应用数学和力学, 第 19 卷 第 12 期(1998 年 12 月)
Applied Mathematics and Mechanics
应用数学和力学编委会编
重 庆 出 版 社 出 版

具有主动约束层阻尼板的振动杂交控制*

张希农[①] 张景绘[①]

(1997 年 4 月 8 日收到)

摘 要

本文研究了局部附加主动约束阻尼层(LACL)板的主、被动杂交控制问题. 基于弹性、粘弹性、压电材料的本构关系, 建立了系统的控制方程. 利用 Galerkin 方法和 GHM 方法将偏微分方程转换为维数不高的常微分方程组, 应用 LQR 方法进行了数值模拟. 计算结果表明, 这种主动和被动结合的杂交控制方式具有良好的控制效果.

关键词 振动控制 主动约束 薄板
中图分类号 V215, O327, TH113

符 号 表

t_b, t_s, t_c 板, 粘弹性层, 压电层的厚度
x_1, x_2, y_1, y_2 粘弹性层, 压电层的边界坐标
u_b, v_b 板的中面在 x, y 方向的位移
u_c, v_c 压电层中面在 x, y 方向的位移
w 板, 粘弹性层, 压电层在 z 方向的位移
ρ_b, ρ_s, ρ_c 板, 粘弹性层, 压电层材料的质量密度
ε_b 板的应变向量
σ_b 板的应力向量
D 板的弹性矩阵
E 板材料的弹性模量
ν 板材料的泊松比
ϕ, φ 粘弹性层在 x, y 方向的剪切应变
τ_x, τ_y 粘弹性层在 x, y 方向的剪应力
$g(t)$ 粘弹性材料的松驰函数
ε_c 压电层的应变向量
σ_c 压电层的应力向量
c 压电层的弹性矩阵
E_z 电场强度
$V(t)$ 控制电压
M^b, T^b, Q^b 板的内力
Q^s 粘弹性层的内力
M^c, T^c, Q^c 压电层的内力
f_x^b, f_y^b, q_b 作用在板上的干扰力
q_s 作用在粘弹性层上的干扰力
f_x^c, f_y^c, q_c 作用在压电层上的干扰力
Q 输出加权矩阵
R 域函数或输入加权矩阵

§1. 引 言

各种粘弹性材料, 由于其良好的阻尼性能, 被广泛地用于抑制结构振动和振动噪声, 一个重要的方式是使用约束阻尼层来被动地控制结构的振动. 振动能量, 主要是通过粘弹性层的剪切而耗散. 这方面的研究已有大量的工作.

* 国家自然科学基金(19632001) 和西安交通大学科研基金资助课题
① 西安交通大学建筑与力学学院, 西安 710049

然而, 仅当振动频率比较高时, 这种方式才有效• 另外, 也难于适应复杂多变的环境载荷• Liao[1]和Shen[2]建议了主动约束层的概念, 并且研究了附加主动约束层阻尼梁的振动杂交控制问题, 得到一些感兴趣的结果• Shen[3]也研究了全部附加主动约束阻尼层复合材料板的振动控制• 在其工作中, 使用了比例加导数控制方式, 设电压是坐标的分布函数, 这在实际中是很难实现的•

本文研究了局部附加粘弹性层和压电约束层板的控制问题, 建立了这种夹层板的偏微分控制方程• 然后利用 Galerkin[4]方法和 GHM[5]方法, 将偏微分方程转换为阶数较低的常微分方程组• 线性最优控制理论的 LQR 方法用于进行模拟计算• 结果表明, 这种杂交控制方式对于板的振动控制是一种较好的方式•

§2. 基 本 方 程

考虑一局部附加主动约束阻尼层的板, 如图1所示• 在系统建模中, 有如下基本假设:

(a) 系统的变形服从克希霍夫理论•

(b) 转动惯量忽略不计•

(c) 仅粘弹性材料的阻尼被考虑•

(d) 每层在 z 方向的位移是相等的•

(e) 层间的位移是完全连续的•

(f) 所施加的控制电压沿板平面是均匀的•

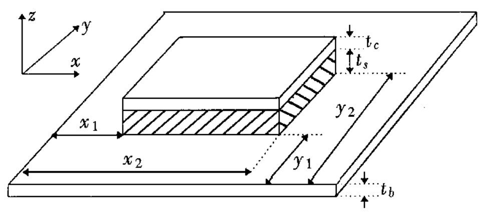


图1　局部附加主动约束阻尼层板

2.1　本构方程

对于弹性板, 考虑弯曲和面内变形, 则

$$\varepsilon_b = \varepsilon_{b0} + z\mathscr{K} \tag{2.1}$$

式中

$$\varepsilon_b = [\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}]_b^T$$

$$\varepsilon_{b0} = [\partial u_b/\partial x \quad \partial v_b/\partial y \quad \partial u_b/\partial y + \partial v_b/\partial x]_b^T$$

$$\mathscr{K} = -[\partial^2 w/\partial x^2 \quad \partial^2 w/\partial y^2 \quad 2\partial^2 w/\partial x\partial y]_b^T$$

本构方程为

$$\sigma_b = D\varepsilon_b \tag{2.2}$$

式中

$$\sigma_b = [\sigma_x \quad \sigma_y \quad \tau_{xy}]_b^T,$$

$$D = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

被广

(2.1)式代入(2.2)式, 得

$$\sigma_b = D\varepsilon_{b0} + zD\mathscr{K} \tag{2.3}$$

对于粘弹性层, 根据假设, 几何关系和变形如图2所示• 粘弹性层的剪应变可以表示为

$$\psi(x, y, t) = \left(1 + (t_b + t_c)/2t_s\right)\partial w/\partial x + (u_c - u_b)/t_s \tag{2.4}$$

$$\varphi(x, y, t) = \left(1 + (t_b + t_c)/2t_s\right)\partial w/\partial y + (v_c - v_b)/t_s \tag{2.5}$$

线性粘弹性本构关系为

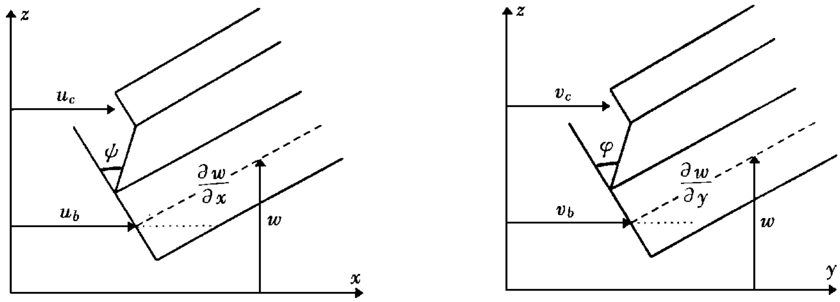


图 2　夹层板的几何关系与变形

$$\tau_x(x,y,t) = \int_{-\infty}^{t} g(t-\xi)\,\dot{\psi}(x,y,\xi)\,d\xi = g*\dot{\psi} \tag{2.6}$$

$$\tau_y(x,y,t) = \int_{-\infty}^{t} g(t-\xi)\,\dot{\varphi}(x,y,\xi)\,d\xi = g*\dot{\varphi} \tag{2.7}$$

