应用数学和力学，第16卷 第6期（1995年6月）
Applied Mathematics and Mechanics

应用数学和力学编委会编
重庆出版社出版

一类具耦合的化学反应扩散系统的奇摄动*

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（戴世强推荐 1994年11月24日收到）

摘 要

本文研究一类带耦合项的化学反应扩散方程组解的性态，利用上下解理论证得解的存在性，然后在一定的参数环境下考虑了相应的奇摄动问题，并给出一致有效的渐近解.

关键词 奇摄动 耦合系统 反应与扩散 迭代法 上下解

一、引 言

有关椭圆型、抛物型方程（组）的奇摄动已有不少结果[1,2]. 对于反应扩散方程组的奇摄动研究多数是在无耦合项的情形下进行的[3]. 然而实际应用中，如具有扩散现象和群体防卫能力的简单食物链模型[4]，化学反应扩散过程[5]等，常出现带耦合项的反应扩散方程组，这给问题的研究增添了复杂性. 对同一个模型，在不同的参数环境下，解可有不同的性态.

本文我们将研究一类化学反应扩散系统初边值问题的奇摄动：

$$\sigma_1 u_t-(\varepsilon\nabla^2 u+L_1 u)=u(a_1-b_1 u-c_1 v)+q_1 v \tag{1.1}$$

$$\sigma_2 v_t-(\varepsilon\nabla^2 v+L_2 v)=-c_2 uv+d_2 w-q_2 v \tag{1.2}$$

$$\sigma_3 w_t-(\varepsilon\nabla^2 w+L_3 w)=b_3 u-d_2 w \qquad (t,x)\in[0,T]\times\Omega \tag{1.3}$$

$$\frac{\partial u}{\partial \nu}=0,\ \frac{\partial v}{\partial \nu}=0,\ \frac{\partial w}{\partial \nu}=0 \qquad (t,x)\in[0,T]\times\partial\Omega \tag{1.4}$$

$$u(0,x)=u_0(x),\ v(0,x)=v_0(x),\ w(0,x)=w_0(x), \qquad x\in\Omega \tag{1.5}$$

其中u，v，w分别表反应物的浓度，a_i，b_i，c_i和q_i取正常数，$\varepsilon>0$是小参数，$\partial/\partial\nu$表$\partial\Omega$的外法向导数，有界区域Ω即扩散区域，L_i为线性算子：

$$L_i=\sum_{\alpha=1}^{n} b_\alpha^i(t,x)\,\frac{\partial}{\partial x_\alpha}-b^i(t,x)$$

$$b^i(t,x)\geqslant b_0>0,\ \sum_{\alpha=1}^{n} b_\alpha^i n_\alpha\leqslant\delta<0 \qquad (i=1,2,3)$$

n_α 为 $\partial\Omega$ 上的外法线方向素. 这类问题在充分搅拌的反应扩散系统中出现. 我们先将针对该

问题给出上下解定义和问题解的存在性结果，然后在一定的条件下，构造问题的形式渐近解，并利用所给结果证得形式解的一致有效性.

二、存在性定理

定义 称光滑函数偶$(\tilde{u},\tilde{v},\tilde{w})$，$(\hat{u},\hat{v},\hat{w})$为下述问题

$$\sigma_1 u_t-(\nabla^2 u+L_1 u)=u(a_1-b_1 u-c_1 v)+q_1 v \tag{2.1}$$

$$\sigma_2 v_t-(\nabla^2 v+L_2 v)=-c_2 uv+d_2 w-q_2 v \tag{2.2}$$

$$\sigma_3 w_t-(\nabla^2 w+L_3 w)=b_3 u-d_3 w \qquad (t,x)\in[0,T]\times\Omega \tag{2.3}$$

$$\frac{\partial u}{\partial \nu}=0,\ \frac{\partial v}{\partial \nu}=0,\ \frac{\partial w}{\partial \nu}=0 \qquad (t,x)\in[0,T]\times\partial\Omega \tag{2.4}$$

$$u(0,x)=u_0(x),\ \ v(0,x)=v_0(x),\ \ w(0,x)=w_0(x) \qquad x\in\Omega \tag{2.5}$$

