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三种群 Lotka_Volterra 非周期食饵_捕食系统的持久性*

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摘要: 研究了三种群非周期食饵_捕食系统的持久性, 得到了系统持久的判据·

关 键 词: 持久性; 食饵_捕食系统; 非周期
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引 言

众所周知, 对三种群食饵_捕食系统的研究不多· 文[1] 中, 作者得到了下列三种群周期食饵_捕食系统

$$\left\{\begin{aligned} \dot{x}_1 &= x_1[b_1(t) - a_{11}(t)x_1 - a_{12}(t)x_2], \\ \dot{x}_2 &= x_2[-b_2(t) + a_{21}(t)x_1 - a_{22}(t)x_2 - a_{23}(t)x_3], \\ \dot{x}_3 &= x_3[-b_3(t) + a_{32}(t)x_2 - a_{33}(t)x_3] \end{aligned}\right\} \tag{1}$$

存在全局渐近稳定的严格正周期解的充分条件·

本文中, 我们研究非自治非周期系统(1)· 系统(1) 中, 连续函数 $b_i(t)$、$a_{ij}(t)$ $(i, j = 1, 2, 3)$ 在半区间$[0, +\infty)$ 上有正的上下界· 系统(1) 表明, 第三个种群是第二个种群的捕食者, 而第二个种群是第一个种群的捕食者·

对实有界函数$f(t)$, 定义

$$f^L = \inf_{t\geq 0} f(t), \quad f^M = \sup_{t\geq 0} f(t)·$$

又记

$$x_1^M = \frac{b_1^M}{a_{11}^L}, \quad x_2^M = \frac{a_{21}^M x_1^M - b_2^L}{a_{22}^L}, \quad x_3^M = \frac{a_{32}^M x_2^M - b_3^L}{a_{33}^L},$$

$$x_1^L = \frac{b_1^L - a_{12}^M x_2^M}{a_{11}^M}, \quad x_2^L = \frac{a_{21}^L x_1^L - a_{23}^M x_3^M - b_2^M}{a_{22}^M}, \quad x_3^L = \frac{a_{32}^L x_2^L - b_3^M}{a_{33}^M}·$$

1 持 久 性

引理 1 R^3 中正的和非负的锥是不变的, 即如果 $\Gamma(t) = (x_1(t), x_2(t), x_3(t))$ 是系统(1) 满足 $\Gamma(0) > 0$ 的任何解, 则 $\Gamma(t) > 0$; 如果 $\Gamma(t)$ 是系统(1) 满足 $\Gamma(0) \geqslant 0$ 的任何解, 则

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$\Gamma(t) \geqslant 0,\ t \in [0, +\infty)$•

证明　因为

$$x_1(t) = x_1(0)\exp\left\{\int_0^t [b_1(s) - a_{11}(s)x_1(s) - a_{12}(s)x_2(s)]\,\mathrm{d}s\right\},$$

$$x_2(t) = x_2(0)\exp\left\{\int_0^t [-b_2(s) + a_{21}(s)x_1(s) - a_{22}(s)x_2(s) - a_{23}(s)x_3(s)]\,\mathrm{d}s\right\},$$

$$x_3(t) = x_3(0)\exp\left\{\int_0^t (-b_3(s) + a_{32}(s)x_2(s) - a_{33}(s)x_3(s)]\,\mathrm{d}s\right\},$$

故立刻可得引理 1 成立•

我们在下列条件下分析系统(1):

$$a_{32}^L a_{21}^L b_1^L - a_{32}^M a_{11}^M b_2^M - (a_{21}^M a_{12}^M + a_{11}^M a_{22}^M) b_3^M > 0, \tag{2}$$

$$\begin{aligned} &a_{11}^L a_{21}^L a_{22}^L a_{33}^L a_{32}^L b_1^L - (a_{33}^L a_{12}^M a_{21}^L + a_{11}^M a_{23}^M a_{32}^M) a_{21}^M a_{32}^L b_1^M + (a_{32}^M a_{11}^M a_{23}^M + \\ &\quad a_{21}^L a_{33}^L a_{12}^M) a_{11}^L a_{32}^L b_2^L + a_{11}^L a_{22}^L a_{11}^M (a_{32}^L a_{23}^M b_3^L - a_{33}^L a_{22}^M b_3^M) - \\ &\quad a_{11}^L a_{22}^L a_{33}^L a_{32}^L a_{11}^M b_2^M > 0• \end{aligned} \tag{3}$$

