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广义经典力学中的广义WHITTAKER方程和场方法*

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摘　要

本文首先利用能量积分降阶广义经典力学的正则方程并得到广义 Whittaker方程．其次，将场方法应用于求广义经典力学方程的积分．最后，举例说明新方程和新方法的应用．

一、引　言

许多物理问题的微分方程是作为变分学问题而出现的．1848年～1858年由Остроградский和 Jacobi 开创的广义经典力学理论，近 40 年得到了很大发展，取得了许多重要结果．在物理学方面，特别在力学和场论中研究带高阶导数的 Lagrange 力学，例如关于带二阶导数的电磁理论[1]．在数学方面，近代几何方法的描述已有较好的结果[2]．但是，有关广义经典力学方程的积分理论还很少研究．本文涉及广义经典力学的积分理论．首先，建立广义经典力学中的正则方程并给出能量积分，利用这个积分降阶正则方程而得到广义Whittaker方程；其次，将场方法推广应用于广义经典力学，建立一种新的积分方法；最后，举例说明新方程和新方法的应用．

二、广义经典力学中的正则方程和能量积分

1．正则方程

令$L(t,y,\dot{y},\cdots,\overset{(m)}{y},z,\dot{z},\cdots,\overset{(n)}{z})$是独立变量$t$，不独立变量$y$,$z$及其分别直至$m$，$n$阶导数的函数。积分

$$\int L(t,y,\dot{y},\cdots,\overset{(m)}{y},z,\dot{z},\cdots,\overset{(n)}{z})dt$$

取稳定值的条件写成形式

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$$\left.\begin{aligned}&\frac{\partial L}{\partial y}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)+\cdots+(-1)^m\frac{d^m}{dt^m}\left(\frac{\partial L}{\partial \overset{(m)}{y}}\right)=0\\&\frac{\partial L}{\partial z}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right)+\cdots+(-1)^n\frac{d^n}{dt^n}\left(\frac{\partial L}{\partial \overset{(n)}{z}}\right)=0\end{aligned}\right\} \tag{2.1}$$

方程(2.1)就是广义经典力学的Lagrange方程。取广义动量

$$\left.\begin{aligned}&p_1=\frac{\partial L}{\partial \dot{y}}-\frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{y}}\right)+\cdots+(-1)^{m-1}\frac{d^{m-1}}{dt^{m-1}}\left(\frac{\partial L}{\partial \overset{(m)}{y}}\right)\\&p_2=\frac{\partial L}{\partial \ddot{y}}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dddot{y}}\right)+\cdots+(-1)^{m-2}\frac{d^{m-2}}{dt^{m-2}}\left(\frac{\partial L}{\partial \overset{(m)}{y}}\right)\\&\cdots\cdots\\&p_m=\partial L/\partial \overset{(m)}{y}\\&p_{m+1}=\frac{\partial L}{\partial \dot{z}}-\frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{z}}\right)+\cdots+(-1)^{n-1}\frac{d^{n-1}}{dt^{n-1}}\left(\frac{\partial L}{\partial \overset{(n)}{z}}\right)\\&p_{m+2}=\frac{\partial L}{\partial \ddot{z}}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dddot{z}}\right)+\cdots+(-1)^{n-2}\frac{d^{n-2}}{dt^{n-2}}\left(\frac{\partial L}{\partial \overset{(n)}{z}}\right)\\&\cdots\cdots\\&p_{m+n}=\partial L/\partial \overset{(n)}{z}\end{aligned}\right\} \tag{2.2}$$

并令

$$q_1=y,\ q_2=\dot{y},\ \cdots,\ q_m=\overset{(m-1)}{y},\ q_{m+1}=z,\ q_{m+2}=\dot{z},\cdots,q_{m+n}=\overset{(n-1)}{z} \tag{2.3}$$

取Hamilton函数$H=H(t,q_1,\cdots,q_{m+n},p_1,\cdots,p_{m+n})$为

$$\begin{aligned}H=&-L+p_1q_2+p_2q_3+\cdots+p_{m-1}q_m+p_m\overset{(m)}{y}+p_{m+1}q_{m+2}\\&+\cdots+p_{m+n-1}q_{m+n}+p_{m+n}\overset{(n)}{z}\end{aligned} \tag{2.4}$$

