

水平刚性基础上弹性层表面受垂直集中力的精确解*

文 丕 华

(中南矿冶学院, 1985年2月7日收到)

摘 要

在文献[1]的基础上, 推出了水平刚性地基上弹性层表面受垂直集中载荷作用下的精确解, 并给出了数值计算结果, 可供土木工程师参考。

一、引 言

Boussinesq于1885年给出了半空间表面受垂直集中载荷作用下的解, 但是在土木建筑工程中, 往往土层下面存在刚性岩层(图1所示), 这时Boussinesq解不再适用, 只能作为粗略估计。对图1问题尚未获得精确解答。文献[2]采用线载荷积分方程方法, 建立了求解这一问题的Fredholm第一类积分方程, 并进行了近似的数值计算。文献[1]提出了分层求逆子阵的方法, 成功地解决了多层半无限弹性体在圆形分布载荷作用下的理论解。本文则在此基础上, 采用Hankel变换, 推出此问题的精确解, 并进行了数值计算, 给出了接触应力和位移的计算结果, 以便工程人员参考。将这些精确解作为某个积分的积分核, 对整个受载面进行积分, 这样轴对称问题便很容易的推广到非轴对称问题中去。

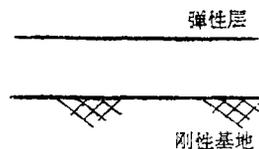


图1 弹性层与刚性地基示意图

二、基本方程推导

R. V. Southwell提出了轴对称问题基本方程:

* 云天铨推荐。

$$\left. \begin{aligned}
 \sigma_r + \sigma_\theta &= \frac{1}{r} \cdot \frac{\partial^2 \lambda}{\partial r \partial z^2} \\
 \sigma_r - \sigma_\theta &= r \cdot \frac{\partial}{\partial r} \cdot \frac{1}{r^2} \left(\frac{\partial^2 \lambda}{\partial z^2} - 2\mu \vartheta^2 \lambda \right) \\
 \sigma_z &= -\frac{1}{r} \cdot \frac{\partial}{\partial r} \cdot \left[\frac{\partial^2 \lambda}{\partial z^2} - (1+\mu) \vartheta^2 \lambda \right] \\
 \tau_{rz} &= \frac{1}{r} \cdot \frac{\partial}{\partial z} \left[\frac{\partial^2 \lambda}{\partial z^2} - (1+\mu) \vartheta^2 \lambda \right]
 \end{aligned} \right\} \quad (2.1)$$

应力函数 λ 应满足柱坐标四阶微分方程:

$$\vartheta^2(\vartheta^2 \lambda) = 0 \quad (2.2)$$

式中
$$\vartheta^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

令
$$\left. \begin{aligned}
 \phi(r, z) &= \vartheta^2 \lambda \\
 \psi(r, z) &= \frac{\partial^2 \lambda}{\partial z^2} - (1+\mu) \vartheta^2 \lambda
 \end{aligned} \right\} \quad (2.3)$$

由(2.2)式, ϕ 、 ψ 需满足

$$\vartheta^2 \phi(r, z) = 0 \quad (2.4)$$

$$\vartheta^2 \psi(r, z) = \frac{\partial^2 \phi}{\partial z^2} \quad (2.5)$$

将(2.3)、(2.4)、(2.5)代入(2.1), 有:

$$\sigma_r + \sigma_\theta = \frac{1}{r} \cdot \frac{\partial}{\partial r} (\psi + \phi) + \frac{\mu}{r} \cdot \frac{\partial \phi}{\partial r} \quad (2.6a)$$

$$\sigma_r - \sigma_\theta = \frac{1}{r} \cdot \frac{\partial}{\partial r} (\psi + \phi) - \frac{\mu}{r} \cdot \frac{\partial \psi}{\partial r} - \frac{z}{r^2} [\psi + (1-\mu)\phi] \quad (2.6b)$$

$$\sigma_z = -\frac{1}{r} \cdot \frac{\partial \psi}{\partial r} \quad (2.6c)$$

$$\tau_{rz} = \frac{1}{r} \cdot \frac{\partial \psi}{\partial z} \quad (2.6d)$$

径向位移 u 和垂直移位 w 可写成:

$$\left. \begin{aligned}
 u &= \frac{1+\mu}{E} \cdot \frac{1}{r} [\psi + (1-\mu)\phi] \\
 \frac{\partial w}{\partial r} &= \frac{1+\mu}{E} \cdot \frac{1}{r} \frac{\partial}{\partial z} [\psi - (1-\mu)\phi]
 \end{aligned} \right\} \quad (2.7)$$