我们容易知道条件(2)蕴含 $x_2^M > 0$ 和 $x_3^M > 0$, 在条件(3) 下 $x_3^L > 0$, 因此 $x_2^L > 0$, $x_1^L > 0$•

进一步, 我们易证

$$0 < x_1^L < x_1^M,\ 0 < x_2^L < x_2^M,\ 0 < x_3^L < x_3^M•$$

引理 2　设系统(1)的系数满足条件(2)和(3), 并且 $a_{21}^M < a_{22}^L$, $a_{32}^M < a_{33}^L$, 则对任何满足

$$0 < \alpha \leqslant x_1^L,\ 0 < \frac{a_{12}^M}{a_{11}^M}\beta < \alpha < \min(x_2^L - \gamma,\ x_3^L - \gamma)$$

的正数 α、β, 域

$$K = \left\{x = (x_1, x_2, x_3): x_1^L - \alpha \leqslant x_1 \leqslant x_1^M + \beta,\ x_i^L - \alpha - \gamma \leqslant x_i \leqslant x_i^M + \beta,\ i = 2, 3\right\}$$

为系统(1)的正不变域•　这里 $\gamma = a_{23}^M\beta / a_{22}^M$•

证明　由引理 1 知, 系统(1)的初始条件非负的解仍保持非负•　因此, 我们有

$$\dot{x}_1 \leqslant x_1(b_1^M - a_{11}^L x_1),$$

故

$$x_1(t) \geqslant \frac{b_1^M}{a_{11}^L} = x_1^M \text{ 时 } \dot{x}_1(t) \leqslant 0$$

从而

$$0 < x_1(0) \leqslant x_1^M + \beta \Rightarrow x_1(t) \leqslant x_1^M + \beta• \tag{4}$$

由引理 1 和(4), 我们有

$$\dot{x}_2 \leqslant x_2[-b_2^L + a_{21}^M(x_1^M + \beta) - a_{22}^L x_2],$$

因此

$$0 < x_2(0) \leqslant \frac{a_{21}^M x_1^M - b_2^L + a_{21}^M\beta}{a_{22}^L} < x_2^M + \beta \Rightarrow x_2(t) \leqslant x_2^M + \beta• \tag{5}$$

由引理 1 和(5), 我们有

$$\dot{x}_3 \leqslant x_3[-b_3^L + a_{32}^M(x_2^M + \beta) - a_{33}^L x_3],$$

因此,

$$0 < x_3(0) \leqslant \frac{a_{32}^M x_2^M - b_3^L}{a_{33}^L} + \frac{a_{32}^M}{a_{33}^L}\beta < x_3^M + \beta \Rightarrow x_3(t) \leqslant x_3^M + \beta• \tag{6}$$

由(5)有

$$\dot{x}_1 \geqslant x_1[b_1^L - a_{11}^M x_1 - a_{12}^M(x_2^M + \beta)] = x_1[b_1^L - a_{11}^M x_1 - a_{12}^M x_2^M - a_{12}^M\beta] \geqslant x_1(b_1^L - a_{11}^M x_1 - a_{12}^M x_2^M - a_{11}^M\alpha),$$

因此

$$x_1(0) \geqslant \frac{b_1^L - a_{12}^M x_2^M}{a_{11}^M} - \alpha = x_1^L - \alpha \Rightarrow x_1(t) \geqslant x_1^L - \alpha \cdot \tag{7}$$

由(6)与(7)有

$$\dot{x}_2 \geqslant x_2[-b_2^M + a_{21}^L(x_1^L - \alpha) - a_{22}^M x_2 - a_{23}^M(x_3^M + \beta)],$$

