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刚塑性材料塑性动力学问题中的一般方程和通解*

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摘 要

本文是文[1～2]的继续．本文讨论了塑性流动理论中的理想刚塑性材料的动力学问题．在引入Dirac-Pauli表象的复变函数理论后，我们可以得到用流函数和理论比例系数表示的一组(两个)所谓“一般方程”．本文还证明了塑性动力学问题的时间发展方程既非耗散型的，又非弥散型的，而其本征方程却是以应力增量的偏张量为本征函数，以理论比例系数为本征值的定态Schrödinger方程．于是，我们使非线性塑性动力学问题成为线性定态Schrödinger方程的求解，由此可以得到刚塑性材料塑性动力学问题的通解．

一、前 言

塑性动力学问题的研究在工程技术中和地球物理学中有重要的实际应用．

塑性动力学的理论模型有若干种[3~18]．为了便于理论分析，本文以塑性流动理论中的理想刚塑性材料为研究对象．

如果我们假定理想刚塑性材料的整个物体都处于塑性状态中，则以塑性流动理论为基础的塑性动力学问题表现为下列一组非线性方程组[3]：

Navier方程（这里暂不研究外场力问题）

$$\partial_k\sigma_{ik}=\rho\partial_t v_i \tag{1.1}$$

de St. Venant-Levy-von Mises方程

$$e_{ji}=\frac{1}{2}(\partial_j v_i+\partial_i v_j)=\lambda\left(\sigma_{ji}-\frac{1}{3}\Theta\delta_{ji}\right) \tag{1.2}$$

von Mises 屈服条件[19]

$$\left(\sigma_{ji}-\frac{1}{3}\Theta\delta_{ji}\right)^2=2k^2 \tag{1.3}$$

式中

$$\Theta=\sigma_{kk} \tag{1.4}$$

σ_{ji}为应力增量，e_{ji}为应变增量，v_i为位移增量(i,j,k=1、2、3)，λ为理论比例系数，k为剪

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切屈服极限，而$\partial_k=\partial/\partial x_k$。

(1.2)式中还隐含着不可压缩条件

$$e_{kk}=\partial_k v_k=0 \tag{1.5}$$

本文中凡重复的角标按Einstein约定求和。

方程组(1.1)式、(1.2)式和(1.3)式共十六个标量方程，未知函数σ_{jl}，e_{jl}，v_i和λ也是十六个。

直接求解以上塑性动力学方程组是比较困难的。困难的来源除了文[1]所指出的存在两组非线性方程组之外，还存在着时间发展的因素。而且，由方程组(1.1)和(1.2)式可以看出，位移增量对时间的依赖关系，实际上与理论比例系数有关。这样，就便增添了塑性动力学问题的非线性程度。由于这些困难的存在，限制了塑性动力学问题的长足发展。

求塑性动力学问题精确解或通解的方法基本上也有三种。第一种方法是将塑性动力学问题诸方程缩并成经典的数理方程。这种方法适宜于简单的塑性动力学问题。在处理这种问题时，我们可以利用Dirac矩阵或Pauli矩阵[20~21]使方程降阶的性质。关于这种方法，可参阅文[1～2]，本文暂不予以讨论。本文将着重讨论下面两种方法。

第二种方法是将塑性动力学诸方程归并为所谓"一般方程"[22~23]，然后找出这些方程的一些特解[24]。文[22]将轴对称塑性变形问题归结为由五个方程决定五个未知函数；文[24]进一步将上述问题归结为由三个方程决定三个未知函数；文[23]更进一步将此问题归结为由两个方程决定两个未知函数。可以证明，在通常应用数学范围内用所谓"一般方程"处理轴对称塑性静力学或塑性动力学问题时，最少也要出现两个方程和两个未知函数。如果方程数多于两个，则多于两个的未知函数必须由其他方程来协调。推而广之，对于三维塑性动力学问题而言，一般来说最少也要出现四个方程和四个未知函数。当然，如果我们利用Dirac-Pauli表象的复变函数理论[25~28]，则又当别论，可以证明，在这种方法中，方程和未知函数的数目，都可以减少到两个。关于这一点，本文将展开讨论。

第三种方法着眼于刚塑性材料塑性动力学诸方程中的特殊结构。在方程组(1.1)式、(1.2)式和(1.3)式中，最引人注目的是应力增量的偏张量。由于有这样的特殊结构，使我们有可能采用一种极巧妙的办法，将非线性的塑性动力学问题，纳入线性的本征值问题之内，而其形式又与量子力学中的定态Schrödinger方程[29~33]相类似。当然，非线性塑性动力学问题的时间发展，与线性弹性动力学问题的时间发展相比，是完全不同的。可以看出，第三种方法是最为方便的方法。它是本文的重点。

在塑性增量理论的解求得后，通过按加载过程的程序积分，就能求得塑性的大变形的解。

本文的结果，可以推广到塑性流动理论中的其他各类材料的研究中去。其原则是相同的。

二、刚塑性材料塑性动力学问题中的一般方程——Dirac-Pauli表象的复变函数理论在塑性动力学中的应用

为了能利用Dirac-Pauli表象的复变函数理论，使刚塑性材料塑性动力学问题中的一般方程的数目减少到最低限度，我们必须引入除时间t和三维空间$x_k(k=1、2、3)$之外的第五个

坐标，即 Kaluza “鬼”坐标[25~28]。当到最后将第五维取为某一常数（此常数通常取为 0）时，就表示问题是三维（四维时空）的，而五维方程同时还原为四维方程，犹如第三维取为常数时，就表示问题是二维的，而方程同时还原为二维方程一样。第五维取为常数的哲学意义，就表示不存在任何“鬼”、“神”。但是在化简我们的方程时，先不急于取第五维为常数。借助鬼斧神工，可以给我们带来方便。