对于压电约束层, 几何方程为

$$\varepsilon_c = \varepsilon_{c0} + z\mathscr{K} \tag{2.8}$$

式中

$$\varepsilon_c = [\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}]_c^T,\quad \varepsilon_{c0} = [\partial u_c/\partial x \quad \partial v_c/\partial y \quad \partial u_c/\partial y + \partial v_c/\partial x]_c^T$$

对于薄板, 通常压电层的极化方向为厚度方向, 且电场仅沿 z 方向作用, 所以, 本构关系为

$$\sigma_c = c\varepsilon_c - eE_z \tag{2.9}$$

式中

$$\sigma_c = [\sigma_x \quad \sigma_y \quad \tau_{xy}]_c^T,\quad c = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{21} & c_{22} & 0 \\ 0 & 0 & c_{66} \end{bmatrix},\quad e = [e_{31} \quad e_{32} \quad 0]^T$$

2.2　控制方程

从多层板中取一微分单元, 分离体图如图 3 所示, 内力可表示如下:

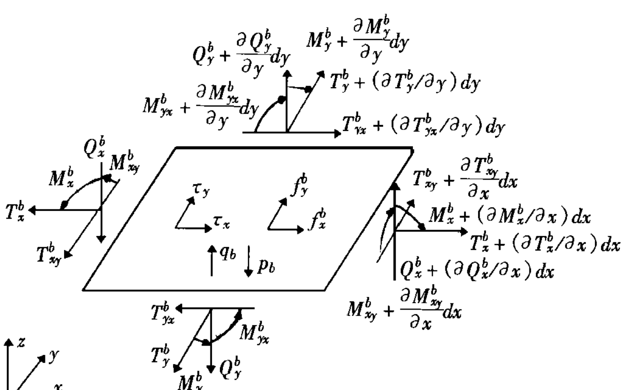


图 3a　板的分离体图

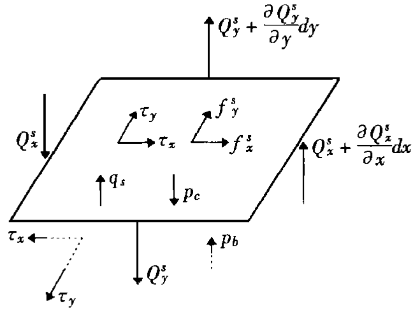


图 3b 剪切层的分离体图

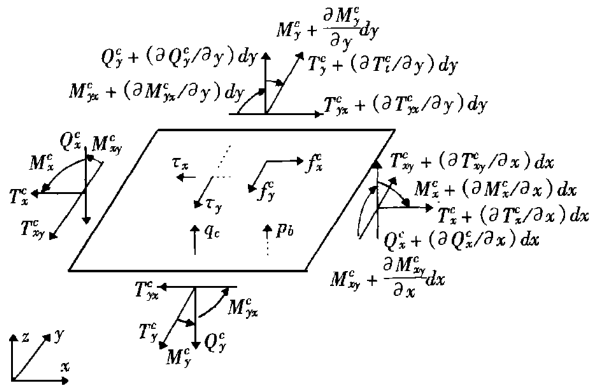


图 3c 约束层的分离体图

$$M_b = [M_x^b \quad M_y^b \quad M_{xy}^b]^T = \int_{-t_b/2}^{t_b/2} \sigma_b z\, dz \tag{2.10}$$

(2.2) 式代入(2.10) 式, 得

$$\text{于薄}\begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -d_0\left(\dfrac{\partial^2 w}{\partial x^2} + \nu\dfrac{\partial^2 w}{\partial y^2}\right) \\ -d_0\left(\dfrac{\partial^2 w}{\partial y^2} + \nu\dfrac{\partial^2 w}{\partial x^2}\right) \\ -d_0(1-\nu)\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{2.11}$$

式中

$$d_0 = Et_b^3/12(1-\nu^2)$$

$$T_b = [T_x^b \quad T_y^b \quad T_{xy}^b]^T = \int_{-t_b/2}^{t_b/2} \sigma_b dz$$

(2.2) 式代入上式, 得

$$\begin{Bmatrix} T_x^b \\ T_y^b \\ T_{xy}^b \end{Bmatrix} = \begin{Bmatrix} d_1\left(\dfrac{\partial u_b}{\partial x} + \nu\dfrac{\partial v_b}{\partial y}\right) \\ d_1\left(\dfrac{\partial v_b}{\partial y} + \nu\dfrac{\partial u_b}{\partial x}\right) \\ g_1\left(\dfrac{\partial u_b}{\partial y} + b\dfrac{\partial v_b}{\partial x}\right) \end{Bmatrix} \tag{2.12}$$

式中

$$d_1 = \frac{Et_b}{1-\nu^2}, \quad g_1 = \frac{Et_b}{2(1+\nu)}$$

类似地, 我们可写出约束层的内力如下:

$$M_c = [M_x^c \quad M_y^c \quad M_{xy}^c]^T = \int_{-t_c/2}^{t_c/2} \sigma_c z\, dz$$

$$T_c = [T_x^c \quad T_y^c \quad T_{xy}^c]^T = \int_{-t_c/2}^{t_c/2} \sigma_c dz$$

将(2.9) 式代入上面两式, 得

$$\begin{Bmatrix} M_x^c \\ M_y^c \\ M_{xy}^c \end{Bmatrix} = \begin{Bmatrix} -\dfrac{t_c^3}{12}\left(c_{11}\dfrac{\partial^2 w}{\partial x^2} + c_{12}\dfrac{\partial^2 w}{\partial y^2}\right) \\ -\dfrac{t_c^3}{12}\left(c_{21}\dfrac{\partial^2 w}{\partial y^2} + c_{22}\dfrac{\partial^2 w}{\partial x^2}\right) \\ -\dfrac{t_c^3}{6}c_{66}\dfrac{\partial^2 w}{\partial x\partial y} \end{Bmatrix} \tag{2.13}$$

$$\begin{Bmatrix} T_x^c \\ T_y^c \\ T_{xy}^c \end{Bmatrix} = \begin{Bmatrix} t_c\left(c_{11}\dfrac{\partial u_c}{\partial x} + c_{12}\dfrac{\partial v_c}{\partial y}\right) - t_c e_{13}E_z \\ t_c\left(c_{21}\dfrac{\partial u_c}{\partial x} + c_{22}\dfrac{\partial v_c}{\partial y}\right) - t_c e_{23}E_z \\ t_c\left(\dfrac{\partial u_c}{\partial y} + \dfrac{\partial v_c}{\partial x}\right) \end{Bmatrix} \tag{2.14}$$