对(2.4)式进行一阶Hankel变换,

$$\bar{\phi} = H_1[\phi(r, z); r \rightarrow \xi] = \int_0^\infty \xi \phi(\xi, z) J_1(r\xi) d\xi \quad (2.8)$$

有
$$\left(\frac{d^2}{dz^2} - \xi^2 \right) \bar{\phi} = 0 \quad (2.9)$$

解微分方程(2.9),

$$\bar{\phi} = A \cdot \exp(\xi z) + B \cdot \exp(-\xi z) \quad (2.10)$$

由(2.5)式, 同理有:

$$\bar{\psi} = \frac{1}{2} (2C + \xi z A) \cdot \exp(\xi z) + \frac{1}{2} (2D - \xi z B) \exp(-\xi z) \quad (2.11)$$

方程(2.10)、(2.11)中 A 、 B 、 C 、 D 均为待定系数, 由特定的边界条件来确定.

对方程(2.6a)、(2.6b)、(2.6c)和(2.6d)分别进行逆变换:

$$\sigma_z = - \int_0^\infty \xi J_0(\xi r) \cdot \bar{\psi} d\xi \quad (2.12)$$

$$\tau_{rz} = \int_0^\infty J_1(\xi r) \cdot \frac{\partial \bar{\psi}}{\partial z} d\xi \quad (2.13)$$

$$\sigma_r + \sigma_\theta = \int_0^\infty \xi J_0(\xi r) \cdot [\bar{\psi} + (1 + \mu)\bar{\phi}] d\xi \quad (2.14)$$

$$u = \int_0^\infty J_1(\xi r) \cdot \frac{1 + \mu}{E} \cdot [\bar{\psi} + (1 - \mu)\bar{\phi}] d\xi \quad (2.15)$$

$$w = - \int_0^\infty \frac{J_0(\xi r)}{\xi} \cdot \frac{1 + \mu}{E} \cdot \frac{\partial}{\partial z} [\bar{\psi} - (1 - \mu)\bar{\phi}] d\xi \quad (2.16)$$

由静力平衡方程

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (2.17)$$

将(2.13)、(2.14)代入上式, 并整理有:

$$\frac{\partial \sigma_r}{\partial r} + 2 \frac{\sigma_r}{r} = f \quad (2.18)$$

$$\text{式中} \quad f = \frac{1}{r} \int_0^\infty \xi J_0(\xi r) \cdot [\bar{\psi} + (1 + \mu)\bar{\phi}] d\xi - \int_0^\infty J_1(\xi r) \frac{\partial^2 \bar{\psi}}{\partial z^2} d\xi \quad (2.19)$$

方程(2.18)式的通解为

$$\sigma_r = \frac{a}{r^2} + \frac{1}{r^2} \int r^2 f dr \quad (2.20)$$

由应力条件, σ_r 在 $r=0$ 处不可能均为无限, 故应有 $a=0$.

由贝塞尔函数积分关系, 分部积分有:

$$\sigma_r = \int_0^\infty J_0(\xi r) \frac{d^2 \bar{\psi}}{\xi d z^2} d\xi - \int_0^\infty \frac{J_1(\xi r)}{r} \left[\frac{2}{\xi^2} \cdot \frac{d^2 \bar{\psi}}{d z^2} - \bar{\psi} - (1 + \mu)\bar{\phi} \right] d\xi \quad (2.21)$$