的上、下解，如果$(\tilde{u},\tilde{v},\tilde{w})\geqslant(\hat{u},\hat{v},\hat{w})$，且满足如下的微分不等式和初边值不等式

$$\sigma_1\tilde{u}_t-(\nabla^2\tilde{u}+L_1\tilde{u})\geqslant\tilde{u}(a_1-b_1\tilde{u}-c_1\hat{v})+q_1\tilde{v}$$
$$\sigma_2\tilde{v}_t-(\nabla^2\tilde{v}+L_2\tilde{v})\geqslant-c_2\hat{u}\tilde{v}+d_2\tilde{w}-q_2\tilde{v}$$
$$\sigma_3\tilde{w}_t-(\nabla^2\tilde{w}+L_3\tilde{w})\geqslant b_3\tilde{u}-d_3\tilde{w}$$
$$\sigma_1\hat{u}_t-(\nabla^2\hat{u}+L_1\hat{u})\leqslant\hat{u}(a_1-b_1\hat{u}-c_1\tilde{v})+q_1\hat{v}$$
$$\sigma_2\hat{v}_t-(\nabla^2\hat{v}+L_2\hat{v})\leqslant-c_2\tilde{u}\hat{v}+d_2\hat{w}-q_2\hat{v}$$
$$\sigma_3\hat{w}_t-(\nabla^2\hat{w}+L_3\hat{w})\leqslant b_3\hat{u}-d_3\hat{w}$$
$$\frac{\partial\tilde{u}}{\partial\nu}\geqslant0\geqslant\frac{\partial\hat{u}}{\partial\nu},\ \ \frac{\partial\tilde{v}}{\partial\nu}\geqslant0\geqslant\frac{\partial\hat{v}}{\partial\nu},\ \ \frac{\partial\tilde{w}}{\partial\nu}\geqslant0\geqslant\frac{\partial\hat{w}}{\partial\nu} \qquad (t,x)\in[0,T]\times\partial\Omega$$
$$\tilde{u}\geqslant u_0\geqslant\hat{u},\ \ \tilde{v}\geqslant v_0\geqslant\hat{v},\ \ \tilde{w}\geqslant w_0\geqslant\hat{w}, \qquad t=0,x\in\Omega$$

由于该类方程组右端的函数不满足通常的拟单调性($q>0$)，故这里定义的上、下解函数较为复杂，其目的是为了生成单调迭代序列.

定理 若问题(2.1)～(2.5)存在如上定义的上、下解$(\tilde{u},\tilde{v},\tilde{w})$，$(\hat{u},\hat{v},\hat{w})$，则该问题存在解$(u,v,w)$且满足：

$$(\hat{u},\hat{v},\hat{w})\leqslant(u,v,w)\leqslant(\tilde{u},\tilde{v},\tilde{w}) \qquad (t,x)\in[0,T]\times\Omega$$

证明 构造迭代过程如下

$$L_1'\bar{u}^{(k)}=\gamma_1\bar{u}^{(k-1)}+\bar{u}^{(k-1)}(a_1-b_1\bar{u}^{(k-1)}-c_1\underline{v}^{(k-1)})+q_1\bar{v}^{(k-1)}$$
$$L_2'\bar{v}^{(k)}=(\gamma_2-q_2)\bar{v}^{(k-1)}-c_2\underline{u}^{(k-1)}\bar{v}^{(k-1)}+d_2\bar{w}^{(k-1)}$$
$$L_3'\bar{w}^{(k)}=b_3\bar{u}^{(k-1)}$$
$$L_1'\underline{u}^{(k)}=\gamma_1\underline{u}^{(k-1)}+\underline{u}^{(k-1)}(a_1-b_1\underline{u}^{(k-1)}-c_1\bar{v}^{(k-1)})+q_1\underline{v}^{(k-1)}$$
$$L_2'\underline{v}^{(k)}=(\gamma_2-q_2)\underline{v}^{(k-1)}-c_2\bar{u}^{(k-1)}\underline{v}^{(k-1)}+d_2\underline{w}^{(k-1)}$$
$$L_3'\underline{w}^{(k)}=b_3\underline{u}^{(k-1)}$$
$$L_i'=\left(\sigma_i\frac{\partial}{\partial t}-(\nabla^2+L_i)+\gamma_i\right) \qquad (i=1,2,3)$$

