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多层弹性导电板在磁场中的运动方程*

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摘　要

本文得到了多层弹性导电板在磁场中的运动方程．著名的Амбарцумян薄板方程是本文的特殊情形．本文还对多层板的横向振动问题进行了讨论．

关键词　磁弹性　导电　多层　板

一、引　　言

导电板作为器件和结构组元有着广泛的应用[1]．对于均质材料的导电板在磁场中的运动问题已有许多研究[2]、[3]、[4]．多层导电板的问题比均质薄板情形复杂．然而，结构的复杂也可能带来结构优化的机会．

本文先讨论分区均匀物体的磁弹性场方程和界面条件，利用Амбарцумян等人提出的薄体磁弹性假设考察了多层板在运动时的内部电磁场，进而给出多层弹性导电板在磁场中的运动方程．作为例子，本文讨论了多层板在纵向磁场中的自由振动问题．本文使用SI单位制．

二、电磁场方程和板内电磁场

设多层薄板是由若干薄层材料组成，层与层之间紧密粘合，没有相对滑动，各层材料都是均匀、各向同性的弹性导电材料．第k层材料的物性参数为：质量密度ρ_k，电导率σ_k，杨氏模量E_k，泊松比ν_k．本文不计热效应．假定所有的材料都有非极化、非磁化的良导体．对于绝大多数的这类材料而言，其介电常数ε、磁导率μ都与自由空间的介电常数ε_0，磁导率μ_0相差无几．因此，我们假设电磁本构方程

$$\mathbf{D}=\varepsilon_0\mathbf{E},\quad \mathbf{H}=\frac{1}{\mu_0}\mathbf{B} \tag{2.1}$$

对所有材料和自由空间都适用，式中 $\mathbf{E}$，$\mathbf{D}$，$\mathbf{H}$，$\mathbf{B}$ 分别为电场强度、电位移、磁场强度和磁感应强度．

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自由空间中的电磁场方程为[5]

$$\left.\begin{aligned}&\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0,\ \nabla\cdot\mathbf{D}=0,\\&\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t},\ \nabla\cdot\mathbf{B}=0\end{aligned}\right\}\tag{2.2}$$

第k层导电材料中的电磁场方程为[6]

$$\left.\begin{aligned}&\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0,\ \nabla\cdot\mathbf{B}=0\\&\nabla\times\mathbf{H}=\mathbf{J}=\sigma_k\left(\mathbf{E}+\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{B}\right)\end{aligned}\right\}\tag{2.3}$$

其中 $\mathbf{u}$是位移矢量，$\mathbf{J}$为感应电流密度矢量。

满足本构方程（2.1）的两种非磁化、非极化导电材料间的界面两侧的电磁学量满足条件[7]、[8]

$$\left.\begin{aligned}&\mathbf{n}\cdot[\mathbf{B}]=0,\ \mathbf{n}\cdot[\mathbf{D}]=0,\ \mathbf{n}\times[\mathbf{E}]=0\\&\mathbf{n}\times[\mathbf{H}]=0,\ \mathbf{n}\cdot[\mathbf{J}]=0\end{aligned}\right\}\tag{2.4}$$

其中 $\mathbf{n}$是界面的单位法矢量，其指向为由材料1指向材料2，而符号[ ]表示量f在界面两侧的改变量$[f]=f_2-f_1$。由(2.1)和(2.4)知，在两种非极化、非磁化的导体界面两侧的电磁学量是连续的。

设板在未受扰动的静止状态，在自由空间和板内的初始电磁场为

$$\mathbf{E}=\mathbf{E}_0=0,\ \mathbf{B}=\mathbf{B}_0(B_1,\ B_2,\ B_3)\tag{2.5}$$

