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矩形网格扁壳结构的非线性振动*

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摘 要

本文运用作者已建立的矩形网格扁壳的非线性弹性理论，求解了该类结构的非线性振动问题．通过采用横向挠度（网格节点横向位移）和力函数的某种（广义）Fourier 级数形式的设定解，由试函数的加权得到解中系数之间的关系和决定时间未知函数的振动方程，然后利用正则摄动法和迦辽金法推导出结构自由振动和谐和激励作用下结构非线性受迫振动的幅频关系，并给出了计算实例．

关键词 矩形网格扁壳 非线性 振动 幅频特征关系 梁振动特征函数

一、引 言

近年来，由杆系、梁元等离散构件组成的空间网格结构，由于具有刚度大、重量轻、受力合理、造型美观等独特的优点，在宇航、机械、建筑、土木等领域得到了十分广泛的应用．

对网格结构进行研究有两类方法．一类是各种离散化方法，例如：Ellington 和 McCallion[1]，Renton[2]用差分法分析了网格梁、板结构的振动和弯曲问题；McDaniel 和 Chang[3]，Williams[4][5]利用有限元和转换矩阵相结合的途径讨论了周期性（对称）空间网格结构；基于离散元件的精确刚度分析，Anderson[6]~[8]等人考虑了简支条件的周期性网格梁型结构的屈曲和振动问题．此外，Dean[9]，Wah[10]等人研究网格结构时，运用了离散场方法，即先对网格结构推导出特征节点的差分方程，然后直接求解或者采用级数展开将差分方程转化为微分方程来分析．另一类方法是连续化方法，其基本思想是，从网格离散结构的应力、应变关系角度或能量角度[11]~[13]建立一个等效的连续模型，通过对连续模型的分析获得原网格结构的力学特性．

采用合理的等效连续化模型能够准确地了解整个离散结构的宏观特征，尤其是对结构发生失稳的临界载荷以及结构振动频率的预测是十分有效的．在文[14]中，本作者通过分析梁元双向正交布置的矩形网格扁壳结构的内力、变形，建立连续化模型，利用变分原理导出了基本控制方程和边界条件，并且还具体研究了正方形网格圆底扁球壳的轴对称屈曲问题[15]．

本文在文献[14]的基础上，讨论了矩形网格扁壳的非线性振动问题．对于边界可移的简

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支和夹支情况，运用摄动法和迦辽金法求解得到了网格结构对应于(m, n)型振动模态的振动幅值与频率之间的非线性关系式，并给出了算例分析．

二、矩形网格扁壳非线性振动问题的提出

如图1所示，设矩形底面的网格扁壳其两向布置的梁元材料相同，截面一样．通过分析网格结构内力与变形，运用虚功原理可导出由横向挠度（节点横向位移）W和力函数ϕ表示的平衡方程与协调关系式[14]

$$\frac{EI}{L_1}W_{,1111}+GJ\left(\frac{1}{L_1}+\frac{1}{L_2}\right)W_{,1122}+\frac{EI}{L_2}W_{,2222}$$
$$=W_{,11}\phi_{,22}-2W_{,12}\phi_{,12}+W_{,22}\phi_{,11}+K_1\phi_{,22}+K_2\phi_{,11}+f_z \tag{2.1}$$

$$\frac{L_1}{EA}\phi_{,2222}+C\phi_{,1122}+\frac{L_2}{EA}\phi_{,1111}$$
$$=(W_{,12})^2-W_{,11}W_{,22}-K_1W_{,22}-K_2W_{,11}+C_1W_{,1122}$$
$$+C_2(W_{,11}W_{,12})_{,12}+C_3(W_{,22}W_{,12})_{,12} \tag{2.2}$$

这里只考虑法向载荷的作用．对于结构的横向振动问题，应有：

$$f_z=g_z(x_1,x_2,t)-\rho\ddot{W}(x_1,\ x_2,t)$$
$$\phi=\phi(x_1,x_2,t)$$

式中，g_z为外载．上面表达式中，

已记 $\qquad (\ )_{,1}=\frac{\partial}{\partial x_1}(\ )$，$(\ )_{,2}=\frac{\partial}{\partial x_2}(\ )$，$\ddot{W}=\frac{\partial^2 W}{\partial t^2}$

