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横观各向同性电磁弹性介质中裂纹对 SH 波的散射

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(我刊编委沈亚鹏来稿)

摘要: 研究横观各向同性电磁弹性介质中裂纹和反平面剪切波之间的相互作用 根据电磁弹性介质的平衡运动微分方程、电位移和磁感应强度微分方程,得到 SH 波传播的控制场方程 引入线性变换,将控制场方程简化为 Helmholtz 方程和两个 Laplace 方程 通过 Fourier 变换,并采用非电磁渗透型裂面边界条件,得到了柯西奇异积分方程组 利用 Chebyshev 多项式求解积分方程,得到应力场、电场和磁场以及动应力强度因子的表达,并给出了数值算例

关 键 词: 电磁弹性; SH 波; 应力强度因子; 积分方程
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引 言

由于在无损检测、信号处理以及地震工程中的广泛应用,弹性波与压电介质、非压电弹性介质中各种缺陷的相互作用,一直是令人关注的课题[1~5] 对压电介质, Zhou[6] 等采用 Schmidt 方法,求解了压电介质中两共线裂纹与 SH 波的相互作用 Narita 和 Shindo[7] 研究了含压电层的复合材料层板中裂纹对 SH 波的散射,利用傅利叶变换,将问题转化为求解对偶积分方程 Wang 和 Meguid[8] 利用积分变换和 Chebyshev 多项式展开的方法,研究了压电介质中多裂纹与 SH 波的相互作用 在文献[9] 中, Wang 等采用同样的方法,研究了压电介质界面多裂纹与 SH 波的相互作用

近来,同时具有压电、压磁和电磁耦合效应的电磁弹性介质力学行为逐渐成为人们研究的一个有意义的课题 Wang 和 Shen[10] 基于 Eshelby 所引入的能量_动量张量的概念推导了电磁弹性介质中的守恒率和路径无关积分 Huang 和 Kuo[11] 借助于三维电磁弹性介质的 Green 函数,研究了电磁弹性介质中的夹杂问题 Pan[12] 获得了三维、各向异性、线性电磁弹性、简支和多层的矩形板在静载荷作用下的精确解答 Wang 和 Shen[13] 得到了电磁弹性介质三维问题的

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势函数通解

本文的目的是研究电磁弹性介质中的裂纹与反平面剪切波即 SH 波的相互作用规律　根据电磁弹性介质的平衡运动微分方程、电位移和磁感应强度控制微分方程, 得到 SH 波传播的控制场方程, 引入线性变换, 将控制场方程简化为 Helmholtz 方程和两个 Laplace 方程　通过 Fourier 变换, 得到了奇异积分方程组　利用 Chebyshev 多项式展开求解积分方程, 得到了应力场、电场和磁场以及动应力强度因子的表达, 并给出了数值算例

1　基 本 方 程

如图 1 所示, 无限大电磁弹性介质中, 有一长度为 $2a$ 的裂纹　xOy 坐标面是横观各向同性面, 极化方向沿 z 轴

在无体力和电荷密度时, 电磁弹性介质的运动微分方程和电位移以及磁感应强度所满足的微分方程为

$$ {}_{xx,x} + {}_{xy,y} + {}_{xz,z} = u_{,tt}, \tag{1a}$$

$$ {}_{xy,x} + {}_{yy,y} + {}_{yz,z} = v_{,tt}, \tag{1b}$$

$$ {}_{xz,x} + {}_{yz,y} + {}_{zz,z} = w_{,tt}, \tag{1c}$$

$$ D_{x,x} + D_{y,y} + D_{z,z} = 0, \tag{2}$$

$$ B_{x,x} + B_{y,y} + B_{z,z} = 0 \tag{3}$$

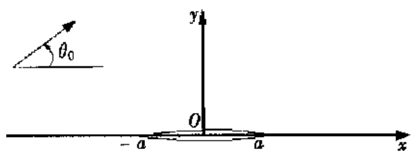

图 1　电磁弹性介质中的裂纹对 SH 波散射

对于各向同性面垂直于 z 轴的横观各向同性电磁弹性介质, 其本构方程表达为

$$\begin{cases} {}_{xx} = c_{11}u_{,x} + c_{12}v_{,y} + c_{13}w_{,z} + e_{31}\ {}_{,z} - f_{31}\ {}_{,z}, \\ {}_{yy} = c_{12}u_{,x} + c_{11}v_{,y} + c_{13}w_{,z} + e_{31}\ {}_{,z} - f_{31}\ {}_{,z}, \\ {}_{zz} = c_{13}u_{,x} + c_{13}v_{,y} + c_{33}w_{,z} + e_{33}\ {}_{,z} - f_{33}\ {}_{,z}, \\ {}_{yz} = c_{44}(v_{,z} + w_{,y}) + e_{15}\ {}_{,y} - f_{15}\ {}_{,y}, \\ {}_{xz} = c_{44}(u_{,z} + w_{,x}) + e_{15}\ {}_{,x} - f_{15}\ {}_{,x}, \\ {}_{xy} = c_{66}(u_{,y} + v_{,x}), \end{cases} \tag{4}$$

