

修正迭代法在波纹圆板非线性 振动问题中的应用

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摘 要

在本文中, 我们将修正迭代法成功地推广运用于全波纹圆板的非线性振动问题的研究, 获得了全波纹圆板的非线性振频和振幅的解析关系式. 本文还讨论了波纹圆板的几何参量对其振动特性的影响. 本文结果对精密仪器弹性元件的设计具有一定的实际意义.

一、引 言

波纹圆板是一种压有同心折皱的圆板, 常用来作为精密仪器的弹性元件. 由于波纹圆板的形状复杂和几何参数众多, 便给理论和实验的研究都带来了巨大的困难.

近年来, 刘人怀^[1~9]依照正交各向异性圆薄板的非线性弯曲理论^[10], 使用修正迭代法^[11~13], 成功地解决了波纹圆板和波纹环形板的一系列非线性弯曲问题. 接着, 我们^[14]又用Галеркин方法进一步研究了波纹圆板的非线性自由振动问题, 获得了满意的结果. 据我们所知, 迄今仅有王新志^[15]对这一问题作过初步研究.

本文是前文^[14]的继续. 按照我们建立的波纹圆板的非线性振动理论^[14], 我们改用修正迭代法重新处理了这个问题, 所得结果与文献[14]结果十分吻合. 显而易见, 本文的解不仅精度高, 而且较以前的解简单. 因此, 把修正迭代法用于研究其它板壳的非线性振动问题将是十分有意义的.

二、非线性特征值问题

我们考虑图1所示周边固定的全波纹圆板. 这里, 板半径为 a , 板厚为 h , 径向坐标为 r , 波纹波长为 l , 波幅为 H , 材料弹性模量为 E , 泊松比为 ν , 板单位面积质量为 m , 横向挠度为 w , 径向位移为 u , 径向薄膜力为 N_r .

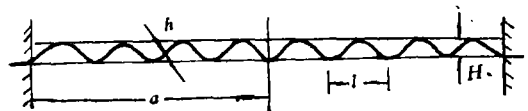


图1 全波纹圆板

将这个全波纹圆板等效为正交各向异性圆薄板后, 便可得到下列波纹圆板的非线性自由振动问题的偏微分方程组^[14]:

$$\left. \begin{aligned} m \frac{\partial^2 w}{\partial t^2} + D k_i^{-1} \left(\frac{\partial^4 w}{\partial r^4} + \frac{2}{r} \frac{\partial^3 w}{\partial r^3} - \frac{k_i k_i'}{r^2} \frac{\partial^2 w}{\partial r^2} + \frac{k_i k_i'}{r^3} \frac{\partial w}{\partial r} \right) \\ - \frac{1}{r} \frac{\partial}{\partial r} \left(r N_r \frac{\partial w}{\partial r} \right) = 0 \\ r \frac{\partial^2}{\partial r^2} (r N_r) + \frac{\partial}{\partial r} (r N_r) - k_r k_i N_r = -\frac{1}{2} k_i E h \left(\frac{\partial w}{\partial r} \right)^2 \end{aligned} \right\} \quad (2.1)$$

其中 k_r , k_i 和 k_i' 分别是与波纹圆板的径向和周向刚度有关的参数^[1~9], D 是抗弯刚度,

$$D = \frac{E h^3}{12 \left(1 - \frac{\nu^2}{k_i k_i'} \right)} \quad (2.2)$$

关于径向位移 u 和径向横剪力 Q_r , 我们还有下列公式

$$\left. \begin{aligned} u = \frac{r}{E h k_i} \left[\frac{\partial}{\partial r} (r N_r) - \nu N_r \right] \\ Q_r = -D k_i^{-1} \left(\frac{\partial^3 w}{\partial r^3} + \frac{1}{r} \frac{\partial^2 w}{\partial r^2} - \frac{k_i k_i'}{r^2} \frac{\partial w}{\partial r} \right) \end{aligned} \right\} \quad (2.3)$$

