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非对称载荷作用的外部圆形裂纹问题

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(我刊编委王银邦来稿)

摘要: 使用边界积分方程方法, 研究了三维无限弹性体中受非对称载荷作用的外部圆形裂纹问题 通过使用 Fourier 级数和超几何函数, 将问题的二维边界奇异积分方程简化为 Abel 型方程, 获得了一般非对称载荷作用的外部圆形裂纹问题的应力强度因子精确解, 比用 Hankel 变换法得到的结果更为一般 结果表明: 边界积分方程法在解析分析方面还有很大的潜力

关 键 词: 外部圆形裂纹; 非对称载荷; 边界积分方程; Abel 方程; 应力强度因子; 精确解

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引 言

边界积分方程方法应用于裂纹分析时, 一般只能求数值解, 例如, Snyder 和 Cruse(1975)[1]、Crouch(1976)[2]、Blandford 等(1981)[3]、Portela 等(1992)[4]、Bui(1977)[5]、Weaver(1977)[6]、Wang 等(1993, 1993, 1997)[7~9]的代表性工作 如何将边界积分方程方法用于裂纹问题的解析分析, 一直是个难题 本文作者和 Chau 等在这方面作了一些探索, 获得了有意义的结果[10~14]

本文在作者三维裂纹问题边界积分方程方法已有工作(1993)[7]的基础上, 研究三维无限弹性体中受非对称载荷作用的外部圆形裂纹问题 通过使用 Fourier 级数和超几何函数, 将问题的二维边界奇异积分方程简化为 Abel 型方程, 获得了一般非对称载荷作用的外部圆型裂纹问题的应力强度因子精确解 在对称载荷作用的特殊情形, 本文的解与文献上用 Hankel 变换法得到的结果(1975)[15]一致

1 边界积分方程

考虑无限弹性体中带有一个外部圆形裂纹 $= \{ (\ , \ , 0) \mid a < \ < \ , 0 \ \ 2 \}$ 的三维问题, 见图 1 假设体积力不存在, 无限远处的应力为零, 裂纹上、下表面作用任意非对称载荷 $_{33}(\ , \ , \ 0)$, $_{3}(\ , \ , \ 0)$ 和 $_{3}(\ , \ , \ 0)$ 利用作者对一般三维裂纹问题导出的边界积分方程(1993)[7], 可得本问题有关裂纹位错 $F(\ , \ )$ 和位错密度 $W_2(\ , \ )$、$W_3(\ , \ )$ 的如下二维边界积分方程:

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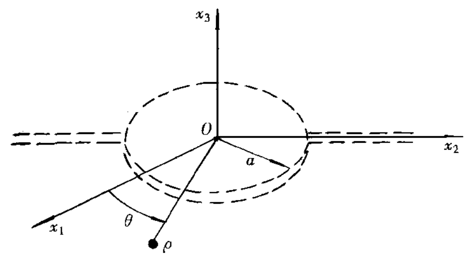

图 1 无限体中的一个外部圆形裂纹

$$_a \mathrm{d} \ {}^2_0 \left\{ \frac{}{}_0 \left(\frac{_0}{r} \right) \left[\frac{F(\ , \ )}{} - t(\ , \ ) \right] + M(\ , \ ) \right\} \mathrm{d} \ = 2 \ P_3(\ _0, \ _0),$$

$$\left(\frac{}{_0} + \frac{\mathrm{i}}{_0} \frac{}{_0} \right) \ _a \mathrm{d} \ {}^2_0 \frac{W(\ , \ )}{r} \mathrm{d} \ = 2 \ [R(\ _0, \ _0) + \mathrm{i}T(\ _0, \ _0)],$$

$$(\mathrm{i} = \sqrt{-1}, a < \ _0 < \ , 0 \ \ _0 \ \ 2 \ ) \qquad (1)$$

其中:

$$M(\ , \ ) = \frac{1}{r} \left[\frac{^2F(\ , \ )}{^2} - \frac{t(\ , \ )}{} \right],$$

