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壳体的非线性应变分量*

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摘要

壳体的非线性应变分量是非线性壳体力学的基础，在壳体的稳定以及各种大变形问题中须要用到它们．由于壳体几何复杂，在现有文献中，还未见到较全面地表示此种非线性应变分量的一般公式，现在导出六个用正交曲线坐标表示的包括线性与非线性部分的壳体应变分量的公式，其中三个为拉伸应变分量，另三个为剪切应变分量．

设P为壳体内一任意点，Q为P的邻点，PQ代表变形前长度为ds的微线元，$P'Q'$代表变形后同一线元，其长度为ds'．各点的位置由下列正交曲线坐标表示：

$P(\xi,\ \eta,\ \zeta)$，$Q(\xi+d\xi,\ \eta+d\eta,\ \zeta+d\zeta)$

$P'(\xi',\ \eta',\ \zeta')$，$Q'(\xi'+d\xi',\ \eta'+d\eta',\ \zeta'+d\zeta')$

l，m，n为ds分别对与正交曲线坐标ξ，η，ζ在P点相切的直角坐标x，y，z的夹角的方向余弦；

l'，m'，n'相应地为ds'分别对在P'点切于ξ'，η'，ζ'的直角坐标x'，y'，z'的夹角的方向余弦；

U，V，W为位移PP'分别在x，y，z方向的投影；

A，B，C为在P点将曲线坐标增量转为线性距离的拉梅（Lamé）系数．

于是，ds在x，y，z方向的投影分别为：

$$lds=Ad\xi,\quad mds=Bd\eta,\quad nds=Cd\zeta \tag{1}$$

将上式平方后相加得：

$$(ds)^2=(Ad\xi)^2+(Bd\eta)^2+(Cd\zeta)^2 \tag{2}$$

P'点的曲线坐标与P点的曲线坐标的关系由下式表示：

$$\left.\begin{aligned}\xi'&=\xi+\lambda\\ \eta'&=\eta+\mu\\ \zeta'&=\zeta+\gamma\end{aligned}\right\} \tag{3}$$

其中，λ，μ，γ为ξ，η及ζ的函数，即

$$\left.\begin{aligned}\lambda&=\lambda(\xi,\ \eta,\ \zeta)\\ \mu&=\mu(\xi,\ \eta,\ \zeta)\\ \gamma&=\gamma(\xi,\ \eta,\ \zeta)\end{aligned}\right\} \tag{4}$$

* 钱伟长推荐．

故

$$\left.\begin{aligned} d\xi'=d\xi+d\lambda=d\xi+\frac{\partial\lambda}{\partial\xi}d\xi+\frac{\partial\lambda}{\partial\eta}d\eta+\frac{\partial\lambda}{\partial\zeta}d\zeta \\ d\eta'=d\eta+d\mu=d\eta+\frac{\partial\mu}{\partial\xi}d\xi+\frac{\partial\mu}{\partial\eta}d\eta+\frac{\partial\mu}{\partial\zeta}d\zeta \\ d\zeta'=d\zeta+d\gamma=d\zeta+\frac{\partial\gamma}{\partial\xi}d\xi+\frac{\partial\gamma}{\partial\eta}d\eta+\frac{\partial\gamma}{\partial\zeta}d\zeta \end{aligned}\right\} \tag{5}$$

相应地，P'点的拉梅系数A'，B'，C'与P点的拉梅系数的关系由下式表示：

$$\left.\begin{aligned} A'=A+dA=A+\frac{\partial A}{\partial\xi}\lambda+\frac{\partial A}{\partial\eta}\mu+\frac{\partial A}{\partial\zeta}\gamma \\ B'=B+dB=B+\frac{\partial B}{\partial\xi}\lambda+\frac{\partial B}{\partial\eta}\mu+\frac{\partial B}{\partial\zeta}\gamma \\ C'=C+dC=C+\frac{\partial C}{\partial\xi}\lambda+\frac{\partial C}{\partial\eta}\mu+\frac{\partial C}{\partial\zeta}\gamma \end{aligned}\right\} \tag{6}$$