引入 Kaluza “鬼”坐标 x_4，我们将方程组(1.1)式至(1.5)式改写为

$$\partial_\beta\sigma_{\alpha\beta}=\rho\partial_t v_\alpha \tag{2.1}$$

$$e_{\alpha\beta}=\frac{1}{2}(\partial_\alpha v_\beta+\partial_\beta v_\alpha)=\lambda\left(\sigma_{\alpha\beta}-\frac{1}{4}\Theta\delta_{\alpha\beta}\right) \tag{2.2}$$

$$\left(\sigma_{\alpha\beta}-\frac{1}{4}\Theta\delta_{\alpha\beta}\right)^2=2k^2 \tag{2.3}$$

$$\Theta=\sigma_{\beta\beta} \tag{2.4}$$

$$e_{\beta\beta}=\partial_\beta v_\beta=0 \tag{2.5}$$

$$(\alpha、\beta=1,2,3,4)$$

在(2.1)式中，我们对每一个标量方程都再微分一次，然后依次相减，可得

$$\partial_1\partial_2(\sigma_{11}-\sigma_{22})-(\partial_1^2-\partial_2^2)\sigma_{12}-\partial_1\partial_3\sigma_{23}+\partial_2\partial_4\sigma_{41}$$
$$+\partial_2\partial_3\sigma_{13}-\partial_4\partial_1\sigma_{24}=\rho\partial_t(\partial_2 v_1-\partial_1 v_2) \tag{2.6a}$$

$$\partial_2\partial_3(\sigma_{22}-\sigma_{33})-(\partial_2^2-\partial_3^2)\sigma_{23}-\partial_2\partial_4\sigma_{34}+\partial_1\partial_3\sigma_{12}$$
$$+\partial_3\partial_4\sigma_{24}-\partial_1\partial_2\sigma_{13}=\rho\partial_t(\partial_3 v_2-\partial_2 v_3) \tag{2.6b}$$

$$\partial_3\partial_4(\sigma_{33}-\sigma_{44})-(\partial_3^2-\partial_4^2)\sigma_{34}-\partial_1\partial_3\sigma_{41}+\partial_2\partial_4\sigma_{23}$$
$$+\partial_4\partial_1\sigma_{13}-\partial_2\partial_3\sigma_{24}=\rho\partial_t(\partial_4 v_3-\partial_3 v_4) \tag{2.6c}$$

$$\partial_4\partial_1(\sigma_{44}-\sigma_{11})-(\partial_4^2-\partial_1^2)\sigma_{41}-\partial_2\partial_4\sigma_{12}+\partial_1\partial_3\sigma_{34}$$
$$+\partial_1\partial_2\sigma_{24}-\partial_3\partial_4\sigma_{13}=\rho\partial_t(\partial_1 v_4-\partial_4 v_1) \tag{2.6d}$$

将(2.6a)式至(2.6d)式连加，我们有

$$\partial_1\partial_2(\sigma_{11}-\sigma_{22})+\partial_2\partial_3(\sigma_{22}-\sigma_{33})+\partial_3\partial_4(\sigma_{33}-\sigma_{44})+\partial_4\partial_1(\sigma_{44}-\sigma_{11})$$
$$+(\partial_2^2-\partial_1^2+\partial_1\partial_3-\partial_2\partial_4)\sigma_{12}+(\partial_3^2-\partial_2^2+\partial_2\partial_4-\partial_1\partial_3)\sigma_{23}$$
$$+(\partial_4^2-\partial_3^2+\partial_1\partial_3-\partial_2\partial_4)\sigma_{34}+(\partial_1^2-\partial_4^2+\partial_2\partial_4-\partial_1\partial_3)\sigma_{41}$$
$$+(-\partial_1\partial_2+\partial_2\partial_3-\partial_3\partial_4+\partial_4\partial_1)\sigma_{13}+(\partial_1\partial_2-\partial_2\partial_3+\partial_3\partial_4-\partial_4\partial_1)\sigma_{24}$$
$$=\rho\partial_t(\partial_2 v_1+\partial_3 v_2+\partial_4 v_3+\partial_1 v_4-\partial_1 v_2-\partial_2 v_3-\partial_3 v_4-\partial_4 v_1) \tag{2.7}$$

同时，方程(2.3)式可以被改写成：

$$(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{44})^2+(\sigma_{44}-\sigma_{11})^2$$
$$+(\sigma_{11}-\sigma_{33})^2+(\sigma_{22}-\sigma_{44})^2+8(\sigma_{12}^2+\sigma_{23}^2+\sigma_{34}^2$$
$$+\sigma_{41}^2+\sigma_{13}^2+\sigma_{24}^2)=8k^2 \tag{2.8}$$

由文[25]，我们设

$$|z\rangle=\frac{1}{2}\gamma_\alpha|x_\alpha\rangle \tag{2.9a}$$

$$(\alpha=1,2,3,4)$$

$$|u\rangle=\frac{1}{2}\gamma_\alpha|v_\alpha\rangle \tag{2.9b}$$

式中 γ_α 为 Flügge 标准矩阵[21]，$|\rangle$ 为 Dirac 符号[20]。由(2.9b)式我们可反解出

$$\left.\begin{aligned} v_1&=i(u_1+u_2-\bar{u}_1-\bar{u}_2)/2 \\ v_2&=(-u_1+u_2-\bar{u}_1+\bar{u}_2)/2 \\ v_3&=i(u_1-u_2-\bar{u}_1+\bar{u}_2)/2 \\ v_3&=(u_1+u_2+\bar{u}_1+\bar{u}_2)/2 \end{aligned}\right\} \tag{2.10}$$

