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立方准晶材料的剪切裂纹问题*

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(王彪推荐)

摘要: 通过引入位移函数, 使得立方准晶的轴对称弹性问题归结为求解一个高阶偏微分方程• 在此基础上, 研究了立方准晶含有圆盘状裂纹的剪切问题, 得到了此问题弹性场的精确解析解, 以及应力强度因子与应变能释放率•

关 键 词: 立方准晶; 剪切裂纹; 应力强度因子
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引 言

准晶作为一种新的固态物质结构[1,2] 不仅给凝态聚物理带来了新的思想, 而且对此新的结构材料物理与力学性能的理论与实验研究, 包括弹性理论研究也带来了新的挑战, 取得了一系列很有价值的成果[3~5]•

准晶发现不久, 就观测到此材料存在缺陷[6,7]• 裂纹就是缺陷之一, 它的存在严重影响准晶材料的物理与力学性能• 研究准晶的裂纹问题无疑具有重要的意义•

立方准晶是一重要的三维准晶, 由于其弹性基本方程的复杂性, 解析解很难得到• 发展求解准晶弹性复杂边值问题系统和直接的方法是一基本的任务[5,8~11]• 本文在立方准晶轴对称弹性理论的基础上, 运用 Hankel 变换与对偶积分方程理论研究剪应力作用下的圆盘状裂纹问题, 得到了其弹性场的解析解, 并由此确定了应力强度因子与应变能释放率, 为研究准晶的变形与断裂提供了重要的信息•

1 位移函数与轴对称问题

立方准晶的应力_应变关系(广义 Hooke 定律) 在轴对称条件下可表示如下(其中(r, θ, z) 为柱坐标)[5]:

$$\sigma_{rr} = C_{11}\varepsilon_{rr} + C_{12}(\varepsilon_{\theta\theta} + \varepsilon_{zz}) + R_{11}E_{rr} + R_{12}(E_{\theta\theta} + E_{zz}), \tag{1}$$

$$\sigma_{\theta\theta} = C_{11}\varepsilon_{\theta\theta} + C_{12}(\varepsilon_{rr} + \varepsilon_{zz}) + R_{11}E_{\theta\theta} + R_{12}(E_{rr} + E_{zz}), \tag{2}$$

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$$\sigma_{zz} = C_{11}\varepsilon_{zz} + C_{12}(\varepsilon_{\theta\theta} + \varepsilon_{rr}) + R_{11}E_{zz} + R_{12}(E_{\theta\theta} + E_{rr}), \tag{3}$$

$$\sigma_{zr} = \sigma_{rz} = 2C_{44}\varepsilon_{rz} + 2R_{44}E_{rz}, \tag{4}$$

$$H_{rr} = R_{11}\varepsilon_{rr} + R_{12}(\varepsilon_{\theta\theta} + \varepsilon_{zz}) + K_{11}E_{rr} + K_{12}(E_{\theta\theta} + E_{zz}), \tag{5}$$

$$H_{\theta\theta} = R_{11}\varepsilon_{\theta\theta} + R_{12}(\varepsilon_{rr} + \varepsilon_{zz}) + K_{11}E_{\theta\theta} + K_{12}(E_{rr} + E_{zz}), \tag{6}$$

$$H_{zz} = R_{11}\varepsilon_{zz} + R_{12}(\varepsilon_{\theta\theta} + \varepsilon_{rr}) + K_{11}E_{zz} + K_{12}(E_{\theta\theta} + E_{rr}), \tag{7}$$

$$H_{zr} = H_{rz} = 2R_{44}\varepsilon_{rz} + 2K_{44}E_{rz}, \tag{8}$$

其中 σ_{ij} 是声子场应力分量, H_{ij} 是相位子场应力分量; ε_{ij} 是声子场应变分量, E_{ij} 是相位子场应变分量; C_{ij} 是声子场弹性常数, K_{ij} 是相位子场弹性常数, R_{ij} 是声子场_相位子场耦合弹性常数• 应变分量定义如下:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r},\quad \varepsilon_{\theta\theta} = \frac{u_r}{r},\quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z},\quad \varepsilon_{rz} = \varepsilon_{zr} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \tag{9}$$

$$E_{rr} = \frac{\partial w_r}{\partial r},\quad E_{\theta\theta} = \frac{w_r}{r},\quad E_{zz} = \frac{\partial w_z}{\partial z},\quad E_{rz} = E_{zr} = \frac{1}{2}\left(\frac{\partial w_r}{\partial z} + \frac{\partial w_z}{\partial r}\right), \tag{10}$$