定义一矩形函数表示局部主动约束层的区域, 亦称之为 R 域,

$$R(x, y) = [H(x - x_1) - H(x - x_2)][H(y - y_1) - H(y - y_2)] \tag{2.15}$$

式中, H 为 Heaveyside' s 函数•

令

$$R_x = \partial R/\partial x = [\delta(x - x_1) - \delta(x - x_2)][H(y - y_1) - H(y - y_2)] \tag{2.16}$$

$$R_y = \partial R/\partial y = [H(x - x_1) - H(x - x_2)][\delta(y - y_1) - \delta(y - y_2)] \tag{2.17}$$

由图 3, 基于牛顿定律, 运动方程推导如下:

基板:

$$\partial T_x^b/\partial x + \partial T_{xy}^b/\partial y + \tau_x R + f_x^b = \rho_b t_b \ddot{u}_b \tag{2.18}$$

$$\partial T_y^b/\partial y + \partial T_{xy}^b/\partial x + \tau_y R + f_y^b = \rho_b t_b \ddot{v}_b \tag{2.19}$$

$$\partial Q_x^b/\partial x + \partial Q_y^b/\partial y - p_b R + q_b = \rho_b t_b \ddot{w} \tag{2.20}$$

$$\frac{\partial M_y^b}{\partial y} + \frac{\partial M_{xy}^b}{\partial x} + \frac{t_b}{2}\tau_y R - Q_y^b = 0 \tag{2.21}$$

$$\frac{\partial M_x^b}{\partial x} + \frac{\partial M_{xy}^b}{\partial y} + \frac{t_b}{2}\tau_x R - Q_x^b = 0 \tag{2.22}$$

(2.21) 、(2.22) 式和(2.11) 式代入(2.20) 式, 得

$$d_e - d_0\nabla^2\nabla^2 w + \frac{t_b}{2}\left(\frac{\partial \tau_x}{\partial x}R + \tau_x R_x + \frac{\partial \tau_y}{\partial y}R + \tau_y R_y\right) - p_b R + q_b = \rho_b t_b \ddot{w} \tag{2.23}$$

式中, ∇^2 为拉普拉氏算子•

(2.12) 式代入(2.18) 、(2.19) 式, 得

$$d_1\frac{\partial^2 u_b}{\partial x^2} + g_1\frac{\partial^2 u_b}{\partial y^2} + (d_1\nu + g_1)\frac{\partial^2 v_b}{\partial x\partial y} + \tau_x R + f_x^b = \rho_b t_b \ddot{u}_b \tag{2.24}$$

$$d_1\frac{\partial^2 v_b}{\partial y^2} + g_1\frac{\partial^2 v_b}{\partial x^2} + (d_1\nu + g_1)\frac{\partial^2 u_b}{\partial x\partial y} + \tau_y R + f_y^b = \rho_b t_b \ddot{v}_b \tag{2.25}$$

粘弹性层:

因为粘弹性层仅作用在 R 域内, 所以

$$\partial Q_x^s/\partial x + \partial Q_y^s/\partial y - p_c + p_b + q_s = \rho_s t_s \ddot{w}, \qquad x, y \in R \tag{2.26}$$

$$Q_x^s = \tau_x t_s, \qquad x, y \in R \tag{2.27}$$

$$Q_y^s = \tau_y t_s, \qquad x, y \in R \tag{2.28}$$

(2.27) 和(2.28) 式代入(2.26) 式, 得

$$t_s\left(\partial\tau_x/\partial x + \partial\tau_y/\partial y\right) - p_c + p_b + q_s = \rho_s t_s \ddot{w}, \qquad x, y \in R \tag{2.29}$$

约束层:

$$\frac{\partial T_x^c}{\partial x} + \frac{\partial T_{xy}^c}{\partial y} - \tau_x + f_x^c = \rho_c t_c \ddot{u}_c, \qquad x, y \in R \tag{2.30}$$

$$\frac{\partial T_y^c}{\partial y} + \frac{\partial T_{xy}^c}{\partial x} - \tau_y + f_y^c = \rho_c t_c \ddot{v}_c, \qquad x, y \in R \tag{2.31}$$

$$\frac{\partial Q_x^c}{\partial x} + \frac{\partial Q_y^c}{\partial y} - p_c + q_c = \rho_c t_c \ddot{w}, \qquad x, y \in R \tag{2.32}$$

$$\frac{\partial M_y^c}{\partial y} + \frac{\partial M_{xy}^c}{\partial x} + \frac{t_c}{2}\tau_y - Q_y^c = 0, \qquad x, y \in R \tag{2.33}$$

$$\frac{\partial M_x^c}{\partial x} + \frac{\partial M_{xy}^c}{\partial y} + \frac{t_c}{2}\tau_x - Q_x^c = 0, \qquad x, y \in R \tag{2.34}$$

(2.33) 、(2.34) 和(2.13) 式代入(2.32) 式, 得

$$\text{基于}\frac{t_c^3}{12}\left[c_{11}\frac{\partial^4 w}{\partial x^4} + (c_{12} + c_{21} + 4c_{66})\frac{\partial^4 w}{\partial x^2\partial y^2} + c_{22}\frac{\partial^4 w}{\partial y^4}\right] + \frac{t_c}{2}\left(\frac{\partial\tau_x}{\partial x} + \frac{\partial\tau_y}{\partial y}\right) + q_c + p_c = \rho_c t_c \ddot{w}, \qquad x, y \in R \tag{2.35}$$

(2.13) 式代入(2.30) 和(2.31), 得

$$t_c\left[c_{11}\frac{\partial^2 u_c}{\partial x^2} + (c_{12} + c_{66})\frac{\partial^2 v_c}{\partial x\partial y} + c_{66}\frac{\partial^2 u_c}{\partial y^2}\right] - \tau_x + f_x^c - t_c e_{31}\partial E_z/\partial x = \rho_c t_c \ddot{u}_c, \qquad x, y \in R \tag{2.36}$$

2 1

$$t_c\left[c_{66}\partial^2 v_c/\partial x^2 + (c_{21} + c_{66})\partial^2 u_c/\partial x\partial y + c_{22}\partial^2 v_c/\partial y^2\right] - \tau_y + f_y^c - t_c e_{32}\partial E_z/\partial y = \rho_c t_c \ddot{v}_c, \qquad x, y \in R \tag{2.37}$$

做(2.23) 式+ (2.29) 式× R+ (2.35) 式× R, 得

$$\begin{aligned}&(\rho_b t_b + (\rho_s t_s + \rho_c t_c)R)\ddot{w} + d_0\nabla^2\nabla^2 w \\ &\quad + \frac{t_c^3}{12}\left[c_{11}\frac{\partial^4 w}{\partial x^4} + (c_{12} + c_{21} + 4c_{66})\frac{\partial^4 w}{\partial x^2\partial y^2} + c_{22}\frac{\partial^4 w}{\partial y^4}\right]\cdot R \\ &\quad - \{(t_b/2 + t_s + t_c/2)(\partial\tau_x/\partial x + \partial\tau_y/\partial y)R - t_b(\tau_x R_x + \tau_y R_y)/2\} \\ &= q_b + (q_s + q_c)R\end{aligned} \tag{2.38}$$

