega l/c$ 变化情况

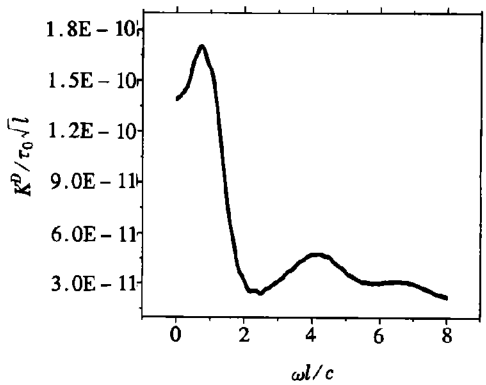

图 3　界面裂纹电位移强度因子随 $\omega l/c$ 变化情况

从这些图中可以得出如下结论:

(i) 对于压电压磁复合材料中界面裂纹对反平面剪切简谐波的散射问题, 其动应力强度因子与材料性质有关• 这与一般的弹性材料中界面裂纹对反平面剪切简谐波的散射问题的性质相同• 其电_磁_弹耦合特性可以从方程(40) 、(41) 中获得• 电位移强度因子和磁通量强度因子不仅与裂纹长度、波速和入射波频率有关, 还与材料性质有关, 这可从方程(40) 、(41) 中看出•

(ii) 如图 2 所时, 动应力强度因子将随着入射波频率的增加而增加, 进而在 $\omega l/c\approx 0.8$ 时, 达到最大值, 随后随入射波频率的增加而震荡减小• 而对于电位移和磁通量强度因子, 他们有与应力相同的变化趋势, 只是数值不同(如图 3 所示)• 这一点可以从方程(40) 、(41) 中看出, 这里不再描述•

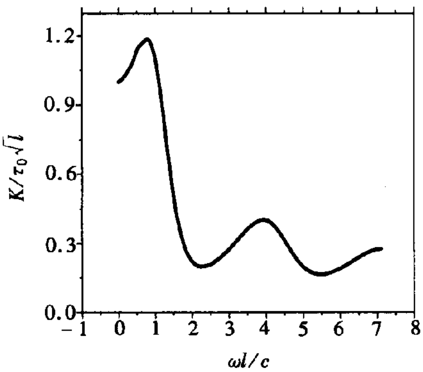

图 4　非界面裂纹应力强度因子随 $\omega l/c$ 变化情况

(iii) 当 $\omega l/c=0$ 时, 本文的解可以返回到静态问题的解, 从结果中可以看出, 应力强度因子的静态解是 1• 这与一般弹性材料中的反平面剪切断裂问题的结果一样•

(iv) 当上下两边平面材料相同时, 有关结果如图 4 所示, 应力强度因子随入射波频率变化趋势与界面裂纹的情况相同, 且当 $\omega l/c=0$ 时, 其值也等于 1• 但由图 2 和图 4 中结果可以看出, 界面裂纹应力强度因子的最大值大于非界面裂纹应力强度因子•

附　　录

$$\boldsymbol{X}_1=\begin{bmatrix}1&0&0\\ \dfrac{a_1}{a_0}&1&0\\ \dfrac{a_2}{a_0}&0&1\end{bmatrix},\quad \boldsymbol{X}_2=\begin{bmatrix}-1&0&0\\ -\dfrac{a_4}{a_3}&-1&0\\ -\dfrac{a_5}{a_3}&0&-1\end{bmatrix},$$

$$\boldsymbol{X}_3=\begin{bmatrix}\left(c_{44}^{(1)}+\dfrac{a_1e_{15}^{(1)}}{a_0}+\dfrac{a_2q_{15}^{(1)}}{a_0}\right)\gamma_1 & -se_{15}^{(1)} & -sq_{15}^{(1)}\\ 0 & \varepsilon_{11}^{(1)} & d_{11}^{(1)}\\ 0 & d_{11}^{(1)} & \mu_{11}^{(1)}\end{bmatrix},$$

