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# 线性随机参变振动的谱分解法*

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**摘 要**

本文是[1]文的一个发展．考虑如下的随机方程：$\ddot{Z}(t)+2\beta\dot{Z}(t)+\omega_0^2Z(t)=(a_0+a_1Z(t))\cdot I(t)+c$，激励$I(t)$和响应$Z(t)$都是随机过程，并设它们相互独立．如[1]，设$I(t)=a(t)I^\circ(t)$，$a(t)$是已知的时间函数，$I^\circ(t)$是平稳随机过程．本文考虑了以上随机方程的谱分解形式，数值求解方法以及一些特殊情况的解式．

## 一、随机参变振动方程及谱分解

在随机振动中常常会遇到如下形式的方程：

$$\ddot{Z}(t)+2\beta\dot{Z}(t)+\omega_0^2Z(t)=(a_0+a_1Z(t))I(t)+c \tag{1.1}$$

例如见[2]．在(1.1)式中：$\beta,\omega_0,a_0,a_1$和$c$都是常数，$I(t)$是激励，可以表示为：

$$I(t)=a(t)I^\circ(t) \tag{1.2}$$

$a(t)$是已知的时间函数，$I^\circ(t)$是平稳随机过程，且有：

$$E[I^\circ(t)]=0,\ E[(I^\circ(t))^2]=\sigma_I^2. \tag{1.3}$$

$E$表示取期望值，$\sigma_I^2$。表示过程$I^\circ$的均方值．$Z(t)$是响应的随机过程．今设$I^\circ(t)$和$Z(t)$相互独立，例如当$I^\circ(t)$是白噪声过程时，则$I^\circ(t)$和$Z(t)$确实是相互独立的，例如见[3]．这时有：

$$E[Z(t)I(t)]=E[a(t)Z(t)I^\circ(t)]=a(t)E[Z(t)]E[I^\circ(t)]=0 \tag{1.4}$$

将(1.1)式进行谱分解得：

$$\begin{aligned}(-\omega^2+2\beta i\omega+\omega_0^2)\tilde{Z}(\omega)&=a_0\tilde{I}(\omega)+a_1\tilde{Z}(\omega)*\tilde{I}(\omega)+c\delta(\omega)\\&=a_0\tilde{a}(\omega)*\tilde{I}^\circ(\omega)+a_1\tilde{a}(\omega)*\tilde{I}^\circ(\omega)*\tilde{Z}(\omega)+c\delta(\omega)\end{aligned} \tag{1.5}$$

由于$E[I^\circ(t)]=0$，得：

$$\left.\begin{aligned}&E[\tilde{I}^\circ(\omega)]=0,\ E[\tilde{a}(\omega)*\tilde{I}^\circ(\omega)]=\tilde{a}(\omega)*E[\tilde{I}^\circ(\omega)]=0\\&E[\tilde{a}(\omega)*\tilde{I}^\circ(\omega)*\tilde{Z}(\omega)]=\tilde{a}(\omega)*E[\tilde{I}^\circ(\omega)]*E[\tilde{Z}(\omega)]=0\end{aligned}\right\} \tag{1.6}$$

对(1.5)式取期望值，且用(1.6)式得：

$$(\omega_0^2+2\beta i\omega-\omega^2)\bar{\tilde{Z}}(\omega)=c\delta(\omega)$$

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或：
$$\left.\begin{aligned}&\bar{\tilde{Z}}(\omega)=E[\tilde{Z}(\omega)]=\frac{c}{\omega_0^2}\delta(\omega)\\&\bar{Z}(t)=E[Z(t)]=\int\bar{\tilde{Z}}(\omega)\exp(i\omega t)d\omega=\frac{c}{\omega_0^2}\end{aligned}\right\}\tag{1.7}$$

令：
$$Y=Z-\bar{Z},\ Z=Y+\bar{Z}\tag{1.8}$$
$$E[Y]=E[Z-\bar{Z}]=0\tag{1.9}$$

将(1.8)式代入(1.1)式得：
$$\left.\begin{aligned}&\ddot{Y}(t)+2\beta\dot{Y}(t)+\omega_0^2Y(t)=(b_0+b_1Y(t))I(t)\\&b_0=a_0+\frac{a_1c}{\omega_0^2},\ b_1=a_1\end{aligned}\right\}\tag{1.10}$$