其余的应变分量为零• 这里 u_i 是声子场位移分量, w_i 是相位子场位移分量• 不计体力的平衡方程为:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0,\quad \frac{\partial \sigma_{zr}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zr}}{r} = 0, \tag{11}$$

$$\frac{\partial H_{rr}}{\partial r} + \frac{\partial H_{rz}}{\partial z} + \frac{H_{rr} - H_{\theta\theta}}{r} = 0,\quad \frac{\partial H_{zr}}{\partial r} + \frac{\partial H_{zz}}{\partial z} + \frac{H_{zr}}{r} = 0• \tag{12}$$

方程(1) ~ (12)为立方准晶轴对称弹性理论的基本方程• 为了简化方程, 引入位移函数 $F(r, z)$, 使得位移可用此函数表示如下[3]:

$$u_r = -\frac{\partial^2}{\partial r\partial z}\left[A_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 - A_2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^2}{\partial z^2} + A_3\frac{\partial^4}{\partial z^4}\right]F, \tag{13}$$

$$u_z = -\left[B_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^3 - B_2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2\frac{\partial^2}{\partial z^2} + B_3\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^4}{\partial z^4} - B_4\frac{\partial^6}{\partial z^6}\right]F, \tag{14}$$

$$w_r = -\frac{\partial^2}{\partial r\partial z}\left[C_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 - C_2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^2}{\partial z^2} + C_3\frac{\partial^4}{\partial z^4}\right]F, \tag{15}$$

$$w_z = -\left[D_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^3 - D_2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2\frac{\partial^2}{\partial z^2} + D_3\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^4}{\partial z^4} - D_4\frac{\partial^6}{\partial z^6}\right]F, \tag{16}$$

其中 A_i、B_i、C_i 和 D_i 是由弹性常数表示的已知常数(见[3] 附录)• $F(r, z)$ 满足下面的偏微分方程:

$$\left[\frac{\partial^8}{\partial z^8} - b\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{\partial^6}{\partial z^6} + c\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2\frac{\partial^4}{\partial z^4} - d\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^3\frac{\partial^2}{\partial z^2} + e\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^4\right]F = 0, \tag{17}$$

其中 b、c、d 与 e 是由弹性常数组成的常数[3]•

把式(13) ~ (16) 代入(9) ~ (10) 再代入(1) ~ (8), 就得到应力 σ_{ij} 和H_{ij} 用$F(r, z)$ 表示的式

子(因太长,这里省略)• 这样,立方准晶的轴对称弹性问题就归结为求解方程(17)•

引进零阶Hankel变换

$$F(\xi, z) = \int_0^\infty rF(r, z)\mathrm{J}_0(\xi r)\mathrm{d}r,$$

这里 $\mathrm{J}_0(\xi r)$ 是第一类零阶Bessel函数• 在方程(17)两端施行Hankel变换,得

$$\left[\frac{\mathrm{d}^8}{\mathrm{d}z^8} + b\xi^2\frac{\mathrm{d}^6}{\mathrm{d}z^6} + c\xi^4\frac{\mathrm{d}^4}{\mathrm{d}z^4} + d\xi^6\frac{\mathrm{d}^2}{\mathrm{d}z^2} + e\xi^8\right]F = 0, \tag{18}$$

其中 ξ 表示Hankel变换参数•

利用应力、位移与函数 $F(r, z)$ 的关系,应力和位移可用 $F(\xi, z)$ 表示如下:

$$u_r = \int_0^\infty \left(A_1\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + A_2\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + A_3\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)F(\xi, z)\mathrm{J}_1(\xi r)\mathrm{d}\xi, \tag{19}$$

$$u_z = \int_0^\infty \left(B_1\xi^7 + B_2\xi^5\frac{\mathrm{d}^2}{\mathrm{d}z^2} + B_3\xi^3\frac{\mathrm{d}^4}{\mathrm{d}z^4} + B_4\xi\frac{\mathrm{d}^6}{\mathrm{d}z^6}\right)F(\xi, z)\mathrm{J}_0(\xi r)\mathrm{d}\xi, \tag{20}$$

$$w_r = \int_0^\infty \left(C_1\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + C_2\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + C_3\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)F(\xi, z)\mathrm{J}_1(\xi r)\mathrm{d}\xi, \tag{21}$$

$$w_z = \int_0^\infty \left(D_1\xi^7 + D_2\xi^5\frac{\mathrm{d}^2}{\mathrm{d}z^2} + D_3\xi^3\frac{\mathrm{d}^4}{\mathrm{d}z^4} + D_4\xi\frac{\mathrm{d}^6}{\mathrm{d}z^6}\right)F(\xi, z)\mathrm{J}_0(\xi r)\mathrm{d}\xi, \tag{22}$$