材料常数C,C_1,C_2,C_3表达如下

$$\left.\begin{aligned}&C=\frac{L_1L_2(L_1+L_2)}{12EI_0},\ C_1=\frac{L_1L_2}{24EI_0}\left[\frac{K_1}{L_1}(L_1+2L_2)+\frac{K_2}{L_2}(L_2+2L_1)\right]GJ\\&C_2=\frac{L_1L_2}{24EI_0}\frac{L_1+2L_2}{L_1}(GJ-EI),\ C_3=\frac{L_1L_2}{24EI_0}\frac{L_2+2L_1}{L_2}(GJ-EI)\end{aligned}\right\} \tag{2.3}$$

上式中，K_1．K_2分别为x_1，x_2方向的曲率，GJ为扭转刚度．

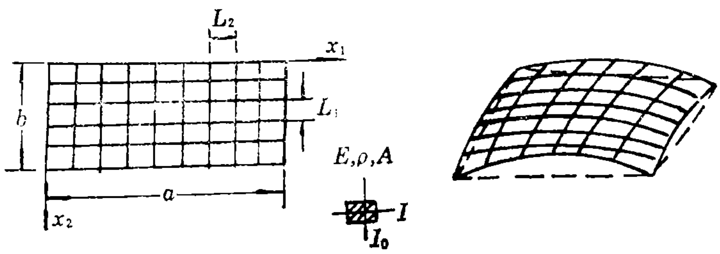

图1 网格扁壳基本几何图

对于可移的底面边界，对应的边界条件可写成

（1） 简支

$$\left.\begin{aligned}&W=0,\ W_{,11}=0,\ \phi_{,22}=0,\ \phi_{,12}=0 \qquad 当x_1=0,a时\\&W=0,\ W_{,22}=0,\ \phi_{,11}=0,\ \phi_{,12}=0 \qquad 当x_2=0,b时\end{aligned}\right\} \tag{2.4}$$

（2） 夹支

$$\left.\begin{array}{ll} W=0,\ W_{,1}=0,\ \phi_{,22}=0,\ \phi_{,12}=0 & \text{当}x_1=0,a\text{时} \\ W=0,\ W_{,2}=0,\ \phi_{,11}=0,\ \phi_{,12}=0 & \text{当}x_2=0,b\text{时} \end{array}\right\} \tag{2.5}$$

初始条件为 $\dot{W}(x_1,x_2,0)=0,\ \dot{\phi}(x_1,x_2,0)=0$ (2.6)

三、非线性振动问题的求解与幅频特征关系

我们首先考虑简支情况．为了满足边界条件(2.4)，设W和ϕ有如下形式的解

$$\left.\begin{array}{l} W(x_1,x_2,t)=\sum\limits_{m=1}^{\infty}\sum\limits_{n=1}^{\infty}A_{mn}f(t)\sin m\alpha x_1\sin n\beta x_2 \\ \phi(x_1,x_2,t)=\sum\limits_{p=1}^{\infty}\sum\limits_{q=1}^{\infty}B_{pq}f(t)X_p(x_1)Y_q(x_2)+\sum\limits_{p=1}^{\infty}\sum\limits_{q=1}^{\infty}C_{pq}f^2(t)X_p(x_1)Y_q(x_2) \end{array}\right\} \tag{3.1a,b}$$

上式中，A_{mn}，B_{pq}，C_{pq}为待定常数，$f(t)$为未知时间函数．

且 $\alpha=\pi/a$，$\beta=\pi/b$，$X_p(x_1)$，$Y_q(x_2)$为梁振动特征函数，即

$$X_p(x_1)=\mathrm{ch}\,\alpha_p\frac{x_1}{a}-\cos\alpha_p\frac{x_1}{a}-\nu_p\left(\mathrm{sh}\,\alpha_p\frac{x_1}{a}-\sin\alpha_p\frac{x_1}{a}\right)$$

$$Y_q(x_2)=\mathrm{ch}\,\alpha_q\frac{x_2}{b}-\cos\alpha_q\frac{x_2}{b}-\nu_q\left(\mathrm{sh}\,\alpha_q\frac{x_2}{b}-\sin\alpha_q\frac{x_2}{b}\right)$$

其中系数α_p，ν_p；α_q,ν_q有下列关系

$$\nu_p=\frac{\cos\alpha_p-\mathrm{ch}\,\alpha_p}{\sin\alpha_p-\mathrm{sh}\,\alpha_p},\quad \cos\alpha_p\cdot\mathrm{ch}\,\alpha_p=1$$

$$\nu_q=\frac{\cos\alpha_q-\mathrm{ch}\,\alpha_q}{\sin\alpha_q-\mathrm{sh}\,\alpha_q},\quad \cos\alpha_q\cdot\mathrm{ch}\,\alpha_q=1$$