(1.10)式的谱分解式是：
$$\begin{aligned}(\omega_0^2+2\beta i\omega-\omega^2)\tilde{Y}(\omega)&=b_0\tilde{I}(\omega)+b_1\tilde{Y}(\omega)*\tilde{I}(\omega)\\&=b_0\tilde{a}(\omega)*\tilde{I}^\circ(\omega)+b_1\tilde{a}(\omega)*\tilde{I}^\circ(\omega)*\tilde{Y}(\omega)\end{aligned}\tag{1.11}$$

令：
$$H(\omega)=\frac{1}{\omega_0^2+2\beta i\omega-\omega^2}\tag{1.12}$$

将(1.12)式代入(1.11)式得：
$$\tilde{Y}(\omega)=b_0H(\omega)[\tilde{a}(\omega)*\tilde{I}^\circ(\omega)]+b_1H(\omega)[\tilde{a}(\omega)*\tilde{I}^\circ(\omega)*\tilde{Y}(\omega)]\tag{1.13}$$

在(1.13)式中分别令$\omega=\omega_1,\omega_2$，所得两式相乘得：
$$\begin{aligned}\tilde{Y}(\omega_1)\tilde{Y}(\omega_2)=&b_0^2[H(\omega_1)(\tilde{a}(\omega_1)*\tilde{I}^\circ(\omega_1))][H(\omega_2)(\tilde{a}(\omega_2)*\tilde{I}^\circ(\omega_2))]\\&+b_0b_1[H(\omega_1)(\tilde{a}(\omega_1)*\tilde{I}^\circ(\omega_1)][H(\omega_2)(\tilde{a}(\omega_2)*\tilde{I}^\circ(\omega_2)*\tilde{Y}(\omega_2))]\\&+b_0b_1[H(\omega_2)(\tilde{a}(\omega_2)*\tilde{I}^\circ(\omega_2))][H(\omega_1)(\tilde{a}(\omega_1)*\tilde{I}^\circ(\omega_1)*\tilde{Y}(\omega_1))]\\&+b_1^2[H(\omega_1)(\tilde{a}(\omega_1)*\tilde{I}^\circ(\omega_1)*\tilde{Y}(\omega_1))][H(\omega_2)(\tilde{a}(\omega_2)*\tilde{I}^\circ(\omega_2)*\tilde{Y}(\omega_2))]\end{aligned}\tag{1.14}$$

$$\begin{aligned}&b_0^2[H(\omega_1)(\tilde{a}(\omega_1)*\tilde{I}^\circ(\omega_1))][H(\omega_2)(\tilde{a}(\omega_2)*\tilde{I}^\circ(\omega_2))]\\&=b_0^2H(\omega_1)H(\omega_2)\iint\tilde{I}^\circ(\omega_3)\tilde{I}^\circ(\omega_4)\tilde{a}(\omega_1-\omega_3)\tilde{a}(\omega_2-\omega_4)d\omega_3d\omega_4\end{aligned}$$

由于$I^\circ(t)$是平稳过程，它的谱量适合正交增量关系，即有[1]：
$$E[\tilde{I}^\circ(\omega)\tilde{I}^\circ(\omega')]=S_{I^\circ}(\omega)\delta(\omega+\omega')\tag{1.15}$$

这里$S_{I^\circ}(\omega)$是$I^\circ$的功率谱函数，$\delta$是Dirac函数，则有：
$$\begin{aligned}&E\{b_0^2[H(\omega_1)(\tilde{a}(\omega_1)*\tilde{I}^\circ(\omega_1))][H(\omega_2)(\tilde{a}(\omega_2)*\tilde{I}^\circ(\omega_2))]\}\\&=b_0^2H(\omega_1)H(\omega_2)\int S_{I^\circ}(\omega_3)\tilde{a}(\omega_1-\omega_3)\tilde{a}(\omega_2+\omega_3)d\omega_3\end{aligned}\tag{1.16}$$