其中$\overset{(m)}{y}$和$\overset{(n)}{z}$借助关系

$$p_m=\partial L/\partial \overset{(m)}{y},\quad p_{m+n}=\partial L/\partial \overset{(n)}{z}$$

而消去。如δ表记量$q_1,q_2,\cdots,q_{m+n}$，p_1，$p_2,\cdots,p_{m+n}$的微小改变，则有

$$\begin{aligned}\delta H=&-\sum_{r=0}^{m-1}\frac{\partial L}{\partial \overset{(r)}{y}}\delta q_{r+1}-\frac{\partial L}{\partial \overset{(m)}{y}}\delta\overset{(m)}{y}-\sum_{r=0}^{n-1}\frac{\partial L}{\partial \overset{(r)}{z}}\delta q_{m+r+1}-\frac{\partial L}{\partial \overset{(n)}{z}}\delta\overset{(n)}{z}\\&+\sum_{r=1}^{m-1}p_r\delta q_{r+1}+p_m\delta\overset{(m)}{y}+\sum_{r=1}^{m-1}q_{r+1}\delta p_r+\overset{(m)}{y}\delta p_m\\&+\sum_{r=m+1}^{m+n-1}p_r\delta q_{r+1}+p_{m+n}\delta\overset{(n)}{z}+\sum_{r=m+1}^{m+n-1}q_{r+1}\delta p_r+\overset{(n)}{z}\delta p_{m+n}\end{aligned} \tag{2.5}$$

由(2.2)得到

$$\left.\begin{array}{l}\dfrac{\partial L}{\partial y}=\dot{p}_1,\quad \dfrac{\partial L}{\partial \dot{y}}=\dot{p}_2+p_1,\quad \dfrac{\partial L}{\partial \ddot{y}}=\dot{p}_3+p_2,\quad \cdots,\quad \dfrac{\partial L}{\partial \overset{(m)}{y}}=p_m \\ \dfrac{\partial L}{\partial z}=\dot{p}_{m+1},\quad \dfrac{\partial L}{\partial \dot{z}}=\dot{p}_{m+2}+p_m,\ \cdots,\quad \dfrac{\partial L}{\partial \overset{(n)}{z}}=p_{m+n}\end{array}\right\} \tag{2.6}$$

将(2.6)代入(2.5)，得到

$$\delta H=-\sum_{r=1}^{m+n}\dot{p}_r\delta q_r+\sum_{r=1}^{m+n}\dot{q}_r\delta p_r \tag{2.7}$$

由此得[3]

$$\frac{dq_r}{dt}=\frac{\partial H}{\partial p_r},\quad \frac{dp_r}{dt}=-\frac{\partial H}{\partial q_r}\qquad (r=1,\cdots,m+n) \tag{2.8}$$

这就是广义经典力学中的Hamilton正则方程。

对于多于两个不独立变量的情形，Hamilton正则方程有类似(2.8)的形式。

2. 能量积分

利用(2.8)求Hamilton函数H对时间t的全导数，有

$$\frac{dH}{dt}=\frac{\partial H}{\partial t}+\sum_{r=1}^{m+n}\frac{\partial H}{\partial q_r}\frac{\partial H}{\partial p_r}+\sum_{r=1}^{m+n}\frac{\partial H}{\partial p_r}\left(-\frac{\partial H}{\partial q_r}\right)=\frac{\partial H}{\partial t} \tag{2.9}$$

因此，当H不显含t，即

$$\partial H/\partial t=0 \tag{2.10}$$

则有

$$H=-h=\text{const} \tag{2.11}$$

我们称(2.11)为能量积分。条件(2.10)等价于条件

$$\partial L/\partial t=0 \tag{2.12}$$

特别地，当$L=L(y,\dot{y},z,\dot{z})$时，积分(2.11)成为通常经典力学中的能量积分

$$-L(y,\dot{y},z,\dot{z})+\frac{\partial L}{\partial \dot{y}}\dot{y}+\frac{\partial L}{\partial \dot{z}}\dot{z}=-h \tag{2.13}$$