$$\boldsymbol{X}_4=\begin{bmatrix}-\left(c_{44}^{(2)}+\dfrac{a_4e_{15}^{(2)}}{a_3}+\dfrac{a_5q_{15}^{(2)}}{a_3}\right)\gamma_2 & -se_{15}^{(2)} & -sq_{15}^{(2)}\\ 0 & \varepsilon_{11}^{(2)} & d_{11}^{(2)}\\ 0 & d_{11}^{(2)} & \mu_{11}^{(2)}\end{bmatrix},\quad \boldsymbol{X}_5=\boldsymbol{X}_1-\boldsymbol{X}_2\boldsymbol{X}_4^{-1}\boldsymbol{X}_3,$$

$$\boldsymbol{X}_6=\begin{bmatrix}x_{11}(s)&x_{12}(s)&x_{13}(s)\\ x_{21}(s)&x_{22}(s)&x_{23}(s)\\ x_{31}(s)&x_{32}(s)&x_{33}(s)\end{bmatrix}=\boldsymbol{X}_3\boldsymbol{X}_5^{-1},$$

$$g_1(s)=x_{11}(s),\ g_2(s)=x_{21}(s),\ g_3(s)=x_{31}(s).$$

$$H_1=-2d_{11}^{(1)}e_{15}^{(1)}c_{44}^{(2)}q_{15}^{(1)}+{e_{15}^{(2)}}^2{q_{15}^{(1)}}^2+c_{44}^{(2)}\varepsilon_{11}^{(1)}{q_{15}^{(1)}}^2-2e_{15}^{(1)}e_{15}^{(2)}q_{15}^{(1)}q_{15}^{(2)}+{e_{15}^{(1)}}^2{q_{15}^{(2)}}^2-2e_{15}^{(1)}c_{44}^{(2)}q_{15}^{(1)}d_{11}^{(2)},$$

$$H_2=c_{44}^{(2)}{q_{15}^{(1)}}^2\varepsilon_{11}^{(2)}+{e_{15}^{(1)}}^2c_{44}^{(2)}(\mu_{11}^{(2)}+\mu_{11}^{(1)}),$$

$$H_3=c_{44}^{(1)}[-{d_{11}^{(1)}}^2c_{44}^{(2)}-2e_{15}^{(2)}q_{15}^{(2)}d_{11}^{(2)}-c_{44}^{(2)}{d_{11}^{(2)}}^2-2d_{11}^{(1)}(e_{15}^{(2)}q_{15}^{(2)}+c_{44}^{(2)}d_{11}^{(2)})+{q_{15}^{(2)}}^2\varepsilon_{11}^{(2)}],$$

$$H_4=c_{44}^{(1)}\{{e_{15}^{(2)}}^2\mu_{11}^{(2)}+c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(2)}+{e_{15}^{(2)}}^2\mu_{11}^{(1)}+c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(1)}+\varepsilon_{11}^{(1)}[{q_{15}^{(2)}}^2+c_{44}^{(2)}(\mu_{11}^{(2)}+\mu_{11}^{(1)})]\},$$

$$R_1=-{d_{11}^{(1)}}^2c_{44}^{(2)}+\varepsilon_{11}^{(1)}{q_{15}^{(1)}}^2+2\varepsilon_{11}^{(1)}q_{15}^{(1)}q_{15}^{(2)}+\varepsilon_{11}^{(1)}{q_{15}^{(2)}}^2-2e_{15}^{(1)}q_{15}^{(1)}d_{11}^{(2)}-2e_{15}^{(2)}q_{15}^{(1)}d_{11}^{(2)},$$

$$R_2=-2e_{15}^{(1)}q_{15}^{(2)}d_{11}^{(2)}-2e_{15}^{(2)}q_{15}^{(2)}d_{11}^{(2)}-c_{44}^{(2)}{d_{11}^{(2)}}^2,$$

