应用数学和力学, 第 19 卷 第 2 期(1998 年 2 月)
Applied Mathematics and Mechanics 应用数学和力学编委会编
重 庆 出 版 社 出 版

弹性厚矩形板受迫振动的功的互等定理法

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(钱伟长推荐; 1995 年 6 月 26 日收到; 1997 年 6 月 9 日收到修改稿)

摘 要

本文将功的互等定理法(RTM)推广应用于求解基于 Reissner 理论的厚矩形板受迫振动问题•

本文导出了厚矩形板动力基本解; 给出了三边固定一边自由厚矩形板在均布简谐干挠力作用下稳态响应的精确解析解• 这是计算厚矩形板受振动稳态响应的一个简便通用的方法•

关键词 功的互等定理法 动力基本解 厚矩形板 受迫振动

§1. 引 言

解决弹性厚矩形板受迫振动问题具有重要的理论意义和实用价值•

由于厚矩形板的振动控制方程比薄板的控制方程复杂很多, 因此, 数学上求解很困难• 许多学者对这类问题进行过研究, 提出了各种特殊的近似求解方法• 例如, 叠加法、初始函数法、利用相应边界条件深梁振型函数组合级数法、能量法、最佳平方近似法等等•

本文中采用 Reissner 理论, 我们将推广功的互等定理法于求解厚矩形板的受迫振动问题•

与广泛应用的叠加法比较, 应用功的互等定理法求解简谐干挠力作用下厚矩形板的稳态响应有两个优点: 第一, 克服了叠加法将复杂边界条件的问题分解为若干简单边界条件的问题并叠加起来的困难; 第二, 没有多次求解被叠加的边值问题所带来的烦琐• 因此, 该方法是一个简便通用的方法•

§2. 基 本 方 程

根据 Reissner 厚板理论, 厚板受迫振动控制方程为

$$\nabla^4 W - \frac{kh^2}{10}\frac{\rho}{D}\frac{\partial^2}{\partial t^2}\nabla^2 W + \frac{\rho}{D}\frac{\partial^2 W}{\partial t^2} = \frac{1}{D}\left(F(x,y,t) - \frac{kh^2}{10}\nabla^2 F(x,y,t)\right) \quad (2.1)$$

$$\nabla^2 \varphi - \frac{10}{h^2}\varphi = 0 \quad (2.2)$$

若不计阻尼, 板在简谐干扰力 $F(x,y,t) = q(x,y)\sin\omega t$ 作用下, 可以假设板的受迫振动稳态响应为

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$$W(x, y, t) = w(x, y)\sin\omega t \tag{2.3}$$

将式(2.3)代入(2.1),我们有

$$\nabla^4 w + \frac{kh^2}{10}\lambda^2\nabla^2 w - \lambda^2 w = \frac{1}{D}\left(q - \frac{kh^2}{10}\nabla^2 q\right) \tag{2.4}$$

式中 $\lambda^2 = \rho\omega^2/D$, $k = (2-\nu)/(1-\nu)$, ρ为板单位面积的质量; $w(x, y)$为振幅挠曲面方程•

板的切力、弯矩、扭矩和转角幅值分别为

$$Q_x = -D\frac{\partial}{\partial x}\nabla^2 w - \frac{kh^2}{10}\frac{\partial}{\partial x}(q + D\lambda^2 w) + \frac{\partial\varphi}{\partial y} \tag{2.5}$$

$$Q_y = -D\frac{\partial}{\partial y}\nabla^2 w - \frac{kh^2}{10}\frac{\partial}{\partial y}(q + D\lambda^2 w) - \frac{\partial\varphi}{\partial x} \tag{2.6}$$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) + \frac{h^2}{5}\frac{\partial Q_x}{\partial x} - \frac{h^2}{10}\frac{\nu}{1-\nu}(q + D\lambda^2 w) \tag{2.7}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) + \frac{h^2}{5}\frac{\partial Q_y}{\partial y} - \frac{h^2}{10}\frac{\nu}{1-\nu}(q + D\lambda^2 w) \tag{2.8}$$

$$M_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x\partial y} + \frac{h^2}{10}\left(\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x}\right) \tag{2.9}$$

$$\omega_x = -\frac{\partial w}{\partial x} + \frac{1}{D}\frac{h^2}{5(1-\nu)}Q_x \tag{2.10}$$

$$\omega_y = -\frac{\partial w}{\partial y} + \frac{1}{D}\frac{h^2}{5(1-\nu)}Q_y \tag{2.11}$$

§3. 动 力 基 本 解

我们取图1所示四边简支厚矩形板为基本系统• 在流动坐标(ξ, η)处作用单位横向二维Dirac-δ函数$\delta(x-\xi, y-\eta)$, 称之为拟单位集中载荷• 定义:四边简支厚矩形板仅在$\delta(x-\xi, y-\eta)$作用下,方程

$$\nabla^4 W_1 + \frac{kh^2}{10}\lambda^2\nabla^2 W_1 - \lambda^2 W_1 = \frac{1}{D}\delta(x-\xi, y-\eta) \tag{3.1}$$

所对应的解为动力基本解•

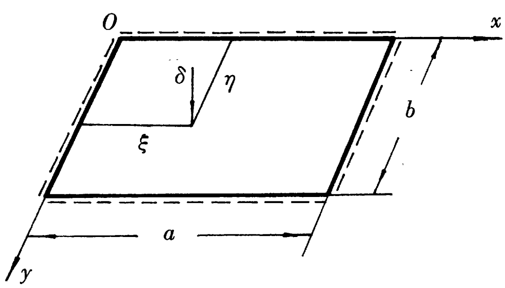

图　1

求解方程(3.1),我们得厚矩形板动力基本解

$$W_1(x, y; \xi, \eta) = \frac{4}{abD}\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{\sin k_m\xi\sin k_n\eta}{k_{mn}}\sin k_m x\sin k_n y \tag{3.2}$$

式中　$k_m = \dfrac{m\pi}{a}$, $k_n = \dfrac{n\pi}{b}$, $k_{mn} = (k_m^2 + k_n^2)^2 - \lambda^2 - \dfrac{kh^2}{10}\lambda^2(k_m^2 + k_n^2)$

为加快级数收敛,避免板的位移边界和力矩边界出现齐次性,我们给出如下形式动力基本解:

对于 $k_m^2 > \dfrac{kh^2}{20}\lambda^2 + \sqrt{\lambda^2 + \left(\dfrac{kh^2}{20}\lambda^2\right)^2}$, $k_n^2 > \dfrac{kh^2}{20}\lambda^2 + \sqrt{\lambda^2 + \left(\dfrac{kh^2}{20}\lambda^2\right)^2}$,

$$W_1(x,y;\xi,\eta)=\frac{2}{Db}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_n^2-\beta_n^2}\left[-\frac{\operatorname{sh}\alpha_n(a-x)\operatorname{sh}\alpha_n\xi}{\alpha_n\operatorname{sh}\alpha_n a}+\frac{\operatorname{sh}\beta_n(a-x)\operatorname{sh}\beta_n\xi}{\beta_n\operatorname{sh}\beta_n a}\right]$$
$$\cdot\sin k_n\eta\sin k_n y\qquad \xi\leqslant x\leqslant a \tag{3.3}$$

当 $0\leqslant x\leqslant\xi$ 时,(3.3)式中 $a-x$ 代以 x, ξ 代以 $a-\xi$•

$$W_1(x,y;\xi,\eta)=\frac{2}{Da}\sum_{m=1,2}^{\infty}\frac{1}{\alpha_m^2-\beta_m^2}\left[-\frac{\operatorname{sh}\alpha_m(b-y)\operatorname{sh}\alpha_m\eta}{\alpha_m\operatorname{sh}\alpha_m b}+\frac{\operatorname{sh}\beta_m(b-y)\operatorname{sh}\beta_m\eta}{\beta_m\operatorname{sh}\beta_m b}\right]$$
$$\cdot\sin k_m\xi\sin k_m x\qquad \eta\leqslant y\leqslant b \tag{3.4}$$

当 $0\leqslant y\leqslant\eta$ 时,(3.4)式中 $b-y$ 代以 y, η 代以 $b-\eta$•

对于 $k_m^2<\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}$, $k_n^2<\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}$,

$$W_1(x,y;\xi,\eta)=\frac{2}{Db}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_n^2-\beta_n^2}\left[-\frac{\operatorname{sh}\alpha_n(a-x)\operatorname{sh}\alpha_n\xi}{\alpha_n\operatorname{sh}\alpha_n a}+\frac{\sin\beta_n(a-x)\sin\beta_n\xi}{\beta_n\sin\beta_n a}\right]$$
$$\cdot\sin k_n\eta\sin k_n y\qquad \xi\leqslant x\leqslant a \tag{3.5}$$