三、广义WHITTAKER方程

现在利用能量积分(2.11)降阶正则方程(2.8)。设由(2.11)可解出广义动量p_1，并写成

$$K(p_2,\cdots,p_{m+n},\ q_1,\cdots,q_{m+n},h)+p_1=0 \tag{3.1}$$

微分形式

$$p_1dq_1+\cdots+p_{m+n}dq_{m+n}+hdt \tag{3.2}$$

可写成形式

$$p_2dq_2+\cdots+p_{m+n}dq_{m+n}+hdt-K(p_2,\cdots,p_{m+n},q_1,\cdots,q_{m+n},h)dq_1 \tag{3.3}$$

其中将$(q_1,\cdots,q_{m+n},p_2,\cdots,p_{m+n},h,t)$当作$2m+2n+1$个变量. 与形式(3.3) 相应的微分方程为

$$\left.\begin{array}{l} \dfrac{dq_r}{dq_1}=\dfrac{\partial K}{\partial p_r},\quad \dfrac{dp_r}{dq_1}=-\dfrac{\partial K}{\partial q_r} \qquad (r=2,\cdots,m+n) \\ \dfrac{dt}{dq_1}=\dfrac{\partial K}{\partial h},\quad \dfrac{dh}{dq_1}=0 \end{array}\right\} \tag{3.4}$$

(3.4)中最后两个可由方程组分出，因前面$(2m+2n-2)$个方程不含t，而h为一常数.于是，原方程(2.8)可用下述方程替代

$$\frac{dq_r}{dq_1}=\frac{\partial K}{\partial p_r},\quad \frac{dp_r}{dq_1}=-\frac{\partial K}{\partial q_r} \qquad (r=2,\cdots,m+n) \tag{3.5}$$

我们称方程(3.5)为广义经典力学中的广义Whittaker方程. 原方程(2.8)的阶为 $2(m+n)$，而广义Whittaker方程(3.5)的阶为$(2m+2n-2)$阶. 因此，方程降低2阶.

特别地，如$m=1$，$n=1$，则方程(3.5)成为经典力学中通常的Whittaker方程[3].

进而，如果K中不显含q_1，则(3.5)有能量积分

$$K+h'=\text{const} \tag{3.6}$$

类似于上述方法，方程(3.5)可借助积分(3.6)再降低2阶.

四、场方法对积分广义经典力学方程的应用

文献[4]给出了积分经典完整非保守系统方程的场方法. 下面将这个方法推广 到 广义经典力学中.

将方程(2.8)作为场方程来研究，其中q_r称为坐标，p_r称为场动量. 将一 个 场动量，例如p_1，当作依赖于时间t，坐标q_r和其余动量$p_2,\cdots,p_{m+n}$的函数，即

$$p_1=u(t,q_r,p_\alpha) \qquad (r=1,\cdots,m+n;\ \alpha=2,\cdots,m+n) \tag{4.1}$$

将其对 t 求导数并利用方程(2.8)，得到

$$\frac{\partial u}{\partial t}+\sum_{r=1}^{m+n}\frac{\partial u}{\partial q_r}\frac{\partial H}{\partial p_r}-\sum_{\alpha=2}^{m+n}\frac{\partial u}{\partial p_\alpha}\frac{\partial H}{\partial q_\alpha}+\frac{\partial H}{\partial q_1}=0 \tag{4.2}$$

我们称拟线性偏微分方程(4.2)为基本偏微分方程. 若方程(4.2)的完全解表为形式

$$p_1=u(t,q_r,p_\alpha,C_A) \qquad (r=1,\cdots,m+n;\ \alpha=2,\cdots,m+n;\ A=1,\cdots,2(m+n)) \tag{4.3}$$

