应用数学和力学编委会编
重 庆 出 版 社 出 版

文章编号: 1000_0887(2001) 05_0483_10

夹杂和裂纹的相互作用及端点相交的奇性性态分析*

陶 敏, 汤任基

(上海交通大学 工程力学系, 上海 200030)

(我刊编委汤任基来稿)

摘要: 利用单根裂纹和单根夹杂的基本解, 通过弹性力学的线性叠加原理, 将平面裂纹和夹杂相互作用的问题归结为解一组带有柯西型奇异积分的积分方程组, 计算了裂纹和夹杂端点的应力强度因子· 给出了一些数值例子· 并对夹杂和裂纹水平接触时的情形作了奇性分析· 结果可作为研究夹杂尖端引起的裂纹及其扩展的工程分析的计算模型·

关 键 词: 裂纹; 夹杂; 相互作用; 应力强度因子; 水平接触; 奇性分析
中图分类号: O346.1 文献标识码: A

引 言

在研究工程结构元件的强度和断裂时, 材料缺陷对结构的影响必须考虑· 以断裂力学的观点, 材料的缺陷可简化为平面夹杂和平面裂纹· 此外, 在研究短纤维复合材料如基体材料的增韧及其综合弹性模量时, 亦可简化为平面夹杂和平面裂纹相互影响的问题· 已有一些文献对此问题作了研究, 但似乎还不够全面· 本文利用了弹性力学中的叠加原理, 将裂纹和夹杂相互作用的问题分解为两个问题的叠加, 一是带一根裂纹的无限区域, 在去掉夹杂处, 施加一荷载 $q(x), p(x)$; 二是一根受纵向载荷和横向载荷作用的孤立的条形夹杂· 本文中夹杂是不可以退化为裂纹的, 这对于夹杂的刚度大于基体刚度的问题是合乎实际的· 因此, 前一个问题利用单裂纹的基本解求得裂纹引起的应力场和位移场; 后一个问题利用单夹杂的基本解得由夹杂引起的应力场和位移场, 并由线性叠加原理可以得到裂纹和夹杂相互作用引起的应力场和位移场, 然后利用裂纹的边界条件和夹杂的联结条件, 将问题归结为解一组带有柯西型奇异积分的积分方程组· 所得到的积分方程组可利用 Gauss_Jacobi 求积分式进行数值求解, 文中给出了裂纹和夹杂端点的应力强度因子并对不同的情况作了数值计算· 本文还对裂纹和夹杂相交时的性态作了分析·

1 裂纹和夹杂相互影响的应力和位移场计算模型

1) 计算模型

为了便于工程计算, 这里提出如下的工程计算模型:

* 收稿日期: 2000_03_03; 修订日期: 2001_01_05
作者简介: 陶 敏(1969—), 硕士, 讲师.

考虑图1 无限区域内的裂纹 $L_2 = L_2(c, d)$ 和夹杂 $L_1 = L_1(a, b)$ 相互影响的平面问题•在无穷远处作用荷载$(\sigma_{yy}^{\infty}, \sigma_{xx}^{\infty}, \sigma_{xy}^{\infty})$• 根据弹性力学叠加原理, 图 1问题可分解为图 2 问题和图3 问题的叠加• 图2是带一根裂纹的无限区域, 去掉夹杂处施加以分布荷载 $q(x_1), p(x_1)$, 且在无穷远处作用荷载$(\sigma_{yy}^{\infty}, \sigma_{xx}^{\infty}, \sigma_{xy}^{\infty})$• 图 3 是一根其上下两侧作用有分布荷载 $q^+(x_1)$, $q^-(x_1), p^+(x_1)$ 和 $p^-(x_1)$ 的孤立夹杂, 其中

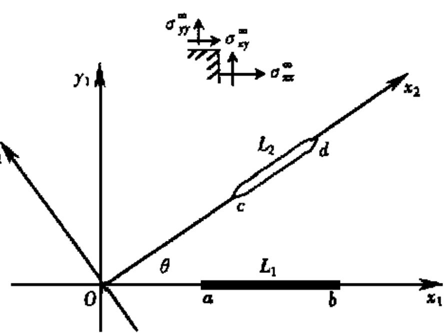

图 1

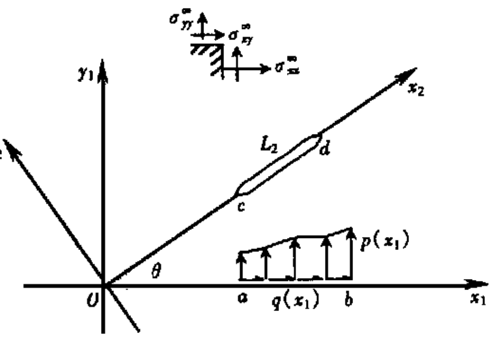

图 2

$$\left.\begin{aligned} p(x_1) &= p^+(x_1) - p^-(x_1), \\ q(x_1) &= q^+(x_1) - q^-(x_1)\text{•} \end{aligned}\right\} \tag{1}$$