又将(2.10)、(2.11)代入(2.12)、(2.13)、(2.15)、(2.16)、(2.21)等各式, 整理后有:

$$\begin{aligned} \sigma_r = & \frac{1}{2} \int_0^\infty \xi J_0(\xi r) \{ \exp(\xi z) [2C + (2 + \xi z) \cdot A] \\ & + \exp(-\xi z) \cdot [2D + (2 - \xi z) B] \} d\xi - I \end{aligned} \quad (2.22a)$$

$$\sigma_\theta = \mu \int_0^\infty \xi J_0(\xi r) [\exp(\xi z) \cdot A + \exp(-\xi z) \cdot B] d\xi + I \quad (2.22b)$$

$$\sigma_z = -\frac{1}{2} \int_0^\infty \xi J_0(\xi r) \{ \exp(\xi z) (2C + \xi z A) + \exp(-\xi z) \cdot (2D - \xi z B) \} d\xi \quad (2.22c)$$

$$\tau_{rz} = \frac{1}{2} \int_0^\infty \xi J_1(\xi r) \{ \exp(\xi z) \cdot [2C + (1 + \xi z) A] - \exp(-\xi z) \cdot [2D + (1 - \xi z) B] \} d\xi \quad (2.22d)$$

$$u = -\frac{1+\mu}{2E} \int_0^\infty J_1(\xi r) \{ \exp(\xi z) [2C + (2-2\mu + \xi z) A] + \exp(-\xi z) [2D + (2-2\mu - \xi z) B] \} d\xi \quad (2.22e)$$

$$w = -\frac{1+\mu}{2E} \int_0^\infty J_0(\xi r) \{ \exp(\xi z) [2C - (1-2\mu - \xi z) A] - \exp(-\xi z) [2D - (1-2\mu + \xi z) B] \} d\xi \quad (2.22f)$$

方程(2.22a)、(2.22b)中

$$I = \frac{1}{2} \int_0^\infty \frac{J_1(\xi r)}{r} \{ \exp(\xi z) [2C + (2-2\mu + \xi z) A] + \exp(-\xi z) [2D + (2-2\mu - \xi z) B] \} d\xi \quad (2.22g)$$

以上方程(2.22)和文献[1]相同。

三、刚性基础弹性层精确解

首先考虑在弹性层表面($z=-h$)载荷为圆形分布压力的情形(图2所示)。此时,边界条件可写成:

$$\sigma_z(r, -h) = \begin{cases} -q_0 & (r \leq R) \\ 0 & (r > R) \end{cases} \quad (3.1)$$

$$\tau_{rz}(r, -h) = 0 \quad (3.2)$$

对(3.1)式两边乘以 $rJ_0(\xi r)$,并对 r 积分

$$\int_0^\infty r J_0(\xi r) \sigma_z(r, -h) dr = \int_0^R r J_0(\xi r) (-q_0) dr = -\frac{q_0 R}{\xi} J_1(\xi R) \quad (3.3)$$

由(2.12)的逆变换, $\bar{\varphi} = -\int_0^\infty r J_0(\xi r) \sigma_z(r, -h) dr$, 故

$$\bar{\varphi}|_{z=-h} = \frac{q_0 R}{\xi} J_1(\xi R) \quad (3.4)$$

对于表面作用集中力问题,取极限:

$$\lim_{R \rightarrow 0} \pi R^2 q_0 = P \quad (3.5)$$

这时(3.4)有

$$\bar{\varphi}|_{z=-h} = \frac{P}{2\pi} \quad (3.6)$$

由(2.11)式

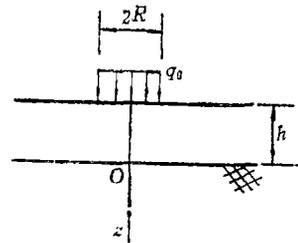


图2 弹性层表面受圆形分布载荷

$$(2C - \xi h A) \cdot \exp(-\xi h) + (2D + \xi h B) \cdot \exp(\xi h) = \frac{P}{\pi} \quad (3.7)$$

又由(3.2)式, 同样得到

$$[2C + (1 - \xi h) A] \cdot \exp(-\xi h) - [2D + (1 + \xi h) B] \cdot \exp(\xi h) = 0 \quad (3.8)$$