在周边夹紧固定的情况下, 其边界条件为

$$\left. \begin{aligned} \text{当 } r=0 \text{ 时, } w \text{ 有限, } \frac{\partial w}{\partial r} = 0, N_r \text{ 有限, } Q_r \text{ 有限} \\ \text{当 } r=a \text{ 时, } w=0, \frac{\partial w}{\partial r} = 0, u=0 \end{aligned} \right\} \quad (2.4)$$

为使以下运算简便, 引入下列无量纲量

$$\left. \begin{aligned} \bar{w} = \frac{w}{h}, \quad \bar{N}_r = -\frac{a}{D} k_i r N_r, \quad \tau = t \left(\frac{D}{m a^4 k_i} \right)^{\frac{1}{2}}, \quad \rho = \frac{r}{a} \\ \alpha^2 = k_i k_i', \quad \beta^2 = k_r k_i, \quad \lambda = 6 k_i' \left(1 - \frac{\nu^2}{\alpha^2} \right) \end{aligned} \right\} \quad (2.5)$$

再利用(2.3), 则方程(2.1)和边界条件(2.4)化为如下无量纲形式

$$\left. \begin{aligned} \frac{\partial^2 \bar{w}}{\partial \tau^2} + \frac{\partial^4 \bar{w}}{\partial \rho^4} + \frac{2}{\rho} \frac{\partial^3 \bar{w}}{\partial \rho^3} - \frac{\alpha^2}{\rho^2} \frac{\partial^2 \bar{w}}{\partial \rho^2} + \frac{\alpha^2}{\rho^3} \frac{\partial \bar{w}}{\partial \rho} \\ + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\bar{N}_r \frac{\partial \bar{w}}{\partial \rho} \right) = 0 \\ \rho^2 \frac{\partial^2 \bar{N}_r}{\partial \rho^2} + \rho \frac{\partial \bar{N}_r}{\partial \rho} - \beta^2 \bar{N}_r = \lambda \rho \left(\frac{\partial \bar{w}}{\partial \rho} \right)^2 \end{aligned} \right\} \quad (2.6a, b)$$

$$\text{当 } \rho=0 \text{ 时, } \bar{w} \text{ 有限, } \frac{\partial \bar{w}}{\partial \rho} = 0, \bar{N}_r = 0, \lim_{\rho \rightarrow 0^+} \left(\rho \frac{\partial^3 \bar{w}}{\partial \rho^3} + \frac{\partial^2 \bar{w}}{\partial \rho^2} - \frac{\alpha^2}{\rho} \frac{\partial \bar{w}}{\partial \rho} \right) = 0 \quad (2.7)$$

$$\text{当 } \rho=1 \text{ 时, } \bar{w}=0, \frac{\partial \bar{w}}{\partial \rho} = 0, \frac{\partial}{\partial \rho} \bar{N}_r - \nu \bar{N}_r = 0 \quad (2.8)$$

为求得全波纹圆板的非线性自由振动基频, 将 \bar{w} 取为谐振动形式的函数

$$\bar{w} = W(\rho) \cos \omega \tau \quad (2.9)$$

在此情况下, 由方程(2.6b), 我们把 \bar{N}_r 取为如下形式

$$\bar{N}_r = N(\rho) \cos^2 \omega \tau \quad (2.10)$$

其中 $W(\rho)$ 和 $N(\rho)$ 为未知函数, ω 为无量纲振频。

将式(2.9)和(2.10)代入方程(2.6a)的左端可得到

$$R(\rho, \tau) = \left[\frac{d^4 W}{d\rho^4} + \frac{2}{\rho} \frac{d^3 W}{d\rho^3} - \frac{\alpha^2}{\rho^2} \frac{d^2 W}{d\rho^2} + \frac{\alpha^2}{\rho^3} \frac{dW}{d\rho} - \omega^2 W \right] \cos \omega \tau + \frac{1}{\rho} \frac{d}{d\rho} \left(N \frac{dW}{d\rho} \right) \cos^3 \omega \tau \neq 0 \quad (2.11)$$