$\mathbf{B}_0$满足静磁场方程

$$\nabla\times\mathbf{B}_0=0,\ \nabla\cdot\mathbf{B}_0=0\tag{2.6}$$

若多层板由n层材料所组成，板静止时的第k层材料的两界面的坐标为z_{k-1}和z_k，坐标面Oxy平行于板面，第k层的厚度为（z_k-z_{k-1}），多层板的总厚度为（z_n-z_0）。按Love-Kirchhoff假设，薄板变形时板上坐标为(x,y,z)的点的位移$\mathbf{u}$的分量为

$$u_1=u-z\frac{\partial w}{\partial},\ u_2=v-z\frac{\partial w}{\partial y},\ u_3=w(x,y,t)\tag{2.7}$$

其中 u，v，w是点$(x,y,0)$的位移分量。

板受扰动而运动时电磁场发生变化。设扰动后的电磁场为

$$\mathbf{E}=\mathbf{e},\ \mathbf{B}=\mathbf{B}_0+\mathbf{b}\tag{2.8}$$

按薄体磁弹性假设[5]，可设第k层材料中的电磁场增量为

$$\mathbf{b}^{(k)}=\mathbf{b}^{(k)}(b_1^{(k)},\ b_2^{(k)},\ f),\ \mathbf{e}^{(k)}=\mathbf{e}^{(k)}(\varphi,\psi,e_3^{(k)})\tag{2.9}$$

其中 φ，ψ，f是x，y，t的函数而与坐标z无关。又从电磁场在界面上的连续条件知，它们还与材料的层数k无关。

对于所考虑的良导体材料，在略去自由电荷和位移电流情形[8]，第k层材料中的扰动电磁场方程为

$$\left.\begin{aligned}&\nabla\times\mathbf{b}^{(k)}=\mu_0\sigma_k\left(\mathbf{e}^{(k)}+\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{B}_0\right)\\&\nabla\cdot\mathbf{b}^{(k)}=0,\ \nabla\times\mathbf{e}^{(k)}=-\frac{\partial\mathbf{b}^{(k)}}{\partial t}\end{aligned}\right\}\tag{2.10}$$

由(2.10)的第一式可得

$$\left.\begin{aligned}\frac{\partial b_1^{(k)}}{\partial z}-\frac{\partial f}{\partial x}&=\mu_0\sigma_k\left(\psi-B_3\frac{\partial u}{\partial t}+B_1\frac{\partial w}{\partial t}+B_3z\frac{\partial^2 w}{\partial x\partial t}\right)\\ \frac{\partial f}{\partial y}-\frac{\partial b_2^{(k)}}{\partial z}&=\mu_0\sigma_k\left(\varphi+B_3\frac{\partial v}{\partial t}-B_2\frac{\partial w}{\partial t}-B_3z\frac{\partial^2 w}{\partial y\partial t}\right)\end{aligned}\right\}\tag{2.11}$$

将上列二式从z_{k-1}到z_k积分，并将其结果对k从1至n求和，可得

$$\left.\begin{aligned}\frac{\partial f}{\partial x}+\mu_0\sigma\left(\psi-B_3\frac{\partial u}{\partial t}+B_1\frac{\partial w}{\partial t}\right)+\frac{\mu_0\Gamma_{\rm I}}{z_n-z_0}B_3\frac{\partial^2 w}{\partial x\partial t}&=\frac{b_1^+-b_1^-}{z_n-z_0}\\ \frac{\partial f}{\partial y}-\mu_0\sigma\left(\varphi+B_3\frac{\partial v}{\partial t}-B_2\frac{\partial w}{\partial t}\right)+\frac{\mu_0\Gamma_{\rm I}}{z_n-z_0}B_3\frac{\partial^2 w}{\partial y\partial t}&=\frac{b_2^+-b_2^-}{z_n-z_0}\end{aligned}\right\}\tag{2.12}$$

其中 $b_i^+=b_i(z_n)$，$b_i^-=b_i(z_0)$ $(i=1,2)$，且

$$\left.\begin{aligned}&\sigma=\frac{\Gamma_{\rm I}}{z_n-z_0}\quad,\Gamma_{\rm I}=\sum_{k=1}^{n}\sigma_k(z_k-z_{k-1})\\ &\Gamma_{\rm I}=\frac{1}{2}\sum_{k=1}^{n}z_k(z_k^2-z_{k-1}^2)\end{aligned}\right\}\tag{2.13}$$