注意：在以上各式及以下各式中积分都是从$-\infty$到$\infty$进行，为书写简单，我们略去了这些符号。今有：
$$\begin{aligned}&b_1^2[H(\omega_1)(\tilde{a}(\omega_1)*\tilde{I}^\circ(\omega_1)*\tilde{Y}(\omega_1))][H(\omega_2)(\tilde{a}(\omega_2)*\tilde{I}^\circ(\omega_2)*\tilde{Y}(\omega_2))]\\&=b_1^2\iiiint\tilde{Y}(\omega_5)\tilde{I}^\circ(\omega_3-\omega_5)\tilde{a}(\omega_1-\omega_3)\tilde{Y}(\omega_6)\tilde{I}^\circ(\omega_4-\omega_6)\tilde{a}(\omega_2\\&\quad-\omega_4)H(\omega_1)H(\omega_2)d\omega_3d\omega_4d\omega_5d\omega_6\end{aligned}$$

将上式求期望值，应用统计规律(1.15)式可得：
$$\begin{aligned}&E\{b_1^2[H(\omega_1)(\tilde{a}(\omega_1)*\tilde{I}^\circ(\omega_1)*\tilde{Y}(\omega_1))][H(\omega_2)(\tilde{a}(\omega_2)*\tilde{I}^\circ(\omega_2)*\tilde{Y}(\omega_2))]\}\\&=b_1^2H(\omega_1)H(\omega_2)\iiint E[\tilde{Y}(\omega_6)\tilde{Y}(\omega_6)]S_{I^\circ}(\omega_4)\tilde{a}(\omega_1-\omega_5+\omega_4)\tilde{a}(\omega_2\\&\quad+\omega_6-\omega_4)d\omega_4d\omega_5d\omega_6\end{aligned}\tag{1.17}$$

对(1.14)式求期望值，应用(1.16)、(1.17)式，注意到(1.14)式右方第二、三项的期望值为零，可得：

$$E[\tilde{Y}(\omega_1)\tilde{Y}(\omega_2)]=b_0^2H(\omega_1)H(\omega_2)\int S_{I^\circ}(\omega_3)\tilde{a}(\omega_1-\omega_3)\tilde{a}(\omega_2+\omega_3)d\omega_3 +b_1^2H(\omega_1)H(\omega_2)\iiint E[\tilde{Y}(\omega_5)\tilde{Y}(\omega_6)]S_{I^\circ}(\omega_4)\tilde{a}(\omega_1-\omega_5+\omega_4)\tilde{a}(\omega_2+\omega_6-\omega_4)d\omega_4d\omega_5d\omega_6 \tag{1.18}$$

或写成等价形式：

$$E[\tilde{Y}(\omega_1)\tilde{Y}(\omega_2)]=b_0^2H(\omega_1)H^*(\omega_2)\int S_{I^\circ}(\omega_3)\tilde{a}(\omega_1-\omega_3)\tilde{a}^*(\omega_2-\omega_3)d\omega_3 +b_1^2H(\omega_1)H^*(\omega_2)\iiint E[\tilde{Y}(\omega_5)\tilde{Y}^*(\omega_6)]S_{I^\circ}(\omega_4)\tilde{a}(\omega_1-\omega_5+\omega_4)\tilde{a}^*(\omega_2-\omega_6+\omega_4)d\omega_4d\omega_5d\omega_6 \tag{1.19}$$

根据谱分解变换原理可得：

$$E[Y(t)Y^*(t)]=\iint E[\tilde{Y}(\omega_1)\tilde{Y}^*(\omega_2)]\exp(i(\omega_1-\omega_2)t)d\omega_1d\omega_2 =b_0^2\int|b(t,\omega)|^2S_{I^\circ}(\omega)d\omega+b_1^2\iiint b(t,\omega_4-\omega_5)b^*(t,\omega_4-\omega_6)S_{I^\circ}(\omega_4)E[\tilde{Y}(\omega_5)\tilde{Y}^*(\omega_6)]d\omega_4d\omega_5d\omega_6 \tag{1.20}$$

