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圆柱薄壳在外压作用下屈曲的边界层理论*

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摘　要

本文依据文[14]提供的圆柱薄壳屈曲的边界层理论，以挠度为摄动参数，采用奇异摄动方法，研究了固支圆柱薄壳在侧向外压和静水外压作用下的屈曲和后屈曲性态．本文同时考虑了初始几何缺陷的影响．计算结果与实验结果的比较表明二者是一致的．

一、引　　言

弄清圆柱薄壳在外压作用下的屈曲和后屈曲性态具有十分重要的意义。因此，这一问题早就引起了研究者的注意。

早在本世纪初，von Mises (1914, 1929)[11][12]依据小挠度理论，先后导得了简支圆柱薄壳在侧向外压和静水外压作用下屈曲问题的经典解。Batdorf(1947)[3]为Mises解导得了一种简化形式。Nash(1954)[13]采用能量法，Bijlaard(1954)[5]引入柱壳有效长度，分别求得了固支圆柱薄壳在静水外压作用下的临界载荷。计算结果表明，要比按Mises公式的计算值高出50%左右。

Kempner等(1957)[9]用非线性大挠度理论研究了简支圆柱薄壳在静水外压作用下的后屈曲性态。Donnell(1956, 1958)[7][8]在大挠度分析中考虑了初始缺陷的影响。

Yamaki(1969)[19]研究了前屈曲变形对外压圆柱薄壳屈曲的影响，发现只有当几何参数$Z>100$时，非线性前屈曲的影响才可忽略。近代，Yamaki等(1974)[21]采用Galerkin法对固支完善圆柱薄壳在静水外压作用下的屈曲和后屈曲性态作了更为细致的分析。

此外，Budiansky等(1968)[6]，Amazigo等(1971)[2]，Amazigo(1974)[1]用Koiter理论研究了简支圆柱薄壳在外压作用下的初始后屈曲性态，分析表明，短柱壳对初始缺陷是敏感的。

Batdorf (1947)[3]引进屈曲载荷参数$c_p=qRL^2/\pi^2D$。目前一般认为，圆柱薄壳在静水外压作用下屈曲载荷的理论曲线可近似地表为

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* 钱伟长推荐．

$$c_\gamma=\begin{cases}1.04\,Z^{\frac{1}{2}} & \text{(简支)}\\ 1.56\,Z^{\frac{1}{2}} & \text{(固支)}\end{cases}\quad 当 Z>100 \tag{1.1}$$

即认为固支条件下的屈曲载荷为简支条件下的1.5倍。但是实验数据[15][17][18]，包括Yamaki等(1973)[20]近代实验数据，都无法证明在固支和简支屈曲载荷间有如此之大的差别。反倒是，根据固支边界条件得到的实验数据和简支的理论曲线比较一致。由于二者的条件不同，我们有理由怀疑过去的结论。

此种情况说明，对于外压柱壳的屈曲问题，特别是对于固支圆柱薄壳在外压作用下的屈曲问题，理论分析工作应当也必须进一步深入下去。

本文将依据文[14]提供的圆柱薄壳屈曲的边界层理论及其分析方法，来研究固支完善和非完善圆柱薄壳在外压作用下的屈曲和后屈曲性态。

二、外压圆柱薄壳屈曲的边界层方程

根据文[14]，我们将首先建立圆柱薄壳在外压作用下屈曲问题边界层理论的数学描述。引进

$$\left.\begin{aligned}&\bar{x}=\frac{\pi}{L}x,\ \bar{y}=\frac{y}{R},\ \beta=\frac{L}{\pi R},\ Z=\frac{L^2}{Rt}\sqrt{1-\nu^2},\ \varepsilon=\frac{\pi^2}{\sqrt{12}Z}\\ &\overline{W}=\frac{W}{t}\varepsilon\sqrt{12(1-\nu^2)},\ \overline{W}^*=\frac{W^*}{t}\varepsilon\sqrt{12(1-\nu^2)},\ \overline{F}=\frac{F}{D}\varepsilon^2\\ &\lambda_q=q/q_{cl},\ \delta_q=\frac{\Delta_x E}{L\sigma_{cl}},\ \overline{\Delta V}=\frac{(\Delta V)E}{\pi R^2 L\sigma_{cl}},\ \gamma=n/n_{cl}\end{aligned}\right\} \tag{2.1}$$

其中q_{cl}，σ_{cl}，n_{cl}分别为简支圆柱薄壳在侧向外压作用下屈曲的经典临界载荷、临界应力和周向波数，即

$$q_{cl}=\frac{\sqrt{2}}{3\sqrt{3}}\frac{\pi E}{(1-\nu^2)^{\frac{3}{4}}}\frac{R}{L}\left(\frac{t}{R}\right)^{\frac{5}{2}} \tag{2.2a}$$

$$\sigma_{cl}=\frac{\sqrt{2}}{3\sqrt{3}}\frac{\pi E}{(1-\nu^2)^{\frac{3}{4}}}\frac{R}{L}\left(\frac{t}{R}\right)^{\frac{3}{2}} \tag{2.2b}$$

$$n_{cl}=\left[\sqrt{6}\,\pi(1-\nu^2)^{\frac{1}{4}}\frac{R}{L}\sqrt{\frac{R}{t}}\right]^{\frac{1}{2}} \tag{2.2c}$$

取坐标系如图1，那么Kármán–Donnell方程可表为如下无量纲形式(略去字母上的“—”号)

$$\varepsilon^2\nabla^4W-\frac{\partial^2F}{\partial x^2}=\beta^2\left[\frac{\partial^2F}{\partial x^2}\frac{\partial^2W}{\partial y^2}-2\frac{\partial^2F}{\partial x\partial y}\frac{\partial^2W}{\partial x\partial y}+\frac{\partial^2F}{\partial y^2}\frac{\partial^2W}{\partial x^2}+\frac{\partial^2F}{\partial x^2}\frac{\partial^2W^*}{\partial y^2}-2\frac{\partial^2F}{\partial x\partial y}\frac{\partial^2W^*}{\partial x\partial y}+\frac{\partial^2F}{\partial y^2}\frac{\partial^2W^*}{\partial x^2}\right]+\frac{4}{3}(3)^{\frac{1}{4}}\lambda_q\varepsilon^{\frac{3}{2}} \tag{2.3}$$

$$\nabla^4F+\frac{\partial^2W}{\partial x^2}=\beta^2\left[\left(\frac{\partial^2W}{\partial x\partial y}\right)^2-\frac{\partial^2W}{\partial x^2}\frac{\partial^2W}{\partial y^2}+2\frac{\partial^2W}{\partial x\partial y}\frac{\partial^2W^*}{\partial x\partial y}-\frac{\partial^2W}{\partial x^2}\frac{\partial^2W^*}{\partial y^2}-\frac{\partial^2W}{\partial y^2}\frac{\partial^2W^*}{\partial x^2}\right] \tag{2.4}$$