2) 由裂纹 L_2 引起的应力场和位移场

记裂纹 $L_2 = L_2(c, d)$ 上的位错密度函数为:

$$\left.\begin{aligned} g(x_2) &= \frac{\partial}{\partial x_2}[v_2(x_2, +0) - v_2(x_2, -0)], \\ h(x_2) &= \frac{\partial}{\partial x_2}[u_2(x_2, +0) - u_2(x_2, -0)], \end{aligned}\right\} \tag{2}$$

这里的 u_2, v_2 是在局部坐标系(x_2, y_2) 中的位移分量•

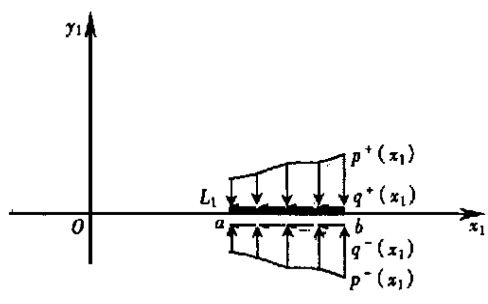

图 3

由基本的位错解[1], 由 g, h 引起的区域内任一点(x_2, y_2) 的应力分量为:

$$\left.\begin{aligned} \sigma_{xx}^{(22)}(x_2, y_2) &= \int_c^d [G_{xx}(x_2, y_2, t)g(t) + H_{xx}(x_2, y_2, t)h(t)]\,\mathrm{d}t, \\ \sigma_{yy}^{(22)}(x_2, y_2) &= \int_c^d [G_{yy}(x_2, y_2, t)g(t) + H_{yy}(x_2, y_2, t)h(t)]\,\mathrm{d}t, \\ \sigma_{xy}^{(22)}(x_2, y_2) &= \int_c^d [G_{xy}(x_2, y_2, t)g(t) + H_{xy}(x_2, y_2, t)h(t)]\,\mathrm{d}t, \end{aligned}\right\} \tag{3}$$

这里

$$\left.\begin{aligned} &G_{xx}(x, y, t) = A(t-x)[(t-x)^2 - y^2],\ H_{xx}(x, y, t) = Ay[3(t-x)^2 + y^2], \\ &G_{xy}(x, y, t) = Ay[-(t-x)^2 + y^2],\ G_{yy}(x, y, t) = A(t-x)[(t-x)^2 + 3y^2], \\ &H_{yy}(x, y, t) = Ay[-(t-x)^2 + y^2],\ H_{xy}(x, y, t) = A(t-x)[(t-x)^2 - y^2], \\ &A(x, y, t) = \frac{2\mu}{\pi(1+\kappa)}\frac{1}{[(t-x)^2 + y^2]^2}, \end{aligned}\right\} \tag{4}$$

式中 μ, κ 是基体的弹性常数, $\mu = E/2(1+\nu)$ 为剪切弹性模量, E 为弹性模量, ν为泊松比, κ

$= 3-4\nu$(平面应变), $\kappa=(3-\nu)/(1+\nu)$(平面应力)•

相应的位移分量为:

$$\left.\begin{aligned} u_2(x_2, y_2) &= 2B_1\int_c^d [G_{gg1}(x_2, y_2, t)g(t) + H_{hh1}(x_2, y_2, t)h(t)]\mathrm{d}t + B^{**}, \\ v_2(x_2, y_2) &= 2B_1\int_c^d [G_{gg2}(x_2, y_2, t)g(t) + H_{hh2}(x_2, y_2, t)h(t)]\mathrm{d}t + C^{**}, \end{aligned}\right\} \tag{5}$$

式中

$$\left.\begin{aligned} G_{gg1}(x, y, t) &= -(\kappa-1)\ln[(t-x)^2+y^2] + \frac{4y^2}{(t-x)^2+y^2}, \\ G_{gg2}(x, y, t) &= \frac{4y(t-x)}{(t-x)^2+y^2} - 2(\kappa+1)\arctan\frac{t-x}{y}, \\ H_{hh1}(x, y, t) &= 2(\kappa+1)\arctan\frac{y}{t-x} - \frac{4y(x-t)}{(t-x)^2+y^2}, \\ H_{hh2}(x, y, t) &= (\kappa-1)\ln[(t-x)^2+y^2] + \frac{4(x-t)^2}{(t-x)^2+y^2}, \\ B_1 &= \frac{1}{2\pi(1+\kappa)}, \end{aligned}\right\} \tag{6}$$

常数 B^{**}, C^{**} 是未定的, 它们可任意置值•

3) 由夹杂 L_1 引起的应力场和位移场

夹杂取出后的无限域问题(图 2), 此处假设夹杂厚度 h_0 很小, 且夹杂与基体连接很好, 因而可假定夹杂端点 a, b 均无集中力和集中力偶作用, 由$[q(x_1), p(x_1)]$ 产生的应力分量[2] 为:

$$\left.\begin{aligned} \sigma_{xx}^{11}(x_1, y_1) &= \int_a^b [I_{xx}(x_1, y_1, t)q(t) + J_{xx}(x_1, y_1, t)p(t)]\mathrm{d}t, \\ \sigma_{yy}^{11}(x_1, y_1) &= \int_a^b [I_{yy}(x_1, y_1, t)q(t) + J_{yy}(x_1, y_1, t)p(t)]\mathrm{d}t, \\ \sigma_{xy}^{11}(x_1, y_1) &= \int_a^b [I_{xy}(x_1, y_1, t)q(t) + J_{xy}(x_1, y_1, t)p(t)]\mathrm{d}t, \end{aligned}\right\} \tag{7}$$