并且，X_p，Y_q满足正交条件

$$\int_0^a X_iX_jdx_1=\begin{cases} a & i=j \\ 0 & i\neq j \end{cases},\quad \int_0^b Y_iY_j\,dx_2=\begin{cases} b & i=j \\ 0 & i\neq j \end{cases}$$

将式(3.1)代入式(2.2)，两边同乘$X_i(x_1)\,Y_j(x_2)$，并在底面的区域上积分，利用上述正交关系，然后合并关于$f(t)$和$f^2(t)$的同类项，得到待定常数之间的关系

$$\left.\begin{array}{l} \dfrac{L_1}{EA}\left(\dfrac{\alpha_j}{b}\right)^4abB_{ij}+C\sum\limits_p\sum\limits_q B_{pq}M_8^{pi}\ N_8^{qj}+\dfrac{L_2}{EA}\left(\dfrac{\alpha_i}{a}\right)^4abB_{ij} \\ \quad=\alpha^2\beta^2\sum\limits_m\sum\limits_n\left(\dfrac{K_1}{\alpha^2}n^2+\dfrac{K_2}{\beta^2}m^2+C_1m^2n^2\right)M_4^{im}N_4^{jn}\ A_{mn} \\ \dfrac{L_1}{EA}\left(\dfrac{\alpha_j}{b}\right)^4abC_{ij}+C\sum\limits_p\sum\limits_q C_{ij}M_8^{pi}\ N_8^{qj}+\dfrac{L_2}{EA}\left(\dfrac{\alpha_i}{a}\right)^4abC_{ij} \\ \quad=\alpha^2\beta^2\sum\limits_m\sum\limits_n\sum\limits_r\sum\limits_s\left(mnrs\ M_6^{imr}\ N_6^{jns}-m^2s^2\ M_1^{imr}\ N_1^{jns}\right)A_{mn}A_{rs} \\ \quad\quad-\dfrac{\alpha^2\beta^2}{4}\sum\limits_m\sum\limits_n\sum\limits_r\sum\limits_s\left(C_2\alpha^2m^2+C_3\beta^2n^2\right)rs\left[(m+r)\ M_7^{i(m+r}\right. \\ \quad\quad\left.+(m-r)M_7^{i,m-r}\right]\left[(n+s)\ N_7^{j\,n+s)}+(n-s)\ N_7^{i\ n-s}\right]A_{mn}A_{rs} \end{array}\right\} \tag{3.2a,b}$$

式中，$i,j=1,2,\cdots$；$p,q,r,s,m,n=1,2,\cdots$；常数$M_1\sim M_8$定义为

$$\left.\begin{aligned}
M_1^{pmi} &=\int_0^a X_p \sin max_1 \sin iax_1 dx_1\\
M_2^{pmi} &=\int_0^a X_{p,11} \sin max_1 \sin iax_1 dx_1\\
M_3^{pmi} &=\int_0^a X_{p,1} \cos max_1 \sin iax_1 dx_1\\
M_4^{pi} &=\int_0^a X_p \sin iax_1 dx_1\\
M_5^{pi} &=\int_0^a X_{p,11} \sin iax_1 dx_1\\
M_6^{imr} &=\int_0^a X_i \cos max_1 \cos rax_1 dx_1\\
M_7^{im} &=\int_0^a X_i \cos max_1 dx_1\\
M_8^{pi} &=\int_0^a X_{p,11} X_i dx_1
\end{aligned}\right\} \qquad (3.3)$$