$$P_3(\ , \ ) = \frac{1}{2}[\ _{33}(\ , \ , +0) + \ _{33}(\ , \ , -0)],$$

$$t_3(\ , \ ) = \ _{33}(\ , \ , -0) - \ _{33}(\ , \ , +0),$$

$$R(\ , \ ) = \frac{1}{2}[\ _3(\ , \ , +0) + \ _3(\ , \ , -0)],$$

$$t(\ , \ ) = \ _3(\ , \ , -0) - \ _3(\ , \ , +0),$$

$$T(\ , \ ) = \frac{1}{2}[\ _3(\ , \ , +0) + \ _3(\ , \ , -0)],$$

$$t(\ , \ ) = \ _3(\ , \ , -0) - \ _3(\ , \ , +0); \qquad (2)$$

$$W(\ , \ ) = W_2(\ , \ ) + \mathrm{i}W_3(\ , \ ) + \ \ t_3(\ , \ ), \qquad (3)$$

$$r = [\ ^2 + \ ^2_0 - 2 \ \ _0\cos(\ - \ _0)]^{1/2}, \qquad (4)$$

$$= (1-2 \ )/(4(1- \ )), \qquad (5)$$

$$W_2(\ , \ ) = \frac{}{2(1- \ )} \left\{ \frac{1}{} \frac{}{}[\ u(\ , \ )] + \frac{1}{} \frac{}{} \ u(\ , \ ) \right\},$$

$$W_3(\ , \ ) = \frac{}{2} \left\{ \frac{1}{} \frac{}{}[\ u(\ , \ )] - \frac{1}{} \frac{}{} \ u(\ , \ ) \right\},$$

$$F(\ , \ ) = \frac{}{2(1- \ )}[u_3(\ , \ , +0) - u_3(\ , \ , -0)],$$

$$u(\ , \ ) = u(\ , \ , +0) - u(\ , \ , -0),$$

$$u(\ , \ ) = u(\ , \ , +0) - u(\ , \ , -0), \qquad (6)$$

u_i 和 $_{ij}$ 分别是位移分量和应力分量; 为 Poisson 比, 是剪切模量

裂纹位错和位错密度还必须满足如下的位移单值性条件(1993)[7]:

$$F(a, \ _0) = 0,$$

$$_a \mathrm{d} \ {}^2_0 [(2- \ )W(\ , \ ) - \overline{W(\ , \ )} + G(\ , \ )] \mathrm{d} \ = 0, \qquad (7)$$

式中

$$G(\ , \ ) = -\frac{1}{2}(1-2 \ ) \ \ t_3(\ , \ )] \left(\frac{}{a} + \frac{\mathrm{i}}{a} \frac{}{_0} \right)$$

$$\ln[a^2+\ {}^2-2a\ \cos(\ -\ _0)]\qquad(0\ \ _0\ \ 2\ )$$

根据文献[16](1979),本问题解的唯一性要求边界积分方程(1)和位移单值性条件(7)中的未知函数函数具有如下性态:

$$\left.\begin{array}{l}F(\ ,\ )=O\left(\frac{1}{\ }\right),\ W_2(\ ,\ )=o\left(\frac{1}{\ }\right)\\ W_3(\ ,\ )=o\left(\frac{1}{\ }\right)\end{array}\qquad(\qquad),\right\}\tag{8}$$

2 Abel 积分方程及其解

二维奇异边界积分方程(1)在位移单值性条件(7)下的求解,文献上没有现成的方法可用,下面建议一种可能的解法

引进未知函数 $F(\ ,\ )$, $W(\ ,\ )$ 和已知载荷的 Fourier 级数:

$$\left\{F(\ ,\ ),W(\ ,\ )\right\}=\sum_{n=-}\left\{f_n(\ ),w_n(\ )\right\}e^{in},\tag{9}$$

$$\left\{P_3(\ ,\ ),R(\ ,\ )+iT(\ ,\ ),\ \ t_3(\ ,\ ),\ \ t(\ ,\ ),\ \ \frac{}{}\ \ t(\ ,\ )\right\}=$$

$$\sum_{n=-}\left\{p_n(\ ),q_n(\ ),g_n(\ ),b_n(\ ),c_n(\ )\right\}e^{in},\tag{10}$$