γ_i按下述约束选取：

$$\gamma_1\geqslant\max\{2b_1u+c_1v-a_1;\ \ \hat{u}\leqslant u\leqslant\tilde{u},\ \ \hat{v}\leqslant v\leqslant\tilde{v}\}$$
$$\gamma_2\geqslant c_2\tilde{u}+q_2,\ \ \gamma_3=d_3$$

选取迭代初值及定解条件如下：

$$(\bar{u}^{(0)},\bar{v}^{(0)},\bar{w}^{(0)})=(\tilde{u},\tilde{v},\tilde{w})$$

$$(\underline{u}^{(0)},\underline{v}^{(0)},\underline{w}^{(0)})=(\hat{u},\hat{v},\hat{w})$$

$$\frac{\partial\bar{u}^{(k)}}{\partial\nu}=\frac{\partial\underline{u}^{(k)}}{\partial\nu}=0,\ \frac{\partial\bar{v}^{(k)}}{\partial\nu}=\frac{\partial\underline{v}^{(k)}}{\partial\nu}=0,\ \frac{\partial\bar{w}^{(k)}}{\partial\nu}=\frac{\partial\underline{w}^{(k)}}{\partial\nu}=0$$

$$\bar{u}^{(k)}=\underline{u}^{(k)}=u_0(x),\quad \bar{v}^{(k)}=\underline{v}^{(k)}=v_0(x),\quad \bar{w}^{(k)}=\underline{w}^{(k)}=w_0(x)$$

令 $(y_1,y_2,y_3)=(\bar{u}^{(0)},\bar{v}^{(0)},\bar{w}^{(0)})-(\bar{u}^{(1)},\bar{v}^{(1)},\bar{w}^{(1)})$

由前面构造的迭代过程有：

$$L_1'y_1=\sigma_1\frac{\partial\bar{u}^{(0)}}{\partial t}-(\nabla^2+L_1)\bar{u}^{(0)}+\gamma_1\bar{u}^{(0)}$$

$$-[\gamma_1\bar{u}^{(0)}+\bar{u}^{(0)}(a_1-b_1\bar{u}^{(0)}-c_1\underline{v}^{(0)})+q_1\bar{v}^{(0)}]$$

$$=\sigma_1\frac{\partial\tilde{u}}{\partial t}-(\nabla^2+L_1)\tilde{u}-[\tilde{u}(a_1-b_1\tilde{u}-c_1\hat{v})+q_1\tilde{v}]\geqslant0$$

$$L_2'y_2=\sigma_2\frac{\partial\bar{v}^{(0)}}{\partial t}-(\nabla^2+L_2)\bar{v}^{(0)}+\gamma_2\bar{v}^{(0)}$$

$$-(\gamma_2-q_2)\bar{v}^{(0)}+c_2\underline{u}^{(0)}\bar{v}^{(0)}-d_2\bar{w}^{(0)}$$

$$=\sigma_2\frac{\partial\tilde{v}}{\partial t}-(\nabla^2+L_2)\tilde{v}-[-c_2\hat{u}\tilde{v}+d_2\tilde{w}-q_2\tilde{v}]\geqslant0$$

$$L_3'y_3=\sigma_3\frac{\partial\bar{w}^{(0)}}{\partial t}-(\nabla^2+L_3)\bar{w}^{(0)}+\gamma_3\bar{w}^{(0)}-b_3\bar{u}^{(0)}$$

$$=\sigma_3\frac{\partial\tilde{w}}{\partial t}-(\nabla^2+L_3)\tilde{w}-[b_3\tilde{u}-d_3\tilde{w}]\geqslant0$$

