应用数学和力学编委会编
重 庆 出 版 社 出 版

文章编号: 1000_0887(1999) 07_0721_08

复合材料多层板壳大挠度非线性问题的迭代解法*

尹邦信

(西南工学院土木系, 四川绵阳 621002)

(张湘伟推荐)

摘要: 在王震鸣等人提出的各向异性多层扁壳的大挠度方程的基础上, 提出了复合材料多层板壳大挠度非线性问题的迭代解法• 分析了四边简支的复合材料多层矩形扁壳, 与小挠度线性理论解析解及有限元非线性解进行了对比• 结果表明, 载荷较小并发生小挠度时, 所得的大挠度解和小挠度解析解非常接近, 载荷较大时, 所得解和有限元非线性解非常接近•

关 键 词: 复合材料多层板壳; 大挠度; 迭代法
中图分类号: O39; TB12 文献标识码: A

引 言

由于复合材料在现代工程技术中的广泛应用以及复合材料力学的高度发展, 精确分析复合材料多层板壳中的应力具有重要的实际意义• 文献[1] 给出了各向异性多层扁壳的大挠度方程, 但对该方程却未能求得解析解• 用有限元程序能分析复合材料多层板壳大挠度非线性问题, 但计算工作量大、计算时间长• 本文提出的迭代法能够简便地求解各向异性多层扁壳的大挠度方程, 计算精度和有限元解相当, 计算工作量和计算时间却大幅度减少•

1 复合材料多层板壳大挠度非线性问题的平衡方程

根据文献[1] , 对于图 1 所示的复合材料多层扁壳, 在正交铺设或 θ 斜交铺设且层数较多时, 可取 $A_{16} = A_{26} = B_{16} = B_{26} = D_{16} = D_{26} = C_{45} = 0$, 当 R_x, R_y 为常数, $1/R_{xy} = 0$ 时, 用 u_0, v_0, γ_x, γ_y, w 作为未知量, 大挠度问题的平衡方程可写为

$$\{a\} + \{b\} + \{d\} = 0, \tag{1}$$

其中

$$\{a\} = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ & L_{22} & L_{23} & L_{24} & L_{25} \\ & & L_{33} & L_{34} & L_{35} \\ & \text{对称} & & L_{44} & L_{45} \\ & & & & L_{55} \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ \gamma_x \\ \gamma_y \\ w \end{Bmatrix}, \tag{2}$$

* 收稿日期: 1996_11_26; 修订日期: 1999_03_10
作者简介: 尹邦信(1945~) , 男, 副教授, 研究方向: 计算结构力学, 已发表论文十多篇.

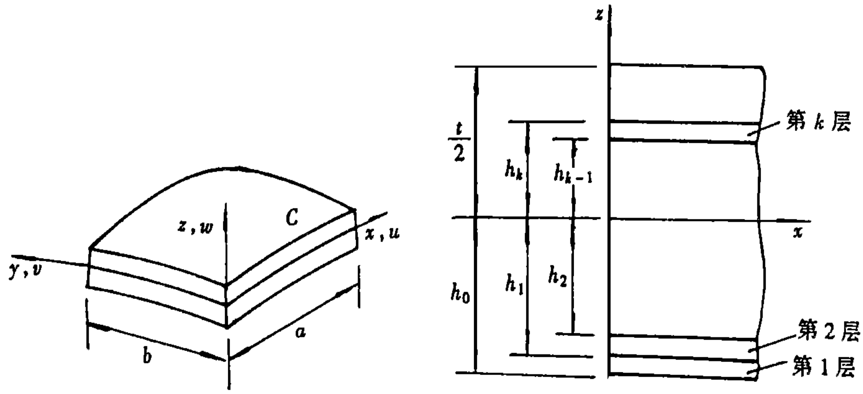

图 1

在式(2)中, 微分算子 L_{ij} 的表达式如下:

$$L_{11} = A_{11} + \frac{\partial^2}{\partial x^2} + A_{66}\frac{\partial^2}{\partial y^2},$$

$$L_{12} = (A_{12} + A_{66})\frac{\partial^2}{\partial x \partial y},$$

$$L_{13} = B_{11}\frac{\partial^2}{\partial x^2} + B_{66}\frac{\partial^2}{\partial y^2},$$