其中:
$$\left\{f_n(\ ),w_n(\ )\right\}=\frac{1}{2}\int_0^2\left\{F(\ ,\ ),W(\ ,\ )\right\}e^{-in}d\ ,\tag{11}$$

$$\left\{p_n(\ ),q_n(\ ),g_n(\ ),b_n(\ ),c_n(\ )\right\}=\frac{1}{2}\int_0^2\left\{P_3(\ ,\ ),R(\ ,\ )+\right.$$
$$\left.iT(\ ,\ ),\ \ t_3(\ ,\ ),\ \ t(\ ,\ ),\ \ \frac{}{}\ \ t(\ ,\ )\right\}e^{-in}d\tag{12}$$

使用积分公式(1980)[17]:

$$\left.\begin{array}{l}\int_0^2(\ ^2+\ _0^2-2\ \ _0\cos\ )^{-1/2}e^{in}d\ =4(\ _0)^{|n|}\int\frac{u^{-2|u|}du}{\sqrt{u^2-\ ^2}\sqrt{u^2-\ _0^2}},\\ \int_0^2\ln(\ ^2+\ _0^2-2\ \ _0\cos\ )d\ =4\ \ln\ ,\\ \int_0^2\ln(\ ^2+\ _0^2-2\ \ _0\cos\ )e^{inU}dU=-\frac{2P}{|\ n\ |}\left(\frac{Q_0}{Q}\right)^{|n|},\quad(n=?1,?2,\ ,\ ;Q_0<\ Q)\end{array}\right\}$$
$$\tag{13}$$

边界积分方程(1)可简化为如下的Abel 积分方程:

$$\left.\begin{array}{l}\frac{5}{5Q_0}\int_{Q_0Q}^{]}\frac{u^{-2|n|}du}{\sqrt{u^2-Q_0^2}}\int_a^u\frac{[Q^{|n|}f_n(Q)]c-Q^{|n|}b_n(Q)}{\sqrt{u^2-Q^2}}dQ=\frac{P}{2}Q_0^{-|n|}p_n(Q_0)+\\ \int_{Q_0}^{]}\frac{u^{-2|n|}du}{\sqrt{u^2-Q_0^2}}\int_a^u\frac{Q^{|n|}[c_n(Q)+|\ n\ |\ b_n(Q)]}{\sqrt{u^2-Q^2}}dQ\qquad(n=0,?1,?2,\ ,\ ),\\ \frac{5}{5Q_0}\int_{Q_0}^{]}\frac{u^{-2|n|}du}{\sqrt{u^2-Q_0^2}}\int_a^u\frac{Q^{1+n}w_n(Q)}{\sqrt{u^2-Q^2}}dQ=\frac{P}{2}Q_0^{-n}q_n(Q_0)\qquad(n=0,1,2,\ ,\ ),\\ \int_{Q_0}^{]}\frac{u^{1-2n}du}{\sqrt{u^2-Q_0^2}}\frac{5}{5u}\int_a^u\frac{Q^{1+n}w_{-n}(Q)}{\sqrt{u^2-Q^2}}dQ=\frac{P}{2}Q_0^{1-n}q_{-n}(Q_0)\qquad(n=1,2,\ ,\ ),\\ \qquad\qquad(a\ [\ \ Q_0<\ \ ]\ )\#\end{array}\right\}\tag{14}$$

位移单值性条件(7)可被简化为:

$$\left.\begin{aligned}&f_n(a)=0\quad(n=0,\pm1,\pm2,\cdots),\\&\int_a^{\infty}\rho^{1-n}\left[(2-\nu)w_{-n}(\rho)-\nu\overline{w_n(\rho)}-\frac{1}{2}(1-2\nu)g_{-n}(\rho)\right]\mathrm{d}\rho=0\quad(n=1,2,\cdots)。\end{aligned}\right\}\tag{15}$$