因此

$$x_2(0) \geqslant \frac{a_{21}^L x_1^L - a_{23}^M x_3^M - b_2^M}{a_{22}^M} - \frac{a_{21}^L\alpha + a_{23}^M\beta}{a_{22}^M} = [a_{11}^L a_{21}^L a_{22}^L a_{33}^L b_1^L - (a_{33}^L a_{12}^M a_{21}^L + a_{11}^M a_{23}^M a_{32}^M) a_{21}^M b_1^M + (a_{32}^M a_{11}^M a_{23}^M + a_{21}^L a_{33}^L a_{12}^M) a_{11}^L b_2^L + a_{11}^L a_{22}^L a_{23}^M a_{11}^M b_3^L - a_{11}^L a_{22}^L a_{33}^L a_{11}^M b_2^M] / a_{11}^L a_{22}^L a_{23}^L a_{11}^M a_{22}^M - \frac{a_{21}^L\alpha + a_{23}^M\beta}{a_{22}^M} \geqslant x_2^L - \alpha - \frac{a_{23}^M}{a_{22}^M}\beta = x_2^L - \alpha - \gamma$$

蕴含 $x_2(t) \geqslant x_2^L - \alpha - \gamma$, 即

$$x_2(0) \geqslant x_2^{\alpha} - \alpha - \gamma \Rightarrow x_2(t) \geqslant x_2^L - \alpha - \gamma \cdot \tag{8}$$

由(8)有

$$\dot{x}_3 \geqslant x_3[-b_3^M + a_{32}^L(x_2^L - \alpha - \gamma) - a_{33}^M x_3],$$

故当

$$x_3(0) \geqslant \frac{a_{32}^L x_2^L - b_3^M}{a_{33}^M} - \frac{a_{32}^L}{a_{33}^M}\alpha - \frac{a_{32}^L}{a_{33}^M}\gamma = [a_{11}^L a_{21}^L a_{22}^L a_{33}^L a_{32}^L b_1^L - (a_{33}^L a_{12}^M a_{21}^L + a_{11}^M a_{23}^M a_{32}^M) a_{21}^M a_{32}^L b_1^M + (a_{32}^M a_{11}^M a_{23}^M + a_{21}^L a_{33}^L a_{12}^M) a_{11}^L a_{32}^L b_2^L + a_{11}^L a_{22}^L a_{11}^M (a_{32}^L a_{23}^M b_3^L - a_{33}^L a_{22}^M b_3^M) - a_{11}^L a_{22}^L a_{33}^L a_{32}^L a_{11}^M b_2^M] / a_{11}^L a_{22}^L a_{33}^L a_{11}^M a_{22}^M a_{33}^M - \frac{a_{32}^L(\alpha + \gamma)}{a_{33}^M} > x_3^L - \alpha - \gamma$$

时 $x_3(t) \geqslant x_3^L - \alpha - \gamma$, 即

$$x_3(0) \geqslant x_3^L - \alpha - \gamma \Rightarrow x_3(t) \geqslant x_3^L - \alpha - \gamma \cdot \tag{9}$$

由(4)~(9)知, 引理2成立, 证毕•

引理3 如果引理2的条件成立, 则存在 T_0, 使得 $t \geqslant T_0$ 时, $x_1(t) \leqslant x_1^M + \beta$•

证明 由(4)知, 如果存在 $t_1 \in [0, +\infty)$, 使得 $x(t_1) \leqslant x_1^M + \beta$• 则引理3成立•

反证法 设对 $t \in I = [0, +\infty)$, $x_1(t) \geqslant x_1^M + \beta$, 因为

$$x_1 > x_1^M + \beta = \frac{b_1^M}{a_{11}^L} + \beta,$$

故

$$\dot{x}_1(t) = x_1[b_1(t) - a_{11}(t)x_1 - a_{12}(t)x_2] \leqslant x_1(b_1^M - a_{11}^L x_1) \leqslant$$

$$-a_{11}^{L}\beta x_1 \leqslant -a_{11}^{L}\beta(x_1^{M}+\beta) \triangleq -\varepsilon_1,$$

与假设矛盾. 因此, 存在 $t_1 \in [0, +\infty)$, 使得 $x_1(t_1) \leqslant x_1^{M}+\beta$. 引理 3 成立.