其中 σ为多层板的平均电导率．又由(2.10)的第三式可得

$$\frac{\partial\psi}{\partial x}-\frac{\partial\varphi}{\partial y}=-\frac{\partial f}{\partial t}\tag{2.14}$$

方程(2.12)和(2.14)即为多层导电板的电磁场方程．

由恒等式$\nabla\cdot\nabla\times\mathbf{A}=0$和(2.10)的第一式有

$$\begin{aligned}\nabla\cdot\left(\mathbf{e}^{(k)}+\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{B}_0\right)=&\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}+\frac{\partial e_3^{(k)}}{\partial z}-B_3\left(\frac{\partial^2 u}{\partial y\partial t}-\frac{\partial^2 v}{\partial x\partial t}\right)\\ &+2B_1\frac{\partial^2 w}{\partial y\partial t}-2B_2\frac{\partial^2 w}{\partial x\partial t}=0\end{aligned}\tag{2.15}$$

上式中除项$\frac{\partial e_3^{(k)}}{\partial z}$之外各项均与$k$无关，故积分上式可得$e_3$(而不只是$e_3^{(k)}$）为

$$\begin{aligned}e_3=\Big[&-\left(\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}\right)+B_3\left(\frac{\partial^2 u}{\partial y\partial t}-\frac{\partial^2 v}{\partial x\partial t}\right)\\ &-2B_1\frac{\partial^2 w}{\partial y\partial t}+2B_2\frac{\partial^2 w}{\partial x\partial t}\Big]z+e_{30}\end{aligned}\tag{2.16}$$

在多层板的两个表面满足法向电流密度为零的条件[5]

$$\left(\mathbf{e}+\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{B}_0\right)\cdot\mathbf{e}_3=0\tag{2.17}$$

其中 $\mathbf{e}_3$是Oz方向的单位矢量，以$z=z_0$和z_n代入上式，得

$$\left.\begin{aligned}e_3(z_0)&=-B_2\frac{\partial u}{\partial t}+B_1\frac{\partial v}{\partial t}+\left(B_2\frac{\partial^2 w}{\partial x\partial t}-B_1\frac{\partial^2 w}{\partial y\partial t}\right)z_0\\ e_3(z_n)&=-B_2\frac{\partial u}{\partial t}+B_1\frac{\partial v}{\partial t}+\left(B_2\frac{\partial^2 w}{\partial x\partial t}-B_1\frac{\partial^2 w}{\partial y\partial t}\right)z_n\end{aligned}\right\}\tag{2.18}$$

将(2.16)与(2.18)进行比较，得

$$\left.\begin{aligned}&\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}=B_3\left(\frac{\partial^2 u}{\partial y\partial t}-\frac{\partial^2 v}{\partial x\partial t}\right)-B_1\frac{\partial^2 w}{\partial y\partial t}+B_2\frac{\partial^2 w}{\partial x\partial t}\\&e_{30}=-B_2\frac{\partial u}{\partial t}+B_1\frac{\partial v}{\partial t}\end{aligned}\right\}\tag{2.19}$$

以(2.19)代入(2.16)可得

$$e_3=\left(B_2\frac{\partial^2 w}{\partial x\partial t}-B_1\frac{\partial^2 w}{\partial y\partial t}\right)z-B_2\frac{\partial u}{\partial t}+B_1\frac{\partial v}{\partial t}\tag{2.20}$$

三、多层导电板的运动方程

由于多层导电板中的各层材料都是均匀、各向同性的弹性体，其运动方程比一般的复合材料多层板简单。第k层材料的运动方程为

$$\frac{\partial\sigma_{ij}^{(k)}}{\partial x_j}+f_i^{(k)}=\rho_k\frac{\partial^2 u_i}{\partial t^2}\tag{3.1}$$

其中 体积力项$f_i^{(k)}$含有电磁体积力和一般体积力.