利用Abel算子的性质,可求得积分方程(14)满足条件(15)和无限远处性态(8)的解为:

$$\left.\begin{aligned}&f_0(\rho_0)=\int_a^{\rho_0}b_0(\rho)\mathrm{d}\rho-\left(1-\frac{2}{\pi}\arcsin\frac{a}{\rho_0}\right)\int_a^{\infty}b_0(\rho)\mathrm{d}\rho-\\&\qquad\frac{2}{\pi}\int_a^{\rho_0}\frac{\mathrm{d}t}{\sqrt{\rho_0^2-t^2}}\int_t^{\infty}\frac{\rho p_0(\rho)\mathrm{d}\rho}{\sqrt{\rho^2-t^2}},\\&f_n(\rho_0)=\rho_0^{-|n|}\int_a^{\rho_0}\rho^{|n|}b_n(\rho)\mathrm{d}\rho-\frac{2}{\pi}\rho_0^{-|n|}\int_a^{\rho_0}\frac{t^{2|n|}\mathrm{d}t}{\sqrt{\rho_0^2-t^2}}\int_t^{\infty}\frac{\rho^{1-|n|}p_n(\rho)}{\sqrt{\rho^2-t^2}}\mathrm{d}\rho-\\&\qquad\frac{2}{\pi}\rho_0^{-|n|}\int_a^{\rho_0}\frac{t^{2|n|}\mathrm{d}t}{\sqrt{\rho_0^2-t^2}}\int_t^{\infty}u^{-2|n|}\mathrm{d}u\int_a^{u}\frac{\rho^{|n|}[c_n(\rho)+|n|b_n(\rho)]}{\sqrt{u^2-\rho^2}}\mathrm{d}\rho\quad(n\neq0),\\&w_n(\rho_0)=-\frac{2}{\pi}\rho_0^{1-n}\frac{\partial}{\partial\rho_0}\int_a^{\rho_0}\frac{t^{2n+2}\mathrm{d}t}{\sqrt{\rho_0^2-t^2}}\int_t^{\infty}\frac{\rho^{-n}q_n(\rho)}{\sqrt{\rho^2-t^2}}\mathrm{d}\rho\quad(n=0,1,2,\cdots),\\&w_{-n}(\rho_0)=\frac{2}{\pi}\frac{B_{-n}\rho_0^{-n}}{\sqrt{\rho_0^2-a^2}}-\frac{2}{\pi}\rho_0^{n}\int_a^{\rho_0}\frac{t^{2n}\mathrm{d}t}{\sqrt{\rho_0^2-t^2}}\frac{\partial}{\partial t}\int_t^{\infty}\frac{\rho^{-n}q_{-n}(\rho)}{\sqrt{\rho^2-t^2}}\mathrm{d}\rho\quad(n=1,2,\cdots),\\&\qquad\qquad(a<\rho_0<\infty)。\end{aligned}\right\}\tag{16}$$

其中:

$$\begin{aligned}B_{-n}=&\frac{1-2\nu}{2(2-\nu)}(2n-1)a^{2n-1}\int_a^{\infty}\rho^{n+1}g_{-n}(\rho)\mathrm{d}\rho\int_\rho^{\infty}\frac{t^{-2n}\mathrm{d}t}{\sqrt{t^2-\rho^2}}-\\&\frac{\nu}{2-\nu}(2n-1)a^{2n-1}\int_a^{\infty}\sqrt{\rho^2-a^2}\,\rho^{-n}\overline{q_n(\rho)}\mathrm{d}\rho-\\&a^{2n-1}\int_a^{\infty}\frac{\rho^{2-n}q_{-n}(\rho)}{\sqrt{\rho^2-a^2}}\mathrm{d}\rho\quad(n=1,2,\cdots)。\end{aligned}\tag{17}$$

将上述解代入Fourier级数(9),即可得二维奇异边界积分方程(1)的级数解。再将(12)式回代,并进行级数的求和运算,又可将级数解表示成有关裂纹上、下表面已知载荷的二重积分形式。但推导过程烦冗,这里不再赘述。