于是，变形后的线元ds'在P'点与正交曲线坐标ξ'相切的直角坐标x'方向的投影为

$$\begin{aligned} l'ds'=A'd\xi'&=\left(A+\frac{\partial A}{\partial\xi}\lambda+\frac{\partial A}{\partial\eta}\mu+\frac{\partial A}{\partial\zeta}\gamma\right)\left(d\xi+\frac{\partial\lambda}{\partial\xi}d\xi+\frac{\partial\lambda}{\partial\eta}d\eta+\frac{\partial\lambda}{\partial\zeta}d\zeta\right) \\ &=Ad\xi+\frac{\partial A}{\partial\xi}\lambda d\xi+\frac{\partial A}{\partial\eta}\mu d\xi+\frac{\partial A}{\partial\zeta}\gamma d\xi+\frac{\partial\lambda}{\partial\xi}Ad\xi \\ &\quad+\frac{\partial\lambda}{\partial\eta}Ad\eta+\frac{\partial A}{\partial\zeta}Ad\zeta+\cdots\cdots \end{aligned} \tag{7}$$

注意上式括号内导数与导数的乘积项属于三级微量，可以省略.

沿x，y，z轴方向的位移分别为：

$$U=A\lambda,\ V=B\mu,\ W=C\gamma \tag{8}$$

经过下列简单变换：

$$\left(\frac{\partial A}{\partial\xi}\lambda+\frac{\partial\lambda}{\partial\xi}A\right)d\xi=\frac{1}{A}\frac{\partial(A\lambda)}{\partial\xi}Ad\xi=\frac{1}{A}\frac{\partial U}{\partial\xi}lds$$

$$\frac{\partial A}{\partial\eta}\mu d\xi=\frac{(B\mu)}{AB}\frac{\partial A}{\partial\eta}Ad\xi=\frac{V}{AB}\frac{\partial A}{\partial\eta}lds$$

$$\frac{\partial A}{\partial\zeta}\gamma d\xi=\frac{(C\gamma)}{AC}\frac{\partial A}{\partial\zeta}Ad\xi=\frac{W}{AC}\frac{\partial A}{\partial\zeta}lds$$

$$\frac{\partial\lambda}{\partial\eta}Ad\eta=\frac{A}{B}\frac{\partial}{\partial\eta}\left(\frac{A\lambda}{A}\right)Bd\eta=\frac{A}{B}\frac{\partial}{\partial\eta}\left(\frac{U}{A}\right)mds$$

$$\frac{\partial\lambda}{\partial\zeta}Ad\zeta=\frac{A}{C}\frac{\partial}{\partial\zeta}\left(\frac{A\lambda}{A}\right)Cd\zeta=\frac{A}{C}\frac{\partial}{\partial\zeta}\left(\frac{U}{A}\right)nds$$

式(7)成为

$$l'ds'=\left(1+\frac{1}{A}\frac{\partial U}{\partial\xi}+\frac{V}{AB}\frac{\partial A}{\partial\eta}+\frac{W}{AC}\frac{\partial A}{\partial\zeta}\right)lds$$

$$+\frac{A}{B}\frac{\partial}{\partial\eta}\left(\frac{U}{A}\right)mds+\frac{A}{C}\frac{\partial}{\partial\zeta}\left(\frac{U}{A}\right)nds \tag{9a}$$

同样地

$$m'ds'=\frac{B}{A}\frac{\partial}{\partial\xi}\left(\frac{V}{B}\right)lds+\left(1+\frac{1}{B}\frac{\partial V}{\partial\eta}+\frac{W}{BC}\frac{\partial B}{\partial\zeta}\right.$$

$$\left.+\frac{U}{AB}\frac{\partial B}{\partial\xi}\right)mds+\frac{B}{C}\frac{\partial}{\partial\zeta}\left(\frac{V}{B}\right)nds \tag{9b}$$

$$n'ds'=\frac{C}{A}\frac{\partial}{\partial\xi}\left(\frac{W}{C}\right)lds+\frac{C}{B}\frac{\partial}{\partial\eta}\left(\frac{W}{C}\right)mds$$

$$+\left(1+\frac{1}{C}\frac{\partial W}{\partial\zeta}+\frac{U}{AC}\frac{\partial C}{\partial\xi}+\frac{V}{BC}\frac{\partial C}{\partial\eta}\right)nds \tag{9c}$$