则将(4.3)代入(4.2)使之成为恒等式. 令初条件为

$$q_r(0)=q_{r0},\quad p_r(0)=p_{r0} \qquad (r=1,\cdots,m+n) \tag{4.4}$$

将(4.4)代入(4.3)，可将一个常数，例如C_1，用q_{r0}，p_{r0}和其余常数$C_B(B=2,\cdots,2(m+n))$表出，这样，(4.3)可写成

$$p_1=u(t,q_r,p_\alpha,C_B) \tag{4.5}$$

容易证明，方程组(2.8)相应初条件(4.4)的解，可由(4.5)以及对任何常数值C_B的[2(m

$+n)-1]$个代数方程

$$\partial u/\partial C_B=0 \qquad (B=2,\cdots,2(m+n)) \tag{4.6}$$

来确定．

场方法比通常的Hamilton-Jacobi方法有如下优越性：1)场方法的基本偏微分方程(4.2)是拟线性的，而Hamilton-Jacobi方法的方程一般是非线性的；2) 场方法可灵活选取场动量或场坐标，基本偏微分方程可建立在任何一个场动量上，也可建立在任何一个场坐标上．对具体问题可选一个较为方便的场动量或场坐标．场方法的主要困难在于求基本偏微分方程(4.2)的完全解.但是，只要求出完全解，不用任何进一步积分，便可直接得到系统的运动．

五、算　　例

设Lagrange函数为

$$L=\frac{1}{2}\alpha_1\dot{y}^2+\frac{1}{2}\alpha_2\ddot{y}^2 \qquad (\alpha_1,\alpha_2>0) \tag{5.1}$$

我们来建立问题的广义Whittaker方程．我们有

$$p_1=\frac{\partial L}{\partial\dot{y}}-\frac{d}{dt}\frac{\partial L}{\partial\ddot{y}}=\alpha_1\dot{y}-\alpha_2\dddot{y},\quad p_2=\frac{\partial L}{\partial\ddot{y}}=\alpha_2\ddot{y} \tag{5.2}$$

令

$$q_1=y,\quad q_2=\dot{y} \tag{5.3}$$

那么，Hamilton函数为

$$H=-L+p_1q_2+p_2\ddot{y}=-(\alpha_1\dot{y}^2/2+\alpha_2\ddot{y}^2/2)+p_1q_2+p_2\ddot{y} \tag{5.4}$$

由(5.2)第二式得

$$\ddot{y}=p_2/\alpha_2 \tag{5.5}$$

将(5.5)和(5.3)代入(5.4)，得

$$H=p_1q_2-\alpha_1q_2^2/2+p_2^2/2\alpha_2 \tag{5.6}$$

正则方程(2.8)给出为

$$\frac{dq_1}{dt}=q_2,\quad \frac{dp_1}{dt}=0,\quad \frac{dq_2}{dt}=\frac{p_2}{\alpha_2},\quad \frac{dp_2}{dt}=-p_1+\alpha_1q_2 \tag{5.7}$$

因H不含t，故有能量积分

$$H=-h$$

即

$$p_1q_2-\alpha_1q_2^2/2+p_2^2/2\alpha_2=-h \tag{5.8}$$

由此解出p_1为

$$p_1=(-h+\alpha_1q_2^2/2-p_2^2/2\alpha_2)/q_2 \tag{5.9}$$

于是

$$K=-p_1=(h-\alpha_1q_2^2/2+p_2^2/2\alpha_2)/q_2 \tag{5.10}$$

广义Whittaker方程(3.5)成为

$$\frac{dq_2}{dq_1}=\frac{\partial K}{\partial p_2}=\frac{p_2}{\alpha_2q_2},\quad \frac{dp_2}{dq_1}=-\frac{\partial K}{\partial q_2}=\frac{1}{q_2^2}\left(h+\frac{1}{2}\alpha_1q_2^2+\frac{p_2^2}{2\alpha_2}\right) \tag{5.11}$$