式中$\bar{u}_k$为u_k的复共轭($k=1,2$)。

将(2.9)式代入(2.5)式，此时不可压缩条件(2.5)式成为

$$\partial_k u_k+\bar{\partial}_k\bar{u}_k=0 \quad (k=1,2) \tag{2.11}$$

式中$\partial_k=\partial/\partial z_k$、$\bar{\partial}_k=\partial/\partial\bar{z}_k(k=1,2)$。而将$\partial/\partial x_\beta$ ($\beta=1,2,3,4$)改记为∂^*_β。同时，我们有

$$\left.\begin{aligned} \partial^*_1&=i(-\partial_1-\partial_2+\bar{\partial}_1+\bar{\partial}_2)/2 \\ \partial^*_2&=(-\partial_1+\partial_2-\bar{\partial}_1+\bar{\partial}_2)/2 \\ \partial^*_3&=i(-\partial_1+\partial_2+\bar{\partial}_1-\bar{\partial}_2)/2 \\ \partial^*_4&=(\partial_1+\partial_2+\bar{\partial}_1+\bar{\partial}_2)/2 \end{aligned}\right\} \tag{2.12}$$

及

$$\left.\begin{aligned} (\partial^*_1)^2=&-\frac{1}{4}(\partial_k\partial_k+\bar{\partial}_k\bar{\partial}_k)+\frac{1}{2}\partial_k\bar{\partial}_k \\ &-\frac{1}{2}(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2-\partial_1\bar{\partial}_2-\partial_2\bar{\partial}_1) \\ (\partial^*_2)^2=&\frac{1}{4}(\partial_k\partial_k+\bar{\partial}_k\bar{\partial}_k)+\frac{1}{2}\partial_k\bar{\partial}_k \\ &-\frac{1}{2}(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2+\partial_1\bar{\partial}_2+\partial_2\bar{\partial}_1) \\ (\partial^*_3)^2=&-\frac{1}{4}(\partial_k\partial_k+\bar{\partial}_k\bar{\partial}_k)+\frac{1}{2}\partial_k\bar{\partial}_k \\ &+\frac{1}{2}(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2-\partial_1\bar{\partial}_2-\partial_2\bar{\partial}_1) \\ (\bar{\partial}^*_4)^2=&\frac{1}{4}(\partial_k\partial_k+\bar{\partial}_k\bar{\partial}_k)+\frac{1}{2}\partial_k\bar{\partial}_k \\ &+\frac{1}{2}(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2+\partial_1\bar{\partial}_2+\partial_2\bar{\partial}_1) \end{aligned}\right\} \tag{2.13}$$

和

$$\left.\begin{aligned} \partial^*_1\partial^*_2&=i(\partial_1^2-\partial_2^2-\bar{\partial}_1^2+\bar{\partial}_2^2-2\partial_1\bar{\partial}_2+2\partial_2\bar{\partial}_1)/4 \\ \partial^*_2\partial^*_3&=i(\partial_1^2+\partial_2^2-\bar{\partial}_1^2-\bar{\partial}_2^2-2\partial_1\partial_2+2\bar{\partial}_1\bar{\partial}_2)/4 \\ \partial^*_3\partial^*_4&=i(-\partial_1^2+\partial_2^2+\bar{\partial}_1^2-\bar{\partial}_2^2-2\partial_1\bar{\partial}_2+2\partial_2\bar{\partial}_1)/4 \\ \partial^*_4\partial^*_1&=i(-\partial_1^2-\partial_2^2+\bar{\partial}_1^2+\bar{\partial}_2^2-2\partial_1\partial_2+2\bar{\partial}_1\bar{\partial}_2)/4 \\ \partial^*_1\partial^*_3&=(-\partial_1^2+\partial_2^2-\bar{\partial}_1^2+\bar{\partial}_2^2+2\partial_1\bar{\partial}_1-2\partial_2\bar{\partial}_2)/4 \\ \partial^*_2\partial^*_4&=(-\partial_1^2+\partial_2^2-\bar{\partial}_1^2+\bar{\partial}_2^2-2\partial_1\bar{\partial}_1+2\partial_2\bar{\partial}_2)/4 \end{aligned}\right\} \tag{2.14}$$

从而可得

$$\begin{aligned} &(\partial^*_2)^2-(\partial^*_1)^2+\partial^*_1\partial^*_3-\partial^*_2\partial^*_4 \\ &\quad=(\partial_k\partial_k+\bar{\partial}_k\bar{\partial}_k)/2-(\partial_1\bar{\partial}_2+\partial_2\bar{\partial}_1)+\partial_1\bar{\partial}_1-\partial_2\bar{\partial}_2 \end{aligned} \tag{2.15a}$$

$$(\partial^*_3)^2-(\partial^*_2)^2+\partial^*_2\partial^*_4-\partial^*_1\partial^*_3$$

$$=(\partial_k\partial_k+\bar\partial_k\bar\partial_k)/2+(\partial_1\partial_2+\bar\partial_1\bar\partial_2)-\partial_1\bar\partial_1+\partial_2\bar\partial_2 \tag{2.15b}$$

$$(\partial_4^*)^2-(\partial_3^*)^2+\partial_1^*\partial_3^*-\partial_2^*\partial_4^*$$
$$=(\partial_k\partial_k+\bar\partial_k\bar\partial_k)/2+(\partial_1\bar\partial_2+\partial_2\bar\partial_1)+\partial_1\bar\partial_1-\partial_2\bar\partial_2 \tag{2.15c}$$

$$(\partial_1^*)^2-(\partial_4)^2+\partial_2^*\partial_4^*-\partial_1^*\partial_3^*$$
$$=-(\partial_k\partial_k+\bar\partial_k\bar\partial_k)/2-(\partial_1\partial_2+\bar\partial_1\bar\partial_2)-\partial_1\bar\partial_1+\partial_2\bar\partial_2 \tag{2.15d}$$

$$-\partial_1^*\partial_2^*+\partial_2^*\partial_3^*-\partial_3^*\partial_4^*+\partial_4^*\partial_1^*$$
$$=i(-\partial_1\partial_2+\bar\partial_1\bar\partial_2-\partial_1\bar\partial_2+\partial_2\bar\partial_1) \tag{2.15e}$$