其中

$$\nabla^4=\frac{\partial^4}{\partial x^4}+2\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+\beta^4\frac{\partial^4}{\partial y^4} \tag{2.5}$$

固支边界条件为

$$x=0,\pi;W=W_{,x}=0 \tag{2.6a}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\beta^2\frac{\partial^2F}{\partial y^2}dy+a\frac{2}{3}(3)^{\frac14}\lambda_q\varepsilon^{\frac32}=0 \tag{2.6b}$$

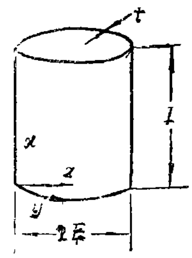

图 1

其中，$a=0$表示侧向外压；$a=1$表示静水外压。

闭合条件为

$$\int_0^{2\pi}\left[\left(\frac{\partial^2F}{\partial x^2}-\nu\beta^2\frac{\partial^2F}{\partial y^2}\right)+W-\frac12\beta^2\left(\frac{\partial W}{\partial y}\right)^2-\beta^2\frac{\partial W}{\partial y}\frac{\partial W^*}{\partial y}\right]dy=0 \tag{2.7}$$

端部缩短为

$$\delta_q=-\frac{\sqrt{3}(3)^{\frac14}}{8\pi^2}\varepsilon^{-\frac32}\int_0^{2\pi}\int_0^{\pi}\left[\left(\beta^2\frac{\partial^2F}{\partial y^2}-\nu\frac{\partial^2F}{\partial x^2}\right)-\frac12\left(\frac{\partial W}{\partial x}\right)^2-\frac{\partial W}{\partial x}\frac{\partial W^*}{\partial x}\right]dxdy \tag{2.8}$$

体积改变量为

$$\Delta V=-\frac{\sqrt{3}(3)^{\frac14}}{8\pi^2}\varepsilon^{-\frac32}\int_0^{2\pi}\int_0^{\pi}\left[\left(\beta^2\frac{\partial^2F}{\partial y^2}-\nu\frac{\partial^2F}{\partial x^2}\right)-\frac12\left(\frac{\partial W}{\partial x}\right)^2\right.$$

$$\left.-\frac{\partial W}{\partial x}\frac{\partial W^*}{\partial x}-2W\right]dxdy \tag{2.9}$$

式(2.3)至(2.9)即为边缘固支完善或非完善圆柱薄壳在外压作用下屈曲问题的控制方程。

当$Z>2.85$时，$\varepsilon<1$。方程组(2.3)、(2.4)即为边界层型方程。下面我们将用奇异摄动方法来构造其渐近解。

三、渐 近 解

设方程(2.3)、(2.4)的解可表为

$$\left.\begin{aligned}W&=w(x,y,\varepsilon)+\widetilde{W}(x,\xi,y,\varepsilon)+\widehat{W}(x,\zeta,y,\varepsilon)\\F&=f(x,y,\varepsilon)+\widetilde{F}(x,\xi,y,\varepsilon)+\widehat{F}(x,\zeta,y,\varepsilon)\end{aligned}\right\} \tag{3.1}$$

其中，$w(x,y,\varepsilon)$，$f(x,y,\varepsilon)$为壳体“外部”正则解；$\widetilde{W}(x,\xi,y,\varepsilon)$，$\widetilde{F}(x,\xi,y,\varepsilon)$及$\widehat{W}(x,\zeta,y,\varepsilon)$，$\widehat{F}(x,\zeta,y,\varepsilon)$分别为$x=0$及$x=\pi$端的边界层解。

且边界层变量

$$\xi=x/\sqrt{\varepsilon},\quad \zeta=(\pi-x)/\sqrt{\varepsilon} \tag{3.2}$$

那么，正则解$w(x,y,\varepsilon)$、$f(x,y,\varepsilon)$满足方程

$$\left.\begin{aligned}&\varepsilon^2\nabla^4w-\frac{\partial^2f}{\partial x^2}=\beta^2\left[\frac{\partial^2f}{\partial x^2}\frac{\partial^2w}{\partial y^2}-2\frac{\partial^2f}{\partial x\partial y}\frac{\partial^2w}{\partial x\partial y}+\frac{\partial^2f}{\partial y^2}\frac{\partial^2w}{\partial x^2}\right.\\&\qquad\left.+\frac{\partial^2f}{\partial x^2}\frac{\partial^2W^*}{\partial y^2}-2\frac{\partial^2f}{\partial x\partial y}\frac{\partial^2W^*}{\partial x\partial y}+\frac{\partial^2f}{\partial y^2}\frac{\partial^2W^*}{\partial x^2}\right]+\frac43(3)^{\frac14}\lambda_q\varepsilon^{\frac32}\\&\nabla^4f+\frac{\partial^2w}{\partial x^2}=\beta^2\left[\left(\frac{\partial^2w}{\partial x\partial y}\right)^2-\frac{\partial^2w}{\partial x^2}\frac{\partial^2w}{\partial y^2}+2\frac{\partial^2w}{\partial x\partial y}\frac{\partial^2W^*}{\partial x\partial y}\right.\\&\qquad\left.-\frac{\partial^2w}{\partial x^2}\frac{\partial^2W^*}{\partial y^2}-\frac{\partial^2w}{\partial y^2}\frac{\partial^2W^*}{\partial x^2}\right]\end{aligned}\right\} \tag{3.3}$$