当电场强度 E_z 沿板平面均匀分布时, 主动约束层仅在边界处有 $\partial E_z/\partial x \neq 0, \partial E_z/\partial y \neq 0$• 此外, 由压电学理论

$$E_z = V(t)/t_c \tag{2.39}$$

式中, $V(t)$ 为控制电压•

所以,

$$\frac{\partial E_z}{\partial x} = \frac{V(t)}{t_c}[\delta(x - x_1) - \delta(x - x_2)] \tag{2.40}$$

$$\frac{\partial E_z}{\partial y} = \frac{V(t)}{t_c}[\delta(y - y_1) - \delta(y - y_2)] \tag{2.41}$$

(2.6) 、(2.7) 、(2.40) 和(2.41) 式相应地代入(2.38) 、(2.24) 、(2.25) 、(2.36) 和(2.37) 式, 并令

$$t_h = t_b/2 + t_s + t_c/2, \quad h = 1 + (t_b + t_c)/2t_s$$

我们得

$$\begin{aligned}
&(\rho_b t_b + (\rho_s t_s + \rho_c t_c)R)\ddot{w} + d_0 \nabla^2 \nabla^2 w \\
&+ \frac{t_c^3}{12}\left[c_{11}\frac{\partial^4 w}{\partial x^4} + (c_{12} + c_{21} + 4c_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + c_{22}\frac{\partial^4 w}{\partial y^4}\right. \\
&- t_h\left[hg * \left(\frac{\partial^3 w}{\partial x^2 \partial t} + \frac{\partial^3 w}{\partial y^2 \partial t}\right) + \frac{1}{t_s} g * \left(\frac{\partial^2 u_c}{\partial x \partial t} - \frac{\partial^2 u_b}{\partial x \partial t}\right)\right. \\
&\left.+ \frac{1}{t_s} g * \left(\frac{\partial^2 v_c}{\partial y \partial t} - \frac{\partial^2 v_b}{\partial y \partial t}\right)\right] R - \frac{t_b}{2}\left[hg * \left(\frac{\partial^2 w}{\partial x \partial t} R_x + \frac{\partial^2 w}{\partial y \partial t} R_y\right)\right. \\
&\left.+ \frac{1}{t_s} g * (\dot{u}_c - \dot{u}_b) R_x + \frac{1}{t_s} g * (\dot{v}_c - \dot{v}_b) R_y\right] = q_b + (q_s + q_c) R
\end{aligned} \tag{2.42}$$

$$\begin{aligned}
&\rho_b t_b \ddot{u}_b - d_1 \partial^2 u_b/\partial x^2 - g_1 \partial^2 u_b/\partial y^2 - (d_1 \nu + g_1)\partial^2 v_b/\partial x \partial y \\
&- hg * \frac{\partial^2 w}{\partial x \partial t} R - \frac{1}{t_s} g * \dot{u}_c R + \frac{1}{t_s} g * \dot{u}_b R = f_x^b
\end{aligned} \tag{2.43}$$

$$\begin{aligned}
&\rho_b t_b \ddot{v}_b - d_1 \partial^2 v_b/\partial y^2 - g_1 \partial^2 v_b/\partial x^2 - (d_1 \nu + g_1)\partial^2 u_b/\partial x \partial y \\
&- hg * \frac{\partial^2 w}{\partial y \partial t} R - \frac{1}{t_s} g * \dot{v}_c R + \frac{1}{t_s} g * \dot{v}_b R = f_y^b
\end{aligned} \tag{2.44}$$

$$\begin{aligned}
&\rho_c t_c \ddot{u}_c - t_c[c_{11}\partial^2 u_c/\partial x^2 - (c_{21} + c_{66})\partial^2 v_c/\partial x \partial y + c_{66}\partial^2 u_c/\partial y^2] \\
&+ hg * \frac{\partial^2 w}{\partial x \partial t} + \frac{1}{t_s} g * \dot{u}_c - \frac{1}{t_s} g * \dot{u}_b \\
&= f_x^c - e_{31} V(t)[\delta(x - x_1) - \delta(x - x_2)], \qquad x, y \in R
\end{aligned} \tag{2.45}$$

$$\begin{aligned}
&\rho_c t_c \ddot{v}_c - t_c[c_{66}\partial^2 v_c/\partial x^2 + (c_{21} + c_{66})\partial^2 u_c/\partial x \partial y + c_{22}\partial^2 v_c/\partial y^2] \\
&+ hg * \frac{\partial^2 w}{\partial y \partial t} + \frac{1}{t_s} g * \dot{v}_c - \frac{1}{t_s} g * \dot{v}_b \\
&= f_y^c - e_{32} V(t)[\delta(y - y_1) - \delta(y - y_2)], \qquad x, y \in R
\end{aligned} \tag{2.46}$$

关于边界条件, 对于基板来说, 同一般情况的边界条件是一样的, 而约束层在 x, y 方向的边界处总是自由的, 即有

$$\left.\frac{\partial u_c}{\partial x}\right|_{\substack{x = x_1 \\ x = x_2}} = 0, \quad \left.\frac{\partial v_c}{\partial y}\right|_{\substack{y = y_1 \\ y = y_2}} = 0$$

(2.42) ~ (2.46) 式即为具有主动约束阻尼层板的控制方程•

§ 3. 离散化与控制方程

具有主动约束阻尼层板的控制方程(2.42) ~ (2.46) 式是偏微分方程, 直接求解是很困难的• 如果仅低频振动需要被控制, 可用 Galerkin[4]方法将偏微分方程转换为常微分方程组:

$$M_g \ddot{r} + K_g r + C_g \cdot g * \dot{r} = Q_g + U_g \tag{3.1}$$

式中, r 为一新的广义坐标; M_g 和 K_g 为广义坐标中的质量和刚度矩阵; C_g 为与粘弹性层有关的系数矩阵; $g(t)$ 为粘弹性材料的松驰函数; Q_g 和 U_g 分别为载荷向量和控制向量• (参看附录)

在(3.1) 式中有卷积积分项, 它表示了粘弹性层的性质• 粘弹性层的动力学特征有许多描述方式• 本文利用 GHM 方法[5] 处理(3.1) 式中的卷积积分项• 在 GHM 方法中, 粘弹性材料

的性质用下式表示

$$sG(s) = G^{\infty}\left[1 + \sum_{k=1}^{n} \lambda_k \frac{s^2 + 2\zeta_k\omega_k s}{s^2 + 2\zeta_k\omega_k s + \omega_k^2}\right] \tag{3.2}$$