另外，由(2.10)式和(2.12)式可得

$$\sigma_{11}-\sigma_{22}=(\partial_1 v_1-\partial_2 v_2)/\lambda$$
$$=\frac{1}{2\lambda}(\partial_2 u_1-\bar\partial_1 u_1+\partial_1 u_2-\bar\partial_2 u_2-\partial_1\bar u_1+\bar\partial_2\bar u_1-\partial_2\bar u_2+\bar\partial_1\bar u_2) \tag{2.16a}$$

$$\sigma_{22}-\sigma_{33}=(\partial_2 v_2-\partial_3 v_3)/\lambda$$
$$=\frac{1}{2\lambda}(\bar\partial_1 u_1-\bar\partial_2 u_1-\bar\partial_1 u_2+\bar\partial_2 u_2+\partial_1\bar u_1-\partial_2\bar u_1-\partial_1\bar u_2+\partial_2\bar u_2) \tag{2.16b}$$

$$\sigma_{33}-\sigma_{44}=(\partial_3 v_3-\partial_4 v_4)/\lambda$$
$$=\frac{1}{2\lambda}(-\partial_2 u_1-\bar\partial_1 u_1-\partial_1 u_2-\bar\partial_2 u_2-\partial_1\bar u_1-\bar\partial_2\bar u_1-\partial_2\bar u_2-\bar\partial_1\bar u_2) \tag{2.16c}$$

$$\sigma_{44}-\sigma_{11}=(\partial_4 v_4-\partial_1 v_1)/\lambda$$
$$=\frac{1}{2\lambda}(\bar\partial_1 u_1+\bar\partial_2 u_1+\bar\partial_1 u_2+\bar\partial_2 u_2+\partial_1\bar u_1+\partial_2\bar u_1+\partial_1\bar u_2+\partial_2\bar u_2) \tag{2.16d}$$

$$\sigma_{11}-\sigma_{33}=(\partial_1 v_1-\partial_3 v_3)/\lambda$$
$$=\frac{1}{2\lambda}(\partial_2 u_1-\bar\partial_2 u_1+\partial_1 u_2-\bar\partial_1 u_2-\partial_2\bar u_1+\bar\partial_2\bar u_1-\partial_1\bar u_2+\bar\partial_1\bar u_2) \tag{2.16e}$$

$$\sigma_{22}-\sigma_{44}=(\partial_2 v_2-\partial_4 v_4)/\lambda$$
$$=\frac{1}{2\lambda}(-\partial_2 u_1-\bar\partial_2 u_1-\partial_1 u_2-\bar\partial_1 u_2-\partial_2\bar u_1-\bar\partial_2\bar u_1-\partial_1\bar u_2-\bar\partial_1\bar u_2) \tag{2.16f}$$

和

$$\sigma_{12}=\frac{1}{2\lambda}(\partial_2 v_1+\partial_1 v_2)=\frac{i}{4\lambda}(\partial_2 u_1-\bar\partial_1 u_1-\partial_1 u_2+\bar\partial_2 u_2$$
$$+\partial_1\bar u_1-\bar\partial_2\bar u_1-\partial_2\bar u_2+\bar\partial_1\bar u_2) \tag{2.17a}$$

$$\sigma_{23}=\frac{1}{2\lambda}(\partial_3 v_2+\partial_2 v_3)=\frac{i}{4\lambda}(-\bar\partial_1 u_1+\bar\partial_2 u_1+\bar\partial_1 u_2-\bar\partial_2 u_2$$
$$+\partial_1\bar u_1-\partial_2\bar u_1-\partial_1\bar u_2+\partial_2\bar u_2) \tag{2.17b}$$

$$\sigma_{34}=\frac{1}{2\lambda}(\partial_4 v_3+\partial_3 v_4)=\frac{i}{4\lambda}(\partial_2 u_1+\bar\partial_1 u_1-\partial_1 u_2-\bar\partial_2 u_2$$
$$-\partial_1\bar u_1-\bar\partial_2\bar u_1+\partial_2\bar u_2+\bar\partial_1\bar u_2) \tag{2.17c}$$

$$\sigma_{41}=\frac{1}{2\lambda}(\partial_1 v_4+\partial_4 v_1)=\frac{i}{4\lambda}(\bar\partial_1 u_1+\bar\partial_2 u_1+\bar\partial_1 u_2+\bar\partial_2 u_2$$
$$-\partial_1\bar u_1-\partial_2\bar u_1-\partial_1\bar u_2-\partial_2\bar u_2) \tag{2.17d}$$

$$\sigma_{13}=\frac{1}{2\lambda}(\partial_3 v_1+\partial_1 v_3)=\frac{1}{4\lambda}(\partial_1 u_1-\bar\partial_1 u_1-\partial_2 u_2+\bar\partial_2 u_2$$

$$-\partial_1\bar{u}_1+\bar{\partial}_1\bar{u}_1+\partial_2\bar{u}_2-\bar{\partial}_2\bar{u}_2) \qquad (2.17e)$$

$$\sigma_{24}=\frac{1}{2\lambda}(\partial_4 v_2+\partial_2 v_4)=\frac{1}{4\lambda}(-\partial_1 u_1-\bar{\partial}_1 u_1+\partial_2 u_2+\bar{\partial}_2 u_2$$

$$-\partial_1\bar{u}_1-\bar{\partial}_1\bar{u}_1+\partial_2\bar{u}_2+\bar{\partial}_2\bar{u}_2) \qquad (2.17f)$$