边界层解 $\widetilde{W}(x,\xi,y,\varepsilon)$、$\widetilde{F}(x,\xi,y,\varepsilon)$ 满足方程

$$\left.\begin{aligned}
&\varepsilon D_{40}\widetilde{W}-D_{20}\widetilde{F}=\beta^2\left[\frac{\partial^2\widetilde{W}}{\partial y^2}D_{20}\widetilde{F}-2D_{10}\widetilde{W}D_{10}\widetilde{F}+\frac{\partial^2\widetilde{F}}{\partial y^2}D_{20}\widetilde{W}\right]\\
&\qquad+\beta^2\left[\frac{\partial^2 f}{\partial y^2}D_{20}\widetilde{W}+\frac{\partial^2 w}{\partial y^2}D_{20}\widetilde{F}-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 f}{\partial x\partial y}D_{10}\widetilde{W}\right.\\
&\qquad-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 w}{\partial x\partial y}D_{10}\widetilde{F}+\varepsilon\frac{\partial^2\widetilde{F}}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\varepsilon\frac{\partial^2 f}{\partial x^2}\frac{\partial^2\widetilde{W}}{\partial y^2}\\
&\qquad\left.+\frac{\partial^2 W^*}{\partial y^2}D_{20}\widetilde{F}-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 W^*}{\partial x\partial y}D_{10}\widetilde{F}+\varepsilon\frac{\partial^2\widetilde{F}}{\partial y^2}\frac{\partial^2 W^*}{\partial x^2}\right]\\
&D_{40}\widetilde{F}+\varepsilon D_{20}\widetilde{W}=\varepsilon\beta^2\left[D_{10}\widetilde{W}D_{10}\widetilde{W}-\frac{\partial^2\widetilde{W}}{\partial y^2}D_{20}\widetilde{W}\right]-\varepsilon\beta^2\left[\frac{\partial^2 w}{\partial y^2}D_{20}\widetilde{W}\right.\\
&\qquad-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 w}{\partial x\partial y}D_{10}\widetilde{W}+\varepsilon\frac{\partial^2\widetilde{W}}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 W^*}{\partial y^2}D_{20}\widetilde{W}\\
&\qquad\left.-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 W^*}{\partial x\partial y}D_{10}\widetilde{W}+\varepsilon\frac{\partial^2\widetilde{W}}{\partial y^2}\frac{\partial^2 W^*}{\partial x^2}\right]
\end{aligned}\right\}\tag{3.4}$$

边界层解 $\widehat{W}(x,\zeta,y,\varepsilon)$，$\widehat{F}(x,\zeta,y,\varepsilon)$ 满足方程

$$\left.\begin{aligned}
&\varepsilon D_{41}\widehat{W}-D_{21}\widehat{F}=\beta^2\left[\frac{\partial^2\widehat{W}}{\partial y^2}D_{21}\widehat{F}-2D_{11}\widehat{W}D_{11}\widehat{F}+\frac{\partial^2\widehat{F}}{\partial y^2}D_{21}\widehat{W}\right]\\
&\qquad+\beta^2\left[\frac{\partial^2 f}{\partial y^2}D_{21}\widehat{W}+\frac{\partial^2 w}{\partial y^2}D_{21}\widehat{F}-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 f}{\partial x\partial y}D_{11}\widehat{W}\right.\\
&\qquad-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 w}{\partial x\partial y}D_{11}\widehat{F}+\varepsilon\frac{\partial^2\widehat{F}}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\varepsilon\frac{\partial^2 f}{\partial x^2}\frac{\partial^2\widehat{W}}{\partial y^2}\\
&\qquad\left.+\frac{\partial^2 W^*}{\partial y^2}D_{21}\widehat{F}-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 W^*}{\partial x\partial y}D_{11}\widehat{F}+\varepsilon\frac{\partial^2\widehat{F}}{\partial y^2}\frac{\partial^2 W^*}{\partial x^2}\right]\\
&D_{41}\widehat{F}+\varepsilon D_{21}\widehat{W}=\varepsilon\beta^2\left[D_{11}\widehat{W}D_{11}\widehat{W}-\frac{\partial^2\widehat{W}}{\partial y^2}D_{21}\widehat{W}\right]-\varepsilon\beta^2\left[\frac{\partial^2 w}{\partial y^2}D_{21}\widehat{W}\right.\\
&\qquad-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 w}{\partial x\partial y}D_{11}\widehat{W}+\varepsilon\frac{\partial^2\widehat{W}}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 W^*}{\partial y^2}D_{21}\widehat{W}\\
&\qquad\left.-2\varepsilon^{\frac{1}{2}}\frac{\partial^2 W^*}{\partial x\partial y}D_{11}\widehat{W}+\varepsilon\frac{\partial^2\widehat{W}}{\partial y^2}\frac{\partial^2 W^*}{\partial x^2}\right]
\end{aligned}\right\}\tag{3.5}$$

其中

$$\left.\begin{aligned}
&D_{40}=\frac{\partial^4}{\partial\xi^4}+4\varepsilon^{\frac{1}{2}}\frac{\partial^4}{\partial x\partial\xi^3}+\varepsilon\left(6\frac{\partial^4}{\partial x^2\partial\xi^2}+2\beta^2\frac{\partial^4}{\partial\xi^2\partial y^2}\right)\\
&\qquad+\varepsilon^{\frac{3}{2}}\left(4\frac{\partial^4}{\partial x^3\partial\xi}+4\beta^2\frac{\partial^4}{\partial x\partial\xi\partial y^2}\right)\\
&\qquad+\varepsilon^2\left(\frac{\partial^4}{\partial x^4}+2\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+\beta^4\frac{\partial^4}{\partial y^4}\right)\\
&D_{20}=\frac{\partial^2}{\partial\xi^2}+2\varepsilon^{\frac{1}{2}}\frac{\partial^2}{\partial x\partial\xi}+\varepsilon\frac{\partial^2}{\partial x^2},\quad D_{10}=\frac{\partial^2}{\partial\xi\partial y}+\varepsilon^{\frac{1}{2}}\frac{\partial^2}{\partial x\partial y}\\
&D_{41}=\frac{\partial^4}{\partial\zeta^4}-4\varepsilon^{\frac{1}{2}}\frac{\partial^4}{\partial x\partial\zeta^3}+\varepsilon\left(6\frac{\partial^4}{\partial x^2\partial\zeta^2}+2\beta^2\frac{\partial^4}{\partial\zeta^2\partial y^2}\right)
\end{aligned}\right\}\tag{3.6}$$

$$-\varepsilon^{\frac{3}{2}}\left(4\frac{\partial^4}{\partial x^3\partial\zeta}+4\beta^2\frac{\partial^4}{\partial x\partial\zeta\partial y^2}\right)$$

$$+\varepsilon^2\left(\frac{\partial^4}{\partial x^4}+2\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+\beta^4\frac{\partial^4}{\partial y^4}\right)$$

$$D_{21}=\frac{\partial^2}{\partial\zeta^2}-2\varepsilon^{\frac{1}{2}}\frac{\partial^2}{\partial x\partial\zeta}+\varepsilon\frac{\partial^2}{\partial x^2},\quad D_{11}=\frac{\partial^2}{\partial\zeta\partial y}-\varepsilon^{\frac{1}{2}}\frac{\partial^2}{\partial x\partial y}$$