引理 4　如果引理 2 的条件成立, 则存在 $T_1 \geqslant T_0$ 和 $T_2 \geqslant T_0$, 使得 $t \geqslant T_1$ 时 $x_2 \leqslant x_2^{M}+\beta$, $t \geqslant T_2$ 时 $x_3 \leqslant x_3^{M}+\beta$.

证明　由引理 2, 如果存在 $t_1 \in [T_0, +\infty)$, 使得 $x_2(t_1) \leqslant x_2^{M}+\beta$, 则 $t \geqslant t_1$ 时 $x_2 \leqslant x_2^{M}+\beta$.

反证法　设 $t \in I = [T_0, +\infty)$ 时 $x_2 > x_2^{M}+\beta$. 由引理 3 知, 当 $t \geqslant T_0$ 时

$$x_1 \leqslant x_1^{M}+\beta = x_1^{M} + \frac{a_{22}^{L}-a_{22}^{L}+a_{21}^{M}}{a_{21}^{M}}\beta =$$
$$\frac{a_{21}^{M}x_1^{M}+a_{22}^{L}\beta+b_2^{L}-b_2^{L}}{a_{21}^{M}} - \frac{a_{22}^{L}-a_{21}^{M}}{a_{21}^{M}}\beta =$$
$$\frac{b_2^{L}}{a_{21}^{M}} + \frac{a_{22}^{L}}{a_{21}^{M}}(x_2^{M}+\beta) - \frac{a_{22}^{L}-a_{21}^{M}}{a_{21}^{M}}\beta,$$

因此

$$-b_2^{L}+a_{21}^{M}x_1 - a_{22}^{L}(x_2^{M}+\beta) \leqslant -(a_{22}^{L}-a_{21}^{M})\beta,$$
$$\dot{x}_2 = x_2[-b_2(t)+a_{21}(t)x_1 - a_{22}(t)x_2 - a_{23}(t)x_3] \leqslant$$
$$x_2(-b_2^{L}+a_{21}^{M}x_1 - a_{22}^{L}x_2) \leqslant x_2[-b_2^{L}+a_{21}^{M}x_1 - a_{22}^{L}(x_2^{M}+\beta)] \leqslant$$
$$-(a_{22}^{L}-a_{21}^{M})\beta x_2 \leqslant -(a_{22}^{L}-a_{21}^{M})\beta(x_2^{M}+\beta) \triangleq -\varepsilon_2,$$

与假设矛盾. 故存在 $t_1 \in I = [T_0, +\infty)$, 使得 $x_2(t_1) \leqslant x_2^{M}+\beta$. 类似可证, 存在 $T_2 \geqslant T_0$, 使得 $t \geqslant T_2$ 时, $x_3(t) \leqslant x_3^{M}+\beta$. 引理 4 成立.

引理 5　如果引理 2 的条件成立, 则存在 $T_1 \geqslant \max(T_1, T_2)$, 使得

$$t \geqslant T_1 \text{ 时 } x_1(t) \geqslant x_1^{L}-\alpha.$$

证明　由引理 2, 如果存在 $t_1 \in [\max(T_1, T_2), +\infty)$, 使得 $x_1(t_1) \geqslant x_1^{L}-\alpha$, 则引理 5 成立.

反证法　设 $t \in I = [\max(T_1, T_2), +\infty)$ 时 $x_1(t) < x_1^{L}-\alpha$, 又由引理 4 可知, $t \geqslant \max(T_1, T_2)$ 时 $x_2(t) \leqslant x_2^{M}+\beta$, 因此

$$\dot{x}_1 = x_1[b_1(t)-a_{11}(t)x_1 - a_{12}(t)x_2] \geqslant$$
$$x_1(b_1^{L}-a_{11}^{M}x_1 - a_{12}^{M}x_2) \geqslant$$
$$x_1[b_1^{L}-a_{11}^{M}(x_1^{L}-\alpha) - a_{12}^{M}(x_2^{M}+\beta)] =$$
$$x_1(a_{11}^{M}\alpha - a_{12}^{M}\beta) \triangleq \varepsilon_1 x_1,$$

与假设矛盾. 故存在 $t_1 \in I = [\max(T_1, T_2), +\infty)$, 使得 $x_1(t_1) \geqslant x_1^{L}-\alpha$. 引理 5 证毕.