另外对于刚性支座还可以得到两个补充方程, 对于光滑接触(接触处无摩擦力)问题, 边界条件可写成:

$$\left. \begin{array}{l} \tau_{rz}|_{z=0} = 0 \\ w|_{z=0} = 0 \end{array} \right\} \quad (3.9)$$

这时由方程(2.22d)、(2.22f)有:

$$2C + A - (2D + B) = 0 \quad (3.10)$$

$$2C - (1 - 2\mu)A - [2D - (1 - 2\mu)B] = 0 \quad (3.11)$$

联立方程(3.7)、(3.8)、(3.10)、(3.11)求解, 确定未知常数 A 、 B 、 C 、 D :

$$A = B = \frac{P}{\pi} \cdot \frac{\text{sh}(\xi h)}{\text{sh}(2\xi h) + 2\xi h \cdot \text{ch}(2\xi h)} \quad (3.12)$$

$$C = D = \frac{P}{2\pi} \frac{\text{sh}(\xi h)}{\text{sh}(2\xi h) + 2\xi h \cdot \text{ch}(2\xi h)} \cdot \left[1 + \xi h \frac{\text{ch}(\xi h)}{\text{sh}(\xi h)} \right] \quad (3.13)$$

若将(3.12)、(3.13)代回方程(2.22)各式, 便获得整个空间应力、位移精确解。

对于非光滑接触(相互粘结在一起)问题, 边界条件为:

$$\left. \begin{array}{l} u|_{z=0} = 0 \\ w|_{z=0} = 0 \end{array} \right\} \quad (3.14)$$

由(2.22c)式:

$$2C + (2 - 2\mu)A + 2D + (2 - 2\mu)B = 0 \quad (3.15)$$

将(3.15)替换(3.10)式, 联立求解:

$$A = -\frac{P}{\pi} \frac{\exp(\xi h)[2\xi h + (3 - 4\mu) \cdot \exp(-2\xi h) + 1]}{(2\xi h)^2 + (3 - 4\mu)^2 + 2(3 - 4\mu)\text{ch}(2\xi h) + 1} \quad (3.16)$$

$$B = -\frac{P}{\pi} \frac{\exp(\xi h)[(3 - 4\mu) + \exp(-2\xi h) - 2\xi h \exp(-2\xi h)]}{(2\xi h)^2 + (3 - 4\mu)^2 + 2(3 - 4\mu)\text{ch}(2\xi h) + 1} \quad (3.17)$$

$$C = -\frac{A + (3 - 4\mu)B}{4} \quad (3.18)$$

$$D = -\frac{B + (3 - 4\mu)A}{4} \quad (3.19)$$

四、数值结果

从接触光滑、非光滑问题结果可以看出, 前者各待定系数均与材料常数 μ (泊松比) 无关, 无疑会给问题带来方便。这时应力、位移关系为:

$$\sigma_r = \int_0^{\infty} \xi J_0(\xi r) \{ 2C \text{ch}(\xi z) + 2A [\text{ch}(\xi z) + \xi z \text{sh}(\xi z) / 2] \} d\xi - I \quad (4.1a)$$

$$\sigma_{\theta} = 2\mu \int_0^{\infty} \xi J_0(\xi r) A \operatorname{ch}(\xi z) d\xi + I \quad (4.1b)$$

$$\tau_{rz} = \int_0^{\infty} \xi J_1(\xi r) \{2C \operatorname{sh}(\xi z) + A[\operatorname{sh}(\xi z) + \xi z \operatorname{ch}(\xi z)]\} d\xi \quad (4.1c)$$

$$\sigma_z = - \int_0^{\infty} \xi J_0(\xi r) [2C \operatorname{ch}(\xi z) + \xi z A \operatorname{sh}(\xi z)] d\xi \quad (4.1d)$$

$$u = \frac{1+\mu}{E} \int_0^{\infty} J_1(\xi r) \{2C \operatorname{ch}(\xi z) + A[2(1-\mu) \operatorname{ch}(\xi z) + \xi z \operatorname{sh}(\xi z)]\} d\xi \quad (4.1e)$$

$$w = - \frac{1+\mu}{E} \int_0^{\infty} J_0(\xi r) \{2C \operatorname{sh}(\xi z) + A[\xi z \operatorname{ch}(\xi z) - (1-2\mu) \operatorname{sh}(\xi z)]\} d\xi \quad (4.1f)$$

$$I = \int_0^{\infty} \frac{J_1(\xi r)}{r} \{2C \operatorname{ch}(\xi z) + A[2(1-\mu) \operatorname{ch}(\xi z) + \xi z \operatorname{sh}(\xi z)]\} d\xi \quad (4.1g)$$