当 $0\leqslant x\leqslant\xi$ 时,(3.5)式中 $a-x$ 代以 x, ξ 代以 $a-\xi$•

$$W_1(x,y;\xi,\eta)=\frac{2}{Da}\sum_{m=1,2}^{\infty}\frac{1}{\alpha_m^2-\beta_m^2}\left[-\frac{\operatorname{sh}\alpha_m(b-y)\operatorname{sh}\alpha_m\eta}{\alpha_m\operatorname{sh}\alpha_m b}+\frac{\sin\beta_m(b-y)\sin\beta_m\eta}{\beta_m\sin\beta_m b}\right]$$
$$\cdot\sin k_m\xi\sin k_m x\qquad \eta\leqslant y\leqslant b \tag{3.6}$$

当 $0\leqslant y\leqslant\eta$ 时,(3.6)式中 $b-y$ 代以 y, η 代以 $b-\eta$•

其中,

$$\alpha_m=\sqrt{k_m^2-\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}},\quad \beta_m=\sqrt{\left|k_m^2-\frac{kh^2}{20}\lambda^2-\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}\right|}$$

$$\alpha_n=\sqrt{k_n^2-\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}},\quad \beta_n=\sqrt{\left|k_n^2-\frac{kh^2}{20}\lambda^2-\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}\right|}$$

动力基本解的边界值见附录•

§4. 均布简谐干挠力作用下的三边固定一边自由厚矩形板

对于图 2 所示的厚矩形板,在均布简谐干挠力作用下,解除固定边的弯曲约束,代以相应的分布弯矩,我们得到如图 3 所示的实际系统•

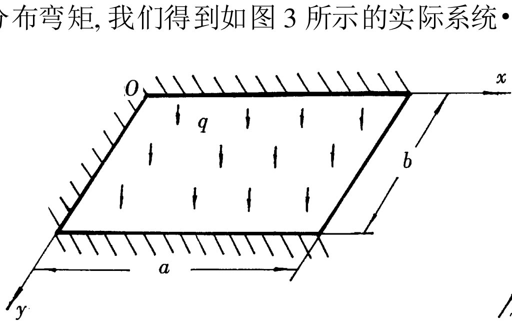

图 2

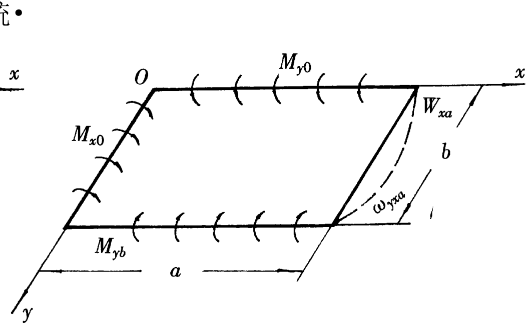

图 3

假设 $$M_{x0}(y)=\sum_{n=1,3}^{\infty}A_n\sin k_n y \tag{4.1}$$

$$M_{y0}(x) = M_{yb}(x) = \sum_{m=1,2}^{\infty} C_m \sin k_m x \tag{4.2a, b}$$

$$w_{xa}(y) = \sum_{n=1,3}^{\infty} b_n \sin k_n y \tag{4.3}$$

$$\omega_{yxa}(y) = \sum_{n=1,3}^{\infty} f_n \cos k_n y \tag{4.4}$$

我们在图 1 基本系统与图 3 实际系统之间应用功的互等定理, 可以得到

$$\begin{aligned} & w(\xi, \eta) + \int_0^b w_{xa} Q_{1xa} dy + \int_0^b \omega_{yxa} M_{1yxa} dy \\ & = \int_0^a \int_0^b \left[q - \frac{kh^2}{10} \nabla^2 q \right] W_1 dx dy - \int_0^b M_{x0} \omega_{1xx0} dy - \int_0^a M_{y0} \omega_{1yy0} dx + \int_0^a M_{yb} \omega_{1yyb} dx \end{aligned} \tag{4.5}$$

将式(4.1)~(4.4)和附录中动力基本解的边界值代入(4.5), 我们得到如下的振幅挠曲面方程:

对于 $k_m^2 > \frac{kh^2}{20}\lambda^2 + \sqrt{\lambda^2 + \left(\frac{kh^2}{20}\lambda^2\right)^2}$, $k_n^2 > \frac{kh^2}{20}\lambda^2 + \sqrt{\lambda^2 + \left(\frac{kh^2}{20}\lambda^2\right)^2}$,

$$\begin{aligned} w(\xi, \eta) = & \frac{4q_0}{\pi D} \sum_{m=1,3}^{\infty} \frac{1}{m} \left(1 + \frac{kh^2}{10} k_m^2\right) \left\{ \frac{1}{\alpha_m^2 - \beta_m^2} \left[\frac{\mathrm{ch}\,\alpha_m (b/2 - \eta)}{\alpha_m^2 \mathrm{ch}\,\alpha_m (b/2)} - \frac{\mathrm{ch}\,\beta_m (b/2 - \eta)}{\beta_m^2 \mathrm{ch}\,\beta_m (b/2)} \right] \right. \\ & \left. + \frac{1}{\alpha_m^2 \beta_m^2} \right\} \sin k_m \xi + \frac{4q_0}{\pi D} \sum_{m=1,3}^{\infty} \frac{1}{m} \frac{kh^2}{10} \frac{1}{\alpha_m^2 - \beta_m^2} \left[- \frac{\mathrm{ch}\,\alpha_m (b/2 - \eta)}{\mathrm{ch}\,\alpha_m (b/2)} \right. \\ & \left. + \frac{\mathrm{ch}\,\beta_m (b/2 - \eta)}{\mathrm{ch}\,\beta_m (b/2)} \right] \sin k_m \xi \\ & \left(\text{或} \frac{4q_0}{\pi D} \sum_{n=1,3}^{\infty} \frac{1}{n} \left(1 + \frac{kh^2}{10} k_n^2\right) \left\{ \frac{1}{\alpha_n^2 - \beta_n^2} \left[\frac{\mathrm{ch}\,\alpha_n (a/2 - \xi)}{\alpha_n^2 \mathrm{ch}\,\alpha_n (a/2)} - \frac{\mathrm{ch}\,\beta_n (a/2 - \xi)}{\beta_n^2 \mathrm{ch}\,\beta_n (a/2)} \right] \right. \right. \\ & \left. + \frac{1}{\alpha_n^2 \beta_n^2} \right\} \sin k_n \eta + \frac{4q_0}{\pi D} \sum_{n=1,3}^{\infty} \frac{1}{n} \frac{kh^2}{10} \frac{1}{\alpha_n^2 - \beta_n^2} \left[- \frac{\mathrm{ch}\,\alpha_n (a/2 - \xi)}{\mathrm{ch}\,\alpha_n (a/2)} \right. \\ & \left. \left. + \frac{\mathrm{ch}\,\beta_n (a/2 - \xi)}{\mathrm{ch}\,\beta_n (a/2)} \right] \sin k_n \eta \right) \\ & + \frac{1}{D} \sum_{n=1,3}^{\infty} \frac{A_n}{\alpha_n^2 - \beta_n^2} \left[- \frac{\mathrm{sh}\,\alpha_n (a - \xi)}{\mathrm{sh}\,\alpha_n a} + \frac{\mathrm{sh}\,\beta_n (a - \xi)}{\mathrm{sh}\,\beta_n a} \right] \sin k_n \eta \\ & + \frac{1}{D} \sum_{m=1,2}^{\infty} \frac{C_m}{\alpha_m^2 - \beta_m^2} \left[- \frac{\mathrm{ch}\,\alpha_m (b/2 - \eta)}{\mathrm{ch}\,\alpha_m (b/2)} + \frac{\mathrm{ch}\,\beta_m (b/2 - \eta)}{\mathrm{ch}\,\beta_m (b/2)} \right] \sin k_m \xi \\ & + \sum_{n=1,3}^{\infty} \frac{b_n}{\alpha_n^2 - \beta_n^2} \left[- (\nu k_n^2 - \alpha_n^2) \frac{\mathrm{sh}\,\alpha_n \xi}{\mathrm{sh}\,\alpha_n a} + (k_n^2 - \beta_n^2) \frac{\mathrm{sh}\,\beta_n \xi}{\mathrm{sh}\,\beta_n a} \right] \sin k_n \eta \\ & + (1 - \nu) \sum_{n=1,3}^{\infty} \frac{f_n k_n}{\alpha_n^2 - \beta_n^2} \left(\frac{\mathrm{sh}\,\alpha_n \xi}{\mathrm{sh}\,\alpha_n a} - \frac{\mathrm{sh}\,\beta_n \xi}{\mathrm{sh}\,\beta_n a} \right) \sin k_n \eta \end{aligned} \tag{4.6}$$