用N，Y，x_2，b，n，q，j，s分别替代M，X，x_1，a，m，p，i，r可相应得到$N_1\sim N_8$。

同样，将式(3.1)代入式(2.1)，两边同乘$\sin iax_1 \sin j\beta x_2$，积分后得到决定时间函数$f(t)$的振动方程

$$\ddot{f}+\omega_0^2 f+a_1 f^2+a_2 f^3+a_3=0 \qquad (3.4)$$

式中，

$$\left.\begin{aligned}
\omega_0^2 &=\frac{1}{\rho}\left[\frac{EI}{L_1} i^4\alpha^4+GJ\left(\frac{1}{L_1}+\frac{1}{L_2}\right)\alpha^2\beta^2 i^2 j^2+\frac{EI}{L_2} j^4\beta^4\right]\\
&\quad -\frac{4}{\rho ab A_{ij}}\sum_p\sum_q (K_1 M_4^{pi} N_5^{qj}+K_2 M_5^{pi} N_4^{qj}) B_{pq}\\
a_1 &= \frac{4}{\rho ab A_{ij}}\left\{\sum_m\sum_n\sum_p\sum_q (m^2\alpha^2 M_1^{pmi} N_2^{qnj}+2mn\alpha\beta M_3^{pmi} N_3^{qnj}\right.\\
&\quad \left.+n^2\beta^2 M_2^{pmi} N_1^{qnj}) A_{mn} B_{pq}-\sum_p\sum_q (K_1 M_4^{pi} N_5^{qj}+K_2 M_5^{pi} N_4^{qj}) C_{pq}\right\}\\
a_2 &= \frac{4}{\rho ab A_{ij}}\sum_m\sum_n\sum_p\sum_q (m^2\alpha^2 M_1^{pmi} N_2^{qnj}+2mn\alpha\beta M_3^{pmi} N_3^{qnj}\\
&\quad +n^2\beta^2 M_2^{pmi} N_1^{qnj}) A_{mn} C_{pq}\\
a_3 &= -\frac{4}{\rho ab A_{ij}}\int_0^a\int_0^b g_v(x_1,x_2,t)\sin iax_1 \sin j\beta x_2 dx_1 dx_2
\end{aligned}\right\} \qquad (3.5)$$

接下来考虑自由振动情况，即$g_v=0$，式(3.4)变成

$$\ddot{f}+\omega_0^2 f+a_1 f^2+a_2 f^3=0 \qquad (3.6)$$

选取$W(x_1,x_2,0)$无量纲化的幅值ε作为摄动参数，设

$$f(\tau)=\sum_{m=0}^{\infty}\varepsilon^m f_m(\tau) \tag{3.7}$$

$$\omega=\sum_{m=0}^{\infty}\varepsilon^m\omega_m \tag{3.8}$$

式中，$\tau=\omega t$，$f(0)A=h\varepsilon$，h为壳体厚度，A为

$$\sum_m\sum_n A_{mn}\sin m\alpha x_1\sin n\beta x_2$$

的幅值。

由式(3.7)和初始条件(2.6)可得到

$$f_1(0)=h/A,\ f_0(0)=f_2(0)=f_3(0)=\cdots=0,\ f'_0(0)=f'_1(0)=f'_2(0)=\cdots=0 \tag{3.9}$$

将方程(3.6)改写成

$$\omega^2f''+\omega_0^2f+\alpha_1f^2+\alpha_2f^3=0 \tag{3.10}$$

式中， $f''=d^2f/d\tau^2$

将式(3.7)、(3.8)代入式(3.10)，由ε^m各次幂的系数为零可得到各阶方程

$$\left.\begin{aligned}
\varepsilon^0:&\quad \omega_0^2f''_0+\omega_0^2f_0+\alpha_1f_0^2+\alpha_2f_0^3=0\\
\varepsilon^1:&\quad \omega_0^2f''_1+\omega_0^2f_1+2\alpha_1f_0f_1+3\alpha_2f_0^2f_1+2\omega_0\omega_1f''_0=0\\
\varepsilon^2,&\quad \omega_0^2f''_2+\omega_0^2f_2+\alpha_1f_1^2+2\omega_0\omega_1f''_1=0\\
\varepsilon^3:&\quad \omega_0^2f''_3+\omega_0^2f_3+2\omega_0\omega_1f''_2+\omega_1^2f''_1+2\omega_0\omega_2f''_1+2\alpha_1f_1f_2+\alpha_2f_1^3=0
\end{aligned}\right\} \tag{3.11a,b,c,d}$$

由式(3.11a)结合条件(3.9)可知$f_0=0$。由此从式(3.11b)可得到

$$f_1=\frac{h}{A}\cos\tau$$

将f_1代入式(3.11c)，克服长期项$\tau\cos\tau$得出$\omega_1=0$，解出f_2为

$$f_2=\frac{\alpha_1}{\omega_0^2}\left(\frac{h}{A}\right)^2\left(\frac{1}{3}\cos\tau+\frac{1}{6}\cos2\tau-\frac{1}{2}\right)$$

再将ω_1，f_1，f_2的结果全部代入式(3.11d)中，整理得到

$$\omega_0^2(f''_3+f_3)=\left[\frac{5}{6}\frac{\alpha_1^2}{\omega_0^2}\left(\frac{h}{A}\right)^3+2\omega_0\omega_2\frac{h}{A}-\frac{3}{4}\alpha_2\left(\frac{h}{A}\right)^3\right]\cos\tau+\cdots$$

为消除长期项，令$\cos\tau$的系数为零，将有

$$\omega_2=\frac{1}{4}\left(\frac{h}{A}\right)^2\left[\frac{3\alpha_2}{2\omega_0}-\frac{5\alpha_1^2}{3\omega_0^3}\right]$$