$$R_3=-2d_{11}^{(1)}[e_{15}^{(1)}(q_{15}^{(1)}+q_{15}^{(2)})+e_{15}^{(2)}(q_{15}^{(1)}+q_{15}^{(2)})+c_{44}^{(2)}d_{11}^{(2)}],$$

$$R_4={q_{15}^{(1)}}^2\varepsilon_{11}^{(2)}+2q_{15}^{(1)}q_{15}^{(2)}\varepsilon_{11}^{(2)}+{q_{15}^{(2)}}^2\varepsilon_{11}^{(2)}+{e_{15}^{(1)}}^2\mu_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}\mu_{11}^{(2)}+{e_{15}^{(2)}}^2\mu_{11}^{(2)}+c_{44}^{(2)}\varepsilon_{11}^{(1)}\mu_{11}^{(1)},$$

$$R_5=c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(2)}+{e_{15}^{(1)}}^2\mu_{11}^{(1)}+2e_{15}^{(1)}e_{15}^{(2)}\mu_{11}^{(1)}+{e_{15}^{(2)}}^2\mu_{11}^{(1)}+c_{44}^{(2)}\varepsilon_{11}^{(1)}\mu_{11}^{(1)}+c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(1)},$$

$$R_6=-c_{44}^{(1)}[{d_{11}^{(1)}}^2+2d_{11}^{(1)}d_{11}^{(2)}+{d_{11}^{(2)}}^2-(\varepsilon_{11}^{(1)}+\varepsilon_{11}^{(2)})(\mu_{11}^{(1)}+\mu_{11}^{(2)})],$$

$$\beta_1=(H_1+H_2+H_3+H_4)/(R_1+R_2+R_3+R_4+R_5+R_6),$$

$$S_1=-e_{15}^{(2)}\varepsilon_{11}^{(1)}{q_{15}^{(1)}}^2-e_{15}^{(2)}\varepsilon_{11}^{(1)}q_{15}^{(1)}q_{15}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}q_{15}^{(1)}d_{11}^{(2)}-c_{44}^{(2)}\varepsilon_{11}^{(1)}q_{15}^{(1)}d_{11}^{(2)}+{e_{15}^{(1)}}^2q_{15}^{(2)}d_{11}^{(2)},$$

$$S_2=2e_{15}^{(1)}e_{15}^{(2)}q_{15}^{(2)}d_{11}^{(2)}+e_{15}^{(1)}c_{44}^{(2)}{d_{11}^{(2)}}^2-e_{15}^{(1)}q_{15}^{(1)}q_{15}^{(2)}\varepsilon_{11}^{(2)}-e_{15}^{(1)}{q_{15}^{(2)}}^2\varepsilon_{11}^{(2)},$$

$$S_3=d_{11}^{(1)}\left\{e_{15}^{(1)}[e_{15}^{(2)}(2q_{15}^{(1)}+q_{15}^{(2)})+c_{44}^{(2)}d_{11}^{(2)}]+q_{15}^{(1)}({e_{15}^{(2)}}^2+c_{44}^{(2)}\varepsilon_{11}^{(2)})\right\},$$

$$S_4 = - e_{15}^{(1)2} e_{15}^{(2)} \mu_{11}^{(2)} - e_{15}^{(1)^2} e_{15}^{(2)^2} \mu_{11}^{(2)} - e_{15}^{(1)} c_{44}^{(2)} \varepsilon_{11}^{(2)} \mu_{11}^{(2)} - e_{15}^{(1)2} e_{15}^{(2)} \mu_{11}^{(1)} - e_{15}^{(1)} e_{15}^{(2)^2} \mu_{11}^{(1)} - e_{15}^{(1)} c_{44}^{(2)} \varepsilon_{11}^{(2)} \mu_{11}^{(1)},$$

$$S_5 = c_{44}^{(1)} \left\{ d_{11}^{(1)^2} e_{15}^{(2)} + d_{11}^{(1)} (e_{15}^{(2)} d_{11}^{(2)} - q_{15}^{(2)} \varepsilon_{11}^{(2)}) + \varepsilon_{11}^{(1)} [q_{15}^{(2)} d_{11}^{(2)} - e_{15}^{(2)} (\mu_{11}^{(2)} + \mu_{11}^{(1)})] \right\},$$