设ϵ为线元ds的应变，

$$\epsilon=\frac{ds'-ds}{ds}$$

$$ds'=(1+\epsilon)ds \tag{10}$$

而 $$(ds')^2=(l'ds')^2+(m'ds')^2+(n'ds')^2$$

将(9)与(10)代入上式，略去微量ϵ^2，

$$1+2\epsilon=\left[\left(1+\frac{1}{A}\frac{\partial U}{\partial\xi}+\frac{V}{AB}\frac{\partial A}{\partial\eta}+\frac{W}{AC}\frac{\partial A}{\partial\zeta}\right)l+\frac{A}{B}\frac{\partial}{\partial\eta}\left(\frac{U}{A}\right)m+\frac{A}{C}\frac{\partial}{\partial\zeta}\left(\frac{U}{A}\right)n\right]^2$$

$$+\left[\frac{B}{A}\frac{\partial}{\partial\xi}\left(\frac{V}{B}\right)l+\left(1+\frac{1}{B}\frac{\partial V}{\partial\eta}+\frac{W}{BC}\frac{\partial B}{\partial\zeta}+\frac{U}{AB}\frac{\partial B}{\partial\xi}\right)m+\frac{B}{C}\frac{\partial}{\partial\zeta}\left(\frac{V}{B}\right)n\right]^2$$

$$+\left[\frac{C}{A}\frac{\partial}{\partial\xi}\left(\frac{W}{C}\right)l+\frac{C}{B}\frac{\partial}{\partial\eta}\left(\frac{W}{C}\right)m\right.$$

$$\left.+\left(1+\frac{1}{C}\frac{\partial W}{\partial\zeta}+\frac{U}{AC}\frac{\partial C}{\partial\xi}+\frac{V}{BC}\frac{\partial C}{\partial\eta}\right)n\right]^2 \tag{11}$$

将式(11)展开，得

$$\epsilon=\epsilon_\xi l^2+\epsilon_\eta m^2+\epsilon_\zeta n^2+\gamma_{\xi\eta}lm+\gamma_{\xi\zeta}ln+\gamma_{\eta\zeta}mn$$

其中

$$\epsilon_\xi=\frac{1}{A}\frac{\partial U}{\partial\xi}+\frac{V}{AB}\frac{\partial A}{\partial\eta}+\frac{W}{AC}\frac{\partial A}{\partial\zeta}+\frac{1}{2}\left\{\left[\frac{1}{A}\frac{\partial U}{\partial\xi}+\frac{V}{AB}\frac{\partial A}{\partial\eta}\right.\right.$$

$$\left.\left.+\frac{W}{AC}\frac{\partial A}{\partial\zeta}\right]^2+\left[\frac{B}{A}\frac{\partial}{\partial\xi}\left(\frac{V}{B}\right)\right]^2+\left[\frac{C}{A}\frac{\partial}{\partial\xi}\left(\frac{W}{C}\right)\right]^2\right\} \tag{12a}$$

$$\epsilon_\eta=\frac{1}{B}\frac{\partial V}{\partial\eta}+\frac{W}{BC}\frac{\partial B}{\partial\zeta}+\frac{U}{AB}\frac{\partial B}{\partial\xi}+\frac{1}{2}\left\{\left[\frac{A}{B}\frac{\partial}{\partial\eta}\left(\frac{U}{A}\right)\right]^2\right.$$

$$\left.+\left[\frac{1}{B}\frac{\partial V}{\partial\eta}+\frac{W}{BC}\frac{\partial B}{\partial\zeta}+\frac{U}{AB}\frac{\partial B}{\partial\xi}\right]^2+\left[\frac{C}{B}\frac{\partial}{\partial\eta}\left(\frac{W}{C}\right)\right]^2\right\} \tag{12b}$$

$$\epsilon_\zeta=\frac{1}{C}\frac{\partial W}{\partial\zeta}+\frac{U}{AC}\frac{\partial C}{\partial\xi}+\frac{V}{BC}\frac{\partial C}{\partial\eta}+\frac{1}{2}\left\{\left[\frac{A}{C}\frac{\partial}{\partial\zeta}\left(\frac{U}{A}\right)\right]^2\right.$$

$$+\left[\frac{B}{C}\frac{\partial}{\partial\zeta}\left(\frac{V}{B}\right)\right]^2+\left[\frac{1}{C}\frac{\partial W}{\partial\zeta}+\frac{U}{AC}\frac{\partial C}{\partial\xi}+\frac{V}{BC}\frac{\partial C}{\partial\eta}\right]^2\Bigg\} \tag{12c}$$

$$\gamma_{\xi\eta}=\frac{A}{B}\frac{\partial}{\partial\eta}\left(\frac{U}{A}\right)+\frac{B}{A}\frac{\partial}{\partial\xi}\left(\frac{V}{B}\right)+\frac{1}{B}\frac{\partial}{\partial\eta}\left(\frac{U}{A}\right)\left[\frac{\partial U}{\partial\xi}+\frac{V}{B}\frac{\partial A}{\partial\eta}\right.$$