对于 $k_m^2 < \frac{kh^2}{20}\lambda^2 + \sqrt{\lambda^2 + \left(\frac{kh^2}{20}\lambda^2\right)^2}$, $k_n^2 < \frac{kh^2}{20}\lambda^2 + \sqrt{\lambda^2 + \left(\frac{kh^2}{20}\lambda^2\right)^2}$,

$$\begin{aligned} w(\xi, \eta) = & \frac{4q_0}{\pi D} \sum_{m=1,3}^{\infty} \frac{1}{m} \left(1 + \frac{kh^2}{10} k_m^2\right) \left\{ \frac{1}{\alpha_m^2 + \beta_m^2} \left[\frac{\mathrm{ch}\,\alpha_m (b/2 - \eta)}{\alpha_m^2 \mathrm{ch}\,\alpha_m (b/2)} + \frac{\cos \beta_m (b/2 - \eta)}{\beta_m^2 \cos \beta_m (b/2)} \right] \right. \\ & \left. - \frac{1}{\alpha_m^2 \beta_m^2} \right\} \sin k_m \xi + \frac{4q_0}{\pi D} \sum_{m=1,3}^{\infty} \frac{1}{m} \frac{kh^2}{10} \frac{1}{\alpha_m^2 + \beta_m^2} \left[- \frac{\mathrm{ch}\,\alpha_m (b/2 - \eta)}{\mathrm{ch}\,\alpha_m (b/2)} \right. \\ & \left. + \frac{\cos \beta_m (b/2 - \eta)}{\cos \beta_m (b/2)} \right] \sin k_m \xi \end{aligned}$$

$$\left(\frac{4q_0}{\pi D}\sum_{n=1,3}^{\infty}\frac{1}{n}\left(1+\frac{kh^2}{10}k_n^2\right)\left\{\frac{1}{\alpha_n^2+\beta_n^2}\left[\frac{\operatorname{ch}\alpha_n(a/2-\xi)}{\alpha_n^2\operatorname{ch}\alpha_n(a/2)}+\frac{\cos\beta_n(a/2-\xi)}{\beta_n^2\cos\beta_n(a/2)}\right]\right.\right.$$

$$\left.-\frac{1}{\alpha_n^2\beta_n^2}\right\}\sin k_n\eta+\frac{4q_0}{\pi D}\sum_{n=1,3}^{\infty}\frac{1}{n}\frac{kh^2}{10}\frac{1}{\alpha_n^2+\beta_n^2}\left[-\frac{\operatorname{ch}\alpha_n(a/2-\xi)}{\operatorname{ch}\alpha_n(a/2)}\right.$$

$$\left.\left.+\frac{\cos\beta_n(a/2-\xi)}{\cos\beta_n(a/2)}\right]\sin k_n\eta\right)$$

$$+\frac{1}{D}\sum_{n=1,3}^{\infty}\frac{A_n}{\alpha_n^2+\beta_n^2}\left[k-\frac{\operatorname{sh}\alpha_n(a-\xi)}{\operatorname{sh}\alpha_n a}+\frac{\sin\beta_n(a-\xi)}{\sin\beta_n a}\right]\sin k_n\eta$$

$$+\frac{1}{D}\sum_{m=1,2}^{\infty}\frac{C_m}{\alpha_m^2+\beta_m^2}\left[-\frac{\operatorname{ch}\alpha_m(b/2-\eta)}{\operatorname{ch}\alpha_m(b/2)}+\frac{\cos\beta_m(b/2-\eta)}{\cos\beta_m(b/2)}\right]\sin k_m\xi$$

$$+\sum_{n=1,3}^{\infty}\frac{b_n}{\alpha_n^2+\beta_n^2}\left[-(k_n^2-\alpha_n^2)\frac{\operatorname{sh}\alpha_n\xi}{\operatorname{sh}\alpha_n a}+(k_n^2+\beta_n^2)\frac{\sin\beta_n\xi}{\sin\beta_n a}\right]\sin k_n\eta$$

$$+(1-\nu)\sum_{n=1,3}^{\infty}\frac{f_n k_n}{\alpha_n^2+\beta_n^2}\left(\frac{\operatorname{sh}\alpha_n\xi}{\operatorname{sh}\alpha_n a}-\frac{\sin\beta_n\xi}{\sin\beta_n a}\right)\sin k_n\eta \tag{4.7}$$

假设厚矩形板的应力函数为如下形式

$$\varphi(\xi,\eta)=\sum_{n=0,1,3}^{\infty}[E_n\operatorname{ch}\delta_n\xi+F_n\operatorname{ch}\delta_n(a-\xi)]\cos k_n\eta$$

$$+\sum_{m=0,1,2}^{\infty}[G_m\operatorname{ch}\gamma_m\eta+H_m\operatorname{ch}\gamma_m(b-\eta)]\cos k_m\xi \tag{4.8}$$

式中 $\delta_n=\sqrt{k_n^2+10/h^2}$, $\gamma_m=\sqrt{k_m^2+10/h^2}$

使内弯矩(2.7)和(2.8)等于三边固定一边自由厚矩形板的边界弯矩, 可求得其应力函数为

$$\varphi(\xi,\eta)=\sum_{n=1,3}^{\infty}\left\{-k_nA_n\operatorname{ch}\delta_n(a-\xi)-D(1-\nu)\left[\frac{5}{h^2}k_nb_n+\left(\frac{5}{h^2}+k_n^2\right)f_n\right]\operatorname{ch}\delta_n\xi\right.$$

$$\left.\cdot\frac{1}{\delta_n\operatorname{sh}\delta_n a}\cos k_n\eta+\sum_{m=1,2}^{\infty}[\operatorname{ch}\gamma_m(b-\eta)-\operatorname{ch}\gamma_m\eta]\frac{k_mC_m}{\gamma_m\operatorname{sh}\gamma_m b}\cos k_m\xi\right.$$

$$-D(1-\nu)\frac{5}{h^2}\frac{\operatorname{ch}\delta_0\xi}{\delta_0\operatorname{sh}\delta_0 a}f_0 \tag{4.9}$$

图2所示三边固定一边自由厚矩形板的所有的边界条件必须满足• 当

$$k_m^2>\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2},\ k_n^2>\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}$$

时:

对于 $\omega_{\xi\xi 0}=0$, 有

$$\frac{A_n}{D}\left\{[s_3-s_2(k_n^2-\alpha_n^2)]\alpha_n\operatorname{cth}\alpha_n a-[s_3-s_2(k_n^2-\beta_n^2)]\beta_n\operatorname{cth}\beta_n a-s_2s_4\frac{k_n^2\operatorname{ch}\delta_n a}{\delta_n\operatorname{sh}\delta_n a}\right.$$

$$+\frac{4}{bD}\sum_{m=1,2}^{\infty}C_ms_4k_mk_n\left[\frac{s_3-s_2(k_m^2+k_n^2)}{k_{mn}}+\frac{s_2}{\gamma_m^2+k_n^2}\right]+b_n\left\{[s_3+s_2(\alpha_n^2-k_n^2)]\right.$$

$$\left.\cdot\frac{\alpha_n(\alpha_n^2-k_n^2)}{\operatorname{sh}\alpha_n a}-[s_3+s_2(\beta_n^2-k_n^2)]\frac{\beta_n(\beta_n^2-k_n^2)}{\operatorname{sh}\beta_n a}-\frac{s_4k_n^2}{\delta_n\operatorname{sh}\delta_n a}\right\}+(1-\nu)f_nk_n$$

$$\cdot\left\{[s_3+s_2(\alpha_n^2-k_n^2)]\frac{\alpha_n}{\operatorname{sh}\alpha_n a}-[s_3+s_2(\beta_n^2-k_n^2)]\frac{\beta_n}{\operatorname{sh}\beta_n a}-\left(\frac{5}{h^2}+k_n^2\right)\frac{s_2s_4}{\delta_n\operatorname{sh}\delta_n a}\right.$$

$$= \frac{4q_0}{n\pi D}\Big\{[(1+s_1k_n^2)(s_3+s_2\alpha_n^2-s_2k_n^2)-s_1\alpha_n^2(s_3+s_2\alpha_n^2)]\frac{\mathrm{th}(\alpha_n a/2)}{\alpha_n}$$

$$-[(1+s_1k_n^2)(s_3+s_2\beta_n^2-s_2k_n^2)-s_1\beta_n^2(s_3+s_2\beta_n^2)]\frac{\mathrm{th}(\beta_n a/2)}{\beta_n}\Big\} \tag{4.10}$$