式中

$$\left.\begin{aligned} I_{xx}(x, y, t) &= B_1\left\{-(\kappa-1)\frac{x-t}{(x-t)^2+y^2} - \frac{4(x-t)^3}{[(x-t)^2+y^2]^2}\right\}, \\ J_{xx}(x, y, t) &= B_1\left\{(\kappa-1)\frac{y}{(x-t)^2+y^2} - \frac{4(x-t)^2}{[(x-t)^2+y^2]^2}\right\}, \\ I_{yy}(x, y, t) &= B_1\left\{(\kappa-5)\frac{x-t}{(x-t)^2+y^2} + \frac{4(x-t)^3}{[(x-t)^2+y^2]^2}\right\}, \\ J_{yy}(x, y, t) &= B_1\left\{-(\kappa+3)\frac{y}{(x-t)^2+y^2} + \frac{4y(x-t)^2}{[(x-t)^2+y^2]^2}\right\}, \\ I_{xy}(x, y, t) &= B_1\left\{-(\kappa-1)\frac{y}{(x-t)^2+y^2} - \frac{4y(x-t)^2}{[(x-t)^2+y^2]^2}\right\}, \\ J_{xy}(x, y, t) &= B_1\left\{-(\kappa+3)\frac{x-t}{(x-t)^2+y^2} + \frac{4(x-t)^3}{[(x-t)^2+y^2]^2}\right\}• \end{aligned}\right\} \tag{8}$$

位移分量为:

$$\left.\begin{aligned} u_1(x_1, y_1) = {} & \frac{1}{2\pi\mu(1+\kappa)}\int_a^b\left[\frac{\kappa}{2}\ln\frac{1}{(x-u)^2+y^2}-\frac{y^2}{(x-t)^2+y^2}\right]q(u)\mathrm{d}u + \\ & \frac{1}{2\pi\mu(1+\kappa)}\int_a^b\frac{y(x-u)}{(x-u)^2+y^2}p(u)\mathrm{d}u, \\ v_1(x_1, y_1) = {} & \frac{1}{2\mu\pi(1+\kappa)}\int_a^b\left[\frac{\kappa}{2}\ln\frac{1}{(x-u)^2+y^2}+\frac{y^2}{(x-t)^2+y^2}\right]p(u)\mathrm{d}u + \\ & \frac{1}{2\pi\mu(1+\kappa)}\int_a^b\frac{y(x-u)}{(x-u)^2+y^2}q(u)\mathrm{d}u \cdot \end{aligned}\right\} \tag{9}$$

4) 无限远处作用应力 $(\sigma_{yy}^{\infty}, \sigma_{xx}^{\infty}, \sigma_{xy}^{\infty})$ 产生的位移场

受到图 1 所示的无限远处应力 $(\sigma_{yy}^{\infty}, \sigma_{xx}^{\infty}, \sigma_{xy}^{\infty})$ 作用时, 域中的应力记为 σ_{xx}^0, σ_{yy}^0, σ_{xy}^0, 容易看出它们为:

$$\sigma_{xx}^0 = \sigma_{xx}^{\infty} = \mathrm{const}, \quad \sigma_{yy}^0 = \sigma_{yy}^{\infty} = \mathrm{const}, \quad \sigma_{xy}^0 = \sigma_{xy}^{\infty} = \mathrm{const} \cdot \tag{10}$$

使用应力_应变关系得到位移场为:

$$\left.\begin{aligned} u_0(x_1, y_1) &= \frac{1+\kappa}{8\mu}\left(\sigma_{xx}^{\infty}-\frac{3-\kappa}{1+\kappa}\sigma_{yy}^{\infty}\right)x_1+\frac{\sigma_{xy}^{\infty}}{2\mu}y_1+B_0, \\ v_0(x_1, y_1) &= \frac{1+\kappa}{8\mu}\left(\sigma_{yy}^{\infty}-\frac{3+\kappa}{1+\kappa}\sigma_{yy}^{\infty}\right)y_1+\frac{\sigma_{xy}^{\infty}}{2\mu}x_1+C_0, \end{aligned}\right\} \tag{11}$$

其中 B_0 和 C_0 为刚体平动, 可任意置值•

5) 夹杂 L_1 的位移计算

夹杂 L_1 由无限平面取出后, 其受力如图 3 所示, 这里使用材料力学讨论, 考虑到变形很小, 故认为轴力与横向力可分开独立计算, 因而变形可按图 4 分别计算•

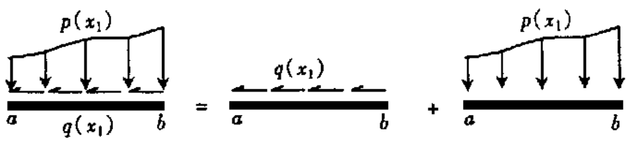

图 4

记夹杂的位移为 $[u_3(x_1), v_3(x_1)]$, 则:

$$\left.\begin{aligned} u_3(x_1) &= \frac{\kappa_1+1}{8\mu_1 h_0}\int_a^{x_1}(x_1-u)q(u)\mathrm{d}u+B^{***}, \\ v_3(x_1) &= \theta_0(x_1-a)-\frac{\kappa_1+1}{4\mu_1 h_0^3}\int_a^{x_1}(x_1-u)^3p(u)\mathrm{d}u+C^{***}, \end{aligned}\right\} \tag{12}$$