$$\begin{cases} D_x = e_{15}(u_{,z} + w_{,x}) - {}_{11}\ {}_{,x} - g_{11}\ {}_{,x}, \\ D_y = e_{15}(v_{,z} + w_{,y}) - {}_{11}\ {}_{,y} - g_{11}\ {}_{,y}, \\ D_z = e_{31}(u_{,x} + v_{,y}) + e_{33}w_{,z} - {}_{33}\ {}_{,z} - g_{33}\ {}_{,z}, \end{cases} \tag{5}$$

$$\begin{cases} B_x = f_{15}(u_{,z} + w_{,x}) + g_{11}\ {}_{,x} - {}_{11}\ {}_{,x}, \\ B_y = f_{15}(v_{,z} + w_{,y}) + g_{11}\ {}_{,y} - {}_{11}\ {}_{,y}, \\ B_z = f_{31}(u_{,x} + v_{,y}) + f_{33}w_{,z} + g_{33}\ {}_{,z} - {}_{33}\ {}_{,z} \end{cases} \tag{6}$$

在以上各式中, ${}_{ij}$, D_i 和 B_i 分别是应力, 电位移和磁感应强度; u, v 和 w 为 3 个位移分量; 和 分别为电势和磁势; $c_{66} = (c_{11} - c_{12})/2$　此横观各向同性电磁介质有 17 个独立的常数, 其中有 5 个弹性常数, 3 个压电常数, 3 个压磁常数, 2 个电磁常数, 2 个介电常数, 2 个介磁常数

将方程(4)、(5) 和(6) 代入(1)、(2) 和(3), 得到如下由位移、电势和磁势所表达的控制方程

$$ c_{11}u_{,xx} + \frac{1}{2}(c_{11} - c_{12})u_{,yy} + c_{44}u_{,zz} + \frac{1}{2}(c_{11} + c_{12})v_{,xy} + (c_{13} + c_{44})w_{,xz} + (e_{15} + e_{31})\ {}_{,xz} - (f_{15} + f_{31})\ {}_{,xz} = u_{,tt}, \tag{7a}$$

$$ \frac{1}{2}(c_{11} - c_{12})v_{,xx} + c_{11}v_{,yy} + c_{44}v_{,zz} + \frac{1}{2}(c_{11} + c_{12})u_{,xy} + (c_{13} + c_{44})w_{,yz} + $$

$$(e_{31}+e_{15})\varphi_{,yz}-(f_{31}+f_{15})\psi_{,yz}=\rho v_{,tt},\tag{7b}$$

$$c_{44}(w_{,xx}+w_{,yy})+c_{33}w_{,zz}+(c_{13}+c_{44})(u_{,xz}+v_{,yz})+e_{15}(\varphi_{,xx}+\varphi_{,yy})-f_{15}(\psi_{,xx}+\psi_{,yy})+e_{33}\varphi_{,zz}-f_{33}\psi_{,zz}=\rho w_{,tt},\tag{7c}$$

$$e_{15}(w_{,xx}+w_{,yy})+e_{33}w_{,zz}+(e_{15}+e_{31})(u_{,xz}+v_{,yz})-\varepsilon_{11}(\varphi_{,xx}+\varphi_{,yy})-g_{11}(\psi_{,xx}+\psi_{,yy})-\varepsilon_{33}\varphi_{,zz}-g_{33}\psi_{,zz}=0,\tag{8}$$

$$f_{15}(w_{,xx}+w_{,yy})+f_{33}w_{,zz}+(f_{15}+f_{31})(u_{,xz}+v_{,yz})+g_{11}(\varphi_{,xx}+\varphi_{,yy})-\mu_{11}(\psi_{,xx}+\psi_{,yy})+g_{33}\varphi_{,zz}-\mu_{33}\psi_{,zz}=0\tag{9}$$

设稳态反平面剪切波的位移场、电势和磁势为

$$\begin{cases} u(x,y)=0,\ v(x,y)=0,\ w(x,y,t)=w(x,y)\mathrm{e}^{-\mathrm{i}\omega t}, \\ \varphi(x,y,t)=\varphi(x,y)\mathrm{e}^{-\mathrm{i}\omega t},\quad \psi(x,y,t)=\psi(x,y)\mathrm{e}^{-\mathrm{i}\omega t}, \end{cases}\tag{10}$$