且由迭代定解条件知

$$\frac{\partial y_1}{\partial\nu}=\frac{\partial\tilde{u}}{\partial\nu}\geqslant0,\ \frac{\partial y_2}{\partial\nu}=\frac{\partial\tilde{v}}{\partial\nu}\geqslant0,\ \frac{\partial y_3}{\partial\nu}=\frac{\partial\tilde{w}}{\partial\nu}\geqslant0$$

$$y_1(0,x)=\tilde{u}(0,x)-u_0(x)\geqslant0,\quad y_2(0,x)=\tilde{v}(0,x)-v_0(x)\geqslant0$$

$$y_3(0,x)=\tilde{w}(0,x)-w_0(x)\geqslant0$$

由最大值原理知$y_i(t,x)\geqslant0$，$(t,x)\in[0,T]\times\Omega$. 故而：

$$(\bar{u}^{(1)},\bar{v}^{(1)},\bar{w}^{(1)})\leqslant(\bar{u}^{(0)},\bar{v}^{(0)},\bar{w}^{(0)})$$

同理还可证明：

$$(\underline{u}^{(1)},\underline{v}^{(1)},\underline{w}^{(1)})\geqslant(\underline{u}^{(0)},\underline{v}^{(0)},\underline{w}^{(0)})$$

若证$(z_1,z_2,z_3)=(\bar{u}^{(1)},\bar{v}^{(1)},\bar{w}^{(1)})-(\underline{u}^{(1)},\underline{v}^{(1)},\underline{w}^{(1)})$，则由迭代过程有

$$L_1'z_1=[\gamma_1+a_1-b_1(\bar{u}^{(0)}+\underline{u}^{(0)})-c_1\underline{v}^{(0)}][\bar{u}^{(0)}-\underline{u}^{(0)}]$$

$$+(c_1\underline{u}^{(0)}+q_1)(\bar{v}^{(0)}-\underline{v}^{(0)})\geqslant0$$

同理可得：

$$L_2'z_2\geqslant0,\quad L_3'z_3\geqslant0$$

并且$\partial z_i/\partial\nu=0$，$z_i(0,x)=0$ $(i=1,2,3)$故$z_i\geqslant0$，即$\bar{u}^{(1)}\geqslant\underline{u}^{(1)}$，$\bar{v}^{(1)}\geqslant\underline{v}^{(1)}$，$\bar{w}^{(1)}\geqslant\underline{w}^{(1)}$，类似可证序列

$$\{\bar{U}^{(k)}\}\equiv\{\bar{u}^{(k)},\bar{v}^{(k)},\bar{w}^{(k)}\},\quad\{\underline{U}^{(k)}\}\equiv\{\underline{u}^{(k)},\underline{v}^{(k)},\underline{w}^{(k)}\}$$

具如下单调有界性

$$\underline{U}^{(k)}\leqslant\underline{U}^{(k+1)}\leqslant\bar{U}^{(k+1)}\leqslant\bar{U}^{(k)},\qquad(k=0,1,2,\cdots)$$

再用文[6]的方法可证

$$\lim_{k\to\infty}\underline{U}^{(k)}=\lim_{k\to\infty}\bar{U}^{(k)}=(u,v,w)$$

即是问题(2.1)～(2.5)的唯一解.