3.1 正则解

设正则解

$$w(x,y,\varepsilon)=\sum_{n=0}\varepsilon^{\frac{n}{2}}w_{\frac{n}{2}}(x,y),\quad f(x,y,\varepsilon)=\sum_{n=0}\varepsilon^{\frac{n}{2}}f_{\frac{n}{2}}(x,y) \tag{3.7}$$

并设屈曲载荷参数渐近展开为

$$\frac{4}{3}(3)^{\frac{1}{4}}\lambda_q\varepsilon^{\frac{3}{2}}=K_y=\sum_{n=0}\varepsilon^{\frac{n}{2}}k_{\frac{n}{2}} \tag{3.8}$$

取无量纲初始缺陷为

$$W^*=\varepsilon^2A_{11}^*\sin mx\sin ny=\varepsilon^2\mu A_{11}^{(2)}\sin mx\sin ny \tag{3.9}$$

其中

$$\mu=A_{11}^*/A_{11}^{(2)} \tag{3.10}$$

为缺陷参数。

将式(3.7)、(3.8)、(3.9)代入方程(3.3)得各级摄动方程，逐级摄动，我们可以得到

$$w(x,y,\varepsilon)=\varepsilon^{3/2}A_{00}^{(3/2)}+\varepsilon^2(A_{00}^{(2)}+A_{11}^{(2)}\sin mx\sin ny)$$
$$+\varepsilon^4(A_{00}^{(4)}+A_{20}^{(4)}\cos 2mx+A_{02}^{(4)}\cos 2ny)+\cdots \tag{3.11}$$

$$f(x,y,\varepsilon)=-\frac{1}{2}B_{00}^{(0)}\left(\beta^2x^2+\frac{1}{2}ay^2\right)+\varepsilon^2\left[-\frac{1}{2}B_{00}^{(2)}\left(\beta^2x^2+\frac{1}{2}ay^2\right)\right.$$
$$\left.+B_{11}^{(2)}\sin mx\sin ny\right]+\varepsilon^4\left[-\frac{1}{2}B_{00}^{(4)}\left(\beta^2x^2+\frac{1}{2}ay^2\right)\right.$$
$$\left.+B_{20}^{(4)}\cos 2mx+B_{02}^{(4)}\cos 2ny\right]+\cdots \tag{3.12}$$

其中

$$\beta^2B_{00}^{(0)}=k_0=\frac{m^4}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)}$$

$$\beta^2B_{00}^{(2)}=k_2=\frac{(m^2n^2\beta^2)^2}{\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)}$$

$$\beta^2B_{00}^{(4)}=k_4=\frac{1}{4}\frac{m^4n^2\beta^2}{(m^2+n^2\beta^2)^2}\left\{2(1+\mu)(2+\mu)\right.$$

$$+\frac{1}{4}\frac{(m^2+n^2\beta^2)^2}{n^2\beta^2\left(n^2\beta^2+\frac{1}{2}am^2\right)}\frac{1+2\mu}{1+\mu} \tag{3.13}$$

$$
\left.\begin{aligned}
&-\frac{n^2\beta^2(m^2+n^2\beta^2)^2}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)-2am^6}\left[2(1+\mu)\right.\\
&+(2+\mu)\frac{(m^2+n^2\beta^2)^2(1+2\mu)+8m^4(1+\mu)}{(m^2+n^2\beta^2)^2}\\
&\left.+\frac{\frac{1}{2}am^2}{n^2\beta^2+\frac{1}{2}am^2}\frac{1+2\mu}{1+\mu}\right]\right\}A_{11}^{(2)}A_{11}^{(2)}
\end{aligned}\right\}
$$

其它系数皆可表为$A_{11}^{(2)}$的形式，如

$$
\left.\begin{aligned}
&B_{11}^{(2)}=\frac{m^2}{(m^2+n^2\beta^2)^2}A_{11}^{(2)}\\
&A_{20}^{(4)}=-\frac{1}{8}n^2\beta^2\left(n^2\beta^2+\frac{1}{2}am^2\right)\frac{(m^2+n^2\beta^2)^2(1+2\mu)+8m^4(1+\mu)}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)-2am^6}(1+\mu)A_{11}^{(2)}A_{11}^{(2)}\\
&A_{02}^{(4)}=\frac{1}{4}\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)^2A_{11}^{(2)}A_{11}^{(2)}\\
&B_{20}^{(4)}=-\frac{1}{4}\frac{m^2n^2\beta^2\left(n^2\beta^2+\frac{1}{2}am^2\right)}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)-2am^6}\left[(1+\mu)^2\right.\\
&\qquad\left.+\frac{1}{2}(1+2\mu)\frac{\frac{1}{2}am^2}{n^2\beta^2+\frac{1}{2}am^2}\right]A_{11}^{(2)}A_{11}^{(2)}\\
&B_{02}^{(4)}=\frac{1}{32}\frac{m^2}{n^2\beta^2}(1+2\mu)A_{11}^{(2)}A_{11}^{(2)}
\end{aligned}\right\}\tag{3.14}
$$

3.2　边界层解

设边界层解为如下渐近展开

$$
\left.\begin{aligned}
&\widetilde{W}(x,\xi,y,\varepsilon)=\sum_{n=0}\varepsilon^{\frac{n}{2}+1}\widetilde{W}_{\frac{n}{2}+1}(x,\xi,y)\\
&\widetilde{F}(x,\xi,y,\varepsilon)=\sum_{n=0}\varepsilon^{\frac{n}{2}+2}\widetilde{F}_{\frac{n}{2}+2}(x,\xi,y)
\end{aligned}\right\}\tag{3.15}
$$

$$
\left.\begin{aligned}
&\widehat{W}(x,\zeta,y,\varepsilon)=\sum_{n=0}\varepsilon^{\frac{n}{2}+1}\widehat{W}_{\frac{n}{2}+1}(x,\zeta,y)\\
&\widehat{F}(x,\zeta,y,\varepsilon)=\sum_{n=0}\varepsilon^{\frac{n}{2}+2}\widehat{F}_{\frac{n}{2}+2}(x,\zeta,y)
\end{aligned}\right\}\tag{3.16}
$$

将式(3.15)代入方程(3.4)有

$$O(\varepsilon^{5/2}):\quad \frac{\partial^4\widetilde{W}_{3/2}}{\partial\xi^4}-\frac{\partial^2\widetilde{F}_{5/2}}{\partial\xi^2}=0,\qquad \frac{\partial^4\widetilde{F}_{5/2}}{\partial\xi^4}+\frac{\partial^2\widetilde{W}_{3/2}}{\partial\xi^2}=0 \tag{3.17}$$