式(3.8)改写成如下形式

$$\omega=\omega_0+\varepsilon^2\omega_2+O(\varepsilon^3)$$

或

$$\frac{\omega}{\omega_0}=\left\{1+\left(\frac{h}{A}\varepsilon\right)^2\left[\frac{3\alpha_2}{4\omega_0^2}-\frac{5\alpha_1^2}{6\omega_0^4}\right]\right\}^{1/2} \tag{3.12}$$

现在处理谐和激励作用下的受迫振动情况。假设

$$f(t)=f(0)\cos\omega t,\ g_v(x_1,x_2,t)=g_0\cos\omega t \tag{3.13}$$

将上面式子代入振动方程(3.4)中，两边同乘$\cos\omega t$，对ωt从$0\sim 2\pi$积分后有

$$\omega^2=\omega_0^2+\frac{3}{4}\alpha_2 f^2(0)+\frac{\alpha_4 g_0}{f(0)}$$

即

$$\frac{\omega}{\omega_0}=\left\{1+\frac{3}{4}\frac{\alpha_2}{\omega_0^2}f^2(0)+\frac{\alpha_4 g_0}{\omega_0^2 f(0)}\right\}^{1/2}$$

或

$$\frac{\omega}{\omega_0}=\left\{1+\frac{3}{4}\frac{\alpha_2}{\omega_0^2}\left(\frac{h}{A}\varepsilon\right)^2+\frac{\alpha_4 g_0}{\omega_0^2}\frac{A}{h\varepsilon}\right\}^{1/2} \tag{3.14}$$

式中，

$$\alpha_4=-\frac{4}{\rho ab A_{ij}}\frac{[1-(-1)^i][1-(-1)^j]}{ij\alpha\beta} \tag{3.15}$$

如果讨论(m,n)型振动模态，可设

$$W(x_1,x_2,t)=hf(t)\sin m\alpha x_1\sin n\beta x_2$$

这样，式(3.12)，(3.14)和(3.15)进一步表达为

$$\frac{\omega_{mn}}{\omega_{mn(0)}}=\left\{1+\varepsilon^2\left[\frac{3\alpha_2}{4\omega_0^2}-\frac{5\alpha_3^2}{6\omega_0^4}\right]\right\}^{1/2} \tag{3.16}$$

$$\frac{\omega_{mn}}{\omega_{mn(0)}}=\left\{1+\frac{3\alpha_2}{4\omega_0^2}\varepsilon^2+\frac{\alpha_4 g_0}{\omega_0^2}\frac{1}{\varepsilon}\right\}^{1/2} \tag{3.17}$$

$$\alpha_4=-\frac{4[1-(-1)^m][1-(-1)^n]}{\pi^2\rho mnh} \tag{3.18}$$

以上考虑的是简支情形．对于由式(2.5)表示的夹支边界条件，可设

$$W(x_1,x_2,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}f(t)(1-\cos 2m\alpha x_1)(1-\cos 2n\beta x_2)$$

和简支情况同样的考虑，可得到类似式(3.4)的振动方程．对于(m,n)型模态，取

$$W(x_1,x_2,t)=hf(t)(1-\cos 2m\alpha x_1)(1-\cos 2n\beta x_2) \tag{3.19}$$

相应于式(3.5)，(3.18)中的系数$\omega_0^2,\alpha_1,\alpha_2,\alpha_3,\alpha_4$表达如下

$$\left.\begin{aligned}
\omega_{mn(0)}^2 &= \frac{16}{9\rho}\left[\frac{3EI}{L_1}m^4\alpha^4+GJ\left(\frac{1}{L_1}+\frac{1}{L_2}\right)\alpha^2\beta^2m^2n^2+\frac{3EI}{L_2}n^4\beta^4\right]\\
&\quad -\frac{4}{9\rho abh}\sum_p\sum_q[K_1U_4^{p(2m)}V_5^{q(2n)}+K_2U_5^{p(2m)}V_4^{q(2n)}]B_{pq}\\
\alpha_1 &= -\frac{16}{9\rho ab}\sum_p\sum_q[m^2\alpha^2U_1^{p(2m)(2m)}V_2^{q(2n)(2n)}-2mn\alpha\beta U_3^{p(2m)(2m)}V_3^{q(2n)(2n)}\\
&\quad +n^2\beta^2U_2^{p(2m)(2m)}V_1^{q(2n)(2n)}]B_{pq}\\
&\quad -\frac{4}{9\rho abh}\sum_p\sum_q[K_1U_4^{p(2m)}V_5^{q(2n)}+K_2U_5^{p(2m)}V_4^{q(2n)}]C_{pq}\\
\alpha_2 &= -\frac{16}{9\rho ab}\sum_p\sum_q[m^2\alpha^2U_1^{p(2m)(2m)}V_2^{q(2n)(2n)}-2mn\alpha\beta U_3^{p(2m)(2m)}V_3^{q(2n)(2n)}\\
&\quad +n^2\beta^2U_2^{p(2m)(2m)}V_1^{q(2n)(2n)}]C_{pq}\\
\alpha_3 &= -\frac{4}{9\rho abh}\int_0^a\int_0^b g_v(x_1,x_2,t)(1-\cos 2m\alpha x_1)(1-\cos 2n\beta x_2)dx_1dx_2\\
\alpha_4 &= -4/9\rho h
\end{aligned}\right\} \tag{3.20}$$