对于 $\omega_{\eta\eta 0}=0$, 有

$$\frac{2}{aD}\sum_{n=1,3}^{\infty}A_n s_4k_mk_n\left[\frac{s_3-s_2(k_m^2+k_n^2)}{k_{mn}}+\frac{s_2}{\delta_n^2+k_m^2}\right]+\frac{C_m}{D}\Big\{[s_3+s_2(\alpha_m^2-k_m^2)]$$

$$\cdot\alpha_m\mathrm{th}\frac{\alpha_m b}{2}-[s_3+s_2(\beta_m^2-k_m^2)]\beta_m\mathrm{th}\frac{\beta_m b}{2}\Big\}+(-1)^m\frac{2}{a}\sum_{n=1,3}^{\infty}b_ns_4k_mk_n$$

$$\cdot\left\{\frac{(k_m^2+k_n^2)[s_2(k_m^2+k_n^2)-s_3]}{k_{mn}}+\frac{1}{\delta_n^2+k_m^2}\right\}+(-1)^m\frac{2(1-\nu)}{a}\sum_{n=1,3}^{\infty}f_ns_4k_m$$

$$\cdot\left\{\frac{k_n^2[s_3-s_2(k_m^2+k_n^2)]}{k_{mn}}+\left(\frac{5}{h^2}+k_n^2\right)\frac{s_2}{\delta_n^2+k_m^2}\right\}$$

$$=\frac{2q_0}{m\pi D}[1-(-1)^m]\Big\{[s_3+s_1s_3(k_m^2-\alpha_m^2)-s_2(k_m^2-\alpha_m^2)-s_1s_2(k_m^2-\alpha_m^2)^2]$$

$$\cdot\frac{\mathrm{th}(\alpha_m b/2)}{\alpha_m}-[s_3+s_1s_3(k_m^2-\beta_m^2)-s_2(k_m^2-\beta_m^2)$$

$$-s_1s_2(k_m^2-\beta_m^2)^2]\frac{\mathrm{th}(\beta_m b/2)}{\beta_m}\Big\} \tag{4.11}$$

对于 $Q_{\xi a}=0$, 有

$$\frac{A_n}{D}\left[(s_1\lambda^2+\alpha_n^2-k_n^2)\frac{\alpha_n}{\mathrm{sh}\,\alpha_n a}-(s_1\lambda^2+\beta_n^2-k_n^2)\frac{\beta_n}{\mathrm{sh}\,\beta_n a}-\frac{s_4k_n^2}{\delta_n\mathrm{sh}\,\delta_n a}\right]$$

$$+\frac{4}{bD}\sum_{m=1,2}^{\infty}(-1)^mC_ms_4k_mk_n\left[\frac{s_1\lambda^2-(k_m^2+k_n^2)}{\delta k_{mn}}+\frac{1}{\gamma_m^2+k_n^2}\right]+b_n\Big[(s_1\lambda^2+\alpha_n^2-k_n^2)$$

$$\cdot(\alpha_n^2-k_n^2)\alpha_n\mathrm{cth}\,\alpha_n a-(s_1\lambda^2+\beta_n^2-k_n^2)(\beta_n^2-k_n^2)\beta_n\mathrm{cth}\,\beta_n a-\frac{s_4k_n^2}{s_2\delta_n}\mathrm{cth}\,\delta_n a\Big]$$

$$+(1-\nu)f_nk_n\Big[(s_1\lambda^2+\alpha_n^2-k_n^2)\alpha_n\mathrm{cth}\,\alpha_n a-(s_1\lambda^2+\beta_n^2-k_n^2)\beta_n\mathrm{cth}\,\beta_n a$$

$$-\left(\frac{5}{h^2}+k_n^2\right)\frac{s_4}{\delta_n}\mathrm{cth}\,\delta_n a\Big]$$

$$=\frac{4q_0}{n\pi D}\Big\{-(s_1\lambda^2+\alpha_n^2-k_n^2)[1-s_1(\alpha_n^2-k_n^2)]\frac{\mathrm{th}(\alpha_n a/2)}{\alpha_n}$$

$$+(s_1\lambda^2+\beta_n^2-k_n^2)[1-s_1(\beta_n^2-k_n^2)]\frac{\mathrm{th}(\beta_n a/2)}{\beta_n}\Big\} \tag{4.12}$$

对于 $M_{\eta\xi a}=0$, 有

$$\frac{A_n}{D}k_n\Big\{\left[1-\nu+\frac{h^2}{5}(s_1\lambda^2+\alpha_n^2-k_n^2)\right]\frac{\alpha_n}{\mathrm{sh}\,\alpha_n a}-\left[1-\nu+\frac{h^2}{5}(s_1\lambda^2+\beta_n^2-k_n^2)\right]\frac{\beta_n}{\mathrm{sh}\,\beta_n a}$$

$$-\frac{h^2}{10}\frac{s_4(\delta_n^2+k_n^2)}{\delta_n\mathrm{sh}\,\delta_n a}\Big\}+\frac{4}{bD}\sum_{m=1,2}^{\infty}(-1)^mC_ms_4k_m\Big\{\frac{k_n^2\left[1-\nu+\frac{h^2}{5}s_1\lambda^2-\frac{h^2}{5}(k_m^2+k_n^2)\right]}{k_{mn}}$$

$$-\frac{h^2}{10}\frac{\gamma_m^2+k_m^2}{\gamma_m^2+k_n^2}\Big\}+b_nk_n\Big\{(\alpha_n^2-k_n^2)\left[1-\nu+\frac{h^2}{5}s_1\lambda^2+\frac{h^2}{5}(\alpha_n^2-k_n^2)\right]\alpha_n\mathrm{cth}\,\alpha_n a$$

$$-(\beta_n^2-k_n^2)\left[1-\nu+\frac{h^2}{5}s_1\lambda^2+\frac{h^2}{5}(\beta_n^2-k_n^2)\right]\beta_n\mathrm{cth}\,\beta_n a-\frac{1-\nu}{2}s_4\left(\delta_n+\frac{k_n^2}{\delta_n}\right)\mathrm{cth}\,\delta_n a\Bigg\}$$

$$+(1-\nu)f_n\Bigg\{k_n^2\left[1-\nu+\frac{h^2}{5}(s_1\lambda^2+\alpha_n^2-k_n^2)\right]\alpha_n\mathrm{cth}\,\alpha_n a-k_n^2\left[1-\nu+\frac{h^2}{5}(s_1\lambda^2\right.$$

$$\left.+\beta_n^2-k_n^2)\right]\beta_n\mathrm{cth}\,\beta_n a-s_4\left(\frac{1}{2}+\frac{h^2}{10}k_n^2\right)\left(\delta_n+\frac{k_n^2}{\delta_n}\right)\mathrm{cth}\,\delta_n a\Bigg\}$$

$$=\frac{4q_0}{bD}\Bigg\{-(1-s_1\alpha_n^2+s_1k_n^2)\left[(1-\nu)+\frac{h^2}{5}(s_1\lambda^2+\alpha_n^2-k_n^2)\right]\frac{\mathrm{th}(\alpha_n a/2)}{\alpha_n}$$

$$+(1-s_1\beta_n^2+s_1k_n^2)\left[(1-\nu)+\frac{h^2}{5}(s_1\lambda^2+\beta_n^2-k_n^2)\right]\frac{\mathrm{th}(\beta_n a/2)}{\beta_n}\Bigg\} \tag{4.13}$$

当 $k_m^2<\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}$, $k_n^2<\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}$

时, 式(4.10)~(4.13)分别成为如下形式:

$$\frac{A_n}{D}\Bigg\{[s_3-s_2(k_n^2-\alpha_n^2)]\alpha_n\mathrm{cth}\,\alpha_n a-[s_3-s_2(k_n^2-\beta_n^2)]\beta_n\mathrm{cth}\,\beta_n a-s_2s_4\frac{k_n^2\mathrm{ch}\,\delta_n a}{\delta_n\mathrm{sh}\,\delta_n a}$$

$$+\frac{4}{bD}\sum_{m=1,2}^{\infty}C_m s_4k_mk_n\left[\frac{s_3-s_2(k_m^2+k_n^2)}{b\,k_{mn}}+\frac{s_2}{\gamma_m^2+k_n^2}\right]+b_n\Bigg\{[s_3+s_2(\alpha_n^2-k_n^2)]$$

$$\cdot\frac{\alpha_n(\alpha_n^2-k_n^2)}{\mathrm{sh}\,\alpha_n a}+[s_3-s_2(\beta_n^2+k_n^2)]\frac{\beta_n(\beta_n^2+k_n^2)}{\sin\beta_n a}-\frac{s_4k_n^2}{\delta_n\mathrm{sh}\,\delta_n a}\Bigg\}+(1-\nu)f_nk_n$$

$$\cdot\Bigg\{[s_3+s_2(\alpha_n^2-k_n^2)]\frac{\alpha_n}{\mathrm{sh}\,\alpha_n a}-[s_3-s_2(\beta_n^2+k_n^2)]\frac{\beta_n}{\sin\beta_n a}-\left(\frac{5}{h^2}+k_n^2\right)\frac{s_2s_4}{2\delta_n\mathrm{sh}\,\delta_n a}\Bigg\}$$