其中，ω 是角频率。为了方便起见，在以下分析中，将时间谐和因子 $\mathrm{e}^{-\mathrm{i}\omega t}$ 略去，只关心各物理量幅值的变化。

将(10)代入方程(7)~(9)，得到 3 个控制方程和本构方程

$$c_{44}\nabla^2 w+e_{15}\nabla^2\varphi-f_{15}\nabla^2\psi+\rho\omega^2 w=0,\tag{11}$$

$$e_{15}\nabla^2 w-\varepsilon_{11}\nabla^2\varphi-g_{11}\nabla^2\psi=0,\tag{12}$$

$$f_{15}\nabla^2 w+g_{11}\nabla^2\varphi-\mu_{11}\nabla^2\psi=0,\tag{13}$$

$$\tau_{xz}=c_{44}w_{,x}+e_{15}\varphi_{,x}-f_{15}\psi_{,x},\quad \tau_{zy}=c_{44}w_{,y}+e_{15}\varphi_{,y}-f_{15}\psi_{,y},\tag{14}$$

$$D_x=e_{15}w_{,x}-\varepsilon_{11}\varphi_{,x}-g_{11}\psi_{,x},\quad D_y=e_{15}w_{,y}-\varepsilon_{11}\varphi_{,y}-g_{11}\psi_{,y},\tag{15}$$

$$B_x=f_{15}w_{,x}+g_{11}\varphi_{,x}-\mu_{11}\psi_{,x},\quad B_y=f_{15}w_{,y}+g_{11}\varphi_{,y}-\mu_{11}\psi_{,y}\tag{16}$$

引入变换

$$\Phi=e_{15}w-\varepsilon_{11}\varphi-g_{11}\psi,\tag{17}$$

$$\Psi=f_{15}w+g_{11}\varphi-\mu_{11}\psi,\tag{18}$$

则方程(11)~(13)简化为

$$\nabla^2 w+k^2 w=0,\quad \nabla^2\Phi=0,\quad \nabla^2\Psi=0\tag{19}$$

同时有

$$\varphi=\frac{1}{\varepsilon_{11}\mu_{11}+g_{11}^2}\left[(\mu_{11}e_{15}-g_{11}f_{15})w-\mu_{11}\Phi+g_{11}\Psi\right],\tag{20a}$$

$$\psi=\frac{1}{g_{11}^2+\varepsilon_{11}\mu_{11}}\left[(g_{11}e_{15}+\varepsilon_{11}f_{15})w-g_{11}\Phi-\varepsilon_{11}\Psi\right],\tag{20b}$$

其中，k 是波数，$k=\omega/c$，c 是波速，

$$c^2=\frac{1}{\rho}\left[c_{44}+\frac{e_{15}}{\varepsilon_{11}\mu_{11}+g_{11}^2}(\mu_{11}e_{15}-g_{11}f_{15})-\frac{f_{15}}{g_{11}^2+\varepsilon_{11}\mu_{11}}(g_{11}e_{15}+\varepsilon_{11}f_{15})\right]\tag{21}$$

显然，当式(20)中的压磁系数 $f_{15}=0$ 时，退化为横观各向同性压电介质中的剪切波波速，进而当压电系数 $e_{15}=0$ 时，退化为弹性介质中剪切波波速。

裂纹表面的力学边界条件是应力自由，因此，散射波场满足下述位移和应力边界条件：

$$w(x,0)=0,\quad |x|\geq a,\tag{22a}$$

$$\tau_{yz}(x,0^+)=\tau_{yz}(x,0^-)=-\tau(x),\quad |x|<a,\tag{22b}$$

其中，$\tau(x)$ 是入射波在裂纹线处形成的剪应力。散射场的电磁边界条件为

$$(x, 0^+) - \quad (x, 0^-) = 0, \qquad |x| \quad a, \tag{23a}$$

$$D_y + \quad [\quad (x, 0^+) - \quad (x, 0^-)] = - D(x), \qquad |x| < a, \tag{23b}$$

$$(x, 0^+) - \quad (x, 0^-) = 0, \qquad |x| \quad a, \tag{24a}$$

$$B_y + \quad [\quad (x, 0^+) - \quad (x, 0^-)] = - B(x), \qquad |x| < a, \tag{24b}$$

其中,　和　是常数,　= 0,　= 0 和　　,　　分别对应非电磁渗透型裂纹和电磁渗透型裂纹两种极限情形　本文只分析非电磁渗透型裂纹, 即　= 0,　= 0 的情形

2　问题的求解

引入傅里叶变换及其逆变换

$$F(s, y) = \int_{-}^{} F(x, y) e^{-isx} dx, \quad F(x, y) = \frac{1}{2} \int_{-}^{+} F(s, y) e^{isx} ds, \tag{25}$$