$$\beta_2 = - (S_1 + S_2 + S_3 + S_4 + S_5) / (R_1 + R_2 + R_3 + R_4 + R_5 + R_6),$$

$$Y_1 = \varepsilon_{11}^{(1)} q_{15}^{(1)^2} q_{15}^{(2)} + \varepsilon_{11}^{(1)} q_{15}^{(1)} q_{15}^{(2)^2} - e_{15}^{(2)} q_{15}^{(1)^2} d_{11}^{(2)} - e_{15}^{(1)} q_{15}^{(1)} q_{15}^{(2)} d_{11}^{(2)} - 2 e_{15}^{(2)} q_{15}^{(1)} q_{15}^{(2)} d_{11}^{(2)} - c_{44}^{(2)} q_{15}^{(1)} d_{11}^{(1)^2},$$

$$Y_2 = q_{15}^{(1)^2} q_{15}^{(2)} \varepsilon_{11}^{(2)} + q_{15}^{(1)} q_{15}^{(2)^2} \varepsilon_{11}^{(2)} + e_{15}^{(1)} e_{15}^{(2)} q_{15}^{(1)} \mu_{11}^{(2)} + e_{15}^{(2)^2} q_{15}^{(1)} \mu_{11}^{(2)} + c_{44}^{(2)} \varepsilon_{11}^{(1)} q_{15}^{(1)} \mu_{11}^{(2)} + c_{44}^{(2)} q_{15}^{(1)} \varepsilon_{11}^{(2)} \mu_{11}^{(2)},$$

$$Y_3 = - d_{11}^{(1)} (2 e_{15}^{(1)} q_{15}^{(1)} q_{15}^{(2)} + e_{15}^{(2)} q_{15}^{(1)} q_{15}^{(2)} + e_{15}^{(1)} q_{15}^{(2)^2} + c_{44}^{(2)} q_{15}^{(1)} d_{11}^{(2)} + c_{44}^{(2)} e_{15}^{(1)} \mu_{11}^{(2)}),$$

$$Y_4 = e_{15}^{(1)2} q_{15}^{(2)} \mu_{11}^{(1)} + e_{15}^{(1)} e_{15}^{(2)} q_{15}^{(2)} \mu_{11}^{(1)} + e_{15}^{(1)} c_{44}^{(2)} d_{11}^{(2)} \mu_{11}^{(1)},$$

$$Y_5 = c_{44}^{(1)} (- d_{11}^{(1)2} q_{15}^{(2)} - d_{11}^{(1)} d_{11}^{(2)} q_{15}^{(2)} + d_{11}^{(1)} e_{15}^{(2)} \mu_{11}^{(2)} + \varepsilon_{11}^{(1)} q_{15}^{(2)} \mu_{11}^{(1)} - e_{15}^{(1)} d_{11}^{(2)} \mu_{11}^{(1)} + q_{15}^{(2)} \varepsilon_{11}^{(2)} \mu_{11}^{(1)}),$$

$$\beta_3 = (Y_1 + Y_2 + Y_3 + Y_4 + Y_5) / (R_1 + R_2 + R_3 + R_4 + R_5 + R_6) \cdot$$

[参　考　文　献]

[1] Wu T L, Huang J H. Closed_form solutions for the magnetoelectric coupling coefficients in fibrous composites with piezoelectric and piezomagnetic phases[J]. International Journal of Solids and Structures, 2000, **37**(21): 2981—3009.

[2] Sih G C, Song Z F. Magnetic and electric poling effects associated with crack growth in $BaTiO_3$_$CoFe_2O_4$ composite[J]. Theoretical and Applied Fracture Mechanics, 2003, **39**(3): 209—227.