三、奇摄动问题

现在来考虑奇摄动问题(1.1)～(1.5)．先构造其外部解$U(t,x,\varepsilon)$，$V(t,x,\varepsilon)$，$W(t,x,\varepsilon)$．其退化问题是：

$$\sigma_1u_t-L_1u=u(a_1-b_1u-c_1v)+q_1v$$
$$\sigma_2v_t-L_2v=-c_2uv+d_2w-q_2v$$
$$\sigma_3w_t-L_3w=b_3u-d_3w$$
$$u(0,x)=u_0(x),\ \ v(0,x)=v_0(x),\ \ w(0,x)=w_0(x)$$

它形式上是一常微分方程组，设它在$[0,T]\times\bar{\Omega}$上有唯一光滑解$U_0(t,x)$，$V_0(t,x)$，$W_0(t,x)$，在此基础上再设外部解有如下展开式

$$U(t,x,\varepsilon)=\sum_{i=0}^{\infty}U_i(t,x)\varepsilon^i \tag{3.1}$$

$$V(t,x,\varepsilon)=\sum_{i=0}^{\infty}V_i(t,x)\varepsilon^i \tag{3.2}$$

$$W(t,x,\varepsilon)=\sum_{i=0}^{\infty}W_i(t,x)\varepsilon^i \tag{3.3}$$

代入(1.1)～(1.3)，(1.5)式，再比较方程两端ε同次幂的系数得：

$$\sigma_1U_{it}-(\nabla^2U_{i-1}+L_1U_i)=a_1U_i-b_1\sum_{k+j=i}U_kU_j-c_1\sum_{k+j=i}U_kV_j+q_1V_i$$

$$\sigma_2V_{it}-(\nabla^2V_{i-1}+L_2V_i)=-c_2\sum_{k+j=i}U_kV_j+d_2W_i-q_2V_i$$

$$\sigma_3W_{it}-(\nabla^2W_{i-1}+L_3W_i)=b_3U_i-d_3W_i$$

$$U_i(0,x)=0,\ \ V_i(0,x)=0,\ \ W_i(0,x)=0,\qquad x\in\partial\Omega$$

由此线性问题可逐次地求解U_i，V_i，W_i，并设其在区域$[0,T]\times\bar{\Omega}$上光滑．从而问题的外部解(3.1)～(3.3)已求得．但它未必满足边界条件(1.4)，为此我们将构造边界层项予以校正．

先在$\partial\Omega$的邻域建立局部坐标系(ρ,φ)．规定在$\partial\Omega$的邻域内每一点Q的坐标按如下方式选取：取$\rho(\leqslant\rho_0')$为从点Q到边界$\partial\Omega$的距离，其中ρ_0'足够小，使得在整个$\partial\Omega$的邻域内，$\partial\Omega$上每一点的内法线互不相交．$\varphi\equiv(\varphi_1,\varphi_2,\cdots,\varphi_{n-1})$为在$(n-1)$维流形$\partial\Omega$上的非奇坐标，同时取点$Q$的$\varphi$坐标等于过$Q$的内法线与边界$\partial\Omega$的交点$P$的$\varphi$坐标．

此时$\partial\Omega$的邻域$\rho\leqslant\rho_0'$内原问题(1.1)，(1.2)，(1,3)具如下表示：

$$\sigma_1u_t-(\varepsilon\bar{L}_1'+\bar{L}_1)u=u(a_1-b_1u-c_1v)+q_1v \tag{1.1$'$}$$

$$\sigma_2v_t-(\varepsilon\bar{L}_2'+\bar{L}_2)v=-c_2uv+d_2w-q_2v \tag{1.2$'$}$$

$$\sigma_3w_t-(\varepsilon\bar{L}_3'+\bar{L}_3)w=b_3u-d_3w \tag{1.3$'$}$$

这里

$$L_i' \equiv a_{nn}(\rho,\varphi)\frac{\partial^2}{\partial\rho^2}+\sum_{i=1}^{n-1}a_{ni}(\rho,\varphi)\frac{\partial^2}{\partial\rho\partial\varphi_i}+\sum_{i,j=1}^{n-1}a_{ij}(\rho,\varphi)\frac{\partial^2}{\partial\varphi_i\partial\varphi_j}$$

$$+b_n(\rho,\varphi)\frac{\partial}{\partial\rho}+\sum_{i=1}^{n-1}b_i(\rho,\varphi)\frac{\partial}{\partial\varphi_i} \tag{3.4}$$

$$\bar{L}_i\equiv\bar{b}_n^i(t,\rho,\varphi)\frac{\partial}{\partial\rho}+\sum_{j=1}^{n-1}\bar{b}_j^i(t,\rho,\varphi)\frac{\partial}{\partial\varphi_j}-\bar{b}^i(t,\rho,\varphi) \tag{3.5}$$