方程(5.11)的阶为2，而原方程的阶为4，因而降了2阶.进而，因K中不含q_1，方程(5.11)仍有能量积分

$$K=-h_1$$

即

$$(h-\alpha_1 q_2^2/2+p_2^2/2\alpha_2)/q_2=-h_1 \tag{5.12}$$

这样，问题可归为求积分. 实际上，由(5.12)得

$$p_2=\pm\{2\alpha_2(\alpha_1 q_2^2/2-h_1 q_2-h)\}^{\frac{1}{2}} \tag{5.13}$$

将(5.13)代入(5.11)第一式并积分，得

$$q_1=\pm\int \alpha_2 q_2\left\{2\alpha_2\left(\frac{1}{2}\alpha_1 q_2^2-h_1 q_2-h\right)\right\}^{-\frac{1}{2}}dq_2 \tag{5.14}$$

将(5.13)代入(5.9)，可将p_1表为q_2的函数.

下面用场方法来求解正则方程(5.7).令

$$q_1=u(t,q_2,p_1,p_2) \tag{5.15}$$

则基本偏微分方程给出为

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial q_2}\frac{p_2}{\alpha_2}+\frac{\partial u}{\partial p_2}(-p_1+\alpha_1 q_2)-q_2=0 \tag{5.16}$$

令(5.16)的解有形式

$$u=f_1(t)+f_2(t)q_2+f_3(t)p_1+f_4(t)p_2 \tag{5.17}$$

将(5.17)代入(5.16)并比较自由项，含q_2,p_1和p_2的项，得到

$$\dot f_1=0,\quad \dot f_2+\alpha_1 f_4-1=0,\quad \dot f_3-f_4=0,\quad \dot f_4+f_2/\alpha_2=0 \tag{5.18}$$

其解为

$$\left.\begin{aligned}
&f_1=C_1\\
&f_2=\frac{1}{2}\left[C_2-(\alpha_1C_4-1)\sqrt{\frac{\alpha_2}{\alpha_1}}\right]\exp\left[\sqrt{\frac{\alpha_1}{\alpha_2}}\,t\right]\\
&\qquad+\frac{1}{2}\left[C_2+(\alpha_1C_4-1)\sqrt{\frac{\alpha_2}{\alpha_1}}\right]\exp\left[-\sqrt{\frac{\alpha_1}{\alpha_2}}\,t\right]\\
&f_3=\frac{1}{\alpha_1}\left\{t-\frac{1}{2}\left[C_2-(\alpha_1C_4-1)\sqrt{\frac{\alpha_2}{\alpha_1}}\right]\exp\left[\sqrt{\frac{\alpha_1}{\alpha_2}}\,t\right]\right.\\
&\qquad\left.-\frac{1}{2}\left[C_2+(\alpha_1C_4-1)\sqrt{\frac{\alpha_2}{\alpha_1}}\right]\exp\left[-\sqrt{\frac{\alpha_1}{\alpha_2}}\,t\right]\right\}\\
&\qquad+C_3+\frac{C_2}{\alpha_1}\\
&f_4=\frac{1}{\alpha_1}\left\{1-\frac{1}{2}\left[C_2\sqrt{\frac{\alpha_1}{\alpha_2}}-(\alpha_1C_4-1)\right]\exp\left[\sqrt{\frac{\alpha_1}{\alpha_2}}\,t\right]\right.\\
&\qquad\left.+\frac{1}{2}\left[C_2\sqrt{\frac{\alpha_1}{\alpha_2}}+(\alpha_1C_4-1)\right]\exp\left[-\sqrt{\frac{\alpha_1}{\alpha_2}}\,t\right]\right\}
\end{aligned}\right\} \tag{5.19}$$

于是

$$\begin{aligned}q_1=u=C_1&+[C_2\mathrm{ch}\sqrt{\alpha_1/\alpha_2}\,t-(\alpha_1C_4-1)\mathrm{sh}\sqrt{\alpha_1/\alpha_2}\,t]q_2\\&+(1/\alpha_1)[t-C_2\mathrm{ch}\sqrt{\alpha_1/\alpha_2}\,t+(\alpha_1C_4-1)\sqrt{\alpha_2/\alpha_1}\end{aligned}$$

$$\cdot \operatorname{sh}\sqrt{\alpha_1/\alpha_2}\,t]p_1+(C_3+C_2/\alpha_1)p_1+\frac{1}{\alpha_1}$$

$$\cdot[1-C_2\sqrt{\alpha_1/\alpha_2}\operatorname{sh}\sqrt{\alpha_1/\alpha_2}\,t+(\alpha_1C_4-1)\operatorname{ch}\sqrt{\alpha_1/\alpha_2}\,t]p_2 \tag{5.20}$$