$$L_{14} = L_{23} = (B_{12} + B_{66})\frac{\partial^2}{\partial x \partial y},$$

$$L_{15} = \left(\frac{A_{11}}{R_x} + \frac{A_{12}}{R_y}\right)\frac{\partial}{\partial x} - L_{13}\frac{\partial}{\partial x} - L_{14}\frac{\partial}{\partial y},$$

$$L_{22} = A_{66}\frac{\partial^2}{\partial x^2} + A_{22}\frac{\partial^2}{\partial y^2},$$

$$L_{24} = B_{66}\frac{\partial^2}{\partial x^2} + B_{22}\frac{\partial^2}{\partial y^2},$$

$$L_{25} = \left(\frac{A_{12}}{R_x} + \frac{A_{22}}{R_y}\right)\frac{\partial}{\partial y} - L_{23}\frac{\partial}{\partial x} - L_{24}\frac{\partial}{\partial y},$$

$$L_{33} = D_{11}\frac{\partial^2}{\partial x^2} + D_{66}\frac{\partial^2}{\partial y^2} - C_{55},$$

$$L_{34} = (D_{12} + D_{66})\frac{\partial^2}{\partial x \partial y},$$

$$L_{35} = \left(\frac{B_{11}}{R_x} + \frac{B_{12}}{R_y} - C_{55}\right)\frac{\partial}{\partial x} - L_{33}\frac{\partial}{\partial x} - L_{34}\frac{\partial}{\partial y},$$

$$L_{44} = D_{66}\frac{\partial^2}{\partial x^2} + D_{22}\frac{\partial^2}{\partial y^2} - C_{44},$$

$$L_{45} = \left(\frac{B_{12}}{R_x} + \frac{B_{22}}{R_y} - C_{44}\right)\frac{\partial}{\partial y} - L_{34}\frac{\partial}{\partial x} - L_{44}\frac{\partial}{\partial y},$$

$$L_{55} = \left(\frac{A_{11}}{R_x^2} + \frac{2A_{12}}{R_xR_y} + \frac{A_{22}}{R_y^2}\right) - C_{55}\frac{\partial^2}{\partial x^2} - C_{44}\frac{\partial^2}{\partial y^2} -$$
$$2L_{35}\frac{\partial}{\partial x} - 2L_{45}\frac{\partial}{\partial y} + L_{33}\frac{\partial^2}{\partial x^2} + 2L_{34}\frac{\partial^2}{\partial x \partial y} + L_{44}\frac{\partial^2}{\partial y^2},$$

$$\left\{b\right\} = \left[q_x, q_y, m_x, m_y, -\left(q_z + \frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y}\right)\right]^{\mathrm{T}}, \qquad (3)$$

$$\{d\} = [d_1, d_2, d_3, d_4, d_5]^{\mathrm{T}} \tag{4}$$

其中

$$\left.\begin{aligned}
&d_1 = \frac{\partial w}{\partial x}L_{11}w + \frac{\partial w}{\partial y}L_{12}w, \quad d_2 = \frac{\partial w}{\partial x}L_{12}w + \frac{\partial w}{\partial y}L_{22}w, \\
&d_3 = \frac{\partial w}{\partial x}L_{13}w + \frac{\partial w}{\partial y}L_{23}w, \quad d_4 = \frac{\partial w}{\partial x}L_{14}w + \frac{\partial w}{\partial y}L_{24}w, \\
&d_5 = \frac{1}{2}\left(\frac{\partial w}{\partial x}L_{51}w + \frac{\partial w}{\partial y}L_{52}w\right) - \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}L_{13}w + \frac{\partial w}{\partial y}L_{23}w\right) - \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}L_{14}w + \right. \\
&\qquad \left.\frac{\partial w}{\partial y}L_{24}w\right) - \frac{\partial w}{\partial x}\left(M_x\frac{\partial w}{\partial x} + N_{xy}\frac{\partial w}{\partial y}\right) - \frac{\partial w}{\partial y}\left(N_{xy}\frac{\partial w}{\partial x} + N_y\frac{\partial w}{\partial y}\right),
\end{aligned}\right\} \tag{5}$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} + \dfrac{w}{R_x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v_0}{\partial y} + \dfrac{w}{R_y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \\ \dfrac{\partial \gamma_x}{\partial x} - \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial \gamma_y}{\partial y} - \dfrac{\partial^2 w}{\partial y^2} \\ \dfrac{\partial \gamma_x}{\partial y} + \dfrac{\partial \gamma_y}{\partial x} - 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} . \tag{6}$$