对于工程实际问题, 往往对接触应力颇感兴趣, 当 $z=0$ 时, (4.1) 各式变为:

$$\sigma_r = \frac{2P}{\pi h^2} [F_{0\sigma} + F_{0A} - F_{1\sigma} - (1-\mu)F_{1A}] \quad (4.2a)$$

$$\sigma_{\theta} = \frac{2P}{\pi h^2} [\mu F_{0A} + F_{1\sigma} + (1-\mu)F_{1A}] \quad (4.2b)$$

$$\tau_{rz} = 0 \quad (4.2c)$$

$$\sigma_z = - \frac{2P}{\pi h^2} F_{0\sigma} \quad (4.2d)$$

$$u = \frac{2(1+\mu)}{E} \cdot \frac{P}{\pi h} [F_{1\sigma} + (1-\mu)F_{1A}] \cdot \beta \quad (4.2e)$$

$$w = 0 \quad (4.2f)$$

$$\text{式中} \quad \left. \begin{aligned} F_{0A}(\beta) &= \int_0^{\infty} x J_0(\beta x) A'(x) dx \\ F_{0\sigma}(\beta) &= \int_0^{\infty} x J_0(\beta x) C'(x) dx \\ F_{1A}(\beta) &= \frac{1}{\beta} \int_0^{\infty} J_1(\beta x) A'(x) dx \\ F_{1\sigma}(\beta) &= \frac{1}{\beta} \int_0^{\infty} J_1(\beta x) C'(x) dx \end{aligned} \right\} \quad (4.3)$$

$$\text{以及} \quad \left. \begin{aligned} A'(x) &= \operatorname{sh}x / (\operatorname{sh}2x + 2x \operatorname{ch}2x)^{\frac{1}{2}}, \quad \beta = r/h \\ C'(x) &= \operatorname{sh}x(1 + x \operatorname{cth}x) / 2(\operatorname{sh}2x + 2x \operatorname{ch}2x) \end{aligned} \right\} \quad (4.4)$$

对于不同的材料, 函数 F_{0A} 、 $F_{0\sigma}$ 、 F_{1A} 、 $F_{1\sigma}$ 均保持不变, 仅与比值 (r/h) 有关, 因此我们仅需计算出 F_{0A} 等的数值结果, 对于不同材料和弹性层厚度 h , 结果均由 (4.2) 各式求出。

另外我们用 $Q(\beta)$ 来定义应力 $\sigma_z(r)$ 密度分布函数,即在 β 处刚性地基的支承反力。

$$Q(\beta) = \int_0^r 2\pi r \sigma_z dr / P$$

$$= -4\beta^2 \int_0^\beta x F_{0a}(x) dx \tag{4.5}$$

由积分关系

$$\int x J_0(x) dx = x J_1(x) + C_0 \tag{4.6}$$

故

$$Q(\beta) = -4\beta^2 F_{1c}(\beta) \tag{4.7}$$

由外力平衡条件,当 β 取无限大时应有

$$\lim_{\beta \rightarrow \infty} Q(\beta) = -1 \tag{4.8}$$

除此之外,我们还可以对应力 τ_{rz} 作出粗略的估计,假设 τ_{rz} 呈抛物线变化,即

$$\tau_{rz}(r, z) \approx \alpha(\beta) \cdot z(z+h)/h^2 \tag{4.9}$$

由平衡条件

$$\int_0^h \tau_{rz} 2\pi r dz = -P(1+Q) \tag{4.10}$$

因此确定 $\alpha(\beta)$ 的大小:

$$\alpha = \frac{3P(1+Q)}{\pi\beta h^2} = \frac{3P(1-4\beta^2 F_{1c})}{\pi\beta h^2} \tag{4.11}$$