$$=\frac{4q_0}{n\pi D}\Bigg\{[(1+s_1k_n^2)(s_3+s_2\alpha_n^2-s_2k_n^2)-s_1\alpha_n^2(s_3+s_2\alpha_n^2)]\frac{\mathrm{th}(\alpha_n a/2)}{\alpha_n}$$

$$-[(1+s_1k_n^2)(s_3-s_2\beta_n^2-s_2k_n^2)+s_1\beta_n^2(s_3-s_2\beta_n^2)]\frac{\tan(\beta_n a/2)}{\beta_n}\Bigg\} \tag{4.14}$$

$$\frac{2}{aD}\sum_{n=1,3}^{\infty}A_ns_4k_mk_n\left[\frac{s_3-s_2(k_m^2+k_n^2)}{k_{mn}}+\frac{s_2}{\delta_n^2+k_m^2}\right]+\frac{C_m}{D}\Bigg\{[s_3+s_2(\alpha_m^2-k_m^2)]$$

$$\cdot\alpha_m\mathrm{th}\frac{\alpha_m b}{2}+[s_3-s_2(\beta_m^2+k_m^2)]\beta_m\tan\frac{\beta_m b}{2}\Bigg\}+(-1)^m\frac{2}{a}\sum_{n=1,3}^{\infty}b_ns_4k_mk_n$$

$$\cdot\Bigg\{\frac{(k_m^2+k_n^2)[s_2(k_m^2+k_n^2)-s_3]}{k_{mn}}+\frac{1}{\delta_n^2+k_m^2}\Bigg\}+(-1)^m\frac{2(1-\nu)}{a}\sum_{n=1,3}^{\infty}f_ns_4k_m$$

$$\cdot\Bigg\{\frac{k_n^2[s_3-s_2(k_m^2+k_n^2)]}{k_{mn}}+\left(\frac{5}{h^2}+k_n^2\right)\frac{s_2}{\delta_n^2+k_m^2}\Bigg\}$$

$$=\frac{2q_0}{m\pi D}[1-(-1)^m]\Bigg\{[s_3+s_1s_3(k_m^2-\alpha_m^2)-s_2(k_m^2-\alpha_m^2)-s_1s_2(k_m^2-\alpha_m^2)^2]$$

$$\cdot\frac{\mathrm{th}(\alpha_m b/2)}{\alpha_m}-[s_3+s_1s_3(k_m^2+\beta_m^2)-s_2(k_m^2+\beta_m^2)$$

$$-s_1s_2(k_m^2+\beta_m^2)^2]\frac{\tan(\beta_m b/2)}{\beta_m}\Bigg\} \tag{4.15}$$

$$\frac{A_n}{D}\left[(s_1\lambda^2+\alpha_n^2-k_n^2)\frac{\alpha_n}{\mathrm{sh}\,\alpha_n a}-(s_1\lambda^2-\beta_n^2-k_n^2)\frac{\beta_n}{\sin\beta_n a}-\frac{s_4k_n^2}{\delta_n\mathrm{sh}\,\delta_n a}\right.$$

$$+\frac{4}{bD}\sum_{m=1,2}^{\infty}(-1)^mC_ms_4k_mk_n\left[\frac{s_1\lambda^2-(k_m^2+k_n^2)}{k_{mn}}+\frac{1}{\gamma_m^2+k_n^2}\right]+b_n\left[(s_1\lambda^2+\alpha_n^2-k_n^2)\right.$$

$$\cdot (\alpha_n^2 - k_n^2)\alpha_n \operatorname{cth}\alpha_n a + (s_1\lambda^2 - \beta_n^2 - k_n^2)(\beta_n^2 + k_n^2)\beta_n \operatorname{ctan}\beta_n a - \frac{s_4 k_n^2}{s_2\delta_n}\operatorname{cth}\delta_n a\Big]$$

$$+ (1-\nu) f_n k_n\Big[(s_1\lambda^2 + \alpha_n^2 - k_n^2)\alpha_n \operatorname{cth}\alpha_n a - (s_1\lambda^2 - \beta_n^2 - k_n^2)\beta_n \operatorname{ctan}\beta_n a$$

$$-\left(\frac{5}{h^2} + k_n^2\right)\frac{s_4}{\delta_n}\operatorname{cth}\delta_n a\Big]$$

$$= \frac{4q_0}{n\pi D}\Big\{-(s_1\lambda^2 + \alpha_n^2 - k_n^2)[1 - s_1(\alpha_n^2 - k_n^2)]\frac{\operatorname{th}(\alpha_n a/2)}{\alpha_n}$$

$$+ (s_1\lambda^2 - \beta_n^2 - k_n^2)[1 + s_1(\beta_n^2 + k_n^2)]\frac{\tan(\beta_n a/2)}{\beta_n}\Big\} \tag{4.16}$$

$$\frac{A_n}{D}k_n\Big\{\left[1-\nu+\frac{h^2}{5}(s_1\lambda^2+\alpha_n^2-k_n^2)\right]\frac{\alpha_n}{\operatorname{sh}\alpha_n a} - \left[1-\nu+\frac{h^2}{5}(s_1\lambda^2-\beta_n^2-k_n^2)\right]\frac{\beta_n}{\sin\beta_n a}$$

$$-\frac{h^2}{10}\frac{s_4(\delta_n^2+k_n^2)}{\delta_n \operatorname{sh}\delta_n a}\Big\} + \frac{4}{bD}\sum_{m=1,2}^{\infty}(-1)^m C_m s_4 k_m\Big\{\frac{k_n^2\left[1-\nu+\frac{h^2}{5}s_1\lambda^2-\frac{h^2}{5}(k_m^2+k_n^2)\right]}{k_{mn}}$$

$$-\frac{h^2}{10}\frac{\gamma_m^2+k_m^2}{\gamma_m^2+k_n^2}\Big\} + b_n k_n\Big\{(\alpha_n^2-k_n^2)\left[1-\nu+\frac{h^2}{5}s_1\lambda^2+\frac{h^2}{5}(\alpha_n^2-k_n^2)\right]\alpha_n\operatorname{cth}\alpha_n a$$

$$+(\beta_n^2+k_n^2)\left[1-\nu+\frac{h^2}{5}s_1\lambda^2-\frac{h^2}{5}(\beta_n^2+k_n^2)\right]\beta_n\operatorname{ctan}\beta_n a - \frac{1-\nu}{2}s_4\Big(\delta_n$$

$$+\frac{k_n^2}{\delta_n}\Big)\operatorname{cth}\delta_n a\Big\} + (1-\nu)f_n\Big\{k_n^2\left[1-\nu+\frac{h^2}{5}(s_1\lambda^2+\alpha_n^2-k_n^2)\right]\alpha_n\operatorname{cth}\alpha_n a - k_n^2\Big[1-\nu$$

$$+\frac{h^2}{5}(s_1\lambda^2-\beta_n^2-k_n^2)\Big]\beta_n\operatorname{ctan}\beta_n a - s_4\left(\frac{1}{2}+\frac{h^2}{10}k_n^2\right)\left(\delta_n+\frac{k_n^2}{\delta_n}\right)\operatorname{cth}\delta_n a\Big\}$$

$$= \frac{4q_0}{bD}\Big\{-(1-s_1\alpha_n^2+s_1k_n^2)\left[(1-\nu)+\frac{h^2}{5}(s_1\lambda^2+\alpha_n^2-k_n^2)\right]\frac{\operatorname{th}(\alpha_n a/2)}{\alpha_n}$$

$$+(1+s_1\beta_n^2+s_1k_n^2)\left[(1-\nu)+\frac{h^2}{5}(s_1\lambda^2-\beta_n^2-k_n^2)\right]\frac{\tan(\beta_n a/2)}{\beta_n}\Big\} \tag{4.17}$$

式中 $s_1 = \frac{kh^2}{10},\ s_2 = \frac{h^2}{5(1-\nu)},\ s_3 = 1 + s_1 s_2\lambda^2,\ s_4 = \alpha_n^2 - \beta_n^2 = \alpha_m^2 - \beta_m^2$

解方程组(4.10)~(4.13)(或(4.14)~(4.17)),即可得到诸常数 A_n, C_m, b_n 和 f_n•

§5. 数 值 分 析

方程(4.10)~(4.13)(或方程(4.14)~(4.17))是具有无穷未知数的联立方程组• 计算结果表明, 对于 m 和 n 各取 30 项时, 我们已经可以得到足够精度的厚板稳态响应•

在本文所列图表中, 我们取 $a/b = 0.5$, $\nu = 1/6$•

对于不同厚跨比 h/a 和不同的频率比 ω/ω_{11}, 表 1~ 表 4 给出了固定边分布弯矩幅值 M_x 和 M_y, 自由边挠度幅值 W_{xa} 和扭角幅值 ω_{yxa}•