对方程(19)进行傅立叶变换, 并利用 $y = 0$ 处的连续条件, 可以得到

$$w(x, y) = \operatorname{sgn}(y) \int_{-}^{} A(s) e^{-\ |y| - isx} ds, \tag{26}$$

$$(x, y) = \operatorname{sgn}(y) \int_{-}^{} B(s) e^{-|sy| - isx} ds, \tag{27}$$

$$(x, y) = \operatorname{sgn}(y) \int_{-}^{} C(s) e^{-|sy| - isx} ds, \tag{28}$$

其中, $A(s)$、$B(s)$ 和 $C(s)$ 是 s 的待定函数　上述方程中, sgn 是符号函数,

$$\operatorname{sgn}(y) = \begin{cases} 1, & y > 0, \\ -1, & y < 0; \end{cases}$$

由下式给出:

$$= \begin{cases} \sqrt{s^2 - k^2}, & |s| \quad k, \\ -i\sqrt{k^2 - s^2}, & |s| \quad k \end{cases} \tag{29}$$

由方程(26)、(27) 和(28) 所给出的散射波场的位移、电势和磁势满足所求解问题的辐射条件, 即在无限远处, 应力和电位移为零

引入下述位错密度函数、电势和磁势密度函数:

$$f(x) = \frac{[w(x, 0^+) - w(x, 0^-)]}{x}, \qquad |x| \quad a, \tag{30}$$

$$g(x) = \frac{[\quad (x, 0^+) - \quad (x, 0^-)]}{x}, \qquad |x| \quad a, \tag{31}$$

$$h(x) = \frac{[\quad (x, 0^+) - \quad (x, 0^-)]}{x}, \qquad |x| \quad a \tag{32}$$

将(26)、(27) 和(28) 代入(30)、(31) 和(32), 得到

$$A(s) = \frac{i}{4\ s} \int_{-a}^{a} f(x) e^{isx} dx, \tag{33}$$

$$B(s) = \frac{i}{4\ s} \frac{1}{c_2 b_1 - c_1 b_2} \int_{-a}^{a} [(c_1 a_2 - c_2 a_1) f(x) + c_2 g(x) - c_1 h(x)] e^{isx} dx, \tag{34}$$

$$C(s) = \frac{i}{4\ s} \frac{1}{b_2 c_1 - b_1 c_2} \int_{-a}^{a} [(b_1 a_2 - b_2 a_1) f(x) + b_2 g(x) - b_1 h(x)] e^{isx} dx, \tag{35}$$

其中

$$a_1 = \frac{_{11} e_{15} - g_{11} f_{15}}{_{11}\ _{11} + g_{11}^2}, \quad b_1 = \frac{-\ _{11}}{_{11}\ _{11} + g_{11}^2}, \quad c_1 = \frac{g_{11}}{_{11}\ _{11} + g_{11}^2}, \tag{36}$$

$$a_2 = \frac{g_{11}e_{15} + \quad {}_{11}f_{15}}{g_{11}^2 + \quad {}_{11} \quad {}_{11}}, \quad b_2 = \frac{- \; g_{11}}{g_{11}^2 + \quad {}_{11} \quad {}_{11}}, \quad c_2 = \frac{- \quad {}_{11}}{g_{11}^2 + \quad {}_{11} \quad {}_{11}} \tag{37}$$

本构关系式(14)~(16)可以表示为

$$_{zx} = m_1 w_{,x} + m_2 \quad _{,x} + m_3 \quad _{,x}, \quad _{zy} = m_1 w_{,y} + m_2 \quad _{,y} + m_3 \quad _{,y}, \tag{38}$$

$$D_x = \quad _{,x}, \quad D_y = \quad _{,y}, \tag{39}$$

$$B_x = \quad _{,x}, \quad B_y = \quad _{,y}, \tag{40}$$

其中 $m_1 = c_{44} + a_1 e_{15} - a_2 f_{15}, \quad m_2 = b_1 e_{15} - b_2 f_{15}, \quad m_3 = c_1 e_{15} - c_2 f_{15},$