令初条件为 $t=0$，$q_1=q_{10}$，$p_1=p_{10}$，$q_2=q_{20}$，$p_2=p_{20}$，则由(5.20)解得

$$C_1=q_{10}-C_2q_{20}-C_3p_{10}-C_4p_{20} \tag{5.21}$$

将(5.21)代入(5.20)，得

$$\begin{aligned}q_1=u=&q_{10}-C_2q_{20}-C_3p_{10}-C_4p_{20}+\Big[C_2\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t\\&-(\alpha_1C_4-1)\sqrt{\frac{\alpha_2}{\alpha_1}}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t\Big]q_2+\frac{1}{\alpha_1}\Big[t-C_2\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t\\&+(\alpha_1C_4-1)\sqrt{\frac{\alpha_2}{\alpha_1}}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t\Big]p_1+\Big(C_3+\frac{C_2}{\alpha_1}\Big)p_1\\&+\frac{1}{\alpha_1}\Big[1-C_2\sqrt{\frac{\alpha_1}{\alpha_2}}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t+(\alpha_1C_4-1)\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t\Big]p_2\end{aligned} \tag{5.22}$$

由(4.6)给出

$$\partial u/\partial C_2=0,\quad \partial u/\partial C_3=0,\quad \partial u/\partial C_4=0$$

即

$$\left.\begin{aligned}&-q_{20}+q_2\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t-\frac{p_1}{\alpha_1}\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t+\frac{p_1}{\alpha_1}-\frac{p_2}{\sqrt{\alpha_1\alpha_2}}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t=0\\&-p_{10}+p_1=0\\&-p_{20}-\sqrt{\alpha_1\alpha_2}\,q_2\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t+\sqrt{\frac{\alpha_2}{\alpha_1}}p_1\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t+p_2\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t=0\end{aligned}\right\} \tag{5.23}$$

由此解得

$$\left.\begin{aligned}&p_1=p_{10}\\&q_2=p_{20}\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t-p_{10}\sqrt{\frac{\alpha_2}{\alpha_1}}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t+q_{20}\sqrt{\alpha_1\alpha_2}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t\\&p_2=\frac{1}{\sqrt{\alpha_1\alpha_2}}p_{20}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t+q_{20}\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t\\&\qquad+\frac{p_{10}}{\alpha_1}\Big(1-\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t\Big)\end{aligned}\right\} \tag{5.24}$$

将(5.24)代入(5.22)，便得

$$\begin{aligned}q_1=&q_{10}+\frac{p_{10}}{\alpha_1}\Big(t-\sqrt{\frac{\alpha_2}{\alpha_1}}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t\Big)+\sqrt{\frac{\alpha_2}{\alpha_1}}q_{20}\operatorname{sh}\sqrt{\frac{\alpha_1}{\alpha_2}}t\\&+\frac{p_{20}}{\alpha_1}\Big(\operatorname{ch}\sqrt{\frac{\alpha_1}{\alpha_2}}t-1\Big)\end{aligned} \tag{5.25}$$

于是，(5.24)和(5.25)是问题的解。

场坐标的取法(5.15)是比较方便的，如取 p_1 或 p_2 则较为复杂。

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Generalized Whittaker Equations and Field Method in Generalized Classical Mechanics

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Abstract

This paper presents the energy integral in generalized classical mechanics. The integral enables us to reduce a given canonical system with $2n$ order to another system with only $(2n-2)$ order and to obtain generalized Whittaker equations. And then, this paper extends a field method integrating the equations of motion for classical mechanics to generalized classical mechanics. Finally, this paper gives an example to illustrate the application of these equations and the field method.