这里：$b(t,\omega)=\int H(\omega')\tilde{a}(\omega'-\omega)\exp(i\omega't)d\omega'$ (1.21)

如果在随机方程中，激励$I(t)$的系数是常数时，即可设$b_0=1$，$b_1=0$，(1.20)式化为：

$$E[Y(t)Y^*(t)]=\int|b(t,\omega)|^2S_{I^\circ}(\omega)d\omega \tag{1.22}$$

在一般情况下，需要先对$E[\tilde{Y}(\omega_1)\tilde{Y}(\omega_2)]$求解，然后代入(1.20)式求$E[Y(t)Y^*(t)]$。

## 二、E[$\tilde{Y}(\omega_1)\tilde{Y}(\omega_2)$]的求解方法

当$S_{I^\circ}(\omega)$，$\tilde{a}(\omega)$可以看作$\omega$的任意函数时，$E[\tilde{Y}(\omega_1)\tilde{Y}(\omega_2)]$可从(1.18)式求解。(1.18)式实际上是$E[\tilde{Y}(\omega_1)\tilde{Y}(\omega_2)]$的积分方程。求解可按两种方式进行：第一种是用离散 Fourier 变换方法，它的谱分解是按照一组离散谱，相应于(1.18)式可以得到一组有关$E[\tilde{Y}(\omega_i)\tilde{Y}(\omega_j)]$的线性联立方程，由此得到一组$E[\tilde{Y}(\omega_i)\tilde{Y}(\omega_j)]$的解，采用离散Fourier变换方法求解线性随机参变振动问题，在本文中不拟作详细讨论。第二种是对积分方程(1.18)式进行求解。以下将参考Fredholm[4]求解方法写出解的形式。

在(1.18)式中令：

$$E[\tilde{Y}(\omega_1)\tilde{Y}(\omega_2)]=u(\omega_1,\omega_2) \tag{2.1a}$$

$$b_0^2H(\omega_1)H(\omega_2)\int S_{I^\circ}(\omega_3)\tilde{a}(\omega_1-\omega_3)\tilde{a}(\omega_2+\omega_3)d\omega_3=f(\omega_1,\omega_2) \tag{2.1b}$$

$$b_1^2H(\omega_1)H(\omega_2)\int S_{I^\circ}(\omega_4)\tilde{a}(\omega_1-\omega_1'+\omega_4)\tilde{a}(\omega_2+\omega_2'-\omega_4)d\omega_4=K(\omega_1,\omega_2;\omega_1',\omega_2') \tag{2.1c}$$

可将(1.18)式改写为：

$$u(\omega_1,\omega_2)=f(\omega_1,\omega_2)+\iint K(\omega_1,\omega_2;\omega_1',\omega_2')u(\omega_1',\omega_2')d\omega_1'd\omega_2' \tag{2.2}$$

上式是Volterra的二独立变量情况下的第二类积分方程。按照Fredholm方法，令：

$$D=1-\iint K(\omega_1,\omega_2)d\omega_1d\omega_2$$

$$+\frac{1}{2!}\iiiint\begin{vmatrix}K(\omega_1,\omega_2;\omega_1,\omega_2) & K(\omega_1,\omega_2;\omega_3,\omega_4)\\ K(\omega_3,\omega_4;\omega_1,\omega_2) & K(\omega_3,\omega_4;\omega_3,\omega_4)\end{vmatrix}d\omega_1 d\omega_2 d\omega_3 d\omega_4$$

$$-\frac{1}{3!}\iiiiiint\begin{vmatrix}K(\omega_1,\omega_2;\omega_1,\omega_2) & K(\omega_1,\omega_2;\omega_3,\omega_4) & K(\omega_1,\omega_2;\omega_5,\omega_6)\\ K(\omega_3,\omega_4;\omega_1,\omega_2) & K(\omega_3,\omega_4;\omega_3,\omega_4) & K(\omega_3,\omega_4;\omega_5,\omega_6)\\ K(\omega_5,\omega_6;\omega_1,\omega_2) & K(\omega_5,\omega_6;\omega_3,\omega_4) & K(\omega_5,\omega_6;\omega_5,\omega_6)\end{vmatrix}d\omega_1 d\omega_2 d\omega_3 d\omega_4 d\omega_5 d\omega_6$$