将式(33)~(35)代入式(26)~(28),再代入(38)~(40),可以得到应力、电位移和磁感应强度的积分表达,其中

$$\begin{cases} w_{,x} = \frac{1}{2} \quad _0 \quad _{-a}^{a} f(u) \mathrm{e}^{- \quad |y|} \cos[s(u - x)] \mathrm{d}u\mathrm{d}s, \\ _{,x} = \frac{1}{2} \frac{1}{c_2 b_1 - c_1 b_2} \quad _0 \; [(c_1 a_2 - c_2 a_1) f(u) + \\ \qquad c_2 g(u) - c_1 h(u)] \mathrm{e}^{-|sy|} \cos[s(u - x)] \mathrm{d}u\mathrm{d}s, \\ _{,x} = \frac{1}{2} \frac{1}{c_1 b_2 - c_2 b_1} \quad _0 \; [(b_1 a_2 - b_2 a_1) f(u) + \\ \qquad b_2 g(u) - b_1 h(u)] \mathrm{e}^{-|sy|} \cos[s(u - x)] \mathrm{d}u\mathrm{d}s, \end{cases} \tag{41}$$

$$\begin{cases} w_{,y} = \frac{1}{2} \quad _0 \quad _{-a}^{a} \frac{}{s} f(u) \mathrm{e}^{- \quad |y|} \sin[s(u - x)] \mathrm{d}u\mathrm{d}s, \\ _{,y} = \frac{1}{2} \frac{1}{c_2 b_1 - c_1 b_2} \quad _0 \; [(c_1 a_2 - c_2 a_1) f(u) + \\ \qquad c_2 g(u) - c_1 h(u)] \mathrm{e}^{-|sy|} \sin[s(u - x)] \mathrm{d}u\mathrm{d}s, \\ _{,y} = \frac{1}{2} \frac{1}{c_1 b_2 - c_2 b_1} \quad _0 \; [(b_1 a_2 - b_2 a_1) f(u) + \\ \qquad b_2 g(u) - b_1 h(u)] \mathrm{e}^{-|sy|} \sin[s(u - x)] \mathrm{d}u\mathrm{d}s \end{cases} \tag{42}$$

在 $y = 0$ 处,当 $|s|$ 时,上述方程中的积分核趋向于常数. 经过适当的渐近分析,利用关系式

$$_{-} \; \mathrm{sgn}(s) \mathrm{e}^{\mathrm{i}s(u - x)} \mathrm{d}s = \frac{2\mathrm{i}}{u - x}, \tag{43}$$

并定义 $P(u, x) = \; _0 \left(\frac{}{s} - 1 \right) \sin[s(u - x)] \mathrm{d}s, \tag{44}$

得到以下应力、电位移和磁感应强度的表达式

$$_{zy}(x, 0) = \frac{c_{44}}{2} \; _{-a}^{a} \frac{f(u)}{u - x} \mathrm{d}u + \frac{e_{15}}{2} \; _{-a}^{a} \frac{g(u)}{u - x} \mathrm{d}u - \frac{f_{15}}{2} \; _{-a}^{a} \frac{h(u)}{u - x} \mathrm{d}u + \\ \frac{m_1}{2} \; _{-a}^{a} f(u) P(u, x) \mathrm{d}u, \tag{45}$$

$$D_y(x, 0) = \frac{e_{15}}{2} \; _{-a}^{a} \frac{f(u)}{u - x} \mathrm{d}u - \frac{E_{11}}{2PQ} \; _{-a}^{a} \frac{g(u)}{u - x} \mathrm{d}u - \frac{g_{11}}{2PQ} \; _{-a}^{a} \frac{h(u)}{u - x} \mathrm{d}u, \tag{46}$$

$$B_y(x, 0) = \frac{f_{15}}{2PQ} \; _{-a}^{a} \frac{f(u)}{u - x} \mathrm{d}u + \frac{g_{11}}{2PQ} \; _{-a}^{a} \frac{g(u)}{u - x} \mathrm{d}u - \frac{L_{11}}{2PQ} \; _{-a}^{a} \frac{h(u)}{u - x} \mathrm{d}u\# \tag{47}$$

将(45)、(46)和(47)式分别代入边界条件(22b)、(23b)和(24b),得到

$$Q \; _{-a}^{a} \frac{f(u)}{u - x} \mathrm{d}u + \frac{e_{15}}{c_{44}Q} \; _{-a}^{a} \frac{g(u)}{u - x} \mathrm{d}u - \frac{f_{15}}{c_{44}Q} \; _{-a}^{a} \frac{h(u)}{u - x} \mathrm{d}u +$$

$$\frac{m_1}{c_{44}}\int_{-a}^{a} f(u)\int_0^{\infty}\left(\frac{\alpha}{s}-1\right)\sin[s(u-x)]\,\mathrm{d}u\,\mathrm{d}s=-\frac{2\pi S(x)}{c_{44}}, \tag{48a}$$

$$\int_{-a}^{a}\frac{f(u)}{u-x}\mathrm{d}u-\frac{E_{11}}{e_{15}}\int_{-a}^{a}\frac{g(u)}{u-x}\mathrm{d}u-\frac{g_{11}}{e_{15}}\int_{-a}^{a}\frac{h(u)}{u-x}\mathrm{d}u=-\frac{2\pi D(x)}{e_{15}}, \tag{48b}$$

$$\int_{-a}^{a}\frac{f(u)}{u-x}\mathrm{d}u+\frac{g_{11}}{f_{15}}\int_{-a}^{a}\frac{g(u)}{u-x}\mathrm{d}u-\frac{L_{11}}{f_{15}}\int_{-a}^{a}\frac{h(u)}{u-x}\mathrm{d}u=-\frac{2\pi B(x)}{f_{15}}, \tag{48c}$$