所以(2.9)和(2.10)不满足方程(2.6a), 应用熟知的 Kantorovich 平均法^[10], 应有

$$\int_0^{2\pi} R(\rho, \tau) \cos \omega \tau d\tau = 0 \quad (2.12)$$

将式(2.11)中的 $R(\rho, \tau)$ 表达式代入(2.12), 则可得

$$\frac{d^4 W}{d\rho^4} + \frac{2}{\rho} \frac{d^3 W}{d\rho^3} - \frac{\alpha^2}{\rho^2} \frac{d^2 W}{d\rho^2} + \frac{\alpha^2}{\rho^3} \frac{dW}{d\rho} - \omega^2 W + \frac{3}{4} \frac{d}{d\rho} \left(N \frac{dW}{d\rho} \right) = 0 \quad (2.13a)$$

再将式(2.9)和(2.10)代入方程(2.6b)又得到

$$\rho^2 \frac{d^2 N}{d\rho^2} + \rho \frac{dN}{d\rho} - \beta^2 N = \lambda \rho \left(\frac{dW}{d\rho} \right)^2 \quad (2.13b)$$

最后, 应用式(2.9)和(2.10), 边界条件(2.7)和(2.8)化为

$$\left. \begin{aligned} \text{当 } \rho=0 \text{ 时, } W \text{ 有限, } \frac{dW}{d\rho} = 0, N = 0 \\ \lim_{\rho \rightarrow 0^+} \left(\rho \frac{d^3 W}{d\rho^3} + \frac{d^2 W}{d\rho^2} - \frac{\alpha^2}{\rho} \frac{dW}{d\rho} \right) = 0 \end{aligned} \right\} \quad (2.14a, b, c, d)$$

$$\text{当 } \rho=1 \text{ 时, } W = 0, \frac{dW}{d\rho} = 0, \frac{dN}{d\rho} - \nu N = 0 \quad (2.15a, b, c)$$

这样, 全波纹圆板的非线性自由振动问题就归结为求解无量纲非线性特征值问题(2.13)~(2.15)。显而易见, 求解这个问题是十分困难的。

三、非线性特征值问题的解

现在, 我们使用修正迭代法求解上述非线性特征值问题(2.13)~(2.15)。在一次近似中, 略去方程(2.13a)中的非线性项, 便得如下线性特征值问题

$$\left. \begin{aligned} \frac{d^4 W_1}{d\rho^4} + \frac{2}{\rho} \frac{d^3 W_1}{d\rho^3} - \frac{\alpha^2}{\rho^2} \frac{d^2 W_1}{d\rho^2} + \frac{\alpha^2}{\rho^3} \frac{dW_1}{d\rho} - \omega_0^2 W_1 = 0 \\ \rho^2 \frac{d^2 N_1}{d\rho^2} + \rho \frac{dN_1}{d\rho} - \beta^2 N_1 = \lambda \rho \left(-\frac{dW_1}{d\rho} \right)^2 \end{aligned} \right\} \quad (3.1a, b)$$

$$\left. \begin{aligned} \text{当 } \rho=0 \text{ 时, } W_1 \text{ 有限, } \frac{dW_1}{d\rho} = 0, N_1 = 0; \\ \lim_{\rho \rightarrow 0^+} \left(\rho \frac{d^3 W_1}{d\rho^3} + \frac{d^2 W_1}{d\rho^2} - \frac{\alpha^2}{\rho} \frac{dW_1}{d\rho} \right) = 0 \end{aligned} \right\} \quad (3.2a, b, c, d)$$

$$\text{当 } \rho=1 \text{ 时, } W_1 = 0, \frac{dW_1}{d\rho} = 0, \frac{dN_1}{d\rho} - \nu N_1 = 0 \quad (3.3a, b, c)$$