$$+\cdots\cdots \tag{2.3}$$

$$D(\omega_1,\omega_2;\omega_3,\omega_4)=K(\omega_1,\omega_2;\omega_3,\omega_4)-\iint\begin{vmatrix}K(\omega_1,\omega_2;\omega_3,\omega_4) & K(\omega_1,\omega_2;\omega_5,\omega_6)\\ K(\omega_5,\omega_6;\omega_3,\omega_4) & K(\omega_5,\omega_6;\omega_5,\omega_6)\end{vmatrix}d\omega_5 d\omega_6$$

$$+\frac{1}{2!}\iiiint\begin{vmatrix}K(\omega_1,\omega_2;\omega_3,\omega_4) & K(\omega_1,\omega_2;\omega_5,\omega_6) & K(\omega_1,\omega_2;\omega_7,\omega_8)\\ K(\omega_5,\omega_6;\omega_3,\omega_4) & K(\omega_5,\omega_6;\omega_5,\omega_6) & K(\omega_5,\omega_6;\omega_7,\omega_8)\\ K(\omega_7,\omega_8;\omega_3,\omega_4) & K(\omega_7,\omega_8;\omega_5,\omega_6) & K(\omega_7,\omega_8;\omega_7,\omega_8)\end{vmatrix}d\omega_5 d\omega_6 d\omega_7 d\omega_8$$

$$+\cdots\cdots \tag{2.4}$$

则$u(\omega_1,\omega_2)$的解可以写为：

$$u(\omega_1,\omega_2)=f(\omega_1,\omega_2)+\frac{1}{D}\iint f(\omega_3,\omega_4)D(\omega_1,\omega_2;\omega_3,\omega_4)d\omega_3 d\omega_4 \tag{2.5}$$

对于实际问题当采用以上公式时一般需要采用数值积分。

## 三、某些特殊情况

［例1］ 考虑如(1.1)式所示的振动方程，假定激励$I(t)$是平稳过程且为白噪声过程，即有：

$$\tilde{a}(\omega)=\delta(\omega) \tag{3.1}$$

$$S_{I^\circ}(\omega)=S_{I^\circ} \tag{3.2}$$

首先假设$I(t)$是平稳过程，则(3.1)式成立。将(3.1)式代入(1.9)式得：

$$E[\tilde{Y}(\omega_1)\tilde{Y}^*(\omega_2)]=b_0^2H(\omega_1)H^*(\omega_2)S_{I^\circ}(\omega_2)\delta(\omega_1-\omega_2)$$
$$+b_1^2H(\omega_1)H^*(\omega_2)\int E[\tilde{Y}(\omega_1+\omega_4)\tilde{Y}^*(\omega_2+\omega_4)]S_{I^\circ}(\omega_4)d\omega_4 \tag{3.3}$$

此时$Y(t)$也是平稳过程，其谱量$\tilde{Y}(\omega)$具有正交增量性质，即有：

$$E[\tilde{Y}(\omega)\tilde{Y}^*(\omega')]=S_Y(\omega)\delta(\omega-\omega') \tag{3.4}$$

将(3.4)式代入(3.3)式得：

$$S_Y(\omega)=b_0^2H(\omega)H^*(\omega)S_{I^\circ}(\omega)+b_1^2H(\omega)H^*(\omega)\int S_Y(\omega+\omega')S_{I^\circ}(\omega')d\omega' \tag{3.5}$$

(3.5)式是$S_Y(\omega)$的第二类积分方程，可用Fredholm方法求解。如果$I(t)$还是白噪声过程，则(3.2)式成立，(3.3)式成为：

$$S_Y(\omega)=b_0^2H(\omega)H^*(\omega)S_{I^\circ}(\omega)+b_1^2H(\omega)H^*(\omega)S_{I^\circ}(\omega)\int S_Y(\omega')d\omega'$$
$$=(b_0^2+b_1^2\sigma_Y^2)H(\omega)H^*(\omega)S_{I^\circ} \tag{3.6}$$