式中, $G(s)$ 为松驰函数 $g(t)$ 的拉普拉氏变换; s 为拉氏变量; G^{∞} 为 $g(t)$ 在平衡状态时的值; λ_k, ζ_k, ω_k 为模型参数. 当 $n = 1$ 时, 令 $\xi_1 = \zeta$, $\omega_1 = \omega$, $\lambda_1 = \lambda$, (3.1) 式可被写为

$$M\ddot{\xi} + C\dot{\xi} + K\xi = \Gamma_\xi + U_\xi \tag{3.3}$$

式中　$\xi = [r^T,\ z^T]^T$

而 z 的拉普拉氏变换为

$$z(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} r(s)$$

以及

$$M = \begin{bmatrix} M_g & 0 \\ 0 & \lambda\dfrac{G^{\infty}}{\omega^2}C_g \end{bmatrix},\quad C = \begin{bmatrix} 0 & 0 \\ 0 & \lambda\dfrac{2\zeta G^{\infty}}{\omega}C_g \end{bmatrix}$$

$$K = \begin{bmatrix} K_g + G^{\infty}(1+\lambda)C_g & -\lambda G^{\infty}C_g \\ -\lambda G^{\infty}C_g & \lambda G^{\infty}C_g \end{bmatrix}$$

$$\Gamma_\xi = \begin{Bmatrix} Q_g \\ 0 \end{Bmatrix},\quad U_\xi = \begin{Bmatrix} U_g \\ 0 \end{Bmatrix}$$

在实际计算中, 广义坐标的维数还进一步降低, 这是因为矩阵 C_g 通常有零特征值, 以致矩阵 M 是非正定的.

在本文中, LQR 最优控制理论被用于确定主动控制增益. 方程(3.3) 以状态方程形式可写为

$$\dot{x} = Ax + Bf + BV(t) \tag{3.4}$$

$$y = Cx \tag{3.5}$$

式中　$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},\quad x = [\xi^T\quad \dot{\xi}^T]^T$

B 和 B 分别为扰动矩阵和控制矩阵; C 是输出矩阵; $V(t)$ 为控制电压.

目标函数为

$$J = \int_0^{\infty}(y^TQy + V^TRV)\,dt \tag{3.6}$$

式中, Q 和 R 分别为关于输出和输入的半正定和正定加权矩阵. 那么控制律为

$$V(t) = -K_c x \tag{3.7}$$

控制增益为

$$K_c = R^T\bar{B}P \tag{3.8}$$

这里, P 满足黎卡提方程

$$A^TP + PA - P\bar{B}R^{-1}\bar{B}^TP + Q = 0 \tag{3.9}$$

§4. 数值算例

考虑一 $0.4\times 0.4\,\mathrm{m}^2$ 的矩形板, 四边简支, 结构参数和材料参数如表 1 所列.

在模拟计算中, 一个脉冲载荷作用在板的 (x_q, y_q) 处, 载荷可以表示为

$$q_b(x, y, t) = F(t)\delta(x - x_q)\delta(y - y_q)$$

式中

$$F(t) = \begin{cases} 1, & 0 \leqslant t < 0.2 \\ 0, & 0.2 \leqslant t \end{cases} \quad ($$

表 1

a 0.4m	E 7.1×10^{10}N/m^2	b 0.4m	ν 0.3
x_1 0.05m	ρ_b 2700kg/m^3	x_2 0.3m	ρ_s 1250kg/m^3
y_1 0.05m	ρ_c 7500kg/m^3	y_2 0.2m	G^∞ 5.6×10^6N/m^2
t_b 0.001m	ω 1000rad/s	t_s 0.001m	λ 1
t_c 0.001m	ξ 4	x_q 0.2m	E_c 6.3×10^{10}N/m^2
y_q 0.2m	d_{31} 254.0pm/V	d_{32} 254.0pm/V	

应用 Galerkin 方法离散偏微分方程, 这里设

$$w(x, y, t) = \sum_{j=1}^{3} W_j(x, y)\alpha_j(t)$$

$$u_b(x, y, t) = \sum_{j=1}^{3} U_{bj}(x, y)\beta_j(t)$$

$$v_b(x, y, t) = \sum_{j=1}^{3} V_{bj}(x, y)\gamma_j(t)$$

$$u_c(x, y, t) = \sum_{j=1}^{3} U_{cj}(x, y)\eta_j(t)$$

$$v_c(x, y, t) = \sum_{j=1}^{3} V_{cj}(x, y)\mu_j(t)$$

式中

$$W_j(x, y) = \sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y, \qquad (m, n) = (1, 1), (1, 2), (2, 1)$$

$$U_{bj}(x, y) = \cos\frac{m\pi}{a}x\sin\frac{n\pi}{b}y, \qquad (m, n) = (1, 1), (1, 2), (2, 1)$$

$$V_{bj}(x, y) = \sin\frac{m\pi}{a}x\cos\frac{n\pi}{b}y, \qquad (m, n) = (1, 1), (1, 2), (2, 1)$$

$$U_{cj}(x, y) = \cos\frac{m\pi}{a}x\sin\frac{n\pi}{b}y, \qquad (m, n) = (1, 1), (1, 2), (2, 1)$$

$$V_{cj}(x, y) = \sin\frac{m\pi}{a}x\cos\frac{n\pi}{b}y, \qquad (m, n) = (1, 1), (1, 2), (2, 1)$$

在 LQR 最优控制中, 加权矩阵取为 $Q = Q_0Q_k$, 其 Q_0 为标量, Q_k 为一已知矩阵· R 在这里也为标量·

当 t_s 趋于零时, 系统成为完全主动控制· 图 4(a)、(b) 表示了响应 $w(2a/3, 2b/3, t)$· 在图 4(a) 中, 虚线是无控制时的响应(t_s 接近于零, 无控制电压); 实线为完全主动控制的结果(t_s 接近于零, $Q_0 = 10^6$, $R = 1$)· 在图 4(b) 中, 虚线表示的是纯被动控制的结果(t_s = 0.001m); 实线表示的是杂交控制的结果(t_s = 0.001m, $Q_0 = 10^6$, $R = 1$)· 图 4(c), (d) 表示的是完全主动控制和杂交控制所需的控制电压, 显然杂交控制比完全主动控制更为有效, 且需