及

$$(\partial_2^* v_1+\partial_3^* v_2+\partial_4^* v_3+\partial_1^* v_4-\partial_1^* v_2-\partial_2^* v_3-\partial_3^* v_4-\partial_4^* v_1)$$
$$=i(-\partial_2 u_1+\bar{\partial}_2 u_1-\partial_1 u_2-\bar{\partial}_1 u_2-\partial_2\bar{u}_1+\bar{\partial}_2\bar{u}_1+\partial_1\bar{u}_2+\bar{\partial}_1\bar{u}_2) \qquad (2.18)$$

现在，由(2.11)式我们引入流函数φ:

$$u_i=\varepsilon_{ik}\partial_k\varphi \qquad (i,k=1、2) \qquad (2.19a)$$

同时有

$$\bar{u}_i=\varepsilon_{ik}\bar{\partial}_k\varphi \qquad (i,k=1、2) \qquad (2.19b)$$

式中φ为实函数,

$$\varepsilon_{ik}=i\sigma_2\delta_{ik} \qquad (2.20)$$

而σ_2为Pauli矩阵[20]，δ_{ik}为 Krönecker 符号[34~35]。

将(2.19)式代入(2.16)式、(2.17)式和(2.18)式，得:

$$\sigma_{11}-\sigma_{22}=(-\partial_1^2+\partial_2^2-\bar{\partial}_1^2+\bar{\partial}_2^2)\varphi/2\lambda \qquad (2.21a)$$
$$\sigma_{22}-\sigma_{33}=(2\partial_1\bar{\partial}_1-2\partial_2\bar{\partial}_2)\varphi/2\lambda \qquad (2.21b)$$
$$\sigma_{33}-\sigma_{44}=(\partial_1^2-\partial_2^2+\bar{\partial}_1^2-\bar{\partial}_2^2)\varphi/2\lambda \qquad (2.21c)$$
$$\sigma_{44}-\sigma_{11}=(-2\partial_1\bar{\partial}_1+2\partial_2\bar{\partial}_2)\varphi/2\lambda \qquad (2.21d)$$
$$\sigma_{11}-\sigma_{33}=(-\partial_1^2+\partial_2^2-\bar{\partial}_1^2+\bar{\partial}_2^2+2\partial_1\bar{\partial}_1-2\partial_2\bar{\partial}_2)\varphi/2\lambda \qquad (2.21e)$$
$$\sigma_{22}-\sigma_{44}=(\partial_1^2-\partial_2^2+\bar{\partial}_1^2-\bar{\partial}_2^2+2\partial_1\bar{\partial}_1-2\partial_2\bar{\partial}_2)\varphi/2\lambda \qquad (2.21f)$$

和

$$\sigma_{12}=i(\partial_1^2+\partial_2^2-\bar{\partial}_1^2-\bar{\partial}_2^2)\varphi/4\lambda \qquad (2.22a)$$
$$\sigma_{23}=i(2\partial_1\bar{\partial}_2-2\partial_2\bar{\partial}_1)\varphi/4\lambda \qquad (2.22b)$$
$$\sigma_{34}=i(\partial_1^2+\partial_2^2-\bar{\partial}_1^2-\bar{\partial}_2^2)\varphi/4\lambda \qquad (2.22c)$$
$$\sigma_{41}=i(-2\partial_1\bar{\partial}_2+2\partial_2\bar{\partial}_1)\varphi/4\lambda \qquad (2.22d)$$
$$\sigma_{13}=(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2-\partial_1\bar{\partial}_2-\partial_2\bar{\partial}_1)\varphi/2\lambda \qquad (2.22e)$$
$$\sigma_{24}=(-\partial_1\partial_2-\bar{\partial}_1\bar{\partial}_2-\partial_1\bar{\partial}_2-\partial_2\bar{\partial}_1)\varphi/2\lambda \qquad (2.22f)$$

及

$$(\partial_2^* v_1+\partial_3^* v_2+\partial_4^* v_3+\partial_1^* v_4-\partial_1^* v_2-\partial_2^* v_3-\partial_3^* v_4-\partial_4^* v_1)$$
$$=i(\partial_1^2-\partial_2^2-\bar{\partial}_1^2+\bar{\partial}_2^2)\varphi \qquad (2.23)$$

将(2.14)式、(2.15)式、(2.21)式、(2.22)式和 (2.23) 式代入方程(2.7)式，我们得到关于φ和λ的非线性方程

$$(\partial_2^2+\bar{\partial}_1^2+\partial_1\bar{\partial}_1-\partial_2\bar{\partial}_2)\left[\frac{1}{\lambda}(\partial_1^2-\bar{\partial}_2^2)\varphi\right]$$

$$+(\partial_1^2+\bar{\partial}_2^2+\partial_1\bar{\partial}_1-\partial_2\bar{\partial}_2)\left[\frac{1}{\lambda}(\partial_2^2-\bar{\partial}_1^2)\varphi\right]$$

$$+(\partial_1^2+\partial_2^2-\bar{\partial}_1^2-\bar{\partial}_2^2)\left[\frac{1}{\lambda}(\partial_1\bar{\partial}_1-\partial_2\bar{\partial}_2)\varphi\right]$$

$$+2(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2)\left[\frac{1}{\lambda}(\partial_1\bar{\partial}_2-\partial_2\bar{\partial}_1)\varphi\right]$$

$$+2(-\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2-\partial_1\bar{\partial}_2+\partial_2\bar{\partial}_1)\left[\frac{1}{\lambda}(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2)\varphi\right]$$

$$=2\rho\partial_t(\partial_1^2-\partial_2^2-\bar{\partial}_1^2+\bar{\partial}_2^2)\varphi \tag{2.24}$$