$$\left.+\frac{W}{C}\frac{\partial A}{\partial\zeta}\right]+\frac{1}{A}\frac{\partial}{\partial\xi}\left(\frac{V}{B}\right)\left[\frac{\partial V}{\partial\eta}+\frac{W}{C}\frac{\partial B}{\partial\zeta}\right.$$

$$\left.+\frac{U}{A}\frac{\partial B}{\partial\xi}\right]+\frac{C^2}{AB}\frac{\partial}{\partial\xi}\left(\frac{W}{C}\right)\frac{\partial}{\partial\eta}\left(\frac{W}{C}\right) \tag{12d}$$

$$\gamma_{\eta\zeta}=\frac{B}{C}\frac{\partial}{\partial\zeta}\left(\frac{V}{B}\right)+\frac{C}{B}\frac{\partial}{\partial\eta}\left(\frac{W}{C}\right)+\frac{A^2}{BC}\frac{\partial}{\partial\eta}\left(\frac{U}{A}\right)\frac{\partial}{\partial\zeta}\left(\frac{U}{A}\right)$$

$$+\frac{1}{C}\frac{\partial}{\partial\zeta}\left(\frac{V}{B}\right)\left[\frac{\partial V}{\partial\eta}+\frac{W}{C}\frac{\partial B}{\partial\zeta}+\frac{U}{A}\frac{\partial B}{\partial\xi}\right]$$

$$+\frac{1}{B}\frac{\partial}{\partial\eta}\left(\frac{W}{C}\right)\left[\frac{\partial W}{\partial\zeta}+\frac{U}{A}\frac{\partial C}{\partial\xi}+\frac{V}{B}\frac{\partial C}{\partial\eta}\right] \tag{12e}$$

$$\gamma_{\zeta\xi}=\frac{C}{A}\frac{\partial}{\partial\xi}\left(\frac{W}{C}\right)+\frac{A}{C}\frac{\partial}{\partial\zeta}\left(\frac{U}{A}\right)+\frac{1}{C}\frac{\partial}{\partial\zeta}\left(\frac{U}{A}\right)\left[\frac{\partial U}{\partial\xi}+\frac{V}{B}\frac{\partial A}{\partial\eta}+\frac{W}{C}\frac{\partial A}{\partial\zeta}\right]$$

$$+\frac{B^2}{AC}\frac{\partial}{\partial\zeta}\left(\frac{V}{B}\right)\frac{\partial}{\partial\xi}\left(\frac{V}{B}\right)+\frac{1}{A}\frac{\partial}{\partial\xi}\left(\frac{W}{C}\right)\left[\frac{\partial W}{\partial\zeta}+\frac{U}{A}\frac{\partial C}{\partial\xi}+\frac{V}{B}\frac{\partial C}{\partial\eta}\right] \tag{12f}$$

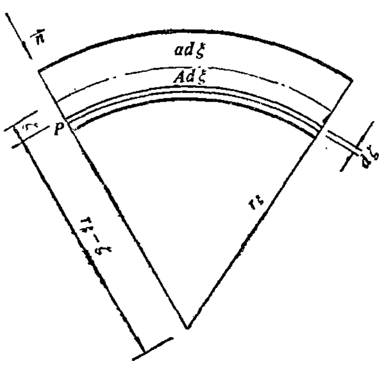

图 1

上六式为弹塑性体发生位移后在P点（ξ，η，ζ）的六个应变分量，前三者为拉伸应变分量，后三者为剪切应变分量，每一分量包括线性与非线性两部分．

现将此等分量用壳体中面系数表示．如图 1 所示，单位法向矢量指向曲面坐标ζ的正方向．

$$\left.\begin{aligned}A&=\alpha\left(1-\frac{\zeta}{r_\xi}\right)\\B&=\beta\left(1-\frac{\zeta}{r_\eta}\right)\\C&=1\end{aligned}\right\} \tag{13}$$

用门拉底—柯打齐（Mainardi-Codazzi）关系

$$\left.\begin{aligned}\frac{1}{r_\eta}\frac{\partial\alpha}{\partial\eta}&=\frac{\partial}{\partial\eta}\left(\frac{\alpha}{r_\xi}\right)\\\frac{1}{r_\xi}\frac{\partial\beta}{\partial\xi}&=\frac{\partial}{\partial\xi}\left(\frac{\beta}{r_\eta}\right)\end{aligned}\right\} \tag{14}$$