$f(u)$、$g(u)$、$h(u)$ 满足下述条件

$$\int_{-a}^{a} f(u)\,\mathrm{d}u=0,\quad \int_{-a}^{a} g(u)\,\mathrm{d}u=0,\quad \int_{-a}^{a} h(u)\,\mathrm{d}u=0. \tag{49}$$

方程(48)是第一类奇异积分方程,其解答可以用 Chebyshev 多项式表示为

$$f(u)=\sum_{k=0}^{\infty}\frac{c_k}{\sqrt{1-(u^2/a^2)}}T_k\left(\frac{u}{a}\right), \tag{50a}$$

$$g(u)=\sum_{k=0}^{\infty}\frac{d_k}{\sqrt{1-(u^2/a^2)}}T_k\left(\frac{u}{a}\right), \tag{50b}$$

$$h(u)=\sum_{k=0}^{\infty}\frac{e_k}{\sqrt{1-(u^2/a^2)}}T_k\left(\frac{u}{a}\right), \tag{50c}$$

其中,T_k 是第一类 Chebyshev 多项式,$T_k(x)=\cos(k\arccos x)$,c_k、d_k、e_k 是未知待定的常数.

根据 Chebyshev 多项式的正交性条件,由(49)式得到,$c_0=d_0=e_0=0$. 将(50a)~(50c)代入方程(48),可以得到关于 c_k、d_k、e_k 的代数方程组,

$$\sum_{k=1}^{\infty}c_kU_{k-1}\left(\frac{x}{a}\right)+\frac{e_{15}}{c_{44}}\sum_{k=1}^{\infty}d_kU_{k-1}\left(\frac{x}{a}\right)-\frac{f_{15}}{c_{44}}\sum_{k=1}^{\infty}e_kU_{k-1}\left(\frac{x}{a}\right)+\frac{m_1}{c_{44}}\sum_{k=1}^{\infty}c_kg_k(x)=-\frac{2}{c_{44}}s(x), \tag{51a}$$

$$\sum_{k=1}^{\infty}c_kU_{k-1}\left(\frac{x}{a}\right)-\frac{E_{11}}{e_{15}}\sum_{k=1}^{\infty}d_kU_{k-1}\left(\frac{x}{a}\right)-\frac{g_{11}}{e_{15}}\sum_{k=1}^{\infty}e_kU_{k-1}\left(\frac{x}{a}\right)=-\frac{2}{e_{15}}D(x), \tag{51b}$$

$$\sum_{k=1}^{\infty}c_kU_{k-1}\left(\frac{x}{a}\right)+\frac{g_{11}}{f_{15}}\sum_{k=1}^{\infty}d_kU_{k-1}\left(\frac{x}{a}\right)-\frac{L_{11}}{f_{15}}\sum_{k=1}^{\infty}e_kU_{k-1}\left(\frac{x}{a}\right)=-\frac{2}{f_{15}}B(x), \tag{51c}$$

其中,U_k 是第二类 Chebyshev 多项式,$U_n(\cos\theta)=\sin[(n+1)\theta]/\sin\theta$,并且

$$g_k(x)=\begin{cases}(-1)^{D}a\int_0^{\infty}\left(\frac{\alpha}{s}-1\right)J_k(sa)\cos(sx)\,\mathrm{d}s, & k=2D+1,\\ (-1)^{D-1}a\int_0^{\infty}\left(\frac{\alpha}{s}-1\right)J_k(sa)\sin(sx)\,\mathrm{d}s, & k=2D,\end{cases} \tag{52}$$

其中,J_k 是第一类 Bessel 函数. 此处利用了第一类和第二类 Chebyshev 多项式之间的关系式

$$\frac{1}{\pi}\int_{-1}^{1}T_k(t)(1-t^2)^{-1/2}\frac{\mathrm{d}t}{t-x}=\begin{cases}0, & k=0,\\ U_{k-1}(x), & k>0.\end{cases} \tag{53a}$$