2 迭代求解的数值方法

对于四边简支的矩形扁壳, 在 $A_{16} = A_{26} = B_{16} = B_{26} = D_{16} = D_{26} = C_{45} = 0$ 时, 可设其广义位移为如下形式, 即可满足边界条件.

$$\left.\begin{aligned}
&u_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u_{0mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \quad v_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} v_{0mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \\
&\gamma_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \gamma_{xmn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \quad \gamma_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \gamma_{ymn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \\
&w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}.
\end{aligned}\right\} \tag{7}$$

将载荷展为如下的级数

$$\left.\begin{aligned}
&q_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{xmn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \quad q_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{ymn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \\
&m_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m_{xmn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \quad m_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m_{ymn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \\
&q_z = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{zmn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}.
\end{aligned}\right\} \tag{8}$$

则式(1)中 $\{a\} + \{b\}$ 之和的各元素可写为

$$\left.\begin{aligned}
&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-T_{11}u_{0mn}-T_{12}v_{omn}-T_{13}\gamma_{xmn}-T_{14}\gamma_{ymn}-T_{15}w_{mn}+q_{xmn})\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b},\\
&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-T_{12}u_{0mn}-T_{22}v_{omn}-T_{23}\gamma_{xmn}-T_{24}\gamma_{ymn}-T_{25}w_{mn}+q_{ymn})\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b},\\
&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-T_{13}u_{0mn}-T_{23}v_{omn}-T_{33}\gamma_{xmn}-T_{34}\gamma_{ymn}-T_{35}w_{mn}+m_{xmn})\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b},\\
&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-T_{14}u_{0mn}-T_{24}v_{omn}-T_{34}\gamma_{xmn}-T_{44}\gamma_{ymn}-T_{45}w_{mn}+m_{ymn})\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b},\\
&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(T_{15}u_{0mn}+T_{25}v_{omn}+T_{35}\gamma_{xmn}+T_{45}\gamma_{ymn}+T_{55}w_{mn}-q_{zmn}+\frac{m\pi}{a}m_{xmn}+\frac{n\pi}{b}m_{ymn}\right)\times\\
&\quad\sin(m\pi x/a)\cos(n\pi y/b),
\end{aligned}\right\}\tag{9}$$