由式(3.17)导得

$$\frac{\partial^4\widetilde{W}_{3/2}}{\partial\xi^4}+\widetilde{W}_{3/2}=0 \tag{3.18}$$

其解可表为

$$\widetilde{W}_{3/2}=(C_{01}^{(3/2)}\cos\phi\xi+C_{10}^{(3/2)}\sin\phi\xi)e^{-\alpha\xi} \tag{3.19}$$

其中

$$\phi=\alpha=\frac{1}{\sqrt{2}} \tag{3.20}$$

计及固支边界条件$x=0$，$W=W_{,x}=0$，我们得到

$$\left.\begin{aligned}\widetilde{W}_{3/2}&=-A_{00}^{(3/2)}(\cos\alpha\xi+\sin\alpha\xi)e^{-\alpha\xi}\\ \widetilde{F}_{5/2}&=A_{00}^{(3/2)}(\cos\alpha\xi-\sin\alpha\xi)e^{-\alpha\xi}\end{aligned}\right\} \tag{3.21}$$

进而，我们导得

$$\left.\begin{aligned}\widetilde{W}_2&=-A_{00}^{(2)}(\cos\alpha\xi+\sin\alpha\xi)e^{-\alpha\xi}\\ \widetilde{F}_3&=A_{00}^{(2)}(\cos\alpha\xi-\sin\alpha\xi)e^{-\alpha\xi}\end{aligned}\right\} \tag{3.22}$$

$$\left.\begin{aligned}\widetilde{W}_{5/2}&=-\sqrt{2}A_{11}^{(2)}m\sin\alpha\xi e^{-\alpha\xi}\sin ny\\ \widetilde{F}_{7/2}&=\sqrt{2}A_{11}^{(2)}m\cos\alpha\xi e^{-\alpha\xi}\sin ny\end{aligned}\right\} \tag{3.23}$$

采用类似的步骤，我们可以求得 $x=\pi$ 端的边界层解。

因此，由式(3.1)我们有

$$\begin{aligned}W&=w(x,y,\varepsilon)+\widetilde{W}(x,\xi,y,\varepsilon)+\widehat{W}(x,\zeta,y,\varepsilon)\\
&=\varepsilon^{3/2}\Big[A_{00}^{(3/2)}-A_{00}^{(3/2)}\Big(\cos\frac{x}{\sqrt{2\varepsilon}}+\sin\frac{x}{\sqrt{2\varepsilon}}\Big)\exp\Big[-\frac{x}{\sqrt{2\varepsilon}}\Big]\\
&\quad-A_{00}^{(3/2)}\Big(\cos\frac{\pi-x}{\sqrt{2\varepsilon}}+\sin\frac{\pi-x}{\sqrt{2\varepsilon}}\Big)\exp\Big[-\frac{\pi-x}{\sqrt{2\varepsilon}}\Big]\Big]\\
&\quad+\varepsilon^2\Big[A_{00}^{(2)}+A_{11}^{(2)}\sin mx\sin ny-A_{00}^{(2)}\Big(\cos\frac{x}{\sqrt{2\varepsilon}}+\sin\frac{x}{\sqrt{2\varepsilon}}\Big)\exp\Big[-\frac{x}{\sqrt{2\varepsilon}}\Big]\\
&\quad-A_{00}^{(2)}\Big(\cos\frac{\pi-x}{\sqrt{2\varepsilon}}+\sin\frac{\pi-x}{\sqrt{2\varepsilon}}\Big)\exp\Big[-\frac{\pi-x}{\sqrt{2\varepsilon}}\Big]\Big]\\
&\quad+\varepsilon^{5/2}\Big[-\sqrt{2}A_{11}^{(2)}m\sin\frac{x}{\sqrt{2\varepsilon}}\exp\Big[-\frac{x}{\sqrt{2\varepsilon}}\Big]\sin ny\\
&\quad-\sqrt{2}A_{11}^{(2)}(-1)^m m\sin\frac{\pi-x}{\sqrt{2\varepsilon}}\exp\Big[-\frac{\pi-x}{\sqrt{2\varepsilon}}\Big]\sin ny\Big]\end{aligned}$$

$$+\varepsilon^4\left[A_{00}^{(4)}+A_{20}^{(4)}\cos 2mx+A_{02}^{(4)}\cos 2ny\right]+\cdots \tag{3.24}$$

$$F=f(x,y,\varepsilon)+\widetilde{F}(x,\xi,y,\varepsilon)+\hat{F}(x,\zeta,y,\varepsilon)$$

$$=-\frac{1}{2}B_{00}^{(0)}\left(\beta^2x^2+\frac{1}{2}ay^2\right)+\varepsilon^2\left[-\frac{1}{2}B_{00}^{(2)}\left(\beta^2x^2+\frac{1}{2}ay^2\right)\right.$$

$$\left.+B_{11}^{(2)}\sin mx\sin ny\right]+\varepsilon^{5/2}\left[A_{00}^{(3/2)}\left(\cos\frac{x}{\sqrt{2\varepsilon}}-\sin\frac{x}{\sqrt{2\varepsilon}}\right)\exp\left[-\frac{x}{\sqrt{2\varepsilon}}\right]\right.$$

$$\left.+A_{00}^{(3/2)}\left(\cos\frac{\pi-x}{\sqrt{2\varepsilon}}-\sin\frac{\pi-x}{\sqrt{2\varepsilon}}\right)\exp\left[-\frac{\pi-x}{\sqrt{2\varepsilon}}\right]\right]$$

$$+\varepsilon^3\left[A_{00}^{(2)}\left(\cos\frac{x}{\sqrt{2\varepsilon}}-\sin\frac{x}{\sqrt{2\varepsilon}}\right)\exp\left[-\frac{x}{\sqrt{2\varepsilon}}\right]\right.$$

$$\left.+A_{00}^{(2)}\left(\cos\frac{\pi-x}{\sqrt{2\varepsilon}}-\sin\frac{\pi-x}{\sqrt{2\varepsilon}}\right)\exp\left[-\frac{\pi-x}{\sqrt{2\varepsilon}}\right]\right]$$

$$+\varepsilon^{7/2}\left[\sqrt{2}A_{11}^{(2)}m\cos\frac{x}{\sqrt{2\varepsilon}}\exp\left[-\frac{x}{\sqrt{2\varepsilon}}\right]\sin ny\right.$$

$$\left.+\sqrt{2}A_{11}^{(2)}(-1)^m m\cos\frac{\pi-x}{\sqrt{2\varepsilon}}\exp\left[-\frac{\pi-x}{\sqrt{2\varepsilon}}\right]\sin ny\right]$$

$$+\varepsilon^4\left[-\frac{1}{2}B_{00}^{(4)}\left(\beta^2x^2+\frac{1}{2}ay^2\right)+B_{20}^{(4)}\cos 2mx+B_{02}^{(4)}\cos 2ny\right]+\cdots \tag{3.25}$$