式中a_{ij}，b_i，$\bar{b}_j^i$，$\bar{b}^i$由原算子的系数和坐标变换系数确定，u,v,w相应地是新变量$(t,\rho,\varphi_1,\cdots,\varphi_{n-1})$的函数，同时定解条件变为

$$\frac{\partial u}{\partial\rho}=\frac{\partial v}{\partial\rho}=\frac{\partial w}{\partial\rho}=0 \tag{1.4$'$}$$

$$\left.\begin{aligned}&u(0,x)=u_0(x)=\bar{u}_0(\rho,\varphi),\quad v(0,x)=v_0(x)=\bar{v}_0(\rho,\varphi)\\&w(0,x)=w_0(x)=\bar{w}_0(\rho,\varphi)\end{aligned}\right\} \tag{1.5$'$}$$

引入多重尺度变量 $(\tilde{\rho}_i=k^i(t,\rho,\varphi)\cdot\varepsilon,\rho,\varphi,t)$，其中的$k^i(t,\rho,\varphi)$将在下面确定．于是由(3.4)、(3.5)有：

$$\sigma_i(\partial/\partial t)-(\varepsilon\bar{L}_i'+\bar{L}_i)=\varepsilon^{-1}K_0^i+K_1^i+\varepsilon K_2^i$$

其中

$$K_0^i=-\left[a_{nn}(k_\rho^i)^2\frac{\partial^2}{\partial\tilde{\rho}_i^2}\bar{b}_n^i k_\rho^i\ \frac{\partial}{\partial\tilde{\rho}_i}\right] \tag{3.6}$$

K_1^i，K_2^i均为线性齐次微分算子，其表示式从略．

设原问题(1.1)～(1.5)的边界层项$(\bar{U},\bar{V},\bar{W})$具展式

$$\bar{U}(t,\tilde{\rho}_i,\rho,\varphi,\varepsilon)\equiv\sum_{j=1}^{\infty}\bar{U}_j(t,\tilde{\rho}_i,\rho,\varphi)\varepsilon^j \tag{3.7}$$

$$\bar{V}(t,\tilde{\rho}_i,\rho,\varphi,\varepsilon)\equiv\sum_{j=1}^{\infty}\bar{V}_j(t,\tilde{\rho}_i,\rho,\varphi)\varepsilon^j \tag{3.8}$$

$$\bar{W}(t,\tilde{\rho}_i,\rho,\varphi,\varepsilon)\equiv\sum_{j=1}^{\infty}\bar{W}_j(t,\tilde{\rho}_i,\rho,\varphi)\varepsilon^j \tag{3.9}$$

令

$$u=U+\bar{U},\quad v=V+\bar{V},\quad w=W+\bar{W} \tag{3.10}$$

将(3.10)代入边界层方程(1.1)′～(1.5)′得到：

$$[\varepsilon^{-1}K_0^1+K_1^1+\varepsilon K_2^1][U+\bar{U}]=[U+\bar{U}][a_1-b_1(U+\bar{U})$$
$$-c_1(V+\bar{V})]+q_1(V+\bar{V}) \tag{3.11}$$

即有

$$[\varepsilon^{-1}K_0^1+K_1^1+\varepsilon K_2^1]\bar{U}=a_1\bar{U}-b_1(2U\bar{U}+\bar{U}^2)-c_1(U\bar{V}+V\bar{V}+\bar{V}^2)+q_1\bar{V} \tag{3.12}$$