其中

$$\left.\begin{aligned}
&T_{11}=A_{11}\frac{m^2\pi^2}{a^2}+A_{66}\frac{n^2\pi^2}{b^2},\quad T_{12}=(A_{12}+A_{66})\frac{mn\pi^2}{ab},\\
&T_{13}=B_{11}\frac{m^2\pi^2}{a^2}+B_{66}\frac{n^2\pi^2}{b^2},\quad T_{14}=T_{23}=(B_{12}+B_{66})\frac{mn\pi^2}{ab},\\
&T_{22}=A_{66}\frac{m^2\pi^2}{a^2}+A_{22}\frac{n^2\pi^2}{b^2},\quad T_{24}=B_{66}\frac{m^2\pi^2}{a^2}+B_{22}\frac{n^2\pi^2}{b^2},\\
&T_{33}=D_{11}\frac{m^2\pi^2}{a^2}+D_{66}\frac{n^2\pi^2}{b^2}+C_{55},\quad T_{34}=(D_{12}+D_{66})\frac{mn\pi^2}{ab},\\
&T_{44}=D_{66}\frac{m^2\pi^2}{a^2}+D_{22}\frac{n^2\pi^2}{b^2}+C_{44},\\
&T_{15}=-\left(\frac{A_{11}}{R_x}+\frac{A_{12}}{R_y}\right)\frac{m\pi}{a}-B_{11}\frac{m^3\pi^3}{a^3}-(B_{12}+2B_{66})\frac{mn^2\pi^3}{ab^2},\\
&T_{25}=-\left(\frac{A_{12}}{R_x}+\frac{A_{22}}{R_y}\right)\frac{n\pi}{b}-B_{22}\frac{n^3\pi^3}{b^3}-(B_{12}+2B_{66})\frac{nm^2\pi^3}{ba^2},\\
&T_{35}=-\left(\frac{B_{11}}{R_x}+\frac{B_{12}}{R_y}\right)\frac{m\pi}{a}-D_{11}\frac{m^3\pi^3}{a^3}-(D_{12}+2D_{66})\frac{mn^2\pi^3}{ab^2},\\
&T_{45}=-\left(\frac{B_{11}}{R_x}+\frac{B_{22}}{R_y}\right)\frac{n\pi}{b}-D_{22}\frac{n^3\pi^3}{b^3}-(D_{12}+2D_{66})\frac{nm^2\pi^3}{ba^2},\\
&T_{55}=\left(\frac{A_{11}}{R_x^2}+\frac{2A_{12}}{R_xR_y}+\frac{A_{22}}{R_y^2}\right)+D_{11}\frac{m^4\pi^4}{a}+2\left(\frac{B_{11}}{R_x}+\frac{B_{12}}{R_y}\right)\frac{m^2\pi^2}{a^2}+\\
&\qquad 2\left(\frac{B_{12}}{R_x}+\frac{B_{22}}{R_y}\right)\frac{n^2\pi^2}{b^2}+2(D_{12}+2D_{66})\frac{n^2m^2\pi^4}{b^2a^2}+D_{22}\frac{n^4\pi^4}{b^4}\cdot
\end{aligned}\right\}\tag{10}$$

迭代的准备阶段, 从小挠度线性理论解开始, 令 $d_1=d_2=d_3=d_4=d_5=0$, 并设

$$\left\{q_{mn}\right\}=\left[q_{xmn},q_{ymn},m_{xmn},m_{ymn},\left(q_{zmn}-\frac{m\pi}{a}m_{xmn}-\frac{n\pi}{b}m_{ymn}\right)\right]^{\mathrm{T}},\tag{11}$$

则从式(9)可以得到

$$\begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15} \\ & T_{22} & T_{23} & T_{24} & T_{25} \\ & & T_{33} & T_{34} & T_{35} \\ & \text{对称} & & T_{44} & T_{45} \\ & & & & T_{55} \end{bmatrix} \begin{Bmatrix} u_{0mn} \\ v_{0mn} \\ \gamma_{xmn} \\ \gamma_{ymn} \\ w_{0mn} \end{Bmatrix} = \begin{Bmatrix} q_{mn} \end{Bmatrix}. \tag{12}$$

根据式(1)~(6)及式(7)~(12)可将迭代步骤整理如下:

1) 对各 m, n 值解方程式(12).

2) 利用求得的 $u_{omn}, v_{0mn}, \gamma_{xmn}, \gamma_{ymn}, w_{mn}$, 用式(5)~(6)及式(7)计算

$$\begin{Bmatrix} d \end{Bmatrix} = [d_1, d_2, d_3, d_4, d_5]^{\mathrm{T}}.$$

3) 利用数值积分将 $\begin{Bmatrix} d \end{Bmatrix}$ 中各元素展为如下级数:

$$\left.\begin{aligned} d_1 &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} d_{1mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \\ d_2 &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} d_{2mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \\ d_3 &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} d_{3mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \\ d_4 &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} d_{4mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \\ d_5 &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} d_{5mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \end{aligned}\right\} \tag{13}$$

其中

$$\left.\begin{aligned} d_{1mn} &= \frac{4}{ab}\int_0^a\int_0^b d_1\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\mathrm{d}x\,\mathrm{d}y, \\ d_{2mn} &= \frac{4}{ab}\int_0^a\int_0^b d_2\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\mathrm{d}x\,\mathrm{d}y, \\ d_{3mn} &= \frac{4}{ab}\int_0^a\int_0^b d_3\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\mathrm{d}x\,\mathrm{d}y, \\ d_{4mn} &= \frac{4}{ab}\int_0^a\int_0^b d_4\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\mathrm{d}x\,\mathrm{d}y, \\ d_{5mn} &= \frac{4}{ab}\int_0^a\int_0^b d_5\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\mathrm{d}x\,\mathrm{d}y. \end{aligned}\right\} \tag{14}$$