在式(3.24)、(3.25)中仅系数$A_{00}^{(3/2)}$，$A_{00}^{(2)}$，$A_{00}^{(4)}$，…倘未确定。将式(3.25)代入边界条件(2.6b)，导得

$$\frac{4}{3}(3)^{\frac{1}{4}}\lambda_q\varepsilon^{3/2}=K_y=\beta^2B_{00}^{(0)}+\varepsilon^2\beta^2B_{00}^{(2)}+\varepsilon^4\beta^2B_{00}^{(4)}+\cdots$$

$$=k_0+\varepsilon^2k_2+\varepsilon^4k_4+\cdots \tag{3.26}$$

将式(3.24)、(3.25)代入闭合条件(2.7)中，计及(3.26)式，我们得到

$$A_{00}^{(3/2)}=\left(1-\frac{1}{2}a\nu\right)\frac{4}{3}(3)^{\frac{1}{4}}\lambda_q,\quad A_{00}^{(2)}=0,\ A_{00}^{(4)}=\frac{1}{8}n^2\beta^2(1+2\mu)A_{11}^{(2)}A_{11}^{(2)} \tag{3.27}$$

至此，我们已经求得了圆柱薄壳在外压作用下，满足固支边界条件的Kármán-Donnell方程的大挠度渐近解。

四、屈曲和后屈曲性态

将式(3.13)代入(3.26)，我们得到

载荷参数

$$\frac{4}{3}(3)^{\frac{1}{4}}\lambda_q\varepsilon^{3/2}=\frac{m^4}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)}$$

$$+\frac{(m^2+n^2\beta^2)^2}{\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)}\varepsilon^2+\frac{1}{4}\frac{m^4n^2\beta^2}{(m^2+n^2\beta^2)^2}\left\{2(1+\mu)(2+\mu)\right.$$

$$+\frac{1}{4}\frac{(m^2+n^2\beta^2)^2}{n^2\beta^2\left(n^2\beta^2+\frac{1}{2}am^2\right)}\frac{1+2\mu}{1+\mu}$$

$$-\frac{n^2\beta^2(m^2+n^2\beta^2)^2}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)-2am^6}$$

$$\cdot\left[2(1+\mu)+(2+\mu)\frac{(m^2+n^2\beta^2)^2(1+2\mu)+8m^4(1+\mu)}{(m^2+n^2\beta^2)^2}\right.$$

$$\left.\left.+\frac{\frac{1}{2}am^2}{n^2\beta^2+\frac{1}{2}am^2}\frac{1+2\mu}{1+\mu}\right]\right\}(A_{11}^{(2)}\varepsilon^2)^2+\cdots \tag{4.1}$$

将式(3.24)、(3.25)代入式(2.8)、(2.9)，我们得到

端部缩短

$$\delta_q=\left[\left(\frac{1}{2}-\nu\right)+\frac{2\sqrt{2}}{\pi}\nu\left(1-\frac{1}{2}a\nu\right)\varepsilon^{1/2}\right]\lambda_q+\left[\frac{1}{3}(3)^{\frac{1}{4}}\frac{\sqrt{2}}{\pi}\left(1-\frac{1}{2}a\nu\right)^2\varepsilon\right]\lambda_q^2$$

$$+\left[\frac{\sqrt{3}}{32}(3)^{\frac{1}{4}}m^2(1+2\mu)\varepsilon^{-3/2}\right](A_{11}^{(2)}\varepsilon^2)^2+\cdots \tag{4.2}$$

以及体积改变量

$$\Delta V=\left[\left(2-\nu-a\nu+\frac{1}{2}a\right)-\frac{2\sqrt{2}}{\pi}(2-\nu)\left(1-\frac{1}{2}a\nu\right)\varepsilon^{1/2}\right]\lambda_q$$

$$+\left[\frac{1}{3}(3)^{\frac{1}{4}}\frac{\sqrt{2}}{\pi}\left(1-\frac{1}{2}a\nu\right)^2\varepsilon\right]\lambda_q^2+\left[\frac{\sqrt{3}}{32}(3)^{\frac{1}{4}}(m^2\right.$$

$$\left.+2n^2\beta^2)(1+2\mu)\varepsilon^{-3/2}\right](A_{11}^{(2)}\varepsilon^2)^2+\cdots \tag{4.3}$$

式中摄动参数$A_{11}^{(2)}\varepsilon^2$具有明显的物理意义。由式(3.24)，当$x=\pi/2m$，$y=\pi/2n$时，最大无量纲挠度

$$w_m=\frac{W_m}{t}\varepsilon\sqrt{12(1-\nu^2)}=\varepsilon^{3/2}A_{00}^{(3/2)}+\varepsilon^2A_{11}^{(2)}+\cdots=\left(1-\frac{1}{2}a\nu\right)K_{\varepsilon}+A_{11}^{(2)}\varepsilon^2+\cdots \tag{4.4a}$$

或者

$$\bar{w}_m=w_m-\left(1-\frac{1}{2}a\nu\right)\left[\frac{m^4}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)}+\frac{(m^2+n^2\beta^2)^2}{\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)}\varepsilon^2\right]$$

$$=A_{11}^{(2)}\varepsilon^2+\frac{1}{4}\left(1-\frac{1}{2}a\nu\right)\frac{m^4n^2\beta^2}{(m^2+n^2\beta^2)^2}$$

$$\cdot\left\{2(1+\mu)(2+\mu)+\frac{1}{4}\frac{(m^2+n^2\beta^2)^2}{n^2\beta^2\left(n^2\beta^2+\frac{1}{2}am^2\right)}\frac{1+2\mu}{1+\mu}\right.$$

$$-\frac{n^2\beta^2(m^2+n^2\beta^2)^2}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)-2am^6}\left[2(1+\mu)\right.$$

$$+(2+\mu)\frac{(m^2+n^2\beta^2)^2(1+2\mu)+8m^4(1+\mu)}{(m^2+n^2\beta^2)^2}$$

$$\left.\left.+\frac{\frac{1}{2}am^2}{n^2\beta^2+\frac{1}{2}am^2}\frac{1+2\mu}{1+\mu}\right]\right\}(A_{11}^{(2)}\varepsilon^2)^2+\cdots \tag{4.4b}$$