$$\begin{aligned}\sigma_{rr} = {} & \int_0^\infty \left(E_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + E_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + E_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + E_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)F(\xi, z)\mathrm{J}_0(\xi r)\mathrm{d}\xi + {} \\ & \frac{1}{r}\int_0^\infty \left(E_5\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + E_6\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + E_7\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)F(\xi, z)\mathrm{J}_1(\xi r)\mathrm{d}\xi,\end{aligned} \tag{23}$$

$$\begin{aligned}\sigma_{\theta\theta} = {} & \int_0^\infty \left(F_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + F_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + F_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + F_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)F(\xi, z)\mathrm{J}_0(\xi r)\mathrm{d}\xi - {} \\ & \frac{1}{r}\int_0^\infty \left(E_5\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + E_6\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + E_7\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)F(\xi, z)\mathrm{J}_1(\xi r)\mathrm{d}\xi,\end{aligned} \tag{24}$$

$$\sigma_{zz} = \int_0^\infty \left(G_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + G_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + G_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + G_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)F(\xi, z)\mathrm{J}_0(\xi r)\mathrm{d}\xi, \tag{25}$$

$$\sigma_{rz} = \sigma_{zr} = \int_0^\infty \left(H_1\xi^8 + H_2\xi^6\frac{\mathrm{d}^2}{\mathrm{d}z^2} + H_3\xi^4\frac{\mathrm{d}^4}{\mathrm{d}z^4} + H_4\xi^2\frac{\mathrm{d}^6}{\mathrm{d}z^6}\right)F(\xi, z)\mathrm{J}_1(\xi r)\mathrm{d}\xi, \tag{26}$$

$$\begin{aligned}H_{rr} = {} & \int_0^\infty \left(I_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + I_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + I_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + I_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)F(\xi, z)\mathrm{J}_0(\xi r)\mathrm{d}\xi + {} \\ & \frac{1}{r}\int_0^\infty \left(I_5\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + I_6\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + I_7\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)F(\xi, z)\mathrm{J}_1(\xi r)\mathrm{d}\xi,\end{aligned} \tag{27}$$

$$\begin{aligned}H_{\theta\theta} = {} & \int_0^\infty \left(J_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + J_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + J_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + J_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)F(\xi, z)\mathrm{J}_0(\xi r)\mathrm{d}\xi - {} \\ & \frac{1}{r}\int_0^\infty \left(I_5\xi^6\frac{\mathrm{d}}{\mathrm{d}z} + I_6\xi^4\frac{\mathrm{d}^3}{\mathrm{d}z^3} + I_7\xi^2\frac{\mathrm{d}^5}{\mathrm{d}z^5}\right)F(\xi, z)\mathrm{J}_1(\xi r)\mathrm{d}\xi,\end{aligned} \tag{28}$$

$$H_{zz} = \int_0^\infty \left(K_1\xi^7\frac{\mathrm{d}}{\mathrm{d}z} + K_2\xi^5\frac{\mathrm{d}^3}{\mathrm{d}z^3} + K_3\xi^3\frac{\mathrm{d}^5}{\mathrm{d}z^5} + K_4\xi\frac{\mathrm{d}^7}{\mathrm{d}z^7}\right)F(\xi, z)\mathrm{J}_0(\xi r)\mathrm{d}\xi, \tag{29}$$

$$H_{rz} = H_{zr} = \int_0^\infty \left(L_1\xi^8 + L_2\xi^6\frac{\mathrm{d}^2}{\mathrm{d}z^2} + L_3\xi^4\frac{\mathrm{d}^4}{\mathrm{d}z^4} + L_4\xi^2\frac{\mathrm{d}^6}{\mathrm{d}z^6}\right)F(\xi, z)\mathrm{J}_1(\xi r)\mathrm{d}\xi, \tag{30}$$