[3] Song Z F, Sih G C. Crack initiation behavior in magnetoelectrioelastic composite under in_plane deformation[J]. Theoretical and Applied Fracture Mechanics, 2003, **39**(3): 189—207.

[4] Van Suchtelen J. Product properties: a new application of composite materials[J]. Phillips Research Reports, 1972, **27**(1): 28—37.

[5] Harshe G, Dougherty J P, Newnham R E. Theoretical modeling of 3_0/0_3 magnetoelectric composites[J]. International Journal of Applied Electromagnetics in Materials, 1993, **4**(1): 161—171.

[6] Avellaneda M, Harshe G. Magnetoelectric effect in piezoelectric/magnetostrictive multiplayer (2_2) composites[J]. Journal of Intelligent Material Systems and Structures, 1994, **5**(3): 501—513.

[7] NAN Ce_wen. Magnetoelectric effect in composites of piezoelectric and piezomagnetic phases[J]. Physical Review B, 1994, **50**(20): 6082—6088.

[8] Benveniste Y. Magnetoelectric effect in fibrous composites with piezoelectric and magnetostrictive phases[J]. Physical Review B, 1995, **51**(8): 16424—16427.

[9] Huang J H, Kuo W S. The analysis of piezoelectric/piezomagnetic composite materials containing ellipsoidal inclusions[J]. Journal of Applied Physics, 1997, **81**(3): 1378—1386.

[10] Li J Y. Magnetoelectroelastic multi_inclusion and inhomogeneity problems and their applications in composite materials[J]. International Journal of Engineering Science, 2000, **38**(18): 1993—2011.

[11] ZHOU Zhen_gong, SHEN Ya_peng. Investigation of the scattering of harmonic shear waves by two collinear cracks using the non_local theory[J]. Acta Mechanica, 1999, **135**(3/4): 169—179.

[12] ZHOU Zhen_gong, LI Hai_chen. Investigation of the scattering of anti_plane shear waves by two collinear cracks in a piezoelectric materials using a new method[J]. Acta Mechanica, 2001, **147**(1_

4): 87—97.

[13] ZHOU Zhen_gong, Wang Biao. Investigation of anti_plane shear behavior of two collinear impermeable cracks in the piezoelectric materials by using the non_local theory[J]. International Journal of Solids and Structures, 2002, **39**(7): 1731—1742.

[14] Morse P M, Feshbach H. Methods of Theoretical Physics[M]. Vol 1. New York: McGraw_Hill, 1958, 926.

[15] Yan W F. Axisymmetric slipless indentation of an infinite elastic cylinder[J]. SIAM Journal on Applied Mathematics, 1967, **15**(2): 219—227.

[16] Gradshteyn I S, Ryzhik I M. Table of Integrals, Series and Products[M]. New York: Academic Press, 1980, 1035—1037.

[17] Erdelyi A. Tables of Integral Transforms[M]. New York: McGraw_Hill, 1954, 34—89.

Scattering of Harmonic Anti_Plane Shear Waves by an Interface Crack in Magneto_Electro_Elastic Composites

ZHOU Zhen_gong, WANG Biao

(Center for Composite Materials, Harbin Institute of Technology,
Harbin 150001, P.R.China)

Abstract: The dynamic behavior of an interface crack in magneto_electro_elastic composites under harmonic elastic anti_plane shear waves is investigated for the permeable electric boundary conditions. By using the Fourier transform, the problem can be solved with a pair of dual integral equations in which the unknown variable was the jump of the displacements across the crack surfaces. To solve the dual integral equations, the jump of the displacements across the crack surface was expanded in a series of Jacobi polynomials. Numerical examples were provided to show the effect of the length of the crack, the wave velocity and the circular frequency of the incident wave on the stress, the electric displacement and the magnetic flux intensity factors of the crack. From the results, it can be obtained that the singular stresses in piezoelectric/piezomagnetic materials carry the same forms as those in a general elastic material for anti_plane shear problem.

Key words: interface crack; elastic wave; magneto_electro_elastic composite; dual integral equation