4) 修正式(12)中的 $\begin{Bmatrix} q_{mn} \end{Bmatrix}$, 即按下式重新计算 $\begin{Bmatrix} q_{mn} \end{Bmatrix}$

$$\begin{Bmatrix} q_{mn} \end{Bmatrix} = \begin{Bmatrix} q_{xmn} + d_{1mn} \\ q_{ymn} + d_{2mn} \\ m_{xmn} + d_{3mn} \\ m_{ymn} + d_{4mn} \\ \left(q_{zmn} - \dfrac{m\pi}{a}m_{xmn} - \dfrac{m\pi}{b}m_{ymn}\right) - d_{5mn} \end{Bmatrix}, \tag{15}$$

反复执行 1) ~ 4) 各步骤, 直至解答满足所定精度要求为止•

3 计 算 结 果

作者使用本文所示的迭代法, 计算了大量的算例, 都得到合理的解答• 限于篇幅, 本文只列举复合材料矩形扁壳在较小载荷和较大载荷两种情况下的解答, 并与小挠度线性理论理论解析解和有限元非线性解对比•

例 1 扁壳四边简支, 8 层, 每层厚 0.5cm, $a = b = 1.0$m, 45°/ − 45°/45°/ − 45°/45°/ − 45°/45°/ − 45° 铺设, $R_x = 8.0$m, $R_y = 6.0$m, $1/R_{xy} = 0$, $E_l = 245.25$GPa, $E_t = 9.81$GPa, $G_{lt} = 4.91$GPa, $G_{tn} = 1.96$GPa, $G_{nl} = 4.91$GPa, $\mu_{lt} = 0.25$, 均布载荷 $q_z = -19.62$kN/m^2(向下)• 在这种小载荷作用下, 本文的大挠度非线性方法计算结果和小挠度线性方法解析解应该接近, 两种结果如表 1 所示•

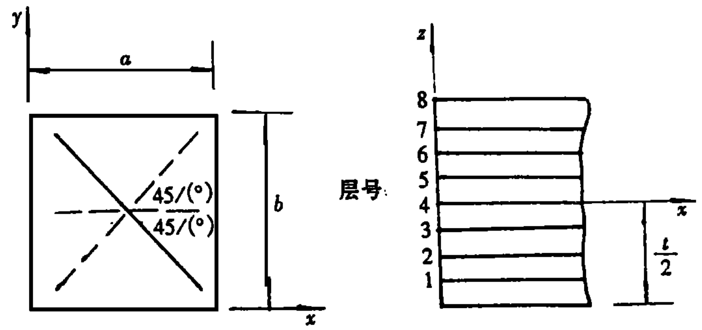

图 2

表 1 点 $x = 25$cm, $y = 50$cm 处各层的 σ_x, τ_{xy}/MPa

层号	z 坐标 cm	σ_x		τ_{xy}	
		小挠度解	本文方法	小挠度解	本文方法
1	− 2.0	0.904	0.910	− 0.797	− 0.801
	− 1.5	0.529	0.533	− 0.461	− 0.464
2	− 1.5	0.529	0.533	0.461	0.464
	− 1.0	0.154	0.157	0.125	0.128
3	− 1.0	0.154	0.157	− 0.125	− 0.128
	− 0.5	− 0.222	− 0.220	0.211	0.209
4	− 0.5	− 0.222	− 0.220	− 0.211	− 0.209
	0.0	− 0.598	− 0.596	− 0.547	− 0.546
5	0.0	− 0.598	− 0.596	0.546	0.546
	0.5	− 0.973	− 0.972	0.883	0.882
6	0.5	− 0.973	− 0.972	− 0.883	− 0.882
	1.0	− 1.349	− 1.348	− 1.218	− 1.218