式中的 B^{***} 为夹杂 a 端的水平位移, C^{***} 为横向刚体位移, θ_0 为端点的转角• μ_1 为剪切弹性模量, ν_1 为泊松比, h_0 为夹杂高度, $\kappa_1 = 3-4\nu_1$(平面应变), $\kappa_1 = (3-\nu_1)/(1+\nu_1)$(平面应力)•

2 积分方程的建立

图 1 问题的积分方程可根据裂纹的边界条件, 夹杂的联结条件及夹杂的平衡条件得到•

裂纹 L_2 表面自由的条件:

$$\left.\begin{aligned}&\sigma_{yy}^{22}(x_2,+0)+\sigma_{yy}^{21}(x_2,+0)+\sigma_{yy}^{20}(x_2,+0)=0\\&\sigma_{xy}^{22}(x_2,+0)+\sigma_{xy}^{21}(x_2,+0)+\sigma_{xy}^{20}(x_2,+0)=0\end{aligned}\quad (c<x_2<d),\right\}\tag{13}$$

式中

$$\left.\begin{aligned}&\sigma_{yy}^{21}(x_2,+0)=\sigma_{yy}^{11}\cos^2\theta+\sigma_{xx}^{11}\sin^2\theta-\sigma_{xy}^{11}\sin2\theta,\\&\sigma_{yy}^{21}(x_2,+0)=(\sigma_{yy}^{11}-\sigma_{xx}^{11})\sin\theta\cos\theta+\sigma_{xy}^{11}\cos2\theta,\\&\sigma_{yy}^{20}(x_2,+0)=\sigma_{yy}^{\infty}\cos^2\theta+\sigma_{xx}^{\infty}\sin^2\theta-\sigma_{xy}^{\infty}\sin2\theta,\\&\sigma_{yy}^{20}(x_2,+0)=(\sigma_{yy}^{\infty}-\sigma_{xx}^{\infty})\sin\theta\cos\theta+\sigma_{xy}^{\infty}\cos2\theta,\\&\qquad(x_1=x_2\cos\theta,\ y_1=x_2\sin\theta)\cdot\end{aligned}\right\}\tag{14}$$

夹杂 L_1 两侧的位移联结条件:

$$\left.\begin{aligned}&u_1(x_1,+0)+u_{12}(x_1,+0)+u_0(x_1,+0)=u_3(x_1)\\&v_1(x_1,+0)+v_{12}(x_1,+0)+v_0(x_1,+0)=v_3(x_1)\end{aligned}\quad(a<x_1<b),\right\}\tag{15}$$

式中

$$\left.\begin{aligned}&u_{12}(x_1,+0)=u_2\cos\theta-v_2\sin\theta\\&v_{12}(x_1,+0)=u_2\sin\theta+v_2\cos\theta\end{aligned}\quad(x_2=x_1\cos\theta,\ y_1=-x_1\sin\theta)\cdot\right\}\tag{16}$$

夹杂孤立体的平衡条件:

$$\int_a^b q(x_1)\,\mathrm{d}x_1=0,\quad \int_a^b p(x_1)\,\mathrm{d}x_1=0,\quad \int_a^b(x_1-a)p(x_1)\,\mathrm{d}x_1=0\cdot\tag{17}$$

将已得到的应力位移表达式(3)、(5)、(7)、(9)、(10)、(11)、(12)代入(13)、(15)、(17)式,得到裂纹与夹杂相互影响问题的积分方程组为:

$$\begin{aligned}&\frac{1}{\pi}\int_c^d\frac{g(t)}{t-x_2}\mathrm{d}t+c_0\int_a^b I_{yy}^{21}(x_2,t)q(t)\,\mathrm{d}t+c_0\int_a^b J_{yy}^{21}(x_2,t)p(t)\,\mathrm{d}t=\\&\qquad-c_0(\sigma_{yy}^{\infty}\cos^2\theta+\sigma_{xx}^{\infty}\sin^2\theta+\sigma_{xy}^{\infty}\sin\theta\cos\theta)\quad(c<x_2<d),\end{aligned}\tag{18}$$

式中

$$\begin{aligned}&I_{yy}^{21}(x_2,t)=I_{yy}\cos^2\theta+I_{xx}\sin2\theta-I_{xy}\sin2\theta\\&J_{yy}^{21}(x_2,t)=J_{yy}\cos^2\theta-J_{xx}\sin2\theta+J_{xy}\cos2\theta\end{aligned}\qquad c_0=(1+\kappa)/2\mu\cdot$$

$$\begin{aligned}&\frac{1}{\pi}\int_c^d\frac{h(t)}{t-x_2}\mathrm{d}t+c_0\int_a^b I_{xy}^{21}(x_2,t)q(t)\,\mathrm{d}t+c_0\int_a^b J_{xy}^{21}(x_2,t)p(t)\,\mathrm{d}t=\\&\qquad-c_0[(\sigma_{yy}^{\infty}-\sigma_{xx}^{\infty})\sin\theta\cos\theta+\sigma_{xy}^{\infty}\cos2\theta]\quad(c<x_2<d),\end{aligned}\tag{19}$$