在略去σ_{13}，σ_{23}，σ_{33}时的应力-位移关系为

$$\left.\begin{aligned}&\sigma_{11}^{(k)}=\frac{E_k}{1-\nu_k^2}\left[\frac{\partial u}{\partial x}+\nu_k\frac{\partial v}{\partial y}-z\left(\frac{\partial^2 w}{\partial x^2}+\nu_k\frac{\partial^2 w}{\partial y^2}\right)\right]\\&\sigma_{22}^{(k)}=\frac{E_k}{1-\nu_k^2}\left[\frac{\partial v}{\partial y}+\nu_k\frac{\partial u}{\partial x}-z\left(\frac{\partial^2 w}{\partial y^2}+\nu_k\frac{\partial^2 w}{\partial x^2}\right)\right]\\&\sigma_{12}^{(k)}=\frac{E_k}{2(1+\nu_k)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2z\frac{\partial^2 w}{\partial x\partial y}\right)\end{aligned}\right\}\tag{3.2}$$

利用类似于复合材料层合板理论的推导方式，可得多层弹性导电板的运动方程为

$$\left.\begin{aligned}&A_{11}\frac{\partial^2 u}{\partial x^2}+A_{66}\frac{\partial^2 u}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y}\\&\quad-B_{11}\frac{\partial}{\partial x}(\nabla^2 w)+X_1=\rho\frac{\partial^2 u}{\partial t^2}-Q\frac{\partial^3 w}{\partial x\partial t^2}\\&(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y}+A_{66}\frac{\partial^2 v}{\partial x^2}+A_{11}\frac{\partial^2 v}{\partial y^2}\\&\quad-B_{11}\frac{\partial}{\partial y}(\nabla^2 w)+X_2=\rho\frac{\partial^2 v}{\partial t^2}-Q\frac{\partial^3 w}{\partial y\partial t^2}\\&D_{11}\nabla^2\nabla^2 w+\rho\frac{\partial^2 w}{\partial t^2}=B_{11}\nabla^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+N_x^0\frac{\partial^2 w}{\partial x^2}\\&\quad+2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y}+N_y^0\frac{\partial^2 w}{\partial y^2}+\frac{\partial m_1}{\partial x}+\frac{\partial m_2}{\partial y}+X_3\end{aligned}\right\}\tag{3.3}$$

其中

$$\left.\begin{aligned}
&A_{11}=\sum_{k=1}^{n}\frac{E_k}{1-\nu_k^2}(z_k-z_{k-1}),\quad A_{12}=\sum_{k=1}^{n}\frac{E_k\nu_k}{1-\nu_k^2}(z_k-z_{k-1})\\
&A_{66}=\sum_{k=1}^{n}\frac{E_k}{2(1+\nu_k)}(z_k-z_{k-1}),\quad B_{11}=\frac{1}{2}\sum_{k=1}^{n}\frac{E_k}{1-\nu_k^2}(z_k^2-z_{k-1}^2)\\
&D_{11}=\frac{1}{3}\sum_{k=1}^{n}\frac{E_k}{1-\nu_k^2}(z_k^3-z_{k-1}^3),\quad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\\
&\rho=\sum_{k=1}^{n}\rho_k(z_k-z_{k-1}),\quad Q=\frac{1}{2}\sum_{k=1}^{n}\rho_k(z_k^2-z_{k-1}^2)\\
&X_i=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}f_i^{(k)}dz\qquad (i=1,2,3)\\
&m_i=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}f_i^{(k)}zdz\qquad (i=1,2)
\end{aligned}\right\}\tag{3.4}$$

而N_x^0，N_y^0、N_{xy}^0为板面内力，满足方程

$$\frac{\partial N_x^0}{\partial x}+\frac{\partial N_{xy}^0}{\partial y}=0,\qquad \frac{\partial N_{xy}^0}{\partial x}+\frac{\partial N_y^0}{\partial y}=0\tag{3.5}$$