我们取 $h/a = 0.2$, 图 4~ 图 7 分别表明了沿 $x = 0$ 的弯矩幅值 M_x, 沿 $y = 0$ 的弯矩幅值 M_y、沿 $x = a$ 的挠度幅值 W 和扭角幅值 ω_{yx} 随 $y/b\,(x/a)$ 与 ω/ω_{11} 的变化曲线, 反映了干扰力频率对厚矩形板稳态响应的影响•

由表 5 可见, 当板的厚跨比 $h/a = 0.01$ 时, 横向剪切变形和挤压变形对于薄板的弯矩幅

值和挠度幅值的影响已经很小•

表 **1** 沿 x= 0 边的弯矩幅值 $M_x(qa^2)$

$\frac{\omega}{\omega_{11}}$	h/a \ y/b (M)	0.0	0.1	0.2	0.3	0.4	0.5
0.1	0.1	0.000000	−0.034890	−0.097821	−0.156215	−0.194584	−0.207756
	0.2	0.000000	−0.045220	−0.102253	−0.156022	−0.192084	−0.204593
	0.3	0.000000	−0.056307	−0.109081	−0.157667	−0.190494	−0.201956
0.3	0.1	0.000000	−0.036138	−0.103438	−0.166977	−0.209187	−0.223756
	0.2	0.000000	−0.047385	−0.108559	−0.167061	−0.206663	−0.220465
	0.3	0.000000	−0.059563	−0.116332	−0.169232	−0.205257	−0.217886
0.5	0.1	0.000000	−0.039451	−0.118451	−0.195820	−0.248376	−0.266718
	0.2	0.000000	−0.053126	−0.125356	−0.196538	−0.245646	−0.262925
	0.3	0.000000	−0.068188	−0.135585	−0.200004	−0.244585	−0.260339
0.8	0.1	0.000000	−0.061623	−0.220449	−0.392854	−0.516836	−0.561281
	0.2	0.000000	−0.091054	−0.237274	−0.393906	−0.507377	−0.548250
	0.3	0.000000	−0.124221	−0.261312	−0.401772	−0.503099	−0.539630

表 **2** 沿 y= 0 边的弯矩幅值 $M_y(qa^2)$

$\frac{\omega}{\omega_{11}}$	h/a \ x/a (M)	0.0	0.2	0.4	0.6	0.8	0.95
0.1	0.1	0.000000	−0.043885	−0.119233	−0.207239	−0.338398	−0.682398
	0.2	0.000000	−0.057482	−0.132511	−0.224135	−0.355801	−0.767543
	0.3	0.000000	−0.075915	−0.156793	−0.257904	−0.417143	−1.067892
0.3	0.1	0.000000	−0.045921	−0.126792	−0.222881	−0.366875	−0.742233
	0.2	0.000000	−0.060651	−0.141408	−0.242301	−0.385259	−0.831483
	0.3	0.000000	−0.080715	−0.167953	−0.277996	−0.451235	−1.153675
0.5	0.1	0.000000	−0.051360	−0.147061	−0.264941	−0.443627	−0.903739
	0.2	0.000000	−0.069096	−0.165183	−0.287298	−0.464353	−1.003333
	0.3	0.000000	−0.093472	−0.197683	−0.331642	−0.542410	−1.383220
0.8	0.1	0.000000	−0.088303	−0.285651	−0.554198	−0.973941	−2.022941
	0.2	0.000000	−0.125393	−0.324651	−0.597486	−0.999927	−2.169338
	0.3	0.000000	−0.176936	−0.393143	−0.685979	−1.146655	−2.906212

表 3 沿 x= a 边的挠度幅值 W(qa^4/D)

$\frac{\omega}{\omega_{11}}$	h/a \ y/b W	0.0	0.1	0.2	0.3	0.4	0.5
0.1	0.1	0.000000	0.005407	0.014632	0.023486	0.029587	0.031741
	0.2	0.000000	0.006656	0.016978	0.028567	0.033100	0.035397
	0.3	0.000000	0.008689	0.020673	0.031333	0.038474	0.040969
0.3	0.1	0.000000	0.005898	0.016013	0.025770	0.032523	0.034913
	0.2	0.000000	0.007247	0.018561	0.029133	0.036368	0.038918
	0.3	0.000000	0.009445	0.022578	0.034336	0.042251	0.045023
0.5	0.1	0.000000	0.007224	0.019736	0.031935	0.040449	0.043476
	0.2	0.000000	0.008837	0.022818	0.036036	0.045164	0.048396
	0.3	0.000000	0.011469	0.027679	0.042387	0.052381	0.055899
0.8	0.1	0.000000	0.016399	0.045504	0.074630	0.095375	0.102832
	0.2	0.000000	0.019618	0.051709	0.082945	0.104981	0.112874
	0.3	0.000000	0.024904	0.061585	0.095973	0.119884	0.128402

表 4 沿 x= a 边的扭角幅值 ω_{yx}(qa^3/D)

$\frac{\omega}{\omega_{11}}$	h/a \ y/b ω	0.025	0.1	0.3	0.5	0.7	0.9	0.975
0.1	0.1	−0.018500	−0.035919	−0.035570	0.000000	0.035570	0.035919	0.018500
	0.2	−0.021479	−0.034270	−0.034100	0.000000	0.034100	0.034270	0.021479
	0.3	−0.028506	−0.032710	−0.032656	0.000000	0.032656	0.032710	0.028506
0.3	0.1	−0.020136	−0.039196	−0.039220	0.000000	0.039220	0.039196	0.020136
	0.2	−0.023292	−0.037383	−0.037590	0.000000	0.037590	0.037383	0.023292
	0.3	−0.030819	−0.035646	−0.035965	0.000000	0.035965	0.035646	0.030819
0.5	0.1	−0.024549	−0.047996	−0.049037	0.000000	0.049037	0.047996	0.024549
	0.2	−0.028164	−0.045728	−0.046962	0.000000	0.046962	0.045728	0.028164
	0.3	−0.037008	−0.043485	−0.044822	0.000000	0.044822	0.043485	0.037008
0.8	0.1	−0.055094	−0.107400	−0.115511	0.000000	0.115511	0.107400	0.055094
	0.2	−0.061195	−0.101577	−0.109934	0.000000	0.109934	0.101577	0.061195
	0.3	−0.078042	−0.094868	−0.103158	0.000000	0.103158	0.094868	0.078042

表 5　　固定边弯矩与自由边振幅

ω/ω_{11}	x/a \ y/b $M(qa^2)$ $W(qa^4/D)$		a/b= 1.0　h/a= 0.01　ν= 1/6						
			0.05	0.15	0.35	0.50	0.70	0.90	0.95
0.3	M_{x0}	本　文	-0.001523	-0.017045	-0.050954	-0.060002	-0.044223	-0.008007	-0.001523
		文献[8]	-0.001391	-0.017020	-0.051020	-0.060080	-0.044270	-0.007943	-0.001391
	M_{y0}	本　文	-0.001542	-0.017528	-0.054433	-0.068765	-0.080063	-0.081697	-0.130296
		文献[8]	-0.001452	-0.017540	-0.054580	-0.068770	-0.079980	-0.080880	-0.130300
	W_{xa}	本　文	0.000116	0.000807	0.002569	0.003101	0.002189	0.000403	0.000116
		文献[8]	0.000115	0.000806	0.002568	0.003101	0.002188	0.000402	0.000115
0.5	M_{x0}	本　文	-0.001444	-0.018377	-0.057445	-0.068164	-0.049544	-0.008368	-0.001444
		文献[8]	-0.001284	-0.018380	-0.057680	-0.068470	-0.049730	-0.008299	-0.001284
	M_{y0}	本　文	-0.001486	-0.018987	-0.062070	-0.080041	-0.095595	-0.099450	-0.160048
		文献[8]	-0.001382	-0.019040	-0.062440	-0.080340	-0.095890	-0.098880	-0.160800
	W_{xa}	本　文	0.000144	0.000999	0.003202	0.003873	0.002725	0.000498	0.000144
		文献[8]	0.000143	0.001003	0.003217	0.003892	0.002738	0.000499	0.000143
0.8	M_{x0}	本　文	-0.000925	-0.026032	-0.095937	-0.116755	-0.080989	-0.010337	-0.000925
		文献[8]	-0.000516	-0.026390	-0.098530	-0.120100	-0.083080	-0.010250	-0.000516
	M_{y0}	本　文	-0.001117	-0.027507	-0.108396	-0.154439	-0.204266	-0.216327	-0.358647
		文献[8]	-0.000951	-0.028140	-0.111800	-0.154300	-0.201400	-0.221600	-0.371900
	W_{xa}	本　文	0.000327	0.002282	0.007411	0.008996	0.006290	0.001135	0.000327
		文献[8]	0.000335	0.002363	0.007686	0.009332	0.006522	0.001173	0.000335