3 应力强度因子

根据作者以前的工作[7],外部圆形裂纹的Ⅰ、Ⅱ、Ⅲ型应力强度因子可直接由下述极限公式计算:

$$\left.\begin{aligned}&K_{\mathrm{I}}(\theta)\equiv\lim_{\rho\to a}\sqrt{2\pi(a-\rho)}\,\sigma_{33}(\rho,\theta,0)=\lim_{\rho\to a}\sqrt{\frac{\pi}{2(\rho-a)}}F(\rho,\theta),\\&K_{\mathrm{II}}(\theta)+\mathrm{i}K_{\mathrm{III}}(\theta)\equiv-\lim_{\rho\to a}\sqrt{2\pi(a-\rho)}[\sigma_{3\rho}(\rho,\theta,0)+\mathrm{i}\sigma_{3\theta}(\rho,\theta,0)]=\\&\qquad-\lim_{\rho\to a}\sqrt{2\pi(\rho-a)}\,W(\rho,\theta)\qquad(0\le\theta\le2\pi)。\end{aligned}\right\}\tag{18}$$

将边界积分方程的级数解(9)、(16) 和(17) 代入以上极限公式, 可求得外部圆形裂纹问题的应力强度因子为:

$$K(\theta)=\frac{-1}{\pi\sqrt{\pi a}}\int_0^{2\pi}\mathrm{d}\varphi\int_a^{\infty}\frac{\sqrt{\rho^2-a^2}\,P_3(\rho,\varphi)\rho\mathrm{d}\rho}{a^2+\rho^2-2a\rho\cos(\varphi-\theta)}-\frac{X}{\pi\sqrt{\pi a}}\int_0^{2\pi}\mathrm{d}\varphi\int_a^{\infty}\Delta t_\rho(\rho,\varphi)\mathrm{d}\rho+$$
$$\frac{X}{\pi\sqrt{\pi a}}\int_0^{2\pi}\mathrm{d}\varphi\int_a^{\infty}\left[\frac{\partial\Delta t_\rho(\rho,\varphi)}{\partial\varphi}H_1\left(\frac{a}{\rho},\varphi-\theta\right)-\frac{\partial\Delta t_\theta(\rho,\varphi)}{\partial\varphi}H_2\left(\frac{a}{\rho},\varphi-\theta\right)\right]\mathrm{d}\rho \tag{19a}$$

$$K(\theta)=\frac{1}{\pi\sqrt{\pi a}}\int_0^{2\pi}\mathrm{d}\varphi\int_a^{\infty}\left\{R(\rho,\varphi)\cos(\varphi-\theta)+N_1(\rho,\theta)\right\}\frac{\sqrt{\rho^2-a^2}\,\rho\mathrm{d}\rho}{a^2+\rho^2-2a\rho\cos(\varphi-\theta)}+$$
$$\frac{1-2\nu}{4(2-\nu)}(\pi a)^{-3/2}\int_0^{2\pi}\mathrm{d}\varphi\int_a^{\infty}\left[\Delta t_3(\rho,\varphi)H_2\left(\frac{a}{\rho},\varphi-\theta\right)+2\frac{\partial\Delta t_3(\rho,\varphi)}{\partial\varphi}H_1\left(\frac{a}{\rho},\varphi-\theta\right)\right]\rho\mathrm{d}\rho \tag{19b}$$

$$K(\theta)=\frac{1}{\pi\sqrt{\pi a}}\int_0^{2\pi}\mathrm{d}\varphi\int_a^{\infty}\left\{T(\rho,\varphi)\cos(\varphi-\theta)+N_2(\rho,\theta)\right\}\frac{\sqrt{\rho^2-a^2}\,\rho\mathrm{d}\rho}{a^2+\rho^2-2a\rho\cos(\varphi-\theta)}+$$
$$\frac{1-2\nu}{4(2-\nu)}(\pi a)^{-3/2}\int_0^{2\pi}\mathrm{d}\varphi\int_a^{\infty}\left[\Delta t_3(\rho,\varphi)\cdot H_1\left(\frac{a}{\rho},\varphi-\theta\right)-2\frac{\partial\Delta t_3(\rho,\varphi)}{\partial\varphi}\cdot H_2\left(\frac{a}{\rho},\varphi-\theta\right)\right]\rho\mathrm{d}\rho\quad(0\le\theta\le 2\pi), \tag{19c}$$