将(3.6)式对$\omega$积分得：

$$\sigma_Y^2=(b_0^2+b_1^2\sigma_Y^2)S_{I^\circ}\int H(\omega)H^*(\omega)d\omega=\frac{\pi}{2\beta\omega_0^2}S_{I^\circ}(b_0^2+b_1^2\sigma_Y^2)$$

由此求得：

$$\sigma_Y^2=\frac{\pi b_0^2 S_{I^\circ}}{2\beta\omega_0^2-\pi b_1^2 S_{I^\circ}} \tag{3.7}$$

代入(3.6)式可用以求$S_Y(\omega)$。

［例 2］ 考虑如下随机振动方程：

$$\left.\begin{array}{l}\ddot{Z}(t)+2\beta\dot{Z}(t)+\omega_0^2 Z(t)=I(t)\\ (t\geqslant 0;\ \dot{X}(0)=X(0)=0)\end{array}\right\} \tag{3.8}$$

其中随机过程$Y(t)$的均值为零，可以表示为下式，其中$Y^\circ(t)$是平稳过程。

$$Y(t)=a(t)Y^\circ(t),\ a(t)=\begin{cases}0, & t<0\\ 1, & t\geqslant 0\end{cases} \tag{3.9}$$

采用文献[1]及本文方法求解此问题。从(3.9)式可见$a(t)$是Heaviside 函数，它的谱分解式可以写成如下极限形式：

$$a(t)=\frac{1}{2\pi}\lim_{\gamma\to 0}\int_{-\infty}^{\infty}\frac{\exp(i\omega t)}{\gamma+i\omega}d\omega \tag{3.10}$$

即有：

$$\tilde{a}(\omega)=\lim_{\gamma\to 0}\frac{1}{2\pi(\gamma+i\omega)},\quad \tilde{a}(\omega'-\omega)=\lim_{\gamma\to 0}\frac{1}{2\pi[\gamma+i(\omega'-\omega)]} \tag{3.11}$$

根据[1]文(1.20b)式，即：

$$b(t,\omega)=\int H(\omega')\tilde{a}(\omega'-\omega)\exp(i\omega' t)\,d\omega' \tag{3.12}$$

将(3.11)式代入(3.12)式得：

$$\begin{aligned}b(t,\omega)&=\lim_{\gamma\to 0}\frac{1}{2\pi}\int\frac{H(\omega')}{\gamma+i(\omega'-\omega)}\exp(i\omega' t)\,d\omega'\\ &=\lim_{\gamma\to 0}\frac{1}{2\pi}\int\frac{1}{-\omega'^2+2\beta i\omega'+\omega_0^2}\cdot\frac{1}{\gamma+i(\omega'-\omega)}\exp(i\omega' t)\,d\omega'\end{aligned} \tag{3.13}$$

在(3.13)式的被积式中，分母具有如下的根：

$$\omega'=\beta i+\omega_{cr},\quad \beta i-\omega_{cr},\quad \gamma i+\omega \tag{3.14a}$$

这里：

$$\omega_{cr}=\sqrt{\omega_0^2-\beta^2} \tag{3.14b}$$

被积式的留数为：

$$\begin{aligned}I=&\frac{\exp((i\omega-\gamma)t)}{-[\omega+i\gamma-(\beta i+\omega_{cr})][\omega+i\gamma-(\beta i-\omega_{cr})]i}+\frac{\exp((i\omega_{cr}-\beta)t)}{-2\omega_{cr}(\beta i+\omega_{cr}-\gamma i-\omega)i}\\ &+\frac{\exp((-i\omega_{cr}-\beta)t)}{2\omega_{cr}(\beta i-\omega_{cr}-\gamma i-\omega)i}\end{aligned} \tag{3.15}$$