令

$$K_0^1\bar{U}_1=-\left[a_{nn}(k_\rho^1)^2\frac{\partial^2}{\partial\tilde{\rho}_1^2}+\bar{b}_n^1k_\rho^1\frac{\partial}{\partial\tilde{\rho}_1}\right]\bar{U}_1=0$$

$$k^1=\int_0^\rho \frac{\bar{b}_n^1(t,s,\varphi)}{a_{nn}(s,\varphi)}ds$$

则可有

$$\bar{U}_1=\beta_1(t,\rho,\varphi)\exp[-k'(t,\rho,\varphi)/\varepsilon]$$

其中$\beta_1(t,\rho,\varphi)$为待定函数，可按[2,3]的方法确定．且当ρ充分小时，$k^1(t,\rho,\varphi)>0$，故$\bar{U}_1$具有边界层函数的性质．同类似的方法可确定$\bar{U}_j$且具如下形式：

$$\bar{U}_j(t,\rho,\varphi)=\beta_j(t,\rho,\varphi)\cdot\exp[-k^1(t,\rho,\varphi)/\varepsilon]$$

令

$$\tilde{U}_j(t,\rho,\varphi)=\psi\bar{U}_j \qquad (j=1,2,3,\cdots)$$

其中ψ为关于ρ的充分光滑函数，满足

$$\psi=\begin{cases}1, & 0\leqslant\rho\leqslant\rho_0'/3\\ 0, & \rho\geqslant 2\rho_0'/2\end{cases}$$

至此已构造出u的形式解，类此可构造v，w的形式解(3.10)．下面来证明(3.10)的一致有效性．

构造上、下解函数偶：

$$(\tilde{U},\tilde{V},\tilde{W})=(X_m+l_1\varepsilon^{m+1},\ Y_m+l_2\varepsilon^{m+1},\ Z_m+l_3\varepsilon^{m+1})$$

$$(\hat{U},\hat{V},\hat{W})=(X_m-l_1\varepsilon^{m+1},Y_m-l_2\varepsilon^{m+1},Z_m-l_3\varepsilon^{m+1})$$

$$X_m=\sum_{j=0}^m (U_j+\varepsilon\bar{U}_j)\varepsilon^j,\ Y_m=\sum_{j=0}^m (V_j+\varepsilon\bar{V}_j)\varepsilon^j,Z_m=\sum_{j=0}^m (W_j+\varepsilon\bar{W}_j)\varepsilon^j$$

其中l_1，l_2，l_3为待定正常数．显然对足够大的l_1，l_2，l_3成立：

$$(\tilde{U},\tilde{V},\tilde{W})|_{t=0}\geqslant(u_0(x),v_0(x),w_0(x))\geqslant(\hat{U},\hat{V},\hat{W})|_{t=0} \tag{3.13}$$

$$\left(\frac{\partial\tilde{U}}{\partial\nu},\frac{\partial\tilde{V}}{\partial\nu},\frac{\partial\tilde{W}}{\partial\nu}\right)\geqslant(0,0,0)\geqslant\left(\frac{\partial\hat{U}}{\partial\nu},\frac{\partial\hat{V}}{\partial\nu},\frac{\partial\hat{W}}{\partial\nu}\right)$$

$$(t,x)\in[0,T]\times\partial\Omega \tag{3.14}$$

继续地证明微分不等式

$$\begin{aligned}\sigma_1\tilde{U}_t-(\varepsilon\nabla^2\tilde{U}+L_1\tilde{U})&=\tilde{U}(a_1-b_1\tilde{U}-c_1\hat{V})+q_1\tilde{V}\\ &=(X_m+l_1\varepsilon^{m+1})[a_1-b_1(X_m+l_1\varepsilon^{m+1})-c_1(Y_m-l_2\varepsilon^{m+1})]+q_1(Y_m+l_2\varepsilon^{m+1})\\ &=a_1X_m-b_1X_m^2-c_1X_mY_m+q_1Y_m\\ &\quad+(-b_1l_1X_m+c_1l_2X_m+a_1l_1-b_1l_1X_m-c_1l_1Y_m+q_1l_2)\varepsilon^{m+1}\\ &\geqslant-M_1\varepsilon^{m+1}+((a_1-2b_1X_m-c_1Y_m)l_1+(c_1X_m+q_1)l_2)\varepsilon^{m+1}\end{aligned} \tag{3.15}$$