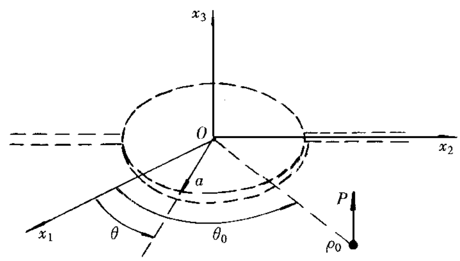

图 2 上表面受集中压力 P 作用的外部圆形裂纹

式中
$$N_1(\rho,\theta)=-T(\rho,\varphi)\sin(\varphi-\theta)+\frac{\nu}{2-\nu}\left[R(\rho,\varphi)-2\frac{\partial T(\rho,\varphi)}{\partial\varphi}\right]\cdot\left[\frac{a}{\rho}-\cos(\varphi-\theta)\right]-\frac{\nu}{2-\nu}\left[T(\rho,\varphi)+2\frac{\partial R(\rho,\varphi)}{\partial\varphi}\right]\sin(\varphi-\theta),$$
$$N_2(\rho,\theta)=R(\rho,\varphi)\sin(\varphi-\theta)-\frac{\nu}{2-\nu}\left[T(\rho,\varphi)+2\frac{\partial R(\rho,\varphi)}{\partial\varphi}\right]\left[\frac{a}{\rho}-\cos(\varphi-\theta)\right]-\frac{\nu}{2-\nu}\left[R(\rho,\varphi)-2\frac{\partial T(\rho,\varphi)}{\partial\varphi}\right]\sin(\varphi-\theta). \tag{19d}$$

$$\left.\begin{aligned}H_1(u,\theta)&=u\sin\theta\int_0^1\frac{(1-t)^{-1/2}\mathrm{d}t}{1-2ut\cos\theta+u^2t^2},\\ H_2(u,\theta)&=\int_0^1\frac{u(\cos\theta-ut)(1-t)^{-1/2}\mathrm{d}t}{1-2ut\cos\theta+u^2t^2}.\end{aligned}\right\} \tag{20}$$

在裂纹上下表面作用对称载荷的特殊情形, 即 $\Delta t_3(\rho,\theta)=\Delta t_\rho(\rho,\theta)=\Delta t_\theta(\rho,\theta)=0$, 本文的解(19) 与文献上用 Hankel 变换法得到的结果(1975)[15] 一致.

4 裂纹上表面受集中压力作用的特例

当外部圆形裂纹的上表面任意点 $(\rho_0,\theta_0,+0)$ 处作用集中压力 P 时(见图 2), 由(19) 式可求得问题的应力强度因子为:

$$\left.\begin{aligned}
K\ \ (H) &= \frac{P}{2P\sqrt{Pa}}\,\frac{\sqrt{Q_0^2-a^2}}{a^2+Q_0^2-2aQ_0\cos(H_0-H)},\\
K\ \ (H) &= \frac{P(1-2M)}{4(2-M)(Pa)^{3/2}}\left[H_2\left(\frac{a}{Q_0},H_0-H\right)-2\frac{5}{5H_0}H_1\left(\frac{a}{Q_0},H_0-H\right)\right],\\
K\ \ (H) &= \frac{P(1-2M)}{4(2-M)(Pa)^{3/2}}\left[H_1\left(\frac{a}{Q_0},H_0-H\right)+2\frac{5}{5H_0}H_2\left(\frac{a}{Q_0},H_0-H\right)\right],\\
&\qquad (0\ [\ \ H\ [\ \ 2P)\#
\end{aligned}\right\}\qquad (21)$$