决定待定常数B_{ij}，C_{ij}的代数方程组为

$$\left.\begin{aligned}&\frac{L_1}{EA}\left(\frac{\alpha_j}{b}\right)^4 abB_{ij}+C\sum_p\sum_q B_{pq}M_8^{pi}N_8^{qj}+\frac{L_2}{EA}\left(\frac{\alpha_i}{a}\right)^4 abB_{ij}\\&\quad=-4\alpha^2\beta^2 h\left[\frac{K_1}{\alpha^2}n^2U_4^{i(2m)}V_6^{j(2n)}+\frac{K_2}{\beta^2}m^2U_6^{i(2m)}V_4^{j(2n)}\right]\\&\qquad+16C_1\alpha^2\beta^2m^2n^2hU_6^{i(2m)}V_6^{j(2n)}\\&\frac{L_1}{EA}\left(\frac{\alpha_j}{b}\right)^4 abC_{ij}+C\sum_p\sum_q C_{pq}M_7^{pi}N_8^{qj}+\frac{L_2}{EA}\left(\frac{\alpha_i}{a}\right)^4 abC_{ij}\\&\quad=16\alpha^2\beta^2m^2n^2h^2[M_1^{i(2m)(2m)}N_2^{j(2n)(2n)}-U_1^{i(2m)(2m)}V_1^{j(2n)(2n)}]\\&\qquad+64C_2\alpha^4\beta^2m^4n^2h^2[V_6^{j(2n)}-V_6^{j(4n)}]U_6^{i(4m)}\\&\qquad+64C_3\alpha^2\beta^4m^2n^4h^2[U_6^{i(2m)}-U_6^{i(4m)}]V_6^{j(4n)}\\&\qquad\qquad(i,j=1,2,\cdots;\ p,q=1,2,\cdots)\end{aligned}\right\}\tag{3.21a,b}$$

上面式子中，已记

$$\left.\begin{aligned}&U_1^{p(2m)(2m)}=\int_0^a X_p\cos 2m\alpha x_1(1-\cos 2m\alpha x_1)dx_1\\&U_2^{p(2m)(2m)}=\int_0^a X_{p,11}(1-\cos 2m\alpha x_1)^2dx_1\\&U_3^{p(2m)(2m)}=\int_0^a X_{p,1}\sin 2m\alpha x_1(1-\cos 2m\alpha x_1)dx_1\\&U_4^{p(2m)}=\int_0^a X_p(1-\cos 2m\alpha x_1)dx_1\\&U_5^{p(2m)}=\int_0^a X_{p,11}(1-\cos 2m\alpha x_1)dx_1\\&U_6^{i(m)}=\int_0^a X_i\cos m\alpha x_1dx_1\end{aligned}\right\}\tag{3.22}$$

同样，用V,Y,x_2,b,n,q,j分别替代U,X,x_1,a,m,p,i可相应得到$V_1\sim V_6$。

由式(3.19)可知，式(3.12)和(3.14)中的$A=4h$，因此夹支条件下自由振动和受迫振动的幅频特征关系式为

$$\frac{\omega_{mn}}{\omega_{mn(0)}}=\left\{1+\frac{1}{16}\varepsilon^2\left[\frac{3\alpha_2}{4\omega_0^2}-\frac{5\alpha_1^2}{6\omega_0^4}\right]\right\}^{1/2}\tag{3.23}$$

$$\frac{\omega_{mn}}{\omega_{mn(0)}}=\left\{1+\frac{3\alpha_2}{64\omega_0^2}\varepsilon^2+\frac{4\alpha_4 g_0}{\omega_0^2}\frac{1}{\varepsilon}\right\}^{1/2}\tag{3.24}$$