考虑中心条件(3.2a, b, d), 则可用下列幂级数表示方程(3.1a)的解

$$W_1 = W_m \left(\sum_{k=0}^{\infty} a_k^{(1)} \rho^{4k} + \varepsilon_1 \sum_{k=0}^{\infty} b_k^{(1)} \rho^{4k+1+\alpha} \right) \quad (3.4)$$

其中 W_m 为波纹圆板的中心振幅, ε_1 为待定系数,

$$\left. \begin{aligned} a_0^{(1)} &= 1, \quad b_0^{(1)} = 1 \\ a_{k+1}^{(1)} &= \frac{\omega_0^2}{8(2k+1)(k+1)(4k+3+\alpha)(4k+3-\alpha)} a_k^{(1)} \\ b_{k+1}^{(1)} &= \frac{\omega_0^2}{8(k+1)(2k+2+\alpha)(4k+3+\alpha)(4k+5+\alpha)} b_k^{(1)} \end{aligned} \right\} \quad (k=0, 1, 2, 3, \dots) \quad (3.5)$$

将解(3.4)代入边界条件(3.3a, b)便得

$$\left. \begin{aligned} \sum_{k=0}^{\infty} a_k^{(1)} + \varepsilon_1 \sum_{k=0}^{\infty} b_k^{(1)} &= 0 \\ \sum_{k=0}^{\infty} 4ka_k^{(1)} + \varepsilon_1 \sum_{k=0}^{\infty} (4k+1+\alpha) b_k^{(1)} &= 0 \end{aligned} \right\} \quad (3.6a, b)$$

从这两个方程消去待定系数 ε_1 , 可得到如下决定线振频 ω_0 的超越方程

$$\sum_{k=0}^{\infty} a_k^{(1)} \left[\sum_{k=0}^{\infty} (4k+1+\alpha) b_k^{(1)} \right] - \sum_{k=0}^{\infty} b_k^{(1)} \left[\sum_{k=0}^{\infty} 4ka_k^{(1)} \right] = 0 \quad (3.7)$$

由此方程解出 ω_0 后, 我们由(3.5)给出诸系数 $a_k^{(1)}$, $b_k^{(1)}$ 的值. 最后, 由(3.6a)得到

$$\varepsilon_1 = - \frac{\sum_{k=0}^{\infty} a_k^{(1)}}{\sum_{k=0}^{\infty} b_k^{(1)}} \quad (3.8)$$

将解(3.4)代入方程(3.1b), 并利用边界条件(3.2c)及(3.3c), 可得到无量纲径向薄膜力的解为

$$N_1 = W_m^2 \left(A \rho^\beta + \sum_{k=0}^{\infty} c_k^{(1)} \rho^{4k-1} + \sum_{k=0}^{\infty} d_k^{(1)} \rho^{4k+\alpha} + \sum_{k=0}^{\infty} e_k^{(1)} \rho^{4k+2\alpha+1} \right) \quad (3.9)$$

其中

$$\begin{aligned} A &= (\nu - \beta)^{-1} \left[\sum_{k=0}^{\infty} (4k-1-\nu) c_k^{(1)} + \sum_{k=0}^{\infty} (4k+\alpha-\nu) d_k^{(1)} \right. \\ &\quad \left. + \sum_{k=0}^{\infty} (4k+2\alpha+1-\nu) e_k^{(1)} \right] \end{aligned}$$

$$\left. \begin{aligned}
 c_k^{(1)} &= \frac{16\lambda \sum_{j=0}^k j(k-j) a_j^{(1)} a_{k-j}^{(1)}}{(4k-1)^2 - \beta^2} \\
 d_k^{(1)} &= \frac{8\lambda \varepsilon_1 \sum_{j=0}^k j(4k-4j+1+\alpha) a_j^{(1)} b_{k-j}^{(1)}}{(4k+\alpha)^2 - \beta^2} \\
 e_k^{(1)} &= \frac{\lambda \varepsilon_1^2 \sum_{j=0}^k (4j+1+\alpha)(4k-4j+1+\alpha) b_j^{(1)} b_{k-j}^{(1)}}{(4k+2\alpha+1)^2 - \beta^2}
 \end{aligned} \right\} \quad (3.10)$$