式中

$$\begin{aligned}&I_{xy}^{21}(x_2,t)=(I_{yy}-I_{xx})\sin\theta\cos\theta+I_{xy}\cos2\theta,\\&J_{xy}^{21}(x_2,t)=(J_{yy}-J_{xx})\sin\theta\cos\theta+J_{xy}\cos2\theta\cdot\end{aligned}$$

$$\begin{aligned}&\frac{1}{\pi}\int_a^b\frac{q(t)}{t-x_1}\mathrm{d}t+c_1\int_c^d G_{xx1}^{12}(x_1,0)g(t)\,\mathrm{d}t+c_1\int_c^d H_{xx1}^{12}(x_1,0)h(t)\,\mathrm{d}t=-c_2\frac{\partial u_3}{\partial x_1}=\\&\qquad-c_2\left(-\frac{1+\kappa}{8\mu}\right)\left(\sigma_{xx}^{\infty}-\sigma_{yy}^{\infty}\frac{3-\kappa}{1-\kappa}\right)\qquad(a<x_1<b),\end{aligned}\tag{20}$$

其中

$$G_{xx1}^{12}(x_1,0)=2A_1\left\{\frac{(\kappa-1)(t-x_2)}{(t-x_2)^2+y_2^2}-\frac{4y_2^2(t-x_2)\cos2\theta+2y_2[y_2^2-(t-x_2)^2]\sin2\theta}{[(t-x_2)^2+y_2^2]^2}\right\},$$

$$H^{12}_{xx1}(x_1, 0) = 2A_1\left\{\frac{(\kappa-1)y_2}{(t-x_2)^2+y_2^2}+\frac{4y_2(t-x_2)^2\cos2\theta-(t-x_2)[-y_2^2+(t-x_2)^2]\sin2\theta}{[(t-x_2)^2+y_2^2]^2}\right\},$$

$$(x_2 = x_1\sin\theta,\ y_2 = -x_1\sin\theta),$$

$$c_1 = \frac{(1+\kappa)}{4\kappa},\quad c_2 = \frac{2\mu(1+\kappa)}{\kappa},\quad A_1 = \frac{2\mu}{\pi(1+\kappa)}\cdot$$

当夹杂为完全刚性时:

$$\frac{\partial u_3(x_1)}{\partial x_1} = 0,\quad \frac{\partial v_3(x_1)}{\partial x_1} = \theta_0(\text{待定常数})\cdot \tag{21}$$

当夹杂为完全弹性时:

$$\left.\begin{aligned}\frac{\partial u_3(x_1)}{\partial x_1} &= \frac{\kappa_1+1}{8\mu_1 h}\int_a^{x_1} q(u)\mathrm{d}u,\\ \frac{\partial v_3(x_1)}{\partial x_1} &= \theta_0-\frac{3(\kappa_1+1)}{4\mu_1 h^3}\int_a^{x_1}(x_1-u)^2 p(u)\mathrm{d}u\cdot\end{aligned}\right\} \tag{22}$$

$$\frac{1}{\pi}\int_a^b \frac{p(t)}{t-x_1}\mathrm{d}t + c_1\int_c^d G^{12}_{yy1}(x_1, 0)g(t)\mathrm{d}t + c_1\int_c^d H^{12}_{yy1}(x_1, 0)h(t)\mathrm{d}t - c_2\frac{\partial v_3}{\partial x_1} = -c_2\frac{\sigma^{\infty}_{xy}}{2\mu}\quad (a < x_1 < b), \tag{23}$$

式中

$$\left.\begin{aligned}G^{12}_{xx1}(x_1, 0) &= 2A_1\left\{\frac{(\kappa+3)y_2}{(t-x_2)^2+y_2^2}-4\frac{y_2^2(t-x_2)\sin2\theta+y_2[y_2^2\sin^2\theta+(t-x_2)^2\cos^2\theta]}{[(t-x_2)^2+y_2^2]}\right\},\\ H^{12}_{yy1}(x_1, 0) &= -4A_1\left\{\frac{\kappa(x_2-t)}{(t-x_2)^2+y_2^2}-2\frac{y_2^2(t-x_2)\sin^2\theta-(t-x_2)^2[y_2\sin2\theta+(t-x_2)^2\cos^2\theta]}{[(t-x_2)^2+y_2^2]^2}\right\},\\ &(x_2 = x_1\sin\theta,\quad y_2 = -x_1\sin\theta)\cdot\end{aligned}\right\} \tag{24}$$