四、算例分析与讨论

在运用式(3.16)和(3.17)或(3.23)和(3.24)进行计算时，通过引入如下无量纲量可对α_1/ω_0^2，α_2/ω_0^2和$\alpha_4 g_0/\omega_0^2$进行无量纲化

$$\left.\begin{aligned}&K_1^*=\frac{K_1a^2}{h},\quad K_2^*=\frac{K_2b^2}{h},\quad g_1=\frac{g_0a^4}{Eh^4}\\&m_1=\frac{a}{b},\quad m_2=\frac{L_1}{L_2},\quad m_3=\frac{h}{L_2},\quad m_4=\frac{a}{L_2}\end{aligned}\right\}\tag{4.1}$$

为便于计算，本文选取梁元截面为圆截面，且取定$m_1=m_2=1$，$m_3=0.05$，$m_4=40$，泊

松比$\nu=0.3$，边界为简支情况.

图2～4显示了各种振型的自由振动幅频关系. 计算结果表明，网格扁壳和网格平板的振动频率随幅值的增加而单调递增. 其中，对应于x_1，x_2方向都为一个全波（$m=n=2$）和都为三个半波($m=n=3$)的振型，频率随幅值增加缓慢. 比较振型$m=1$，$n=1$；$m=1$，$n=2$；$m=1$，$n=3$所对应的幅频频关系可知，网格扁圆柱壳的振动特征较网格扁球壳更接近网格平板的情况.

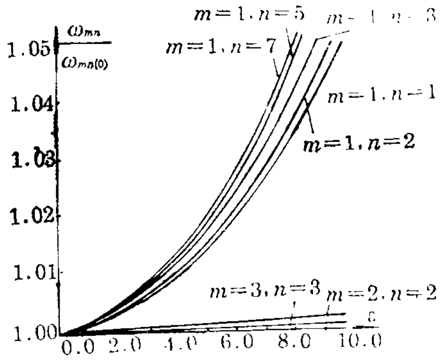

图2 网格扁球壳自由振动幅频关系
($K_1^*=K_2^*=25.0$)

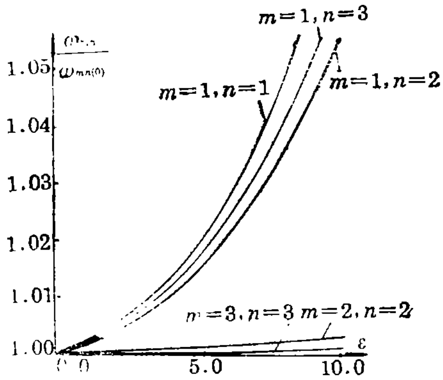

图3 网格扁圆柱壳自由振动幅频关系
($K_1^*=0$，$K_2^*=25.0$)

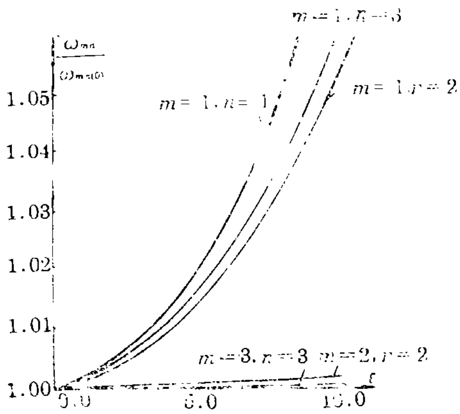

图4 网格平板自由振动幅频关系
($K_1^*=K_2^*=0$)

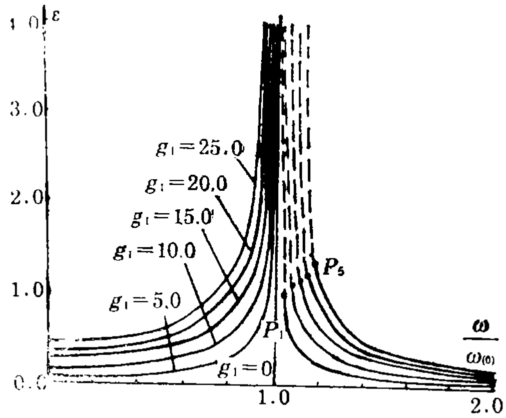

图5 网格扁球壳受迫振动幅频关系
($K_1^*=K_2^*=25.0$)

网格扁球壳和网格平板在对应于振型$m=1$，$n=1$（x_1，x_2方向皆为一个半波）的情况下受迫振动频率计算结果如图5，6所示.计算预示，网格扁壳和网格平板振动时有跳跃现象产生. 图中实线代表结构的稳定振动，虚线表征非稳定振动，点$P_1,\cdots,P_5$是非线性振动跳跃现象的“临界”点.$g_1=0$所代表的曲线是结构非线性自由振动曲线.