($k=0, 1, 2, 3 \dots$)

在二次近似中, 我们有如下线性特征值问题

$$\begin{aligned}
 \frac{d^4 W_2}{d\rho^4} + \frac{2}{\rho} \frac{d^3 W_2}{d\rho^3} - \frac{\alpha^2}{\rho^2} \frac{d^2 W_2}{d\rho^2} + \frac{\alpha^2}{\rho^3} \frac{dW_2}{d\rho} \\
 - \omega^2 W_2 + \frac{3}{4} \frac{d}{d\rho} \left(N_1 \frac{dW_1}{d\rho} \right) = 0
 \end{aligned} \quad (3.11)$$

$$\left. \begin{aligned}
 \text{当 } \rho=0, W_2 \text{ 有限, } \frac{dW_2}{d\rho} = 0 \\
 \lim_{\rho \rightarrow 0^+} \left(\rho \frac{d^3 W_2}{d\rho^3} + \frac{d^2 W_2}{d\rho^2} - \frac{\alpha^2}{\rho} \frac{dW_2}{d\rho} \right) = 0
 \end{aligned} \right\} \quad (3.12a, b, c)$$

$$\text{当 } \rho=1, W_2=0, \frac{dW_2}{d\rho} = 0 \quad (3.13a, b)$$

将解(3.4)和(3.9)代入方程(3.11), 注意到中心条件(3.12), 可将方程(3.11)的解取成如下幂级数的形式

$$\begin{aligned}
 W_2 = & W_m \left(\sum_{k=0}^{\infty} a_k^{(2)} \rho^{4k} + \varepsilon_2 \sum_{k=0}^{\infty} b_k^{(2)} \rho^{4k+1+\alpha} \right) \\
 & + W_m^3 \left(\sum_{k=0}^{\infty} c_k^{(2)} \rho^{4k-4} + \sum_{k=0}^{\infty} d_k^{(2)} \rho^{4k+\alpha-3} + \sum_{k=0}^{\infty} e_k^{(2)} \rho^{4k+\beta-3} \right. \\
 & \left. + \sum_{k=0}^{\infty} f_k^{(2)} \rho^{4k+2\alpha-2} + \sum_{k=0}^{\infty} g_k^{(2)} \rho^{4k+3\alpha-1} + \sum_{k=0}^{\infty} q_k^{(2)} \rho^{4k+\alpha+\beta-2} \right)
 \end{aligned} \quad (3.14)$$

其中 ε_2 为待定系数

$$a_0^{(2)} = 1, b_0^{(2)} = 1$$

$$c_0^{(2)} = d_0^{(2)} = e_0^{(2)} = f_0^{(2)} = g_0^{(2)} = q_0^{(2)} = c_1^{(2)} = d_1^{(2)} = 0$$

$$a_{k+1}^{(2)} = \frac{\omega^2}{8(k+1)(2k+1)(4k+3+\alpha)(4k+3-\alpha)} a_k^{(2)}$$

$$b_{k+1}^{(2)} = \frac{\omega^2}{8(k+1)(2k+2+\alpha)(4k+5+\alpha)(4k+3+\alpha)} b_k^{(2)}$$