其中M_1为常数，满足[2]：

$$|a_1X_m-b_1X_m^2-c_1X_mY_m+q_1Y_m|\leqslant M_1\varepsilon^{m+1} \tag{3.16}$$

$$\begin{aligned}\sigma_2\tilde{V}_t-(\varepsilon\nabla^2\tilde{V}+L_2\tilde{V})&=-c_2\hat{U}\tilde{V}+d_2\tilde{W}-q_2\tilde{V}\\ &=-c_2X_mY_m+d_2Z_m-q_2Y_m+[d_2l_3-q_2l_2-c_2l_2X_m+c_2l_1Y_m]\varepsilon^{m+1}\\ &\geqslant-M_2\varepsilon^{m+1}+[(d_2l_3+c_2l_1Y_m)-(q_2+c_2X_m)l_2]\varepsilon^{m+1}\end{aligned} \tag{3.17}$$

$$\begin{aligned}\sigma_3\tilde{W}_t-[\varepsilon\nabla\tilde{W}+L_3\tilde{W}]&=b_3(X_m+l_1\varepsilon^{m+1})-d_3(Z_m+l_3\varepsilon^{m+1})\\ &=b_3X_m-d_3Z_m+(l_1b_3-l_3d_3)\varepsilon^{m+1}\\ &\geqslant-M_3\varepsilon^{m+1}+(l_1b_3-l_3d_3)\varepsilon^{m+1}\end{aligned} \tag{3.18}$$

这里的M_2，M_3类似(3.16)确定．再由(3.15)，(3.17)，(3.18)解出l_1，l_2，l_3满足的关系

$$\frac{2b_1X_m+c_1Y_m-a_1}{c_1X_m+q_1}<\frac{l_2}{l_1}<\frac{c_2Y_m+d_2\cdot(b_3/d_3)}{c_2X_m+q_2} \tag{3.19}$$

$$l_3<l_1b_3/d_3 \tag{3.20}$$

并选取l_3充分大，可保证(3.15)，(3.17)，(3.18)式均成立．对下解可有类似的讨论，故由第二节定理知问题(1.1)～(1.5)有解，且满足

$$(u,v,w)=(X_m,Y_m,Z_m)+O(\varepsilon^{m+1}) \tag{3.21}$$

注 参数环境(3.19)，(3.20)可通过控制化学反应在一定的条件下进行来实现，由此可见耦合反应的复杂性．

参 考 文 献

[1] 莫嘉琪，一类半线性椭圆型方程Dirichlet问题的奇摄动，数学物理学报，7 (1987)，395—401.

[2] 莫嘉琪，一类非线性反应—扩散方程组的奇摄动，中国科学，A辑，10 (1988)，1041—1049.

[3] 莫嘉琪，许玉兴，一类奇摄动非线性反应扩散积分微分方程组，应用数学学报，17 (1994)，278—286.

[4] 李云，具有扩散和群体防卫的简单食物链的Hopf分支，应用数学学报，17 (1994)，287—299.

[5] W. H. Ruan and C. V. Pao, Asymptotic behavior and positive solutions of a chemical reaction diffusion system, *J. Math. Anal. Appl.*, 169 (1992), 157—178.

[6] C. V. Pao, Reaction diffusion equations with nonlinear boundary conditions, *Nonlinear Analysis, Methods & Applications*, 5 (1981), 1077—1094.

Singular Perturbation for a Class of Coupled Chemical Reaction and Diffusion Systems

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Abstract

In this paper, singular perturbation for a class of coupled chemical reaction diffusion system with initial and Neumann boundary conditions is considered. Under some suitable conditions and restrictions, we obtain a uniformly valid asymptotic solution of the stated system by using the iteration method and the method of upper and lower solutions.

Key words singular perturbation, coupled systems, reaction and diffusion, iteration method, upper and lower solutions