从图4和图6也可看出，网格平板非线性振动特征与各向异性板的情况定性一致[16].

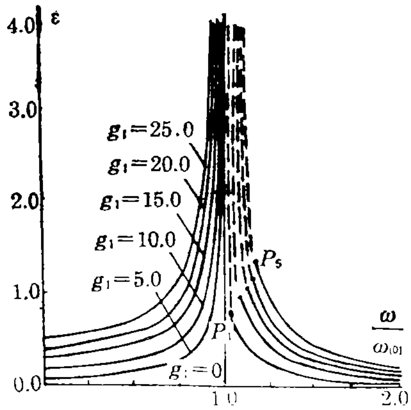

图6 网格平板受迫振动幅频关系
($K_1^*=K_2^*=0$)

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参 考 文 献

[1] Ellington, J. P. and H. McCallion, The free vibration of grillages, *ASME J. Appl. Mech.*, **26** (1959), 603—607.

[2] Renton, J. D., On the analysis of triangular mesh grillages, *Int. J. Solids Structures*, **2** (1966), 307—318.

[3] McDaniel, T. J. and K. J. Chang, Dynamics of rotationally periodic large space structures, *J. Sound and Vibration*, **68**(3) (1980), 351—368.

[4] Williams, F. W., An algorithm for exact eigenvalue calculations for rotationally periodic structures, *Int. J. for Num. Meth. in Eng.*, **23** (1986), 609—622.

[5] Williams, F. W., Exact eigenvalue calculations for structures with rotationally periodic substructures, *Int. J. for Num. Meth. in Eng.*, **23** (1986), 695—706.

[6] Anderson, M. S., Buckling of periodic lattice structures, *AIAA J.*, **19**(6) (1981), 782—788.

[7] Anderson, M. S., Vibration of prestressed periodic lattice structures, *AIAA J.*, **20** (1982), 551—556.

[8] Anderson, M. S. and F. W. Williams, Natural vibration and buckling of general periodic lattice structures, *AIAA J.*, **24**(1) (1986), 163—169.

[9] Dean, D. L., *Discrete Field Analysis of Structural Systems*, Course No. 203, Int. Center for Mech. Sci., Udine, Italy, Pergamon Press (1976).

[10] Wah, T. and L. R. Calcote, *Structural Analysis by Finite Difference Calculus*, von Nostrand Reinhold Co., New York (1970).

[11] Kollar, L., Continuum equations of timber lattice shells, *Acta Technica Academiae Scientiarum Hungaricae*, **94**(3-4) (1982), 133—141.

[12] Noor, A. K., et al., Continuum models for beam-and platelike lattice structures, *AIAA J.*, **16**(12) (1978), 1219—1228.

[13] Noor, A. K. and M. P. Nemeth, Anaysis of spatial beamlike lattices with rigid joints, *Computer Meth. in Appl. Mech. Eng.*, **24** (1980), 35—59.

[14] Nie Guo-hua, A non-linear theory of reticulated shallow shells, *Proceedings of the 22nd Midwestern Mechanics Conference*, Rolla, Missouri, USA (1991).

[15] Liu Ren-huai, Li Dong, Nie Guo-hua and Cheng Zheng-qiang, Non-linear buckling of squarely-latticed shallow spherical shells, *Int. J. Non-Linear Mech.*, **26**(5) (1991), 547—565.

[16] Chia, C. Y., *Nonlinear Analysis of Plates*, McGraw-Hill New York (1980).

Non-Linear Vibration of Rectangular Reticulated Shallow Shell Structures

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Abstract

This paper deals with non-linear vibration of rectangular reticulated shallow shells by applying non-linear elastic theory of such structures established by the author. Using the assumed (generalized) Fourier series solutions for transverse deflection (lattice joint transverse displacement) and force function, weighted means of the trial functions lead to the relations among the coefficients related to the solutions and vibration equation which determines the unknown time function, and then the amplitude-frequency relations for free vibration and forced vibration due to harmonic force are derived with the aid of the regular perturbation method and Galerkin procedure, respectively. Numerical examples are given as well.

Key words rectangular reticulated shallow shells, non-linear, vibration, characteristic amplitude-frequency relation, beam vibration eigenfunctions