$$\begin{aligned}
c_{k+1}^{(2)} &= \frac{\omega^2 c_k^{(2)} - 6(2k-1) \sum_{j=0}^k j a_j^{(1)} c_{k-j}^{(1)}}{8k(2k-1)(4k-\alpha-1)(4k+\alpha-1)} \quad (k \geq 1) \\
d_{k+1}^{(2)} &= \frac{4\omega^2 d_k^{(2)} - 3(4k+\alpha-1) \sum_{j=0}^k [(4j+1+\alpha) \varepsilon_1 b_j^{(1)} c_{k-j}^{(1)} + 4j a_j^{(1)} d_{k-j}^{(1)}]}{32k(2k+\alpha)(4k+\alpha+1)(4k+\alpha-1)} \quad (k \geq 1) \\
e_{k+1}^{(2)} &= \frac{\omega^2 e_k^{(2)} - 3k(4k+\beta-1) A a_k^{(1)}}{(4k+1+\beta)(4k-1+\beta)(4k+\beta-\alpha)(4k+\beta+\alpha)} \\
f_{k+1}^{(2)} &= \frac{2\omega^2 f_k^{(2)} - 3(2k+\alpha) \sum_{j=0}^k [(4j+1+\alpha) \varepsilon_1 b_j^{(1)} d_{k-j}^{(1)} + 4j a_j^{(1)} e_{k-j}^{(1)}]}{8(2k+\alpha)(2k+\alpha+1)(4k+\alpha+1)(4k+3\alpha+1)} \\
g_{k+1}^{(2)} &= \frac{4\omega^2 g_k^{(2)} - 3(4k+3\alpha+1) \sum_{j=0}^k (4j+1+\alpha) \varepsilon_1 b_j^{(1)} e_{k-j}^{(1)}}{16(2k+\alpha+1)(2k+2\alpha+1)(4k+3\alpha+1)(4k+3\alpha+3)} \\
q_{k+1}^{(2)} &= \frac{4\omega^2 q_k^{(2)} - 3(4k+\alpha+\beta)(4k+\alpha+1) A e_1 b_k^{(1)}}{4(4k+\beta+1)(4k+\alpha+\beta)(4k+\alpha+\beta+2)(4k+2\alpha+\beta+1)} \\
&\quad (k=0, 1, 2, 3 \dots)
\end{aligned} \tag{3.15}$$

将解(3.14)代入边界条件(3.13), 即得到如下决定波纹圆板非线性振频 ω 的超越方程组

$$\left. \begin{aligned}
W_m [S_1(\omega) + \varepsilon_2 S_3(\omega)] + W_m^3 S_8(\omega) &= 0 \\
W_m [S_4(\omega) + \varepsilon_2 S_2(\omega)] + W_m^3 S_5(\omega) &= 0
\end{aligned} \right\} \tag{3.16a, b}$$

其中

$$\left. \begin{aligned}
S_1(\omega) &= \sum_{k=0}^{\infty} a_k^{(2)}, \quad S_2(\omega) = \sum_{k=0}^{\infty} (4k+1+\alpha) b_k^{(2)} \\
S_3(\omega) &= \sum_{k=0}^{\infty} b_k^{(2)}, \quad S_4(\omega) = \sum_{k=0}^{\infty} 4k a_k^{(2)} \\
S_5(\omega) &= \sum_{k=0}^{\infty} [4(k-1)c_k^{(2)} + (4k+\alpha-3)d_k^{(2)} + (4k+\beta-2)e_k^{(2)} + 2(2k \\
&\quad + \alpha - 1) f_k^{(2)} + (4k+3\alpha-1) g_k^{(2)} + (4k+\alpha+\beta-2) q_k^{(2)}] \\
S_8(\omega) &= \sum_{k=0}^{\infty} [c_k^{(2)} + d_k^{(2)} + e_k^{(2)} + f_k^{(2)} + g_k^{(2)} + q_k^{(2)}]
\end{aligned} \right\} \tag{3.17}$$

由(3.15)看出, (3.17)中的 $S_1(\omega) \sim S_8(\omega)$ 均是关于 ω 的无穷幂级数, 收敛速度快。从两个方程(3.16a, b)中消去 ε_2 后, 我们得到

$$W_m^2 = \frac{S_1(\omega)S_2(\omega) - S_3(\omega)S_4(\omega)}{S_3(\omega)S_5(\omega) - S_2(\omega)S_8(\omega)} \tag{3.18}$$