(2.24)式实际上是下列方程与其共轭方程的差：

$$(\partial_2^2+\bar{\partial}_1^2+\partial_1\bar{\partial}_1-\partial_2\bar{\partial}_2)\left[\frac{1}{\lambda}(\partial_1^2-\bar{\partial}_2^2)\varphi\right]$$

$$+(\partial_1^2+\partial_2^2)\left[\frac{1}{\lambda}(\partial_1\bar{\partial}_1-\partial_2\bar{\partial}_2)\varphi\right]$$

$$+2(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2)\left[\frac{1}{\lambda}(\partial_1\bar{\partial}_2)\varphi\right]$$

$$+2(\bar{\partial}_1\bar{\partial}_2+\partial_2\bar{\partial}_1)\left[\frac{1}{\lambda}(\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2)\varphi\right]$$

$$=2\rho\partial_t(\partial_1^2+\bar{\partial}_2^2)\varphi \tag{2.25}$$

关于 φ 和 λ 的另一个非线性方程可以从(2.8)式中得到。由(2.21)式可知

$$\left.\begin{aligned}(\sigma_{33}-\sigma_{44})^2=(\partial_{11}-\sigma_{22})^2\\(\sigma_{44}-\sigma_{11})^2=(\sigma_{22}-\sigma_{33})^2\end{aligned}\right\} \tag{2.26}$$

同时，由(2.22)式可知

$$\sigma_{34}^2=\sigma_{12}^2,\ \sigma_{41}^2=\sigma_{23}^2 \tag{2.27}$$

从而，方程(2.8)式可改写成

$$2(\sigma_{11}-\sigma_{22})^2+2(\sigma_{22}-\sigma_{33})^2+(\sigma_{11}-\sigma_{33})^2+(\sigma_{22}-\sigma_{44})^2$$
$$+8(2\sigma_{12}^2+2\sigma_{23}^2+\sigma_{13}^2+\sigma_{24}^2)=8k^2 \tag{2.28}$$

将(2.21)式和(2.22)式代入(2.28)式，我们有

$$[(\partial_1^2-\bar{\partial}_2^2)\varphi][(\bar{\partial}_1^2-\partial_2^2)\varphi]+2(\partial_1\bar{\partial}_1\varphi)^2$$
$$+2(\partial_2\bar{\partial}_2\varphi)^2+4(\partial_1\bar{\partial}_2\varphi)(\partial_2\bar{\partial}_1\varphi)=4k^2\lambda^2 \tag{2.29}$$

其中等号左端每一项都是一个量与其共轭量的乘积。

(2.25)式和(2.29)式组成刚塑性材料塑性动力学问题的所谓“一般方程”。这两个方程中只有两个未知函数，即 φ 和 λ。

如果用分离变量法来解这两个方程，则由(2.29)式可以看出，φ的时间因子 φ_t 与 λ 的时间因子λ_t是相同的，并且很容易从(2.25)式得到φ_t的时间发展是

$$\partial_t\varphi_t=\text{const} \tag{2.30}$$

(2.30)式的结果是与塑性材料的刚塑性条件相吻合的。

三、刚塑性材料塑性动力学的通解

刚塑性材料塑性动力学的通解，可以从刚塑性材料塑性动力学的本征方程中得到；而该本征方程，又可以从刚塑性材料诸方程的特殊结构中导出。为此，首先，我们用分离变量法，设理想刚塑性材料对时间的依赖关系是各向同性的，即

$$v_i=A(t)\tilde{v}_i(x_k) \tag{3.1}$$

$$\sigma_{ji}=B(t)\tilde{\sigma}_{ji}(x_k) \tag{3.2}$$

$$\lambda=C(t)\tilde{\lambda}(x_k) \tag{3.3}$$

并仍用v_i，σ_{ji}，λ来表示$\tilde{v}_i$，$\tilde{\sigma}_{ji}$，$\tilde{\lambda}$，则(1.1)式、(1.2)式和(1.3)式成为

$$\rho v_i\partial_t A=B\partial_k\sigma_{ik} \tag{3.4}$$

$$\frac{1}{2}A(\partial_j v_i+\partial_i v_j)=BC\lambda\left(\sigma_{ji}-\frac{1}{3}\Theta\delta_{ji}\right) \tag{3.5}$$

$$B^2\left(\sigma_{ji}-\frac{1}{3}\Theta\delta_{ji}\right)^2=2k^2 \tag{3.6}$$

要使方程(3.6)式在形式上与(1.3)式保持一致，最方便的方法是令

$$B(t)=1 \tag{3.7}$$

由此，如果方程(3.5)式在形式上要与(1.2)式保持一致，则必定有

$$A(t)=C(t) \tag{3.8}$$

对方程(3.4)式用分离变量法，顾及(3.7)式，我们有以下时间发展方程：

$$\rho\partial_t A=1 \tag{3.9}$$

积分后得到

$$A(t)=t/\rho \tag{3.10}$$

(3.7)式和(3.10)式的结果与(2.30)式一样，是符合材料的刚塑性假设的。

从时间发展方程(3.9)式或(2.30)式可以看出，刚塑性材料塑性动力学问题既非耗散型，又非色散型。力学问题的耗散或色散的唯一标志，就是其时间发展方程。

由于(3.9)式，这时方程(3.4)式、(3.5)式和(3.6)式成为

$$v_i=\partial_k\sigma_{ik} \tag{3.11}$$

$$e_{ji}=\frac{1}{2}(\partial_j v_i+\partial_i v_j)=\lambda\left(\sigma_{ji}-\frac{1}{3}\Theta\delta_{ji}\right) \tag{3.12}$$

$$\left(\sigma_{ji}-\frac{1}{3}\Theta\delta_{ji}\right)^2=2k^2 \tag{3.13}$$

注意其中的δ_i，σ_{ji}，λ实际上是$\tilde{v}_i$，$\tilde{\sigma}_{ji}$，$\tilde{\lambda}$。

将(3.11)式代入(3.12)式，我们有

$$\frac{1}{2}(\partial_i\partial_k\sigma_{jk}+\partial_j\partial_k\sigma_{ik})=\lambda\left(\sigma_{ji}-\frac{1}{3}\Theta\delta_{ji}\right) \tag{3.14}$$