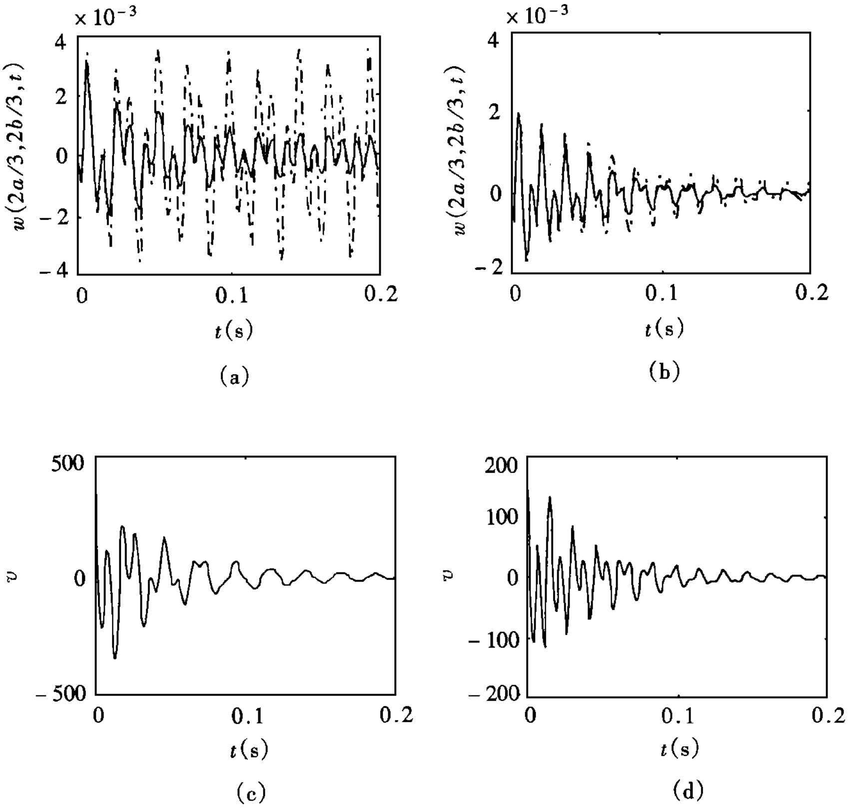
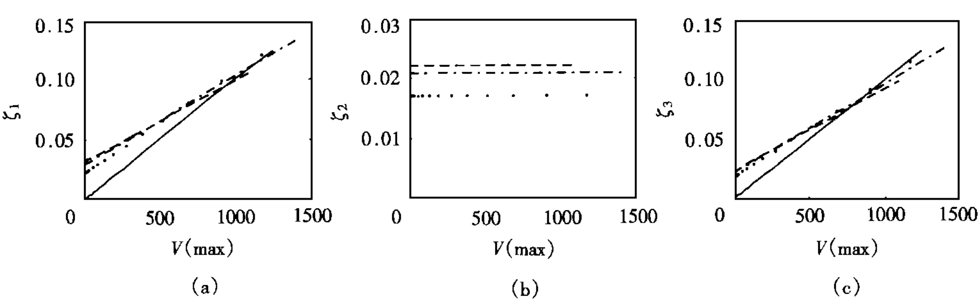


图 4　响应和控制电压曲线

要较低的控制电压•



—: t_s = 0; …: t_s = 0.001m; –•– : t_s = 0.002m; ---: t_s = 0.003m

图 5　阻尼比和最大控制电压和关系

增加 Q_0 的值, 意味着要求抑制更多的振动, 以致更高的控制电压被增加•　图 5 给出的曲线描述在不同的粘弹性厚度 t_s 时, 阻尼比与最大控制电压之间的关系•　由图可见, 阻尼比同最大控制电压之间的关系几乎是线性的•　杂交控制的阻尼比线的斜率比完全主动控制的阻尼比线的斜率小, 因此, 两条线随着最大控制电压的增加将相互交叉•　这说明, 当控制电压比较低时, 杂交控制的效果优于完全主动控制, 杂交控制比完全主动控制产生了更大的阻

尼系数• 虽然当控制电压比较高时, 完全主动控制好于杂交控制, 但对于工程结构来说, 是不实际的, 因为过高的控制电压会使得被控系统的安全性下降• 此外在图 5(b) 中, 随着最大控制电压的变化, ζ_2 没有任何变化, 这是因为在本例中, 第二阶模态和第三阶模态有同样的频率但振型不同, 由于载荷的位置, 第二阶模态未被激起, 系统的响应没有第二阶模态响应, 因而第二阶振动不能被主动控制抑制•

§5. 结 束 语

在本文中给出了局部附加主动约束阻尼层板的偏微分方程, 进而利用 Galerkin 和 GHM 方法将其转换为常微分方程组• 这个工作为进一步建立有限元模型奠定了基础• 数值算例结果表明, 杂交控制不仅具有主动控制功能, 而且具有被动控制功能, 特别是, 杂交控制比完全主动控制, 在达到同样的控制效果时, 需要较小的控制电压• 杂交控制的实际实施, 还需进一步研究•

附 录

使用 Galerkin 方法, 方程(2.42)~(2.46) 式可以被离散化, 设

$$w(x, y, t) = \sum_{j=1}^{N} W_j(x, y)\alpha_j(t) \tag{A1}$$

$$u_b(x, y, t) = \sum_{j=1}^{N} U_{bj}(x, y)\beta_j(t) \tag{A2}$$

$$v_b(x, y, t) = \sum_{j=1}^{N} V_{bj}(x, y)\gamma_j(t) \tag{A3}$$

$$u_c(x, y, t) = \sum_{j=1}^{N} U_{cj}(x, y)\eta_j(t) \tag{A4}$$

$$v_c(x, y, t) = \sum_{j=1}^{N} V_{cj}(x, y)\mu_j(t) \tag{A5}$$

式中, α_j, β_j, γ_j, η_j 和 μ_j 为新的广义坐标; W_j, U_{bj}, V_{bj}, U_{cj} 和 V_{cj} 分别为相应于 w, u_b, v_b, u_c 和 v_c 的展开函数• (2.42) ~ (2.46) 式可转换为

$$M_h\ddot{r} + K_g r + C_g \cdot g^* \dot{r} = Q_g + U_g \tag{A6}$$

式中

$$r = [\alpha \quad \beta \quad \gamma \quad \eta \quad \mu]^T \tag{A7}$$

式中

$$\alpha = [\alpha_1 \cdots \alpha_N], \quad \beta = [\beta_1 \cdots \beta_N], \quad \gamma = [\gamma_1 \cdots \gamma_N]$$

$$\eta = [\eta_1 \cdots \eta_N], \quad \mu = [\mu_1 \cdots \mu_N]$$

$$M_g = \begin{bmatrix} [M_{ij}^{\alpha}] & & & & \\ & [M_{ij}^{\beta}] & & & \\ &) & [M_{ij}^{\gamma}] & & \\ & & & [M_{ij}^{\eta}] & \\ & & & & [M_{ij}^{\mu}] \end{bmatrix} \tag{A8}$$