引理 6　如果引理 2 的条件成立, 则存在 $T_3 \geqslant T_2 \geqslant T_1$, 使得 $t \geqslant T_2$ 时 $x_2 \geqslant x_2^{L}-\alpha-\gamma$, $t \geqslant T_3$ 时 $x_3 \geqslant x_3^{L}-\alpha-\gamma$.

证明　由引理 2, 如果存在 $t_1 \in [T_1, +\infty)$, 使得 $x_2(t_1) \geqslant x_2^{L}-\alpha-\gamma$, 则 $t \geqslant t_1 \geqslant T_1$ 时 $x_2(t) \geqslant x_2^{L}-\alpha-\gamma$.

反证法　设 $t \in I = [T_1, +\infty)$ 时 $x_2(t) < x_2^{L}-\alpha-\gamma$, 则由引理 4 与引理 5 知, 对 $t \in I$ 有

$$\dot{x}_2 = x_2(-b_2^{M}+a_{21}^{L}x_1 - a_{22}^{M}x_2 - a_{23}^{M}x_3) \geqslant$$

$$x_2[-b_2^M+a_{21}^M(x_1^L-\alpha)-a_{22}^M(x_1^L-\alpha-\gamma)-a_{23}^M(x_3^M+\beta)]=$$
$$x_2[(a_{22}^M-a_{21}^L)\alpha+a_{22}^M\gamma-a_{23}^M\beta]=$$
$$x_2(a_{22}^M-a_{21}^L)\alpha\triangleq\varepsilon_2x_2,$$

与假设矛盾• 因此存在 $T_2\geqslant T_1$, 使得当 $t\geqslant T_2$ 时 $x_2(t)\geqslant x_2^L-\alpha-\gamma$•

又如果对 $t\in[T_2,+\infty)$ 有 $x_3(t)<x_3^L-\alpha-\gamma$, 则

$$\dot{x}_3\geqslant x_3(-b_3^M+a_{32}^Lx_2-a_{33}^Mx_3)\geqslant$$
$$x_3[-b_3^M+a_{32}^L(x_2^L-\alpha-\gamma)-a_{33}^M(x_3^L-\alpha-\gamma)]=$$
$$x_3(a_{33}^M-a_{32}^L)(\alpha+\gamma)\triangleq\varepsilon_3,x_3,$$

与假设矛盾• 故存在 $t_1\geqslant T_2$, 使得 $x_3(t_1)\geqslant x_3^L-\alpha-\gamma$, 由引理 2, 存在 $T_3\geqslant T_2$, 使得 $t\geqslant T_3$ 时 $x_3(t)\geqslant x_3^L-\alpha-\gamma$•

综上所述, 引理 6 成立•

由引理 2 至引理 6 知

定理 如果系统(1)的系数满足(2)和(3)且 $a_{21}^M<a_{22}^L$, $a_{32}^M<a_{33}^L$, 则系统(1)是持久的•

[参 考 文 献]

[1] CHEN Lan_sun, PENG Qiu_liang. Persistence in a three species Lotka_Volterra periodic predator_prey system[J]. System Science and Mathematical Science, 1994, **7**(4): 337—343.

[2] Ahmad S. Convergence and ultimate bounds of solution of the nonautonpmmous Volterra_Lotka competition equation[J]. J Math Anal Appl, 1987, **127**(3): 377—387.

[3] Zitan Amine. A periodic prey_predator system[J]. J Math Anal Appl, 1994, **185**(4): 477—489.

[4] Tineo A. On the asymptotic behaviour of some population model[J]. J Math Anal Appl, 1992, **16**(5): 516—529.

[5] Tineo A, Alvarez C. A different consideration about the globally asymptotically stable solution of the periodic *N* _competing species problems[J]. J Math Anal Appl, 1991, **159**(1): 44—50.

Persistence in a Three Species Lotka_Volterra Nonperiodic Predator_Prey System

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Abstract: The predator_prey model for three species in which the right_hand sides are nonperiodic functions in time were considered. It's proved that the model is persistent under appropriate conditions.

Key words: persistence; predator_prey system; nonperiodic