照顾到(3.13)式，我们可以说，(3.14)式是以$\sigma_{ji}-\Theta\delta_{ji}/3$为本征函数，$\lambda$为本征值的定态Schrödinger方程，而(3.13)式是定态Schrödinger方程(3.14)式的归一化条件。

方程(3.14)式等号左端不是$\sigma_{ji}-\Theta\delta_{ji}/3$的显式。为了将它改写成关于$\sigma_{ji}-\Theta\delta_{ji}/3$为本征函数的显式，必须进行一番运算。为此，我们先写出(3.14)式的展开式：

$$\partial_1(\partial_1\sigma_{11}+\partial_2\sigma_{12}+\partial_3\sigma_{31})=e_{11} \tag{3.15a}$$

$$\partial_2(\partial_1\sigma_{12}+\partial_2\sigma_{22}+\partial_3\sigma_{23})=e_{22} \tag{3.15b}$$

$$\partial_3(\partial_1\sigma_{31}+\partial_2\sigma_{23}+\partial_3\sigma_{33})=e_{33} \tag{3.15c}$$

$$[\partial_2\partial_3(\sigma_{22}+\sigma_{33})+(\partial_2^2+\partial_3^2)\sigma_{23}+\partial_1\partial_2\sigma_{31}+\partial_3\partial_1\sigma_{12}]/2=e_{23} \tag{3.15d}$$

$$[\partial_3\partial_1(\sigma_{33}+\sigma_{11})+(\partial_3^2+\partial_1^2)\sigma_{31}+\partial_2\partial_3\sigma_{12}+\partial_1\partial_2\sigma_{23}]/2=e_{31} \tag{3.15e}$$

$$[\partial_1\partial_2(\sigma_{11}+\sigma_{22})+(\partial_1^2+\partial_2^2)\sigma_{12}+\partial_3\partial_1\sigma_{23}+\partial_2\partial_3\sigma_{31}]/2=e_{12} \tag{3.15f}$$

首先对(3.15a)、(3.15b)和(3.15c)三式各再微分两次，然后依次相减；其次对(3.15d)、(3.15e)和(3.15f)三式各再微分一次，然后依次相减；于是我们有

$$\begin{aligned}&\partial_2^2\partial_3^2(\sigma_{22}-\sigma_{33})+\partial_2\partial_3(\partial_3^2-\partial_2^2)\sigma_{23}\\&\quad-\partial_1\partial_2\partial_3(\partial_2\sigma_{31})+\partial_1\partial_2\partial_3(\partial_3\sigma_{12})=\partial_3^2e_{22}-\partial_2^2e_{33}\end{aligned} \tag{3.16a}$$

$$\begin{aligned}&\partial_3^2\partial_1^2(\sigma_{33}-\sigma_{11})+\partial_3\partial_1(\partial_1^2-\partial_3^2)\sigma_{31}\\&\quad-\partial_1\partial_2\partial_3(\partial_3\sigma_{12})+\partial_1\partial_2\partial_3(\partial_1\sigma_{23})=\partial_1^2e_{33}-\partial_3^2e_{11}\end{aligned} \tag{3.16b}$$

$$\begin{aligned}&\partial_1^2\partial_2^2(\sigma_{11}-\sigma_{22})+\partial_1\partial_2(\partial_2^2-\partial_1^2)\sigma_{12}\\&\quad-\partial_1\partial_2\partial_3(\partial_1\sigma_{23})+\partial_1\partial_2\partial_3(\partial_2\sigma_{31})=\partial_2^2e_{11}-\partial_1^2e_{22}\end{aligned} \tag{3.16c}$$

$$\begin{aligned}&\partial_1\partial_2\partial_3(\sigma_{22}-\sigma_{33})+\partial_1(\partial_3^2-\partial_2^2)\sigma_{23}\\&\quad-\partial_1^2(\partial_2\sigma_{31})+\partial_1^2(\partial_3\sigma_{12})=2(\partial_3e_{12}-\partial_2e_{31})\end{aligned} \tag{3.16d}$$

$$\begin{aligned}&\partial_1\partial_2\partial_3(\sigma_{33}-\sigma_{11})+\partial_2(\partial_1^2-\partial_3^2)\sigma_{31}\\&\quad-\partial_2^2(\partial_3\sigma_{12})+\partial_2^2(\partial_1\sigma_{23})=2(\partial_1e_{23}-\partial_3e_{12})\end{aligned} \tag{3.16e}$$

$$\begin{aligned}&\partial_1\partial_2\partial_3(\sigma_{11}-\sigma_{22})+\partial_3(\partial_2^2-\partial_1^2)\sigma_{12}\\&\quad-\partial_3^2(\partial_1\sigma_{23})+\partial_3^2(\partial_2\sigma_{31})=2(\partial_2e_{31}-\partial_1e_{23})\end{aligned} \tag{3.16f}$$

令

$$\left.\begin{aligned}(2\sigma_{11}-\sigma_{22}-\sigma_{33})/3=\sqrt{2}k\psi_1,\quad \sigma_{23}=k\psi_4\\(2\sigma_{22}-\sigma_{33}-\sigma_{11})/3=\sqrt{2}k\psi_2,\quad \sigma_{31}=k\psi_5\\(2\sigma_{33}-\sigma_{11}-\sigma_{22})/3=\sqrt{2}k\psi_3,\quad \sigma_{12}=k\psi_6\end{aligned}\right\} \tag{3.17}$$

则

$$\left.\begin{aligned}\sigma_{22}-\sigma_{33}=\sqrt{2}k(\psi_2-\psi_3)\\\sigma_{33}-\sigma_{11}=\sqrt{2}k(\psi_3-\psi_1)\\\sigma_{11}-\sigma_{22}=\sqrt{2}k(\psi_1-\psi_2)\end{aligned}\right\} \tag{3.18}$$