式中

$$M_{ij}^{\alpha} = \int_0^a\int_0^b \rho_b t_b W_i W_j dxdy + \int_{x_1}^{x_2}\int_{y_1}^{y_2}(\rho_s t_s + \rho_c t_c) W_i W_j dxdy \tag{A9}$$

$$M_{ij}^{\beta} = \int_0^a\int_0^b \rho_b t_b U_{bi} U_{bj} dxdy \tag{A10}$$

$$M_{ij}^{\gamma} = \int_0^a\int_0^b \rho_b t_b V_{bi} V_{bj}\, dx\, dy \tag{A11}$$

$$M_{ij}^{\eta} = \int_{x_1}^{x_2}\int_{y_1}^{y_2} \rho_c t_c U_{ci} U_{cj}\, dx\, dy \tag{A12}$$

$$M_{ij}^{\mu} = \int_{x_1}^{x_2}\int_{y_1}^{y_2} \rho_c t_c V_{ci} V_{cj}\, dx\, dy \tag{A13}$$

$$K_g = \begin{bmatrix} [K_{ij}^{\alpha\alpha}] & 0 & 0 & 0 & 0 \\ 0 & [K_{ij}^{\beta\beta}] & [K_{ij}^{\beta\gamma}] & 0 & 0 \\ 0 & [K_{ij}^{\gamma\beta}] & [K_{ij}^{\gamma\gamma}] & 0 & 0 \\ 0 & 0 & 0 & [K_{ij}^{\eta\eta}] & [K_{ij}^{\eta\mu}] \\ 0 & 0 & 0 & [K_{ij}^{\mu\eta}] & [K_{ij}^{\mu\mu}] \end{bmatrix} \tag{A14}$$

式中

式中

$$K_{ij}^{\alpha\alpha} = d_0\int_0^a\int_0^b W_i\left(\frac{\partial^4 W_j}{\partial x^4} + 2\frac{\partial^4 W_j}{\partial x^2\partial y^2} + \frac{\partial^4 W_j}{\partial y^4}\right) dx\, dy + \frac{t_c^3}{12}\int_{x_1}^{x_2}\int_{y_1}^{y_2} W_i\left(c_{11}\frac{\partial^4 W_i}{\partial x^4}\right. \quad \text{vb}$$

$$\left. +\eta(c_{12} + c_{21} + 4c_{66})\frac{\partial^4 W_j}{\partial x^2\partial y^2} + c_{22}\frac{\partial^4 W_j}{\partial y^4}\right) dx\, dy \tag{A15}$$

$$K_{ij}^{\beta\beta} = -\int_0^a\int_0^b U_{bi}\left(d_1\frac{\partial^2 U_{bj}}{\partial x^2} + g_1\frac{\partial^2 U_{bj}}{\partial y^2}\right) dx\, dy \tag{A16}$$

$$K_{ij}^{\beta\gamma} = -\int_0^a\int_0^b (d_1\nu + g_1)\, U_{bi}\frac{\partial^2 V_{bj}}{\partial x\partial y} dx\, dy \tag{A17}$$

$$K_{ij}^{\gamma\beta} = -\int_0^a\int_0^b (d_1\nu + g_1)\, V_{bi}\frac{\partial^2 U_{bj}}{\partial x\partial y} dx\, dy \tag{A18}$$

$$K_{ij}^{\gamma\gamma} = -\int_0^a\int_0^b V_{bi}\left(d_1\frac{\partial^2 V_{bj}}{\partial x^2} + g_1\frac{\partial^2 V_{bj}}{\partial y^2}\right) dx\, dy \quad , \tag{A19}$$

$$K_{ij}^{\eta\eta} = -t_c\int_{x_1}^{x_2}\int_{y_1}^{y_2} U_{ci}\left(c_{11}\frac{\partial^2 U_{cj}}{\partial x^2} + c_{66}\frac{\partial^2 U_{cj}}{\partial y^2}\right) dx\, dy \tag{A20}$$

$$K_{ij}^{\eta\mu} = -t_c\int_{x_1}^{x_2}\int_{y_1}^{y_2} (c_{12} + c_{66})\, U_{ci}\frac{\partial^2 V_{cj1}}{\partial x\partial y} dx\, dy \qquad 2 \tag{A21}$$

$$K_{ij}^{\mu\eta} = -t_c\int_{x_1}^{x_2}\int_{y_1}^{y_2} (c_{21} + c_{66})\, V_{ci}\frac{\partial^2 U_{cj}}{\partial x\partial y} dx\, dy \tag{A22}$$

$$K_{ij}^{\mu\mu} = -t_c\int_{x_1}^{x_2}\int_{y_1}^{y_2} V_{ci}\left(c_{66}\frac{\partial^2 V_{cj}}{\partial x^2} + c_{22}\frac{\partial^2 V_{cj}}{\partial y^2}\right) dx\, dy \tag{A23}$$

$$C_g = \begin{bmatrix} [C_{ij}^{\alpha\alpha}] & [C_{ij}^{\alpha\beta}] & [C_{ij}^{\alpha\gamma}] & [C_{ij}^{\alpha\eta}] & [C_{ij}^{\alpha\mu}] \\ [C_{ij}^{\beta\alpha}] & [C_{ij}^{\beta\beta}] & 0 & [C_{ij}^{\beta\eta}] & 0 \\ [C_{ij}^{\gamma\alpha}] & 0 & [C_{ij}^{\gamma\gamma}] & 0 & [C_{ij}^{\gamma\mu}] \\ [C_{ij}^{\eta\alpha}] & [C_{ij}^{\eta\beta}] & 0 & [C_{ij}^{\eta\eta}] & 0 \\ [C_{ij}^{\mu\alpha}] & 0 & [C_{ij}^{\mu\gamma}] & 0 & [C_{ij}^{\mu\mu}] \end{bmatrix} \tag{A24}$$

式中

(A

$$C_{ij}^{\alpha\alpha} = -(C_{ij}^{\alpha\alpha} + C_{ij}^{\alpha\alpha}) \tag{A25}$$

$$C_{ij}^{\alpha\alpha} = \frac{t_h h}{2t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2} W_i\left(\frac{\partial^2 W_j}{\partial x^2} + \frac{\partial^2 W_j}{\partial y^2}\right) dx\, dy \tag{A26}$$

$$C_{ij}^{\alpha\alpha} = \frac{t_b h}{4t_s}\int_{y_1}^{y_2}\left[W_i(x_1, y)\frac{\partial W_j}{\partial x}\bigg|_{x=x_1} - W_i(x_2, y)\frac{\partial W_j}{\partial y}\bigg|_{x=x_2}\right] dy \tag{A27}$$

$$C_{ij}^{\alpha\beta} = C_{ij}^{\alpha\beta} + C_{ij}^{\alpha\beta} \tag{A28}$$

$$C_{ij}^{\alpha\beta} = \frac{t_h}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2} W_i\frac{\partial U_{bi}}{\partial x} dx\, dy \tag{A29}$$

$$C_{ij}^{\alpha\beta}=\frac{t_b}{2t_s}\int_{y_1}^{y_2}\left[W_i(x_1,y)U_{bj}(x_1,y)-W_i(x_2,y)U_{bj}(x_2,y)\right]dy \tag{A30}$$