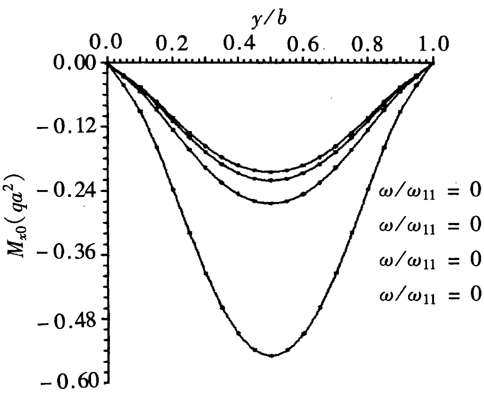

图　4

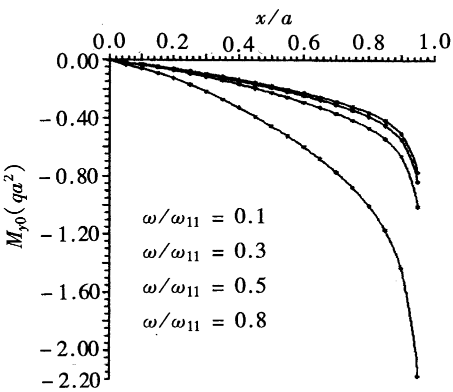

图　5

§6. 结　论

1. 本文首次给出了三边固定一边自由厚矩形板简谐干扰力作用下稳态响应的精确解析

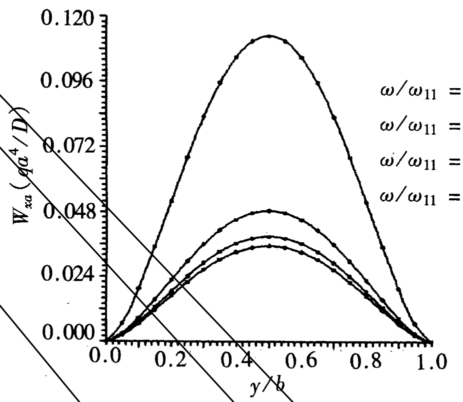

图 6

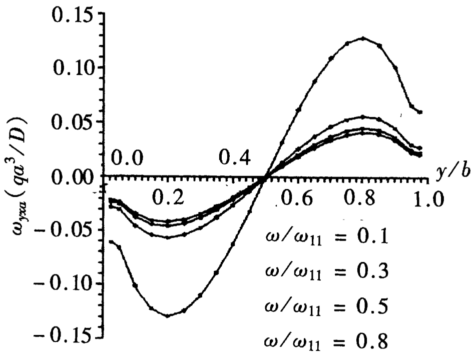

图 7

解•

2. 功的互等定理法是求解厚矩形板受迫振动问题的一种简单、通用、有效的新方法•

附 录

为计算实际系统的振幅挠曲面方程,兹给出厚矩形板动力基本解的诸边界转角、切力和扭矩如下:

对于 $k_m^2 > \frac{kh^2}{20}\lambda^2 + \sqrt{\lambda^2 + \left(\frac{kh^2}{20}\lambda^2\right)^2}$, $k_n^2 > \frac{kh^2}{20}\lambda^2 + \sqrt{\lambda^2 + \left(\frac{kh^2}{20}\lambda^2\right)^2}$,

$$\omega_{1xx0} = \frac{2}{bD}\sum_{n=1}^{\infty}\left\{\frac{1}{\alpha_n^2-\beta_n^2}\left[-\frac{\mathrm{sh}\,\alpha_n(a-\xi)}{\mathrm{sh}\,\alpha_n a}+\frac{\mathrm{sh}\,\beta_n(a-\xi)}{\mathrm{sh}\,\beta_n a}\right]\sin k_n\eta\sin k_n y\right\} \qquad (A.1)$$

$$\omega_{1xxa} = \frac{2}{bD}\sum_{n=1}^{\infty}\left\{\frac{1}{\alpha_n^2-\beta_n^2}\left[\frac{\mathrm{sh}\,\alpha_n\xi}{\mathrm{sh}\,\alpha_n a}-\frac{\mathrm{sh}\,\beta_n\xi}{\mathrm{sh}\,\beta_n a}\right]\sin k_n\eta\sin k_n y\right\} \qquad (A.2)$$

$$\omega_{1yy0} = \frac{2}{aD}\sum_{m=1}^{\infty}\left\{\frac{1}{\alpha_m^2-\beta_m^2}\left[-\frac{\mathrm{sh}\,\alpha_m(b-\eta)}{\mathrm{sh}\,\alpha_m b}+\frac{\mathrm{sh}\,\beta_m(b-\eta)}{\mathrm{sh}\,\beta_m b}\right]\sin k_m\xi\sin k_m x\right\} \qquad (A.3)$$

$$\omega_{1yyb} = \frac{2}{aD}\sum_{m=1}^{\infty}\left\{\frac{1}{\alpha_m^2-\beta_m^2}\left[\frac{\mathrm{sh}\,\alpha_m\eta}{\mathrm{sh}\,\alpha_m b}-\frac{\mathrm{sh}\,\beta_m\eta}{\mathrm{sh}\,\beta_m b}\right]\sin k_m\xi\sin k_m x\right\} \qquad (A.4)$$

$$Q_{1x0} = \frac{2}{b}\sum_{n=1}^{\infty}\left\{\frac{1}{\alpha_n^2-\beta_n^2}\left[\frac{(\alpha_n^2-k_n^2)\,\mathrm{sh}\,\alpha_n(a-\xi)}{\mathrm{sh}\,\alpha_n a}-\frac{(\beta_n^2-k_n^2)\,\mathrm{sh}\,\beta_n(a-\xi)}{\mathrm{sh}\,\beta_n a}\right]\sin k_n\eta\sin k_n y\right\} \qquad (A.5)$$

$$Q_{1xa} = \frac{2}{b}\sum_{n=1}^{\infty}\left\{\frac{1}{\alpha_n^2-\beta_n^2}\left[-\frac{(\alpha_n^2-k_n^2)\,\mathrm{sh}\,\alpha_n\xi}{\mathrm{sh}\,\alpha_n a}+\frac{(\beta_n^2-k_n^2)\,\mathrm{sh}\,\beta_n\xi}{\mathrm{sh}\,\beta_n a}\right]\sin k_n\eta\sin k_n y\right\} \qquad (A.6)$$

$$Q_{1y0} = \frac{2}{a}\sum_{m=1}^{\infty}\left\{\frac{1}{\alpha_m^2-\beta_m^2}\left[\frac{(\alpha_m^2-k_m^2)\,\mathrm{sh}\,\alpha_m(b-\eta)}{\mathrm{sh}\,\alpha_m b}-\frac{(\beta_m^2-k_m^2)\,\mathrm{sh}\,\beta_m(b-\eta)}{\mathrm{sh}\,\beta_m b}\right]\sin k_m\xi\sin k_m x\right\} \qquad (A.7)$$

$$Q_{1yb} = \frac{2}{a}\sum_{m=1}^{\infty}\left\{\frac{1}{\alpha_m^2-\beta_m^2}\left[-\frac{(\alpha_m^2-k_m^2)\,\mathrm{sh}\,\alpha_m\eta}{\mathrm{sh}\,\alpha_m b}-\frac{(\beta_m^2-k_m^2)\,\mathrm{sh}\,\beta_m\eta}{\mathrm{sh}\,\beta_m b}\right]\sin k_m\xi\sin k_m x\right\} \qquad (A.8)$$

$$M_{1yx0} = \frac{2(1-\nu)}{b}\sum_{n=1}^{\infty}\left\{\frac{k_n}{\alpha_n^2-\beta_n^2}\left[\frac{\mathrm{sh}\,\alpha_n(a-\xi)}{\mathrm{sh}\,\alpha_n a}-\frac{\mathrm{sh}\,\beta_n(a-\xi)}{\mathrm{sh}\,\beta_n a}\right]\sin k_n\eta\cos k_n y\right\} \qquad (A.9)$$

$$M_{1yxa} = \frac{2(1-\nu)}{b}\sum_{n=1}^{\infty}\left\{\frac{k_n}{\alpha_n^2-\beta_n^2}\left[-\frac{\mathrm{sh}\,\alpha_n\xi}{\mathrm{sh}\,\alpha_n a}+\frac{\mathrm{sh}\,\beta_n\xi}{\mathrm{sh}\,\beta_n a}\right]\sin k_n\eta\cos k_n y\right\} \qquad (A.10)$$

$$M_{1xy0} = \frac{2(1-\nu)}{a}\sum_{m=1}^{\infty}\left\{\frac{k_m}{\alpha_m^2-\beta_m^2}\left[\frac{\mathrm{sh}\,\alpha_m(b-\eta)}{\mathrm{sh}\,\alpha_m b}-\frac{\mathrm{sh}\,\beta_m(b-\eta)}{\mathrm{sh}\,\beta_m b}\right]\sin k_m\xi\cos k_m x\right\} \qquad (A.11)$$

$$M_{1xyb}=\frac{2(1-\nu)}{a}\sum_{m=1}^{\infty}\left\{\frac{k_m}{\alpha_m^2-\beta_m^2}\left[-\frac{\operatorname{sh}\alpha_m\eta}{\operatorname{sh}\alpha_m b}+\frac{\operatorname{sh}\beta_m\eta}{\operatorname{sh}\beta_m b}\right]\sin k_m\xi\cos k_m x\right. \tag{A.12}$$