采用FORTRAN语言编制定积分程序,积分相对误差均小于1/100,各变量 F_{0a} 、 F_{0c} 、 F_{1a} 、 F_{1c} 数值结果如表1所示。图3、4、5给出了它们的变化曲线。

表1 数值计算结果

β	$F_{0a}(\beta)$	$F_{0c}(\beta)$	$F_{1a}(\beta)$	$F_{1c}(\beta)$	$Q(\beta)$ %	β	$F_{0a}(\beta)$	$F_{0c}(\beta)$	$F_{1a}(\beta)$	$F_{1c}(\beta)$	$Q(\beta)$ %
0.0	0.21730	0.30660	0.10860	0.15330	0.0	2.1	0.03000	0.02609	0.03924	0.04211	74.3
0.1	0.21504	0.30221	0.10807	0.15220	0.6	2.2	0.02665	0.02299	0.03701	0.03945	76.4
0.2	0.20933	0.29015	0.10662	0.14931	2.4	2.3	0.02369	0.02028	0.03493	0.03702	78.3
0.3	0.20040	0.27181	0.10431	0.14431	5.2	2.4	0.02104	0.01791	0.03299	0.03478	80.1
0.4	0.18900	0.24936	0.10127	0.13813	8.8	2.5	0.01872	0.01584	0.03118	0.03271	81.8
0.5	0.17608	0.22465	0.09766	0.13101	13.1	2.6	0.01665	0.01402	0.02950	0.03080	83.3
0.6	0.16205	0.19975	0.09363	0.12335	17.8	2.7	0.01482	0.01242	0.02792	0.02904	84.7
0.7	0.14792	0.17587	0.08934	0.11550	22.6	2.8	0.01319	0.01102	0.02646	0.02741	86.0
0.8	0.13408	0.15379	0.08492	0.10771	27.6	2.9	0.01174	0.00978	0.02509	0.02591	87.2
0.9	0.12086	0.13390	0.08047	0.10017	32.5	3.0	0.01044	0.00833	0.02380	0.02451	88.2
1.0	0.10849	0.11631	0.07600	0.19300	77.2	3.1	0.00932	0.00771	0.02261	0.02301	88.5
1.1	0.09707	0.10094	0.07172	0.08628	41.8	3.2	0.00831	0.00687	0.02148	0.02201	90.2
1.2	0.08664	0.08761	0.06758	0.08001	46.1	3.3	0.00741	0.00615	0.02044	0.02089	91.0
1.3	0.07719	0.07610	0.06363	0.07423	50.2	3.4	0.00661	0.00545	0.01946	0.01984	91.7
1.4	0.06869	0.06619	0.05988	0.06883	54.0	3.5	0.00644	0.00483	0.01854	0.01887	92.5
1.5	0.06108	0.05765	0.05634	0.06394	57.5	3.6	0.00588	0.00431	0.01767	0.01796	93.1
1.6	0.05427	0.05030	0.05300	0.05953	61.0	3.7	0.00531	0.00386	0.01686	0.01711	93.7
1.7	0.04821	0.04397	0.04987	0.05543	64.1	3.8	0.00420	0.00345	0.01610	0.01632	94.3
1.8	0.04282	0.03849	0.04693	0.05167	67.0	3.9	0.00375	0.00308	0.01538	0.01557	94.7
1.9	0.03803	0.03376	0.04419	0.04823	69.6	4.0	0.00336	0.00341	0.01407	0.01488	95.2
2.0	0.03377	0.02965	0.04163	0.04500	72.0						