在第k层材料的体积力$f_i^{(k)}$中所包含的电磁力为洛伦兹力

$$\mathbf{f}^{(k)}=\sigma_k\left(\mathbf{e}+\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{B}_0\right)\times\mathbf{B}_0\tag{3.6}$$

其分量为

$$\left.\begin{aligned}
&f_1^{(k)}=\sigma_k\left(B_3\psi-B_3^2\frac{\partial u}{\partial t}+B_1B_3\frac{\partial w}{\partial t}+B_3^2\frac{\partial^2 w}{\partial x\partial t}z\right)\\
&f_2^{(k)}=\sigma_k\left(-B_3\varphi-B_3^2\frac{\partial v}{\partial t}+B_2B_3\frac{\partial w}{\partial t}+B_3^2\frac{\partial^2 w}{\partial y\partial t}z\right)\\
&f_3^{(k)}=\sigma_k\left[B_2\varphi-B_1\psi+B_1B_3\frac{\partial u}{\partial t}+B_2B_3\frac{\partial v}{\partial t}\right.\\
&\qquad\left.-(B_1^2+B_2^2)\frac{\partial w}{\partial t}-B_1B_3\frac{\partial^2 w}{\partial x\partial t}z-B_2B_3\frac{\partial^2 w}{\partial y\partial t}z\right]
\end{aligned}\right\}\tag{3.7}$$

将(3.7)代入(3.4)，得洛伦兹力的合力和合矩为

$$\left.\begin{aligned}
&X_1=\Gamma_{\rm I}\left(B_3\psi-B_3^2\frac{\partial u}{\partial t}+B_1B_3\frac{\partial w}{\partial t}\right)+\Gamma_{\rm I}B_3^2\frac{\partial^2 w}{\partial x\partial t}\\
&X_2=\Gamma_{\rm I}\left(-B_3\varphi-B_3^2\frac{\partial v}{\partial t}+B_2B_3\frac{\partial w}{\partial t}\right)+\Gamma_{\rm I}B_3^2\frac{\partial^2 w}{\partial y\partial t}\\
&X_3=\Gamma_{\rm I}\left[B_2\varphi-B_1\psi+B_1B_3\frac{\partial u}{\partial t}+B_2B_3\frac{\partial v}{\partial t}-(B_1^2+B_2^2)\frac{\partial w}{\partial t}\right]\\
&\qquad-\Gamma_{\rm I}\left(B_1B_3\frac{\partial^2 w}{\partial x\partial t}+B_2B_3\frac{\partial^2 w}{\partial y\partial t}\right)
\end{aligned}\right\}\tag{3.8}$$

$$
\left.
\begin{aligned}
m_1 &= \Gamma_{\mathrm{I}}\left(B_3\psi - B_3^2\frac{\partial u}{\partial t} + B_1B_3\frac{\partial w}{\partial t}\right) + \Gamma_{\mathrm{II}}B_3^2\frac{\partial^2 w}{\partial x\partial t} \\
m_2 &= \Gamma_{\mathrm{I}}\left(-B_3\varphi - B_3^2\frac{\partial v}{\partial t} + B_2B_3\frac{\partial w}{\partial t}\right) + \Gamma_{\mathrm{II}}B_3^2\frac{\partial^2 w}{\partial y\partial t}
\end{aligned}
\right\}
$$

其中

$$
\Gamma_{\mathrm{II}} = \frac{1}{3}\sum_{k=1}^{n}\left(z_k^3 - z_{k-1}^3\right) \tag{3.9}
$$