对于 $k_m^2<\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}$, $k_n^2<\frac{kh^2}{20}\lambda^2+\sqrt{\lambda^2+\left(\frac{kh^2}{20}\lambda^2\right)^2}$,

$$\omega_{1xx0}=\frac{2}{bD}\sum_{n=1}^{\infty}\left\{\frac{1}{\alpha_n^2+\beta_n^2}\left[-\frac{\operatorname{sh}\alpha_n(a-\xi)}{\operatorname{sh}\alpha_n a}+\frac{\sin\beta_n(a-\xi)}{\sin\beta_n a}\right]\sin k_n\eta\sin k_n y\right. \tag{A.13}$$

$$\omega_{1xxa}=\frac{2}{bD}\sum_{n=1}^{\infty}\left\{\frac{1}{\alpha_n^2+\beta_n^2}\left[\frac{\operatorname{sh}\alpha_n\xi}{\operatorname{sh}\alpha_n a}-\frac{\sin\beta_n\xi}{\sin\beta_n a}\right]\sin k_n\eta\sin k_n y\right. \tag{A.14}$$

$$\omega_{1yy0}=\frac{2}{aD}\sum_{m=1}^{\infty}\left\{\frac{1}{\alpha_m^2+\beta_m^2}\left[-\frac{\operatorname{sh}\alpha_m(b-\eta)}{\operatorname{sh}\alpha_m b}+\frac{\sin\beta_m(b-\eta)}{\sin\beta_m b}\right]\sin k_m\xi\sin k_m x\right. \tag{A.15}$$

$$\omega_{1yyb}=\frac{2}{aD}\sum_{m=1}^{\infty}\left\{\frac{1}{\alpha_m^2+\beta_m^2}\left[\frac{\operatorname{sh}\alpha_m\eta}{\operatorname{sh}\alpha_m b}-\frac{\sin\beta_m\eta}{\sin\beta_m b}\right]\sin k_m\xi\sin k_m x\right. \tag{A.16}$$

$$Q_{1x0}=\frac{2}{b}\sum_{n=1}^{\infty}\left\{\frac{1}{\alpha_n^2+\beta_n^2}\left[\frac{(\alpha_n^2-k_n^2)\operatorname{sh}\alpha_n(a-\xi)}{\operatorname{sh}\alpha_n a}+\frac{(\beta_n^2+k_n^2)\sin\beta_n(a-\xi)}{\sin\beta_n a}\right]\sin k_n\eta\sin k_n y\right. \tag{A.17}$$

$$Q_{1xa}=\frac{2}{b}\sum_{n=1}^{\infty}\left\{\frac{-1}{\alpha_n^2+\alpha_n^2}\left[\frac{(\alpha_n^2-k_n^2)\operatorname{sh}\alpha_n\xi}{\operatorname{sh}\alpha_n a}+\frac{(\beta_n^2+k_n^2)\sin\beta_n\xi}{\sin\beta_n a}\right]\sin k_n\eta\sin k_n y\right. \tag{A.18}$$

$$Q_{1y0}=\frac{2}{a}\sum_{m=1}^{\infty}\left\{\frac{1}{\alpha_m^2+\beta_m^2}\left[\frac{(\alpha_m^2-k_m^2)\operatorname{sh}\alpha_m(b-\eta)}{\operatorname{sh}\alpha_m b}+\frac{(\beta_m^2+k_m^2)\sin\beta_m(b-\eta)}{\sin\beta_m b}\right]\sin k_m\xi\sin k_m x\right. \tag{A.19}$$

$$Q_{1yb}=\frac{2}{a}\sum_{m=1}^{\infty}\left\{\frac{-1}{\alpha_m^2+\beta_m^2}\left[\frac{(\alpha_m^2-k_m^2)\operatorname{sh}\alpha_m\eta}{\operatorname{sh}\alpha_m b}+\frac{(\beta_m^2+k_m^2)\sin\beta_m\eta}{\sin\beta_m b}\right]\sin k_m\xi\sin k_m x\right\} \tag{A.20}$$

$$M_{1yx0}=\frac{2(1-\nu)}{b}\sum_{n=1}^{\infty}\left\{\frac{k_n}{\alpha_n^2+\beta_n^2}\left[\frac{\operatorname{sh}\alpha_n(a-\xi)}{\operatorname{sh}\alpha_n a}-\frac{\sin\beta_n(a-\xi)}{\sin\beta_n a}\right]\sin k_n\eta\cos k_n y\right. \tag{A.21}$$

$$M_{1yxa}=\frac{2(1-\nu)}{b}\sum_{n=1}^{\infty}\left\{\frac{k_n}{\alpha_n^2+\beta_n^2}\left[-\frac{\operatorname{sh}\alpha_n\xi}{\operatorname{sh}\alpha_n a}+\frac{\sin\beta_n\xi}{\sin\beta_n a}\right]\sin k_n\eta\cos k_n y\right. \tag{A.22}$$

$$M_{1xy0}=\frac{2(1-\nu)}{a}\sum_{m=1}^{\infty}\left\{\frac{k_m}{\alpha_m^2+\beta_m^2}\left[\frac{\operatorname{sh}\alpha_m(b-\eta)}{\operatorname{sh}\alpha_m b}-\frac{\sin\beta_m(b-\eta)}{\sin\beta_m b}\right]\sin k_m\xi\cos k_m x\right. \tag{A.23}$$

$$M_{1xyb}=\frac{2(1-\nu)}{a}\sum_{m=1}^{\infty}\left\{\frac{k_m}{\alpha_m^2+\beta_m^2}\left[-\frac{\operatorname{sh}\alpha_m\eta}{\operatorname{sh}\alpha_m b}+\frac{\sin\beta_m\eta}{\sin\beta_m b}\right]\sin k_m\xi\cos k_m x\right. \tag{A.24}$$

参 考 文 献

1 E. Reissner, The effect of transverse shear deformation on the bending of elastic plates, *Journal of Applied Mechanics*, **12**(2) (1945), 69—77.

2 V. L. Salerno and M. A. Goldberg, Effect of shear deformation on the bending of rectangular plates, *Journal of Applied Mechanics*, **27**(1) (1960), 54—58.

3 T. C. Huang, Application of variational methods to the vibration of plate including rotatory inertia and shear, *Developments in Mechanics*, Vol. **1**, Northholland Publ. Co. (1961), 18.

4 . . ±ÇÀ¾À³, ¶ÄÀµ ¿±½ÅÉÊºÇ ¼³ ±µÂ±²º É¶Ã¼ºÇ ÁÂºµ½º, ¶¿º» ³ ÄÀ¶Âºº ÄÀ½ÃÄÍÇ Á½ºÄ, Âº¼½±±¿ ±Ñ ¶Ç±ºº¼±, **9**(10) (1973), 61.

5 K. T. Sundara, R. Iyengar, K. Chandrashekhara, V. K. Sebastian, On the analysis of thick rectangular plates, *Ing. Arch.*, **43**(5) (1974), 317.

6 曹志远、杨升田, 对矩形厚板动力分析精确解一般表达式的探讨, 科学通报, **23**(10) (1978), 627.

7 付宝连, 应用功的互等定理求解复杂边界条件矩形板挠曲面方程, 应用数学和力学, **3**(3) (1982), 315—325.

8 付宝连、李农, 弹性矩形薄板受迫振动的功的互等定理法(Ⅰ)——四边固定矩形板和三边固定的矩形板, 应用数学和力学, **10**(8) (1989), 693—714.

Reciprocal Theorem Method for the Forced Vibration of Elastic Thick Rectangular Plates

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Abstract

In this paper, reciprocal theorem method(RTM) is generalized to solve the problems for the forced vibration of thick rectangular plates based on the Reissner's theory.

The paper derives the dynamic basic solution of thick rectangular plates; and the exact analytical solution of the steady-state responses of thick rectangular plates with three clamped edges and one free edge under harmonic uniformly distributed disturbing forces is found by RTM. It is regarded as a simple, convenient and general method for calculating the steady-state responses of forced vibration of thick rectangular plates.

Key words reciprocal theorem method, dynamic basic solution, thick rectangular plate, forced vibration