续表 1

层号	z 坐标 cm	σ_x		τ_{xy}	
		小挠度解	本文方法	小挠度解	本文方法
7	1.0	-1.349	-1.348	1.218	1.218
	1.5	-1.725	-1.726	1.555	1.556
8	1.5	-1.725	-1.726	-1.555	-1.556
	2.0	-2.100	-2.102	-1.890	-1.891

例 2　扁壳参数同例 1, 均布载荷 $q_z = -981\text{kN/m}^2$(向下)• 在这种较大载荷作用下, 本文的大挠度非线性方法计算结果和小挠度线性方法解析解有较大差别, 和有限元非线性解接近, 三种结果如表 2 所示•

表 2　点 $x = 25\text{cm}, y = 50\text{cm}$ 处各层的 σ_x, τ_{xy}/MPa

层号	z 坐标 cm	σ_x			τ_{xy}		
		小挠度	有限元	本文方法	小挠度解	有限元	本文方法
1	-2.0	45.23	61.87	62.04	-39.84	-54.40	-54.59
	-1.5	26.46	40.64	40.47	-23.03	-35.29	-35.21
2	-1.5	26.46	40.68	40.47	23.03	35.24	35.21
	-1.0	7.67	18.72	18.86	6.24	16.11	15.83
3	-1.0	7.67	18.86	-6.24	-6.24	-16.18	-15.83
	-0.5	-11.10	-2.87	-2.73	10.56	3.85	3.55
4	-0.5	-11.10	-2.89	-2.73	-10.56	-3.90	-3.55
	0.0	-29.89	-24.36	-24.32	-27.35	-22.68	-22.93
5	0.0	-29.89	-24.34	-24.32	27.35	22.74	22.93
	0.5	-48.67	-45.73	-45.91	44.15	42.43	42.31
6	0.5	-48.67	-45.66	-45.91	-44.15	-42.39	-42.31
	1.0	-67.45	-67.62	-67.49	-60.94	-61.71	-61.70
7	1.0	-67.45	-67.60	-67.49	60.94	61.74	61.70
	1.5	-86.23	-88.90	-89.07	77.73	80.97	81.07
8	1.5	-86.23	-88.89	-89.07	-77.73	-81.01	-81.07
	2.0	104.9	-109.8	-110.6	-94.53	-100.9	-100.5

4　结　　论

本文用迭代法求解了复合材料多层扁壳的大挠度非线性问题, 建立了相应的迭代公式• 尽管复合材料多层扁壳的大挠度非线性问题关系复杂, 求解难度很大, 但用本文的迭代法求解却比较简便• 文中的算例通过与小挠度线性解析解和有限元非线性解的对比, 证明了其计算结果的正确性和合理性• 同时也表明, 载荷较小并发生小挠度时, 大挠度非线性解和小挠度线

性解是非常接近的; 载荷较大而发生大挠度时, 大挠度非线性解和小挠度线性解是有较大区别的• 使用本文方法时, 应使用复合材料多层板壳的实际参数, 迭代时收敛速度较快, 若使用量纲一的参数, 容易导致发散•

[参 考 文 献]

[1] 王震鸣, 刘国玺, 吕明身. 各向异性多层扁壳的大挠度方程[J]. 应用数学和力学, 1982, **3**(1): 49~65.

[2] 冯康等. 数值计算方法[M]. 北京: 国防工业出版社, 1978.

[3] 江晓禹, 张相周, 陈百屏. 复合材料层合板的三维非线性分析[J]. 应用数学和力学, 1996, **15**(7): 589~599.

Iterative Method on Large Deflection Nonlinear Problem of Laminated Composite Shallow Shells and Plates

Yin Bangxin

(Department of Civil Engineering, Southwest Institute of Technology, Mianyang, Sichuan 621000, P R China)

Abstract: Based on result by Wang, in this paper the iterative method is presented for the research of large deflection nonlinear problem of laminated composite shallow shells and plates. The rectangular laminated composite shallow shells have been analyzed. The results have been compared with the small deflection linear analytical solution and finite element nonlinear solution. The results proved that the solution coincide with small deflection linear analytical solution in the condition of a little loads and finite element nonlinear solution in the condition of large loads.

Key words: laminated composite shallow shells and plates; large deflection; iterative method