知有下列关系的存在，

$$\left.\begin{aligned}\frac{\partial}{\partial\eta}\left[\alpha\left(1-\frac{\zeta}{r_\xi}\right)\right]&=\frac{\partial\alpha}{\partial\eta}\left(1-\frac{\zeta}{r_\eta}\right)\\\frac{\partial}{\partial\xi}\left[\beta\left(1-\frac{\zeta}{r_\eta}\right)\right]&=\frac{\partial\beta}{\partial\xi}\left(1-\frac{\zeta}{r_\xi}\right)\end{aligned}\right\}\tag{15}$$

将(13)代入(12)，并在需要时引用(15)得：

$$\begin{aligned}\epsilon_\xi=&\frac{1}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\left(\frac{\partial U}{\partial\xi}+\frac{V}{\beta}\frac{\partial\alpha}{\partial\eta}-\alpha\frac{W}{r_\xi}\right)\\&+\frac{1}{2\left[\alpha\left(1-\frac{\zeta}{r_\xi}\right)\right]^2}\left\{\left[\frac{\partial U}{\partial\xi}+\frac{V}{\beta}\frac{\partial\alpha}{\partial\eta}-\alpha\frac{W}{r_\xi}\right]^2\right.\\&\left.+\left[\beta\left(1-\frac{\zeta}{r_\eta}\right)\frac{\partial}{\partial\xi}\left[\frac{V}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\right]\right]^2+\left[\frac{\partial W}{\partial\xi}\right]^2\right\}\end{aligned}\tag{16a}$$

$$\begin{aligned}\epsilon_\eta=&\frac{1}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\left(\frac{\partial V}{\partial\eta}-\beta\frac{W}{r_\eta}+\frac{U}{\alpha}\frac{\partial\beta}{\partial\xi}\right)\\&+\frac{1}{2\left[\beta\left(1-\frac{\zeta}{r_\eta}\right)\right]^2}\left\{\left[\alpha\left(1-\frac{\zeta}{r_\xi}\right)\frac{\partial}{\partial\eta}\left[\frac{U}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\right]\right]^2\right.\\&\left.+\left[\frac{\partial V}{\partial\eta}-\beta\frac{W}{r_\eta}+\frac{U}{\alpha}\frac{\partial\beta}{\partial\xi}\right]^2+\left[\frac{\partial W}{\partial\eta}\right]^2\right\}\end{aligned}\tag{16b}$$

$$\begin{aligned}\epsilon_\zeta=&\frac{\partial W}{\partial\zeta}+\frac{1}{2}\left\{\left[\alpha\left(1-\frac{\zeta}{r_\xi}\right)\frac{\partial}{\partial\zeta}\left[\frac{U}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\right]\right]^2\right.\\&\left.+\left[\beta\left(1-\frac{\zeta}{r_\eta}\right)\frac{\partial}{\partial\zeta}\left[\frac{V}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\right]\right]^2+\left[\frac{\partial W}{\partial\zeta}\right]^2\right\}\end{aligned}\tag{16c}$$

$$\begin{aligned}\gamma_{\xi\eta}=&\frac{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\frac{\partial}{\partial\eta}\left[\frac{U}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\right]+\frac{\beta\left(1-\frac{\zeta}{r_\eta}\right)}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\frac{\partial}{\partial\xi}\left[\frac{V}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\right]\\&+\frac{1}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\frac{\partial}{\partial\eta}\left[\frac{U}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\right]\cdot\left[\frac{\partial U}{\partial\xi}+\frac{V}{\beta}\frac{\partial\alpha}{\partial\eta}-\alpha\frac{W}{r_\xi}\right]\end{aligned}$$

$$+\frac{1}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\frac{\partial}{\partial\xi}\Bigg]\frac{V}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\Bigg]\cdot\left[\frac{\partial V}{\partial\eta}-\beta\frac{W}{r_\eta}+\frac{U}{\alpha}\frac{\partial\beta}{\partial\xi}\right]$$

$$+\frac{1}{\alpha\beta\left(1-\frac{\zeta}{r_\xi}\right)\left(1-\frac{\zeta}{r_\eta}\right)}\frac{\partial W}{\partial\xi}\frac{\partial W}{\partial\eta} \tag{16d}$$