以及积分公式

$$\int_{-1}^{1}(1-x)^{-1/2}T_k(x)\sin(px)\,\mathrm{d}x=\begin{cases}0, & k=2n,\\ (-1)^n\pi J_k(p), & k=2n+1,\end{cases} \tag{53b}$$

$$\int_{-1}^{1}(1-x)^{-1/2}T_k(x)\cos(px)\,\mathrm{d}x=\begin{cases}0, & k=2n+1,\\ (-1)^n\pi J_k(p), & k=2n.\end{cases} \tag{53c}$$

在式(50a)~(50c)中截取 Chebyshev 多项式的前 M 项,并设方程(51a)~(51c)沿裂纹面上的 M 个点成立

$$x_l = a\cos\left(\frac{l}{M+1}\pi\right) \qquad (l = 1, 2, \cdots, M). \tag{54}$$

方程(51a)～(51c)化为具有下述形式的代数方程组

$$\sum_{k=1}^{M} c_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} + \frac{e_{15}}{c_{44}}\sum_{k=1}^{M} d_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} - \frac{f_{15}}{c_{44}}\sum_{k=1}^{M} e_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} + \frac{m_1}{c_{44}}\sum_{k=1}^{M} c_k g_k(x_l) = -\frac{2}{c_{44}}\sigma(x_l) \qquad (k, l = 1, 2, \cdots, M), \tag{55a}$$

$$\sum_{k=1}^{M} c_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} - \frac{\varepsilon_{11}}{e_{15}}\sum_{k=1}^{M} d_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} - \frac{g_{11}}{e_{15}}\sum_{k=1}^{M} e_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} = -\frac{2}{e_{15}}D(x_l) \qquad (k, l = 1, 2, \cdots, M), \tag{55b}$$

$$\sum_{k=1}^{M} c_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} + \frac{g_{11}}{f_{15}}\sum_{k=1}^{M} d_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} - \frac{\mu_{11}}{f_{15}}\sum_{k=1}^{M} e_k \frac{\sin\left(\frac{kl\pi}{M+1}\right)}{\sin\left(\frac{k\pi}{M+1}\right)} = -\frac{2}{f_{15}}B(x_l) \qquad (k, l = 1, 2, \cdots, M). \tag{55c}$$

由(55)式得到 Chebyshev 多项式展开式的系数 c_k、d_k、e_k，则由裂纹引起的散射场的应力、电位移和磁感应强度等物理量随之可以确定.

在裂纹前缘，定义如下的动应力强度因子、电位移强度因子和磁感应强度因子[9]

$$K_\tau = \lim_{x \to a}\left\{\sqrt{\pi(x-a)}\,\tau_{yz}(x, 0)\right\}, \tag{56a}$$

$$K_e = \lim_{x \to a}\left\{\sqrt{\pi(x-a)}\,D_y(x, 0)\right\}, \tag{56b}$$

$$K_m = \lim_{x \to a}\left\{\sqrt{\pi(x-a)}\,B_y(x, 0)\right\}, \tag{56c}$$

其具体表达式为

$$K_\tau = \frac{1}{2}\sqrt{\pi a}\sum_{j=1}^{M}\left[\left(m_1 + \frac{m_2(c_1a_2 - c_2a_1) - m_3(b_1a_2 - b_2a_1)}{c_2b_1 - c_1b_2}\right)c_j + \frac{m_2c_2 - m_3b_2}{c_2b_1 - c_1b_2}d_j - \frac{m_2c_1 - m_3b_1}{c_2b_1 - c_1b_2}e_j\right], \tag{57a}$$

$$K_e = \frac{1}{2}\sqrt{\pi a}\sum_{j=1}^{M}\left[\left(\frac{c_1a_2 - c_2a_1}{c_2b_1 - c_1b_2}\right)c_j + \frac{c_2}{c_2b_1 - c_1b_2}d_j - \frac{c_1}{c_2b_1 - c_1b_2}e_j\right], \tag{57b}$$

$$K_m = \frac{1}{2}\sqrt{\pi a}\sum_{j=1}^{M}\left[\left(\frac{-(b_1a_2 - b_2a_1)}{c_2b_1 - c_1b_2}\right)c_j - \frac{b_2}{c_2b_1 - c_1b_2}d_j + \frac{b_1}{c_2b_1 - c_1b_2}e_j\right]. \tag{57c}$$

当 $f_{15} = 0$, $g_{11} = 0$ 时，以上各式成为只有压电效应时的应力强度因子和电位移强度因子

$$K_\tau = \frac{1}{2}\sqrt{\pi a}(c_{44}c_j + e_{15}d_j), \quad K_e = \frac{1}{2}\sqrt{\pi a}(e_{15}c_j - \varepsilon_{11}d_j). \tag{58}$$