以上积分方程是关于未知函数$p(t)$, $q(t)$, $g(t)$, $h(t)$ 及未知数 θ_0 柯西积分方程, 它们应在夹杂的平衡条件(17) 及裂纹的位移单值条件

$$\int_c^d g(t)\mathrm{d}t = 0,\quad \int_c^d h(t)\mathrm{d}t = 0 \tag{25}$$

下求解·

3 裂纹和夹杂端点的应力强度因子

根据文献[2]、[3]、[4]及柯西型积分在积分曲线端点的性质[5], 得到夹杂端 点(a, b) 及裂纹端点(c, d) 的 Ⅰ 型和 Ⅱ 型应力强度因子分别为:

$$\left.\begin{aligned}
k_1(a) &= \lim_{x_1\to a}\sqrt{2(a-x_1)}\,\sigma_{yy}^{11}(x_1,0) = -\frac{\kappa-1}{2(\kappa+1)}\lim_{x_1\to a}\sqrt{2(x_1-a)}\,q(x_1),\\
k_2(a) &= \lim_{x_1\to a}\sqrt{2(a-x_1)}\,\sigma_{xy}^{11}(x_1,0) = \frac{\kappa-1}{2(\kappa+1)}\lim_{x_1\to a}\sqrt{2(x_1-a)}\,p(x_1),\\
k_1(b) &= \lim_{x_1\to b}\sqrt{2(b-x_1)}\,\sigma_{yy}^{11}(x_1,0) = -\frac{\kappa-1}{2(\kappa+1)}\lim_{x_1\to b}\sqrt{2(b-x_1)}\,q(x_1),\\
k_2(b) &= \lim_{x_1\to b}\sqrt{2(b-x_1)}\,\sigma_{xy}^{11}(x_1,0) = \frac{\kappa-1}{2(\kappa+1)}\lim_{x_1\to b}\sqrt{2(b-x_1)}\,p(x_1),\\
k_1(c) &= \lim_{x_2\to c}\sqrt{2(c-x_2)}\,\sigma_{yy}^{22}(x_2,0) = -\frac{2\mu}{\kappa+1}\lim_{x_2\to c}\sqrt{2(x_1-c)}\,g(x_2),\\
k_2(c) &= \lim_{x_2\to c}\sqrt{2(c-x_2)}\,\sigma_{xy}^{22}(x_2,0) = \frac{2\mu}{\kappa+1}\lim_{x_2\to c}\sqrt{2(x_2-c)}\,h(x_2),\\
k_2(d) &= \lim_{x_2\to d}\sqrt{2(d-x_2)}\,\sigma_{yy}^{22}(x_2,0) = -\frac{2\mu}{\kappa+1}\lim_{x_2\to d}\sqrt{2(d-x_2)}\,g(x_2),\\
k_2(d) &= \lim_{x_2\to d}\sqrt{2(d-x_2)}\,\sigma_{xy}^{22}(x_2,0) = \frac{2\mu}{\kappa+1}\lim_{x_2\to d}\sqrt{2(d-x_2)}\,h(x_2)\cdot
\end{aligned}\right\}\tag{26}$$

4 积分方程组的数值法

积分方程组(18)、(19)、(20)、(23)的未知函数$p(t)$, $q(t)$在夹杂L_1的积分, $g(t)$, $h(t)$在裂纹L_2上的积分都存在柯西奇异核, 按奇异积分方程组的数值法进行离散• 本文利用文献[7]的Gauss_Jacobi求积公式求解• 令:

$$\left.\begin{aligned}
g(t) &= G(t)(t-c)^{-1/2}(d-t)^{-1/2}, \quad h(t) = H(t)(t-c)^{-1/2}(d-t)^{-1/2},\\
q(t) &= Q(t)(t-a)^{-1/2}(b-t)^{-1/2}, \quad p(t) = P(t)(t-a)^{-1/2}(b-t)^{-1/2},
\end{aligned}\right\}\tag{27}$$

代入上述积分方程组, 得到一组封闭的方程组, 然后编制程序求解• 结果通过外推计算代入(26)式可得到裂纹和夹杂两端的应力强度因子•

5 裂纹与夹杂端点相交的奇性性态分析

这里讨论裂纹与夹杂在同一水平线上接触的特殊情况, 即在图1中取以下参数:

$$a = 0, \quad c = 0, \quad \theta = \pi, \tag{28}$$

此时方程组(18)、(19)、(20)、(23)中的积分核都可同时变为无界• 设

$$\left.\begin{aligned}
g(t) &= G(t)(t-c)^{\beta}(d-t)^{\alpha_1}, \quad h(t) = H(t)(t-c)^{\beta}(d-t)^{\alpha_1},\\
q(t) &= Q(t)(t-a)^{\beta}(b-t)^{\alpha_2}, \quad p(t) = P(t)(t-a)^{\alpha}(b-t)^{\alpha_2},
\end{aligned}\right\}\tag{29}$$

式中β为接触处的应力奇性指数, α_1, α_2为d, b端的奇性指数• 根据Cauchy型积分在积分曲线端点附近的性质[8], 对方程组(18)、(19)、(20)、(23)进行处理, 得到这四个方程的主部为:

$$-\cot\beta\pi[G(c)(d-c)^{\alpha_1}(x_2-c)^{\beta}+\cot\alpha_1\pi[G(d)(d-c)^{\beta}(d-x_2)^{\alpha_1}+ \\ \frac{\kappa-1}{4\mu}\frac{1}{\sin\beta\pi}[Q(a)(b-a)^{\alpha_2}(x_2+a)^{\beta}] = f_1^*(x_2) \quad (c<x_2<d), \tag{30}$$