将(3.17)式和(3.18)式代入(3.16)式，并对(3.16d)、(3.16e)和(3.16f)三式各再微分一次，然后写成矩阵形式，有

$$\hat{L}\psi=\hat{M}(\lambda\psi) \tag{3.19}$$

式中

$$\hat{L}=\begin{bmatrix}0 & p_2^2p_3^2 & -p_2^2p_3^2 & p_2p_3(p_3^2-p_2^2) & -p_1p_2^2p_3 & p_1p_2p_3^2\\ -p_3^2p_1^2 & 0 & p_3^2p_1^2 & p_1^2p_2p_3 & p_3p_1(p_1^2-p_3^2) & -p_1p_2p_3^2\\ p_1^2p_2^2 & -p_1^2p_2^2 & 0 & -p_1^2p_2p_3 & p_1p_2^2p_3 & p_1p_2(p_2^2-p_1^2)\\ 0 & p_1^2p_2p_3 & -p_1^2p_2p_3 & p_1^2(p_3^2-p_2^2) & -p_1^3p_2 & p_3p_1^3\\ -p_1p_2^2p_3 & 0 & p_1p_2^2p_3 & p_1p_2^3 & p_2^2(p_1^2-p_3^2) & -p_2^3p_3\\ p_1p_2p_3^2 & -p_1p_2p_3^2 & 0 & -p_3^3p_1 & p_2p_3^3 & p_3^2(p_2^2-p_1^2)\end{bmatrix} \tag{3.20}$$

$$\hat{M}=\begin{bmatrix}0 & -p_3^2 & p_2^2 & 0 & 0 & 0\\ p_3^2 & 0 & -p_1^2 & 0 & 0 & 0\\ -p_2^2 & p_1^2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 2p_1p_2 & -2p_3p_1\\ 0 & 0 & 0 & -2p_1p_2 & 0 & 2p_2p_3\\ 0 & 0 & 0 & 2p_3p_1 & -2p_2p_3 & 0\end{bmatrix} \tag{3.21}$$

$$p=i\partial_k \quad (k=1、2、3) \tag{3.22}$$

$$\psi=(\psi_1,\ \psi_2,\ \psi_3,\ \psi_4,\ \psi_5,\ \psi_6)^T \tag{3.23}$$

而(3.13)式成为 ψ 的归一化条件

$$|\psi|^2=1 \tag{3.24}$$

矩阵$\hat{L}$还可以写成$\hat{M}$左乘另一矩阵$\hat{H}$，即

$$\hat{L}=\hat{M}(\hat{H}) \tag{3.25}$$

由此，方程(3.19)式可以改写成

$$\hat{M}(\hat{H}\psi-\lambda\psi)=0 \tag{3.26}$$

如果 ψ 具有非平凡解，则必定有

$$\hat{H}\psi=\lambda\psi \tag{3.27}$$

式中

$$\hat{H}=\begin{bmatrix} 0 & p_1^2 & p_1^2 & 0 & -p_3p_1 & -p_1p_2 \\ p_2^2 & 0 & p_2^2 & -p_2p_3 & 0 & -p_1p_2 \\ p_3^2 & p_3^2 & 0 & -p_2p_3 & -p_3p_1 & 0 \\ 0 & -p_2p_3/2 & -p_2p_3/2 & -(p_2^2+p_3^2)/2 & -p_1p_2/2 & -p_3p_1/2 \\ -p_3p_1/2 & 0 & -p_3p_1/2 & -p_1p_2/2 & -(p_3^2+p_1^2)/2 & -p_2p_3/2 \\ -p_1p_2/2 & -p_1p_2/2 & 0 & -p_3p_1/2 & -p_2p_3/2 & -(p_1^2+p_2^2)/2 \end{bmatrix} \tag{3.28}$$

而

$$|\psi|^2=1, \qquad p=i\partial_k \quad (k=1、2、3) \tag{3.29}$$

于是，我们重新得到了以 $\sigma_{ji}-\Theta\delta_{ji}/3$ 为本征函数，λ 为本征值的定态 Schrödinger 方程。定态Schrödinger方程(3.27)式中的$\hat{H}$算符是线性的，因此方程(3.27)式具有通解。

将定态 Schrödinger 方程(3.27)式的通解代入(3.11)式，或用文[1～2]中相同的方法，可以求得位移增量 $\tilde{v}_i$ $(i=1,2,3)$。

全部问题即可顺利解决。

从以上分析来看，刚塑性材料塑性动力学问题诸方程的特殊结构是导出定态Schrödinger方程的关键，而该定态Schrödinger方程的求解，又是全部问题的关键。不过，定态Schrödinger方程的求解倒是比其他方法来得简单[36~37]。

至此，我们已将弹塑性力学问题中的一些重要方程都化归为Schrödinger方程乃至Dirac方程的求解[38~42]。

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On the General Equation and the General Solution in Problems for Plastodynamics with Rigid-Plastic Material

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Abstract

This work is the continuation of the discussion of refs. [1-2]. We discuss the dynamics problems of ideal rigid-plastic material in the flow theory of plasticity in this paper. From introduction of the theory of functions of complex variable under Dirac-Pauli representation we can obtain a group of the so-called "general equations" (i. e. have two scalar equations) expressed by the stream function and the theoretical ratio. In this paper we also testify that the equation of evolution for time in plastodynamics problems is neither dissipative nor dispersive, and the eigenequation in plastodynamics problems is a stationary Schrödinger equation, in which we take partial tensor of stress-increment as eigenfunctions and take theoretical ratio as eigenvalues. Thus, we turn nonlinear plastodynamics problems into the solution of linear stationary Schrodinger equation, and from this we can obtain the general solution of plastodynamics problems with rigid-plastic material.