以(3.8)代入(3.3)，得多层弹性导电板在磁场中的运动方程

$$
\left.
\begin{aligned}
& A_{11}\frac{\partial^2 u}{\partial x^2} + A_{66}\frac{\partial^2 u}{\partial y^2} + (A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y} \\
&\quad - B_{11}\frac{\partial}{\partial x}(\nabla^2 w) + \Gamma_{\mathrm{I}}\left(B_3\psi - B_3^2\frac{\partial u}{\partial t} + B_1B_3\frac{\partial w}{\partial t}\right) \\
&\quad + \Gamma_{\mathrm{II}}B_3^2\frac{\partial^2 w}{\partial x\partial t} = \rho\frac{\partial^2 u}{\partial t^2} - Q\frac{\partial^3 w}{\partial x\partial t^2} \\
& (A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y} + A_{66}\frac{\partial^2 v}{\partial x^2} + A_{11}\frac{\partial^2 v}{\partial y^2} \\
&\quad - B_{11}\frac{\partial}{\partial y}(\nabla^2 w) + \Gamma_{\mathrm{I}}\left(-B_3\varphi - B_3^2\frac{\partial v}{\partial t} + B_2B_3\frac{\partial w}{\partial t}\right) \\
&\quad + \Gamma_{\mathrm{II}}B_3^2\frac{\partial^2 w}{\partial y\partial t} = \rho\frac{\partial^2 v}{\partial t^2} - Q\frac{\partial^3 w}{\partial y\partial t^2} \\
& D_{11}\nabla^2\nabla^2 w + \rho\frac{\partial^2 w}{\partial t^2} = N_x^0\frac{\partial^2 w}{\partial x^2} + 2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y} + N_y^0\frac{\partial^2 w}{\partial y^2} \\
&\quad + B_{11}\Delta^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \Gamma_{\mathrm{I}}\Big[B_2\varphi - B_1\psi + B_1B_3\frac{\partial u}{\partial t} \\
&\quad + B_2B_3\frac{\partial v}{\partial t} - (B_1^2+B_2^2)\frac{\partial w}{\partial t}\Big] + \Gamma_{\mathrm{I}}\Big[B_3\left(\frac{\partial\psi}{\partial x} - \frac{\partial\varphi}{\partial y}\right) \\
&\quad - B_3^2\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\Big] + \Gamma_{\mathrm{II}}B_3^2\frac{\partial}{\partial t}(\nabla^2 w) + p
\end{aligned}
\right\} \tag{3.10}
$$

其中　p是除洛伦兹力以外的沿O_z方向的载荷．

多层弹性导电板在磁场中的运动问题即是利用方程(2.12)、(2.14)和(3.10)求解φ，ψ，f，u，v，w，同时利用(2.2)求解自由空间的电磁学量，自由空间与多层板界面两侧的电磁学量满足条件(2.4)，u，v，w应满足相应的力学边界条件．

四、多层导电板的横向振动

与处理一般的复合材料层合板的横向振动问题相似[9]，我们略去方程(3.10)中的惯性力项 $\rho\frac{\partial^2 u}{\partial t^2}$，$\rho\frac{\partial^2 v}{\partial t^2}$，$Q\frac{\partial^3 w}{\partial x\partial t^2}$，$Q\frac{\partial^3 w}{\partial y\partial t^2}$，同时取$N_x^0=N_y^0=N_{xy}^0=0$．在考虑自由振动时，还取$p=0$．由于诸方程中$u$，$v$，$w$与$\varphi$，$\psi$，$f$相互耦合，求解仍较困难．下面讨论几种特殊

情形。

1. 横向磁场中的对称板

若板的几何结构和物性（力学的和电磁学的）都对于板的中面对称，取 Oxz 于中面，有 $B_{11}=0$，$\Gamma_{\rm I}=0$。横向磁场即 $B_1=B_2=0$，$B_3\neq 0$。可得关于挠度 w 的方程

$$D_{11}\nabla^2\nabla^2 w+\rho\frac{\partial^2 w}{\partial t^2}=\Gamma_{\rm I}B_3^2\frac{\partial}{\partial t}(\nabla^2 w) \tag{4.1}$$