$$-\cot\beta\pi[H(c)(d-c)^{\alpha_1}(x_2-c)^{\beta}+\cot\alpha_1\pi[G(d)(d-c)^{\beta}(d-x_2)^{\alpha_1}- \\ \frac{\kappa-1}{4\mu}\frac{1}{\sin\beta\pi}[P(a)(b-a)^{\alpha_2}(x_2+a)^{\beta}] = f_2^*(x_2) \quad (c<x_2<d), \tag{31}$$

$$-\cot\beta\pi Q(a)(b-a)^{\alpha_2}(x_1-a)^{\beta}++\cot\alpha_2\pi Q(b)(b-a)^{\beta}(b-x_1)^{\alpha_2}-$$

$$\frac{\mu(\kappa-1)}{\kappa}\frac{1}{\sin\beta\pi}[G(c)(d-c)^{\alpha_2}(x_1+c)^{\beta}]=f_3^*(x_1)\quad(a<x_1<b),\tag{32}$$

$$-\cot\beta\pi P(a)(b-a)^{\alpha_2}(x_1-a)^{\beta}++\cot\alpha_2\pi P(b)(b-a)^{\beta}(b-x_1)^{\alpha_2}+$$

$$\frac{\mu(\kappa-1)}{\kappa}\frac{1}{\sin\beta\pi}[H(c)(d-c)^{\alpha_2}(x_1+c)^{\beta}]=f_4^*(x_1)\quad(a<x_1<b),\tag{33}$$

同时 $a=c=0$, 并且在(30)、(31) 式两端同乘以$(d-x_2)^{-\alpha_1}$, 并让 $x_2\to d$, 得到:

$$\cot\alpha_1\pi=0\Rightarrow\alpha_1=-\frac{1}{2}\cdot\tag{34}$$

(32)、(33) 式两端同乘以 $(b-x_1)^{-\alpha_1}$, 并让 $x_1\to b$, 得到:

$$\cot\alpha_2\pi=0\Rightarrow\alpha_2=-\frac{1}{2}\cdot\tag{35}$$

然后在(30)、(31) 式两端同乘以 $x_2^{-\beta}$, 并让 $x_2\to 0$, 在(32)、(33) 式两端同乘以 $x_1^{-\beta}$, 并让 $x_1\to 0$, 得到:

$$-\cos\beta\pi[G(0)d^{\alpha_1}/\sin\beta\pi]+\frac{\kappa-1}{4\mu}[Q(0)b^{\alpha_2}/\sin\beta\pi]=0,\tag{36}$$

$$-\cos\beta\pi[H(0)d^{\alpha_1}/\sin\beta\pi]-\frac{\kappa-1}{4\mu}[P(0)b^{\alpha_2}/\sin\beta\pi]=0,\tag{37}$$

$$-\cos\beta\pi[Q(0)d^{\alpha_{21}}/\sin\beta\pi]-\frac{\mu(\kappa-1)}{\kappa}[G(0)d^{\alpha_2}/\sin\beta\pi]=0,\tag{38}$$

$$-\cos\beta\pi[P(0)b^{\alpha_1}/\sin\beta\pi]+\frac{\mu(\kappa-1)}{\kappa}[H(0)d^{\alpha_2}/\sin\beta\pi]=0\cdot\tag{39}$$

根据这四个方程组的特征方程等于零的条件, 求得:

$$\cos\beta\pi=\pm\frac{\kappa-1}{2\sqrt{\kappa}}\mathrm{i},\tag{40}$$

这表明, 相交时应力奇性指数 β 与夹杂的弹性常数有关, 而且是一个复数·

6 数值例子

这里给出夹杂为完全刚性和完全弹性两种情形的结果· 令图 1 的 $\vartheta=\pi$, $\mu_1/\mu\to\infty$即为完全刚性夹杂与裂纹共线相互作用的问题· 并令:

$$a=-c,\quad b=-d,\quad \kappa=\kappa_1=2.08,$$

$$c_1=(b-a)/2,\quad d_1=(b+a)/2,$$

$$k_{a_1}^*=\frac{k_1(a)}{\sigma^{\infty}\sqrt{c_1}},\quad k_{b_1}^*=\frac{k_1(b)}{\sigma^{\infty}\sqrt{c_1}}\cdot$$

无穷远处作用拉伸应务 $\sigma_{yy}^{\infty}=\sigma^{\infty}$, 夹杂两端的应力强度因子 $k_{a_1}^*$, $k_{b_1}^*$ 与$\dfrac{a}{d_1}$ 的关系图绘于图5, 图中可以看出: 当$\dfrac{a}{d_1}=0.8$ 时, $k_{a_1}^*$, $k_{b_1}^*$ 非常接近, 当$\dfrac{a}{d_1}=0.99$ 时, $k_{a_1}^*=0.059\,716$, $k_{b_1}^*=0.059\,704$ 与文[8] 相比, 误差很小, 这说明本文的理论与方法是可靠的·

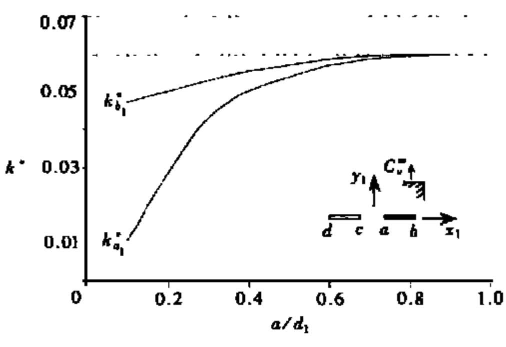

图 5

夹杂为完全弹性时, 令 $\dfrac{h_0}{c_1}=\dfrac{1}{50}$($h_0$ 为夹杂高度, 即弹性夹杂是很细的纤维夹杂), $\dfrac{\mu_1}{\mu}=$