应有：

$$\begin{aligned}b(t,\omega)=&\lim_{\gamma\to 0}\frac{2\pi i}{2\pi}\cdot I=\frac{\exp(i\omega t)}{-[\omega-\omega_{cr}-\beta i][\omega+\omega_{cr}-\beta i]}\\ &+\frac{\exp((i\omega_{cr}-\beta)t)}{2\omega_{cr}(\omega-\omega_{cr}-\beta i)}-\frac{\exp((-i\omega_{cr}-\beta)t)}{2\omega_{cr}(\omega+\omega_{cr}-\beta i)}\end{aligned} \tag{3.16a}$$

$$b^*(t,\omega)=\frac{\exp(-i\omega t)}{-[\omega-\omega_{cr}+\beta i][\omega+\omega_{cr}+\beta i]}+\frac{\exp((-i\omega_{cr}-\beta)t)}{2\omega_{cr}(\omega-\omega_{cr}+\beta i)}$$

$$-\frac{\exp((i\omega_{cr}-\beta)t)}{2\omega_{cr}(\omega+\omega_{cr}+\beta i)} \tag{3.16b}$$

$$b(t,\omega)b^*(t,\omega)=\frac{1}{[(\omega-\omega_{cr})^2+\beta^2][(\omega+\omega_{cr})^2+\beta^2]}+\frac{\exp(-2\beta t)}{4\omega_{cr}^2[(\omega-\omega_{cr})^2+\beta^2]}$$

$$+\frac{\exp(-2\beta t)}{4\omega_{cr}^2[(\omega+\omega_{cr})^2+\beta^2]}-\frac{\exp((2i\omega_{cr}-2\beta)t)}{4\omega_{cr}^2(\omega-\omega_{cr}-\beta i)(\omega+\omega_{cr}+\beta i)}$$

$$-\frac{\exp((-2i\omega_{cr}-2\beta)t)}{4\omega_{cr}^2(\omega+\omega_{cr}-\beta i)(\omega-\omega_{cr}+\beta i)}-\frac{\exp(-\beta t+i(\omega-\omega_{cr})t)}{2\omega_{cr}[(\omega-\omega_{cr})^2+\beta^2](\omega+\omega_{cr}-\beta i)}$$

$$-\frac{\exp(-\beta t-i(\omega-\omega_{cr})t)}{2\omega_{cr}[(\omega-\omega_{cr})^2+\beta^2](\omega+\omega_{cr}+\beta i)}+\frac{\exp(-\beta t+i(\omega+\omega_{cr})t)}{2\omega_{cr}[(\omega+\omega_{cr})^2+\beta^2](\omega-\omega_{cr}-\beta i)}$$

$$+\frac{\exp(-\beta t-i(\omega+\omega_{cr})t)}{2\omega_{cr}[(\omega+\omega_{cr})^2+\beta^2](\omega-\omega_{cr}+\beta i)} \tag{3.17}$$

采用简写符号：

$$z(\omega)=1-\left(\frac{\omega}{\omega_0}\right)^2-\frac{2\beta i\omega}{\omega_0^2} \tag{3.18a}$$

$$z_1(\omega,t)=\left\{\frac{\beta}{\omega_{cr}}\sin\omega_{cr}t+\cos\omega_{cr}t\right\}+i\frac{\omega}{\omega_{cr}}\sin\omega_{cr}t \tag{3.18b}$$

观察(3.14a)式可得：

$$b(t,\omega)=\frac{1}{\omega_0^2 z^*(\omega)}\{\exp(i\omega t)-\exp(-\beta t)z_1(\omega,t)\} \tag{3.19}$$

与文献[5]中结果完全相同。本文所得结果不难按照[1]文方法推广到多自由度振动情况。

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# A Spectral Resolving Method for Analyzing Linear Random Vibrations with Variable Parameters


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### Abstract


This paper is a development of Ref. [1]. Consider the following random equation: $\ddot{Z}(t)+2\beta\dot{Z}(t)+\omega_0^2 Z(t)=(a_0+a_1Z(t))I(t)+c$, in which excitation $I(t)$ and response $Z(t)$ are both random processes, and it is proposed that they are mutually independent. Suppose that $I(t)=a(t)I^*(t)$, $a(t)$ is a known function of time and $I^*(t)$ is a stationary random process. In this paper, the spectral resolving form of the random equation stated above, the numerical solving method and the solutions in some special cases are considered.