此式与单层板方程[2]类似。

2. 纵向磁场中的对称板

对称条件如上。磁场为 $B_1\neq 0$，$B_2=B_3=0$。对本问题和下面两个问题中，我们只考虑多层板作板-条形运动情形，即 $w=w(x,t)$，并设诸量与 y 无关。我们可取 $v=0$ 和 $\varphi=0$，同时，利用由三维理论导出的近似关系[2]

$$\frac{\partial(b_1^+-b_1^-)}{\partial t}=-\frac{2}{\lambda}\psi \tag{4.2}$$

这里 λ 是长度为弹性板振动半波长的特征长度。于是，可得方程

$$\left.\begin{aligned}&\frac{\partial^2\psi}{\partial x^2}-\mu_0\sigma\frac{\partial}{\partial t}\left(\psi+B_1\frac{\partial w}{\partial t}\right)=\frac{z}{\lambda(z_n-z_0)}\psi\\&D_{11}\frac{\partial^4 w}{\partial x^4}+\rho\frac{\partial^2 w}{\partial t^2}=-\Gamma_{\rm I}B_1\left(\psi+B_1\frac{\partial w}{\partial t}\right)\end{aligned}\right\} \tag{4.3}$$

这组方程亦与单层板方程[2]类似。

3. 横向磁场中的非对称板

磁场为 $B_1=B_2=0$，$B_3\neq 0$，且设 $w=w(x,t)$。可得方程

$$\left.\begin{aligned}&\frac{\partial^2\psi}{\partial x^2}-\mu_0\sigma\frac{\partial}{\partial t}\left(\psi-B_3\frac{\partial u}{\partial t}\right)-\frac{\mu_0\Gamma_{\rm I}}{z_n-z_0}B_3\frac{\partial^3 w}{\partial x\partial t^2}=\frac{2}{\lambda(z_n-z_0)}\psi\\&A_{11}\frac{\partial^2 u}{\partial x^2}-B_{11}\frac{\partial^3 w}{\partial x^3}+\Gamma_{\rm I}B_3\left(\psi-B_3\frac{\partial u}{\partial t}\right)+\Gamma_{\rm I}B_3^2\frac{\partial^2 w}{\partial x\partial t}=0\\&D_{11}\frac{\partial^4 w}{\partial x^4}+\rho\frac{\partial^2 w}{\partial t^2}=B_{11}\frac{\partial^3 u}{\partial x^3}+\Gamma_{\rm I}B_3\frac{\partial}{\partial x}\left(\psi-B_3\frac{\partial u}{\partial t}\right)+\Gamma_{\rm I}B_3^2\frac{\partial^3 w}{\partial x^2\partial t}\end{aligned}\right\} \tag{4.4}$$

从上述方程知，与单层板和对称板的情形不同，这里 w 与 u、ψ 是耦合的。

4. 纵向磁场中的非对称板

磁场为 $B_1\neq 0$，$B_2=B_3=0$，仍设 $w=w(x,t)$，可得方程

$$\left.\begin{aligned}&\frac{\partial^2\psi}{\partial x^2}-\mu_0\sigma\frac{\partial}{\partial t}\left(\psi+B_1\frac{\partial w}{\partial t}\right)=\frac{2}{\lambda(z_n-z_0)}\psi\\&\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\frac{\partial^4 w}{\partial x^4}+\rho\frac{\partial^2 w}{\partial t^2}=-\Gamma_{\rm I}B_1\left(\psi+B_1\frac{\partial w}{\partial t}\right)\end{aligned}\right\} \tag{4.5}$$

这组方程与单层板和对称板方程(4.3)类似，但对刚度 D_{11} 有一修正系数 $\left(-\frac{B_{11}^2}{A_{11}}\right)$。对此问题可以用与[10]类似的方法研究其振动性质。

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Motion Equations of Multilayered Elastic Electroconductive Plates in a Magnetic Field

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Abstract

The motion equations of multilayered elastic electroconductive plates in a magnetic field are obtained. The well known Ambartsumian's equations of plates are the special ones of this paper. The equations of transverse vibration of multilayered plate are also discussed here.

Key words magnetoelasticity, electroconductive, multilayer, plate