将上述结果作为外部圆形裂纹问题的 Green 函数,可得到许多有意义的结果#

5 结 论

本文用边界积分方程方法,研究了三维无限弹性体中受非对称载荷作用的外部圆形裂纹问题# 通过将二维奇异边界积分方程简化为 Abel 方程,获得了问题的应力强度因子精确解,比文献上用 Hankel 变换法得到的结果更为一般# 本文的结果表明:边界积分方程方法在解析分析方面还有很大的潜力#

[参 考 文 献]

[1] Snyder M D, Cruse T A. Boundary integral equation analysis of cracked anisotropic plates[J]. Int J Fract, 1975, 11(2): 315) 328.

[2] Crouch S L. Solution of plane elasticity problems by the displacement discontinuity method[J]. Int J Numer Meth Eng, 1976, 10(2): 301) 343.

[3] Blandford G E, Ingraffea A R, Liggett J A. Two_dimensional stress intensity factor computations using the boundary element method[J]. Int J Numer Meth Eng, 1981, 17(2): 387) 404.

[4] Portela A, Aliabadi M H, Rooke D P. The dual boundary element method: effective implementation for crack problems[J]. Int J Numer Meth Eng, 1992, 33(6): 1269) 1287.

[5] Bui H D. An integral equations method for solving the problem of a plane crack of arbitrary shape[J]. J Mech Phys Solids, 1977, 25(1): 29) 39.

[6] Weaver J. Three_dimensional crack analysis[J]. Int J Solids Structures, 1977, 13(4): 321) 330.

[7] WANG Yin_bang. A new boundary integral equation method of three_dimensional crack analysis[J]. Int J Fract, 1993, 63(4): 317) 328.

[8] WANG Yin_bang, CHEN Wei_jiang. Interaction of two equal coplanar square cracks in three_dimensional elasticity[J]. Int J Solids Structures, 1993, 30(23): 3315) 3320.

[9] WANG Yin_bang, Chau K T. A new boundary element method for plane elastic problems involving cracks and holes[J]. Int J Fract, 1997, 87(1): 1) 20.

[10] 王银邦. 外部圆形裂纹轴对称问题的边界积分方程解法[J]. 兰州大学学报, 1993, 29(1): 19) 24.

[11] 王银邦. 双弹性材料界面裂纹平面问题的边界积分程解法[J]. 兰州大学学报, 1995, 31(1): 14) 21.

[12] 王银邦, 王海峰. 圆形裂纹分析的边界积分方程方法[J]. 兰州大学学报, 1997, 33(1): 33) 38.

[13] Chau K T, WANG Yin_bang. A new boundary integral formulation for plane elastic bodies containing cracks and holes[J]. Int J Solids Structures, 1999, 36(14): 2041) 2074.

[14] Chau K T, WANG Yin_bang. Singularity analysis and boundary integral equation method for frictional crack problems in two_dimensional elasticity[J]. Int J Fracture, 1998, 90(3): 251) 274.

[15] Kassir M K, Sih G C. Three Dimensional Crack Problems[M]. Leyden: Noordhoff International

1975.

[16] Kupradze V D. Three_Dimensional Problems of the Mathematical Theory of Elasticity and Thermoelasticity[M]. Amsterdam: North_Holland Publishing Company, 1979.

[17] Gradshteyn I S, Ryzhik I M. Table of Integrals Series and Products[M]. New York: Academic Press, 1980.

The Problem of an External Circular Crack Under Asymmetric Loadings

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Abstract: Using the boundary integral equation method, the problem of an external circular crack in a three_dimensional infinite elastic body under asymmetric loadings is investigated. The two_dimensional singular boundary integral equations of the problem were reduced to a system of Abel integral equations by means of Fourier series and hypergeometric functions. The exact solutions of stress intensity factors are obtained for the problem of an external circular crack under asymmetric loadings, which are even more universal than the results obtained by the use of Hankel transform method. The results demonstrate that the boundary integral equation method has great potential as a new analytic method.

Key words: external circular crack; asymmetric loadings; boundary integral equations; Abel equations; stress intensity factors; exact solutions