当 e_{15}, $g_{11} = 0$ 时，为只有压磁效应时的应力强度因子和磁感应强度因子

$$K_\tau = \frac{1}{2}\sqrt{\pi a}(c_{44}c_j - f_{15}e_j), \quad K_m = \frac{1}{2}\sqrt{\pi a}(f_{15}c_j - \mu_{11}e_j). \tag{59}$$

3 数值算例与结论

如图 1 所示，无限大横观各向同性电磁弹性介质中裂纹受频率为 ω 的反平面剪切波以及

面内电位移D_y和磁感应强度B_y作用，入射波的应力场为

$$S_{xz}^{(in)} = S\cos\theta_0 e^{ik(x\cos\theta_0 + y\sin\theta_0)},\quad S_{yz}^{(in)} = S\sin\theta_0 e^{ik(x\cos\theta_0 + y\sin\theta_0)},$$

其中，S是剪切应力的最大值。为方便起见，本文假定SH波的入射角为$\theta_0 = 90°$，即垂直入射。

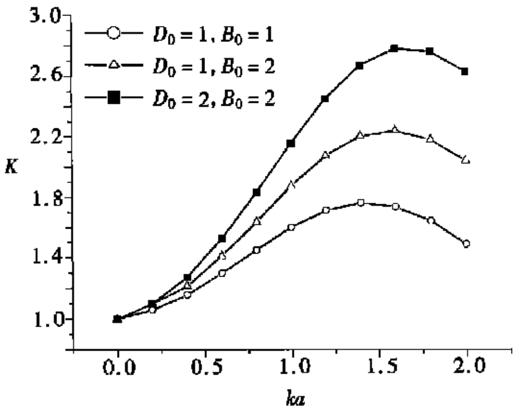

图2　电磁弹性介质裂纹应力强度因子K与ka的关系

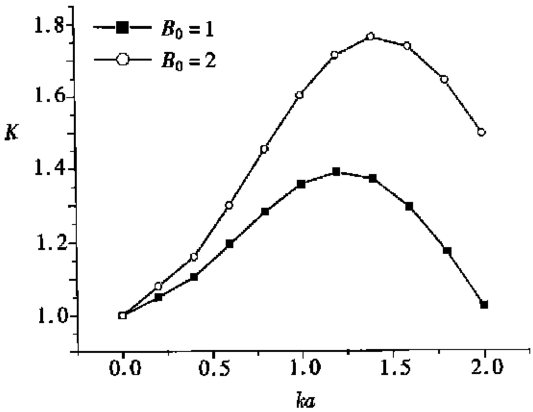

图3　压磁弹性介质中裂纹应力强度因子K与ka的关系

图2表示入射波频率(ka)、外加电位移$(D_0 = D/D, D = (\varepsilon_{11}/e_{15})S)$和磁感应强度$(B_0 = B/B, B = (\mu_{11}/f_{15})S)$与动应力强度因子$(K = K_R/(S\sqrt{\pi a}))$的关系。图3是压磁弹性介质中裂纹与入射波频率以及外加磁感应强度因子的关系。由计算结果可见，动应力强度因子K_R随外加电位移场和磁感应强度场的增加而增大。在外加电场或磁场的作用下，动应力强度因子的幅值比静态值有较大的增加。

本文根据电磁弹性介质的平衡运动微分方程、电位移和磁感应强度控制微分方程，得到SH波传播的耦合波方程，通过Fourier变换，并采用非电磁渗透型裂面条件，得到了描述SH波和裂纹相互作用的奇异积分方程组。利用Chebyshev多项式展开求解所得到的方程组，给出了应力场、电场和磁场以及动应力强度因子、电位移和磁感应强度因子的表达。数值算例表明，动应力强度因子随外加电位移、磁感应强度的增加而增大。

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Scattering of Anti_Plane Shear Waves by a Single Crack in an Unbounded Transversely Isotropic Electro_Magneto_Elastic Medium

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Abstract: A theoretical treatment of the scattering of anti_plane shear(SH) waves is provided by a single crack in an unbounded transversely isotropic electro_magneto_elastic medium. Based on the differential equations of equilibrium, electric displacement and magnetic induction intensity differential equations, the governing equations for SH waves were obtained. By means of a linear transform, the governing equations were reduced to one Helmholtz and two Laplace equations. The Cauchy singular integral equations were gained by making use of Fourier transform and adopting electro_magneto impermeable boundary conditions. The closed form expression for the resulting stress intensity factor at the crack was achieved by solving the appropriate singular integral equations using Chebyshev polynomial. Typical examples are provided to show the loading frequency upon the local stress fields around the crack tips. The study reveals the importance of the electro_magneto_mechanical coupling terms upon the resulting dynamic stress intensity factor.

Key words: electro_magneto_elasticity; SH wave; stress intensity factor; integral equation