5.0, $\kappa=\kappa_1=2.08$, c_1, d_1 规定同前, 应力强度因子和弹性夹杂的初始转角 $\frac{\mu}{p^*}\theta_0$(p^* 与 σ_{yy}^{∞}, σ_{xy}^{∞} 同量级), 在拉伸应力 σ_{yy}^{∞} 和剪切应力 σ_{xy}^{∞} 作用下随 θ, $\frac{a}{d_1}$ 变化的数值列于表 1•

表 1

θ	σ	k	a/d_1				
			1/25	1/10	1/2	9/10	$\to 1$
180°	σ_{yy}^{∞}	$k_{a_1}^*$	−0.022 938	−0.005 546	0.011 456	0.013 194	0.013 230
		$k_{b_1}^*$	0.011 214	0.011 427	0.012 554	0.013 199	0.013 116
		$k_{c_1}^*$	0.998 57	1.007 49	1.000 36	1.000 12	1.000 00
		$k_{d_1}^*$	0.999 98	1.000 15	1.000 05	0.999 91	1.000 0
180°	σ_{xy}^{∞}	$k_{a_2}^*$	−0.0167 44	−0.010 942	−0.00145	-9.112×10^{-6}	0
		$k_{b_2}^*$	0.000 182	0.000 463 4	0.000 911	8.377×10^{-6}	0
		$k_{c_2}^*$	1.016 86	1.011 05	1.001 56	1.000 12	1.000 0
		$k_{d_2}^*$	0.999 72	0.999 44	0.998 99	0.999 90	1.000 0
		$\mu\theta_0/p^*$	0.269 34	0.391 36	0.490 05	0.499 66	0.500 00
30°	σ_{yy}^{∞}	$k_{a_1}^*$	−0.064 694	−0.037 079	−0.003 18	0.012 540	0.013 230
		$k_{b_1}^*$	0.030 348	0.029 492	0.022 852	0.013 730	0.013 116
30°	σ_{xy}^{∞}	$k_{c_2}^*$	0.455 74	0.464 63	0.480 76	0.500 08	—
		$k_{d_2}^*$	0.500 17	0.501 67	0.509 94	0.499 98	—
		$\mu\theta_0/p^*$	1.233 2	1.045 4	0.626 60	0.510 56	0.500 00
90°	σ_{yy}^{∞}	$k_{a_1}^*$	0.013 60	0.013 234	0.022 851	0.013 237	0.013 230
		$k_{b_1}^*$	0.013 320	0.013 223	0.022 846	0.013 234	0.013 116
120°	σ_{xy}^{∞}	$k_{c_1}^*$	0.864 47	0.864 64	0.866 03	0.866 12	—
		$k_{d_1}^*$	0.864 83	0.865 18	0.865 88	0.865 94	—

[参 考 文 献]

[1] Dundurs J. Elastic Interaction of Dislocations with Inhomogeneities[A]. In: T Mura Ed. Mathematical Theory of Dislocations[C]. T Mura Ed. New York: ASME, 1969, 70—115.

[2] Chen Y H, Hahn H G. The stress singularity coefficient at infinite rigid flate inclusion in an orthotropic plane elastic body[J]. Engng Fracture Mech, 1993, **44**(3): 359—362.

[3] 张明焕. 平面与反平面线夹杂力学的工程理论[D]. 博士毕业论文. 上海: 上海交通大学, 1994.

[4] 范天佑. 断裂力学基础[M]. 南京: 江苏科学出版社, 1978.

[5] 路见可. 解析函数边值问题[M]. 上海: 上海科学出版社, 1978.

[6] Erdogan F. Mixed boundary_value problem in mechanics[A]. In: S Nemat_Nasser Ed. Mechanics Today[C]. Illinois: Northwestern University, 1978, 71—73.

[7] Erodogan F, Gupta G D. Stresses near a flat inclusion in bonded dissimilar[J]. International Journal Solids and Structures, 1972, **8**(2): 533—547.

The Crack_Inclusion Interaction and the Analysis of Singularity for the Horizontal Contact

TAO Fang_ming, TANG Ren_ji
(Department of Engineering Mechanics, Shanghai Jiaotong University,
Shanghai 200030, P R China)

Abstract: Using the basic solutions of a single crack and a single inclusion, and making use of the principle of linear superposition of elastic mechanics, the interaction problem between a planar crack and a flat inclusion in an elastic solid is studied. The problem is reduced to solve a set of standard Cauchy_type singular equations. And the stress intensity factors at points of crack and inclusion were obtained. Besides, the singularity for the horizontal contact of crack and inclusion was analyzed. The calculating model put forward can be regarded as a new technique for studying the crack and its expanding caused by inclusion tip. Then several numerical examples are given.

Key words: crack; inclusion; interaction; the horizontal contact; singularity analyse; stress intensity factor