$$C_{ij}^{\alpha\gamma}=C_{ij}^{\alpha\gamma}+C_{ij}^{\alpha\gamma} \tag{A31}$$

$$C_{ij}^{\alpha\gamma}=\frac{t_h}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}W_i\frac{\partial V_{bj}}{\partial y}dxdy \tag{A32}$$

$$C_{ij}^{\alpha\gamma}=\frac{t_b}{2t_s}\int_{x_1}^{x_2}\left[W_i(x,y_1)V_{bj}(x,y_1)-W_i(x,y_2)V_{bj}(x,y_2)\right]dx \tag{A33}$$

$$C_{ij}^{\alpha\eta}=-(C_{ij}^{\alpha\eta}+C_{ij}^{\alpha\eta}) \tag{A34}$$

$$C_{ij}^{\alpha\eta}=\frac{t_h}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}W_i\frac{\partial U_{cj}}{\partial x}dxdy \tag{A35}$$

$$C_{ij}^{\alpha\eta}=\frac{t_b}{2t_s}\int_{y_1}^{y_2}\left[W_i(x_1,y)U_{cj}(x_1,y)-W_i(x_2,y)U_{cj}(x_2,y)\right]dy \tag{A36}$$

$$C_{ij}^{\alpha\mu}=-(C_{ij}^{\alpha\mu}+C_{ij}^{\alpha\mu}) \tag{A37}$$

$$C_{ij}^{\alpha\mu}=\frac{t_h}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}W_i\frac{\partial V_{cj}}{\partial y}dxdy \tag{A38}$$

$$C_{ij}^{\alpha\mu}=\frac{t_b}{2t_s}\int_{x_1}^{x_2}\left[W_i(x,y_1)V_{cj}(x,y_1)-W_i(x,y_2)V_{cj}(x,y_2)\right]dx \tag{A39}$$

$$C_{ij}^{\beta\alpha}=-\frac{t_h}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}U_{bi}\frac{\partial W_j}{\partial x}dxdy \tag{A40}$$

$$C_{ij}^{\beta\beta}=\frac{1}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}U_{bi}U_{bj}dxdy \tag{A41}$$

$$C_{ij}^{\beta\eta}=-\frac{1}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}U_{bi}U_{cj}dxdy \tag{A42}$$

$$C_{ij}^{\gamma\alpha}=-\frac{h}{2t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}V_{bi}\frac{\partial W_j}{\partial y}dxdy \tag{A43}$$

$$C_{ij}^{\gamma\gamma}=\frac{1}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}V_{bi}V_{bj}dxdy \tag{A44}$$

$$C_{ij}^{\gamma\mu}=-\frac{1}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}V_{bi}V_{cj}dxdy \tag{A45}$$

$$C_{ij}^{\eta\alpha}=\frac{t_h}{2t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}U_{ci}\frac{\partial W_j}{\partial x}dxdy \tag{A46}$$

$$C_{ij}^{\eta\beta}=-\frac{1}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}U_{ci}U_{bj}dxdy \tag{A47}$$

$$C_{ij}^{\eta\eta}=\frac{1}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}U_{ci}U_{cj}dxdy \tag{A48}$$

$$C_{ij}^{\mu\alpha}=\frac{h}{2t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}V_{ci}\frac{\partial W_j}{\partial y}dxdy \tag{A49}$$

$$C_{ij}^{\gamma\mu}=-\frac{1}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}V_{ci}V_{bj}dxdy \tag{A50}$$

$$C_{ij}^{\mu\mu}=-\frac{1}{t_s}\int_{x_1}^{x_2}\int_{y_1}^{y_2}V_{ci}V_{cj}dxdy \tag{A51}$$

$$Q_g=\left[[Q_i^{\alpha}]^T\quad[Q_i^{\beta}]^T\quad[Q_i^{\gamma}]^T\quad[Q_i^{\eta}]^T\quad[Q_i^{\mu}]^T\right]^T \tag{A52}$$

式中

$$Q_i^{\alpha}=\int_0^a\int_0^b(q_b+(q_s+q_c)R(x,y))W_idxdy \tag{A53}$$

$$Q_i^{\beta}=\int_0^a\int_0^b f_x^b U_{bi}dxdy \tag{A54}$$

$$Q_i^{\gamma} = \int_0^a\int_0^b f_y^b V_{bi} dx dy \tag{A55}$$

$$Q_i^{\eta} = \int_{x_1}^{x_2}\int_{y_1}^{y_2} f_x^c U_{ci} dx dy \tag{A56}$$

$$Q_i^{\mu} = \int_{x_1}^{x_2}\int_{y_1}^{y_2} f_y^c V_{ci} dx dy \tag{A57}$$

$$U_g = [0 \quad 0 \quad 0 \quad [U_i^{\eta}]^T \quad [U_i^{\mu}]^T]^T \tag{A58}$$

式中

$$U_i^{\eta} = -e_{31} V(t)(y_2 - y_1)\int_{x_1}^{x_2} U_{ci}[\delta(x - x_1) - \delta(x - x_2)]dx \tag{A59}$$

$$U_i^{\mu} = -e_{32} V(t)(x_2 - x_1)\int_{y_1}^{y_2} V_{ci}[\delta(y - y_1) - \delta(y - y_2)]dy \tag{A60}$$

参 考 文 献

1 W. H. Liao and K. W. Wang, On the active_passive hybrid vibration control actions of structures with active constrained layer treatments, Design Engineering Technical Conferences, ASME., **3** Part C (1995), 125—140.

2 I. Y. Shen, Hybrid damping through intelligent constrained layer treatments, Journal of Vibration and Acoustics, **116**(3) (1994), 341—349.

3 I. Y. Shen, Bending vibration control of composite plate structures through intelligent layer treatments, Proceedings SPIE, **2193** (1994), 115—125.

4 D. J. Inman, Vibration with Control, Measurement, and Stability, Prentice Hall (1989).

5 D. J. McTavish and P. C. Hughes, Modeling of linear viscoelastic space structures, Journal of Vibration and Acoustics, **115**(1) (1993), 103—110.

The Hybrid Control of Vibration of Thin Plate with Active Constrained Damping Layer

Zhang Xinong Zhang Jinghui

(Xi'an Jiaotong University, Xi'an 710049, P. R. China)

Abstract

This paper concerns in the active and passive hybrid control of vibration of the thin plate with Local Active Constrained damping Layer (LACL). The governing equations of system are formulated based on the constitutive equations of elastic, viscoelastic, piezoelectric materials. Galerkin method and GHM method are employed to transform partial differential equations into ordinary ones with a lower dimension. LQR method of classical control theory is used in simulating calculation. Numeral results show that the active and passive hybrid control manner obtained in this paper is a better one for vibration control of the plate.

Key words vibration control, active constraint, sandwich plate