反之

$$A_{11}^{(2)}\varepsilon^2=\overline{w}_m-\frac{1}{4}\left(1-\frac{1}{2}a\nu\right)\frac{m^4n^2\beta^2}{(m^2+n^2\beta^2)^2}\left\{2(1+\mu)(2+\mu)\right.$$

$$+\frac{1}{4}\frac{(m^2+n^2\beta^2)^2}{n^2\beta^2\left(n^2\beta^2+\frac{1}{2}am^2\right)}\frac{1+2\mu}{1+\mu}-\frac{n^2\beta^2(m^2+n^2\beta^2)^2}{(m^2+n^2\beta^2)^2\left(n^2\beta^2+\frac{1}{2}am^2\right)(1+\mu)-2am^6}$$

$$\cdot\left[2(1+\mu)+(2+\mu)\frac{(m^2+n^2\beta^2)^2(1+2\mu)+8m^4(1+\mu)}{(m^2+n^2\beta^2)^2}\right.$$

$$\left.\left.+\frac{\frac{1}{2}am^2}{n^2\beta^2+\frac{1}{2}am^2}\frac{1+2\mu}{1+\mu}\right]\right\}\overline{w}_m^2+\cdots \tag{4.5}$$

将式(4.5)代入(4.1)、(4.2)、(4.3)我们可以得到以最大无量纲挠度为摄动参数的载荷参数，端部缩短和体积改变量。

在式(4.2)、(4.3)中小参数ε的分数指数项即为边界层的贡献。在式(4.1)中，当$w_m=0$时，我们得到屈曲载荷。但由式(4.4b)我们看到，当$w_m=0$时，$\overline{w}_m\neq0$，故$A_{11}^{(2)}\varepsilon^2\neq0$，因此，边界层对屈曲载荷亦有贡献。只有当壳体足够长时，此时小参数ε趋于零，边界层效应方可完全忽略。比照 Batdorf 屈曲载荷参数 $c_y=qRL^2/\pi^2D$，利用式(2.1)我们容易得到$c_y=1.042^{1/2}\lambda_q$。因此，当壳体足够长时，固支圆柱薄壳在外压作用下的屈曲载荷趋近 Batdorf 简支解。

五、数值计算结果

依据渐近分析导出的公式，我们分别计算了固支圆柱薄壳在侧向外压和静水外压作用下的屈曲载荷与周向波数。Z 的范围从 5 到 10^4，Poisson 比取 $\nu=0.3$，计算结果如图 2 所示。

图示表明，当几何参数Z比较小时，侧向外压和静水外压屈曲载荷曲线取两种不同的趋

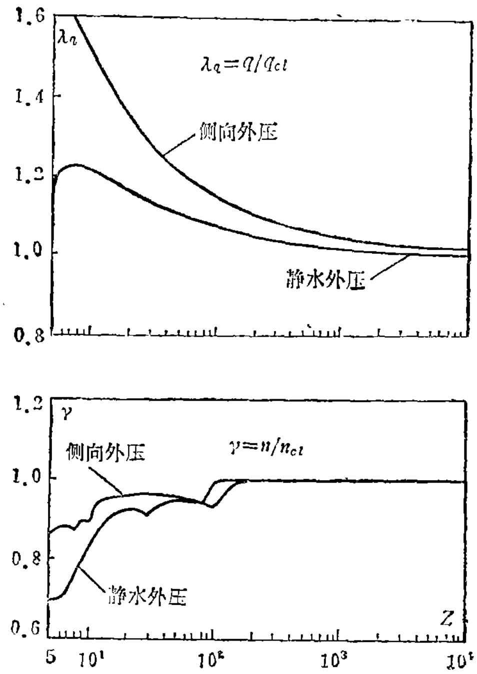

图 2 圆柱薄壳在外压作用下屈曲载荷与周向波数理论曲线

势，这一结论和经典结果是完全不同的。当$Z>100$时，固支圆柱薄壳在外压作用下的屈曲载荷与简支圆柱薄壳受侧向外压的经典值相差不多，而相应的周向波数几乎和经典解相等，或者少许低一点。

图 3 为本文理论曲线与Batdorf(1947)[3]简支圆柱薄壳理论解的比较。当$Z>100$时，两

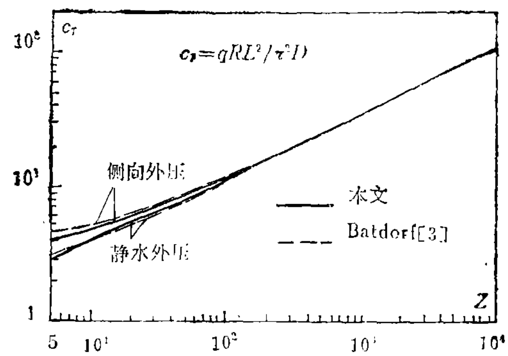

图 3 圆柱薄壳在外压作用下屈曲载荷理论曲线与经典解比较

组曲线非常接近，而当Z较小时，本文的结果低于Batdorf经典解。

图 4 为固支圆柱薄壳在静水外压作用下屈曲的理论曲线与以往实验结果[15,17,18,20]的比较，其中载荷参数$c_p=qRL^2/\pi^2D$，周向波数参数$\eta=nL/\pi R$。可以看出，包括$Z<100$的值在内，在理论和实验结果之间得到了非常合理的符合。

图 5 为对应不同几何参数Z的圆柱薄壳在侧向外压和静水外压作用下的后屈曲平衡路径。图示表明，对应$Z>100$，在后屈曲阶段，随着变形的增加常常要求压力有所增加，且随着Z值的增加，载荷增加逐趋平缓。这一事实曾为Kempner(1957)[9]，Donnell[8]所指出。而对于较小的Z，后屈曲平衡路径呈下降趋势。此时壳体对初始缺陷变得敏感。这一结论与Budiansky和Amazigo(1968)[6]初始后屈曲分析结果是一致的。

图 6 和图 7 显示了典型的圆柱薄壳在侧向外压和静水外压作用下的后屈曲平衡路径。可以看出，由于边界层的贡献，对非完善壳体，当挠度等于零($W_m=0$)时，载荷并不为零。这一结果也是和以往结果完全不同的。

总的看来，圆柱薄壳在外压作用下的后屈曲性态，同样也是主要依赖于壳体本身的特性。

六、结　　语

本文依据圆柱薄壳屈曲的边界层理论，将Karman-Donnell方程化为边界层型方程，采用奇异摄动方法，求得其大挠度渐近解，再以挠度为摄动参数研究了固支完善和非完善圆柱薄壳在外压作用下的屈曲和后屈曲性态，得到了一些新的结果。