$$\gamma_{\eta\zeta}=\left(1-\frac{\zeta}{r_\eta}\right)\frac{\partial}{\partial\zeta}\left(\frac{V}{1-\frac{\zeta}{r_\eta}}\right)+\frac{1}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\frac{\partial W}{\partial\eta}$$

$$+\frac{\alpha\left(1-\frac{\zeta}{r_\xi}\right)^2}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\frac{\partial}{\partial\zeta}\left(\frac{U}{1-\frac{\zeta}{r_\xi}}\right)\frac{\partial}{\partial\eta}\left[\frac{U}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\right]$$

$$+\frac{1}{\beta}\frac{\partial}{\partial\zeta}\left(\frac{V}{1-\frac{\zeta}{r_\eta}}\right)\cdot\left[\frac{\partial V}{\partial\eta}-\beta\frac{W}{r_\eta}+\frac{U}{\alpha}\frac{\partial\beta}{\partial\xi}\right]$$

$$+\frac{1}{\beta\left(1-\frac{\zeta}{r_\eta}\right)}\frac{\partial W}{\partial\eta}\frac{\partial W}{\partial\zeta} \tag{16e}$$

$$\gamma_{\zeta\xi}=\frac{1}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\frac{\partial W}{\partial\xi}+\left(1-\frac{\zeta}{r_\xi}\right)\frac{\partial}{\partial\zeta}\left(\frac{U}{1-\frac{\zeta}{r_\xi}}\right)$$

$$+\frac{1}{\alpha}\frac{\partial}{\partial\zeta}\left(\frac{U}{1-\frac{\zeta}{r_\xi}}\right)\cdot\left[\frac{\partial U}{\partial\xi}+\frac{V}{\beta}\frac{\partial\alpha}{\partial\eta}-\alpha\frac{W}{r_\xi}\right]$$

$$+\frac{\beta\left(1-\frac{\zeta}{r_\eta}\right)^2}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\frac{\partial}{\partial\zeta}\left(\frac{V}{1-\frac{\zeta}{r_\eta}}\right)\frac{\partial}{\partial\xi}\left[\frac{V}{\beta\left(1-\frac{\zeta}{r_\xi}\right)}\right]$$

$$+\frac{1}{\alpha\left(1-\frac{\zeta}{r_\xi}\right)}\frac{\partial W}{\partial\xi}\frac{\partial W}{\partial\zeta} \tag{16f}$$

为了验证式(16)的正确，使

$$\xi=x,\ \eta=y,\ \zeta=z$$

$$\alpha=\beta=1,\ r_\xi=r_\eta=\infty$$

我们得以下六个为众所熟知的表示板的非线性应变分量的公式：

$$\left.\begin{aligned}
\epsilon_x&=\frac{\partial U}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2+\left(\frac{\partial W}{\partial x}\right)^2\right]\\
\epsilon_y&=\frac{\partial V}{\partial y}+\frac{1}{2}\left[\left(\frac{\partial U}{\partial y}\right)^2+\left(\frac{\partial V}{\partial y}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2\right]\\
\epsilon_z&=\frac{\partial W}{\partial z}+\frac{1}{2}\left[\left(\frac{\partial U}{\partial z}\right)^2+\left(\frac{\partial V}{\partial z}\right)^2+\left(\frac{\partial W}{\partial z}\right)^2\right]\\
\gamma_{xy}&=\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}+\frac{\partial U}{\partial x}\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+\frac{\partial W}{\partial x}\frac{\partial W}{\partial y}\\
\gamma_{yz}&=\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}+\frac{\partial U}{\partial y}\frac{\partial U}{\partial z}+\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\frac{\partial W}{\partial z}\\
\gamma_{zx}&=\frac{\partial W}{\partial x}+\frac{\partial U}{\partial z}+\frac{\partial U}{\partial z}\frac{\partial U}{\partial x}+\frac{\partial V}{\partial z}\frac{\partial V}{\partial x}+\frac{\partial W}{\partial z}\frac{\partial W}{\partial x}
\end{aligned}\right\}\qquad (17a\text{-}f)$$

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The Nonlinear Strain Components of Thin Shells

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Abstract

The nonlinear strain components of shells are the foundations of the nonlinear shell mechanics. They are used in the stability and various large deformation analyses of shells. Because of the complication of shell geometry we have not seen the general formulae of nonlinear strain components of shells in the existing references. Here we present a set of formulae including linear and nonlinear strain components of shells with orthogonal curvilinear coordinates. Three of them are tensile, the others are shear strain components.