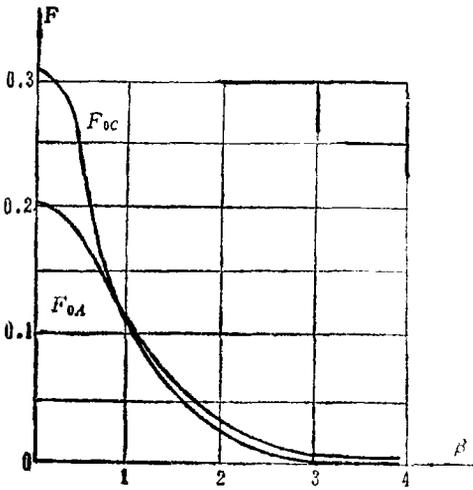


图3 F_{0A} 、 F_{0c} 变化曲线

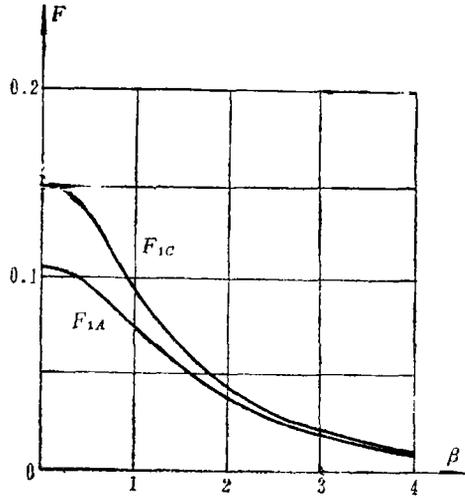


图4 F_{1A} 、 F_{1c} 变化曲线

五、结 束 语

本文采用 Hankel 变换，推导出了刚性支座弹性体表面受集中载荷作用下的精确解，并进行了数值计算。若对于弹性层表面承受任意分布载荷问题，可将这一精确解作为某个积分

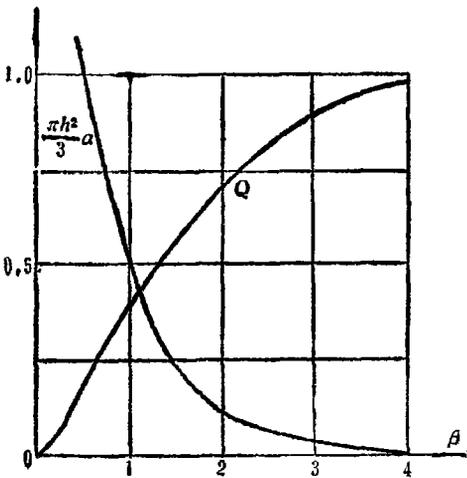


图5 Q 、 α 变化曲线

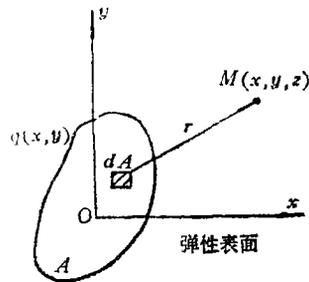


图6 任意分布载荷示意图

的积分核，对整个受载面积进行积分。通过适当的坐标变换，便可获得任意载荷作用下 $M(x, y, z)$ 点各方向的应力、位移大小，如应力 $\sigma_z(M)$ 可以写成

$$\sigma_z(M) = \int_A \sigma_z^*(r, z) q(x, y) dA \tag{5.1}$$

的积分形式，此处 $\sigma_z^*(r, z)$ 为单位集中力作用下的解析解， $q(x, y)$ 为外载荷， r 为坐标 M 点到微元面积 dA 的距离。这样就将轴对称问题推广到非轴对称问题中去。这些工作有待今后补充和完善。

参 考 文 献

- [1] 郭文复, 多层半无限弹性体在圆形载荷下的解析解, 力学学报, 16, 3 (1984), 282.
[2] 云天铨, 水平刚性岩基上的弹性层表面受垂直集中力问题的积分方程解法, 固体力学学报, 3 (1983).

**An Accurate Solution for the Surface of Elastic Layer
under Normal Concentrated Load Acting on a
Rigid Horizontal Base**

Wen Pi-hua

(Central-South Institute of Mining and Metallurgy, Changsha)

Abstract

On the basis of reference^[1], this paper deduces an accurate solution for the surface of elastic layer under normal concentrated load acting on a rigid horizontal base, and gives numerical results, which suit civil engineers for reference.