特别地, 如果在式(3.18)中取 $W_m=0$, 则式(3.18)退化为线性情形(3.7)。

从式(3.18)就可给出 ω - W_m 关系, 或可转化为 T_{NL}/T_L - W_m 关系; 这里, T_{NL} 是非线性

周期

$$T_{NL} = \frac{2\pi}{\omega} \quad (3.19)$$

T_L 是线性周期

$$T_L = \frac{2\pi}{\omega_0} \quad (3.20)$$

我们先由(3.18)解出 ω , 然后由式(3.15)定出诸系数 $a_j^{(2)}, b_j^{(2)} \dots$, 最后由方程(3.16a) 求出待定系数 ε_2

$$\varepsilon_2 = - \frac{\sum_{k=0}^{\infty} a_k^{(2)} + W_m^2 \sum_{k=0}^{\infty} [c_k^{(2)} + d_k^{(2)} + e_k^{(2)} + f_k^{(2)} + g_k^{(2)} + q_k^{(2)}]}{\sum_{k=0}^{\infty} b_k^{(2)}}$$

这样一来, 非线性特征值问题(3.1)~(3.13)的解就完全确定了。

四、算例与讨论

为了便于将现有结果与已有理论结果进行比较, 我们首先研究如图2所示锯齿形波纹圆板的非线性自由振动。这一波纹圆板的有关数据是

$$\begin{aligned} H &= 0.414 \text{ mm}, \quad h = 0.101 \text{ mm}, \\ a &= 24.3 \text{ mm}, \quad \theta_0 = 8^\circ 45', \quad l = 5.4 \text{ mm}, \\ E &= 1.35 \times 10^4 \text{ kg/mm}^2, \quad \nu = 0.3 \end{aligned}$$

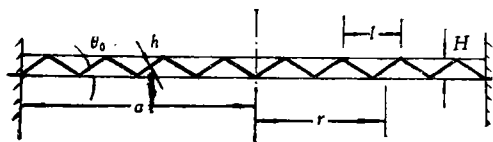


图2 锯齿形波纹圆板

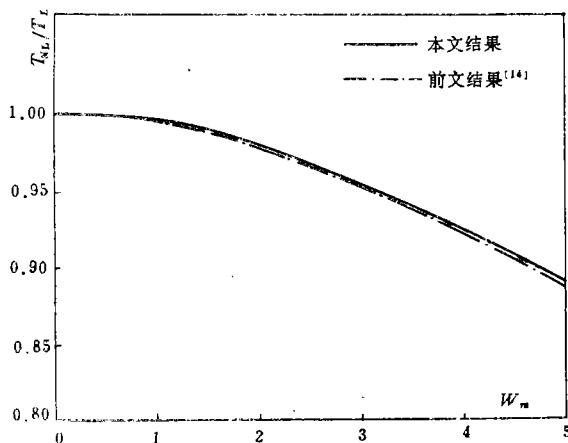


图3 理论结果的比较

将这些数据代入式(3.18), 根据(3.19)和(3.20)可将结果绘在图3中。由此看出, 本文结果与我们以前用 Галеркин 方法所得结果^[14]十分吻合。

下面, 我们讨论全波纹圆板的波幅大小对其振动特性的影响。作为一个例子, 考虑如图1所示正弦波纹圆板。假定

$$\begin{aligned} h &= 0.1 \text{ mm}, \quad a = 25.0 \text{ mm}, \quad l = 5.0 \text{ mm} \\ E &= 1.20 \times 10^4 \text{ kg/mm}^2, \quad \nu = 0.3 \end{aligned}$$