外压柱壳的后屈曲性态主要依赖于壳体本身的特性。对于短壳，边界层效应的影响尤为重要。此时壳体对初始缺陷变得敏感，因而造成屈曲载荷的降低和实验结果的离散。

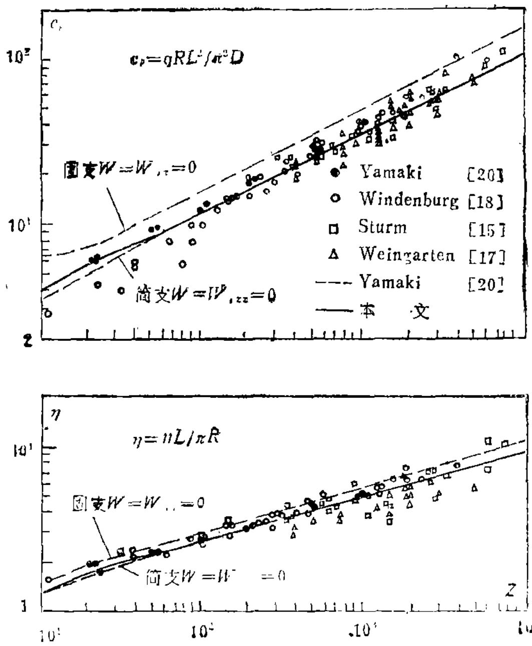

图 4 圆柱薄壳在静水外压作用下理论曲线与实验结果比较

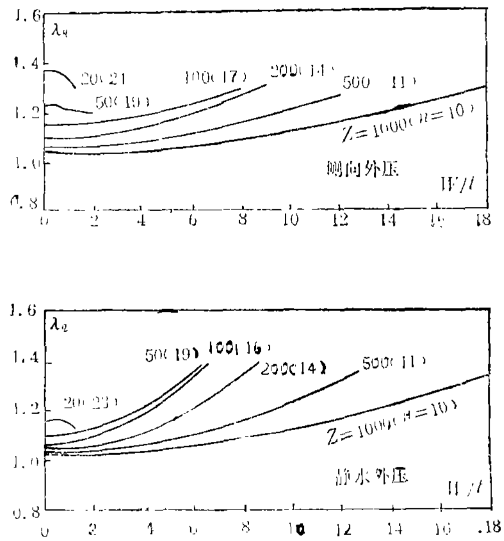

图 5 不同几何参数，后屈载载荷—挠度曲线比较

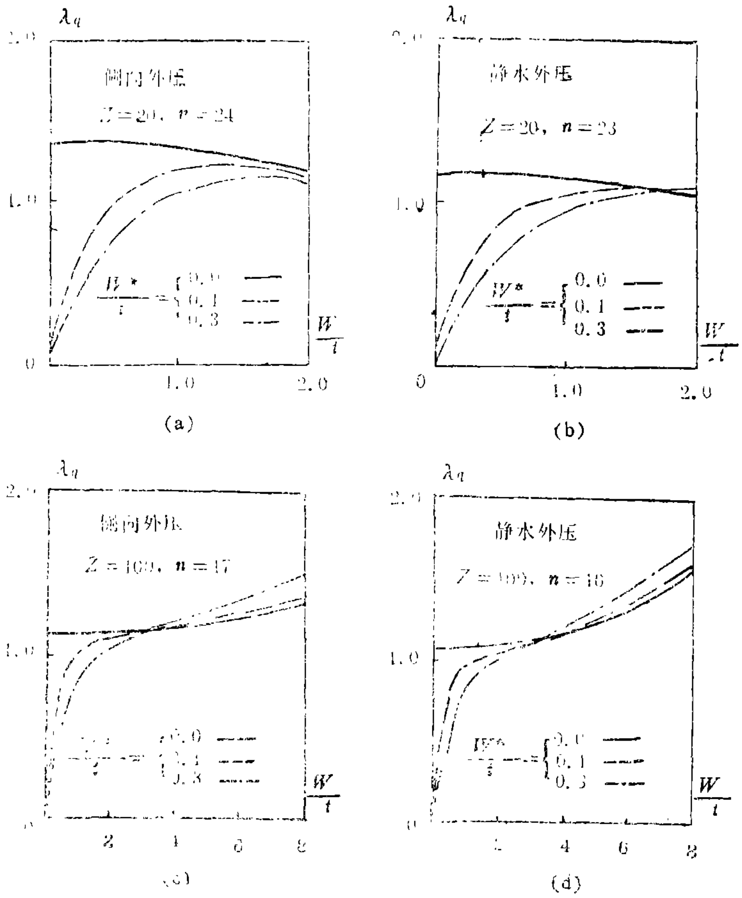

图 6 后屈曲载荷—挠度曲线

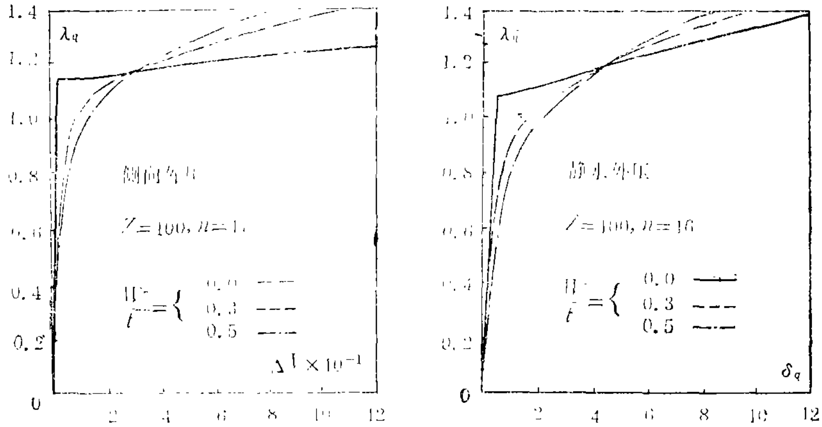

图 7 后屈曲荷载—端部缩短或载荷—体积改变曲线

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A Boundary Layer Theory for the Buckling of Thin Cylindrical Shells under External Pressure

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Abstract

Based on the boundary layer theory for the buckling of thin elastic shells suggested in ref. [14], the buckling and postbuckling behavior of clamped circular cylindrical shells under lateral or hydrostatic pressure is studied applying singular perturbation method by taking deflection as perturbation parameter. The effects of initial geometric imperfection are also considered. Some numerical results for perfect and imperfect cylindrical shells are given. The analytical results obtained are compared with some experimental data in detail, which shows that both are rather coincident.