而 H 分别取为 0.15 mm, 0.2 mm, 0.25 mm, 0.30 mm, 0.35 mm 及 0.40 mm。

按照式(3.18)进行数值计算, 我们将结果绘于图4中。由此图看出, 波纹圆板振动的非线性效应随波幅的增大而减弱。因此, 在实际工程中, 若波纹板波幅与板厚相比较, 我们便可用较简单的线性理论来分析波纹圆板的振动。此时, 仅会引起较小的误差。

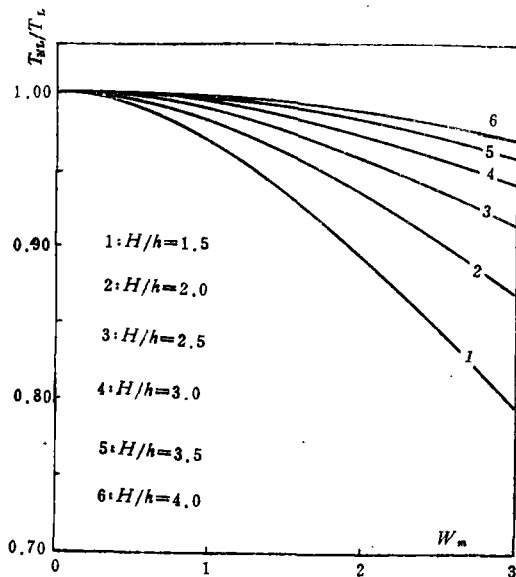


图4 波纹深度对波纹圆板振动周期的影响

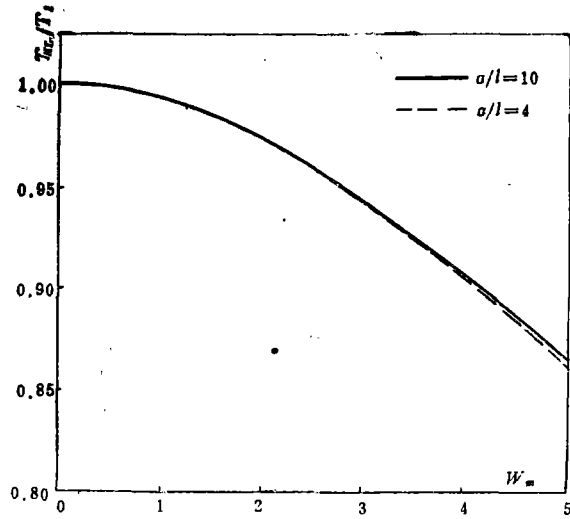


图5 波纹疏密对波纹圆板振动周期的影响

最后,我们来分析全波纹圆板的波纹疏密对其振动特性的影响。仍以如图1所示正弦波纹圆板为例,假定

$$H=0.30\text{mm}, h=0.10\text{mm}, a=25.0\text{mm},$$

$$E=1.20 \times 10^4 \text{kg/mm}^2, \nu=0.3$$

波长 l 分别取为 2.5mm, 3.125mm, 5.0mm, 6.25mm。

应用式(3.18),我们得到有用的数值结果。我们可作一族 T_{NL}/T_L-W_m 曲线,每一曲线对应着一个 a/l 值, a/l 分别取值为4, 5, 8和10。这些曲线绘在图5中。由此看出,因为两条 a/l 为5和8的曲线介于 $a/l=10$ 的实线和 $a/l=4$ 的虚线之间,而它们几乎重合,所以无法画出它们。很显然,波纹圆板疏密对其振动特性的影响是很小的。

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Application of the Modified Iteration Method to Nonlinear Vibration of Corrugated Circular Plates

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Abstract

In this paper, the modified iteration method is successfully extended to investigate the nonlinear free vibration of corrugated circular plates with full corrugations. The analytical relation for the amplitude-frequency response of corrugated circular plates is obtained and discussions on the influences of geometrical parameters on vibration behaviours of corrugated circular plates are made. The present results are practically important in the design of elastic elements in precision instruments.