这里 $E_i, F_i, G_i, H_i, I_i, J_i, K_i$ 和 L_i 是由弹性常数构成的已知常数,

$$E_i = C_{11}A_i + C_{12}B_i + R_{11}C_i + R_{12}D_i,\quad F_i = C_{12}(A_i + B_i) + R_{12}(C_i + D_i),$$

$$G_i = C_{12}A_i + C_{11}B_i + R_{12}C_i + R_{11}D_i,\quad H_i = C_{44}(A_{i-1} - B_i) + R_{44}(C_{i-1} - D_i),$$

$$I_i = R_{11}A_i + R_{12}B_i + K_{11}C_i + K_{12}D_i,\quad J_i = R_{12}(A_i + B_i) + K_{12}(C_i + D_i),$$
$$K_i = R_{12}A_i + R_{11}B_i + K_{12}C_i + K_{11}D_i,\quad L_i = R_{44}(A_{i-1} - B_i) + K_{44}(C_{i-1} - D_i)$$
$$(i = 1, 2, 3, 4;\ A_0 = C_0 = A_4 = C_4 = 0),$$
$$E_{i+4} = (C_{12} - C_{11})A_i + (R_{12} - R_{11})C_i,\quad J_{i+4} = (R_{12} - R_{11})A_i + (K_{12} - K_{11})C_i$$
$$(i = 1, 2, 3),$$

$J_1(\xi r)$ 为第一类一阶 Bessel 函数•

作为典型的轴对称问题, 下面我们研究含有圆盘状裂纹的立方准晶在轴对称剪应力作用下的弹性问题•

2　圆盘状裂纹问题的解

考虑立方准晶材料中央有一半径为 a 的圆盘状裂纹, 裂纹的大小相对于该固体非常小, 可以认为该材料为无穷大•　在无穷远处, 此材料在 xy_平面受到一轴对称剪应力 s•　坐标系的选取如图 1 所示•

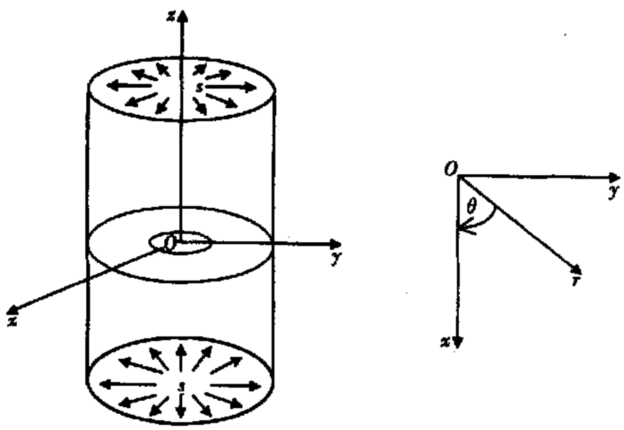

图 1　立方准晶中的圆盘状裂纹和坐标系

由于问题的对称性, 只需研究上半空间 $z > 0$(或下半空间 $z < 0$)•　此时, 问题的边界条件可表示为

$$\sqrt{r^2 + z^2} \to \infty:\ \sigma_{zr} = s,\ H_{zr} = 0, \tag{31}$$
$$z = 0,\ 0 \leqslant r \leqslant a:\ \sigma_{zz} = \sigma_{zr} = 0;\ H_{zz} = H_{rz} = 0, \tag{32}$$
$$z = 0,\ r > a:\ \sigma_{zz} = 0,\ u_r = 0;\ H_{zz} = 0,\ w_r = 0• \tag{33}$$

应用线性叠加原理, 此问题可化为两个问题的叠加•　其中之一具有边界条件

$$\sqrt{r^2 + z^2} \to \infty:\ \sigma_{ij} = H_{ij} = 0, \tag{34}$$
$$z = 0,\ 0 \leqslant r \leqslant a:\ \sigma_{zr} = -s,\ \sigma_{zz} = 0;\ H_{zz} = H_{rz} = 0, \tag{35}$$
$$z = 0,\ r > a:\ \sigma_{zz} = 0,\ u_r = 0;\ H_{zz} = 0,\ w_r = 0• \tag{36}$$

另一问题是没有裂纹的材料在无穷远处受一剪应力 s 作用, 此问题的解是已知的, 即 $\sigma_{zr} = s$, 其余的应力分量为零•　因此, 只需求解偏微分方程的边值问题(18) 与(34)~(36)•

在 Hankel 变换域, 方程(18) 满足条件(34) 的解为

$$F(\xi, z) = \alpha_1(\xi)e^{-\lambda_1\xi z} + \alpha_2(\xi)e^{-\lambda_2\xi z} + \alpha_3(\xi)e^{-\lambda_3\xi z} + \alpha_4(\xi)e^{-\lambda_4\xi z}, \tag{37}$$

其中 $\lambda_i(i=1,2,3,4)$ 是仅与弹性常数有关的常数[3]，$\alpha_i(\xi)\ (i=1,2,3,4)$ 是待定函数•

这里假定 $\lambda_i \neq \lambda_j(i\neq j)$ 且 $\lambda_i>0$(对于 λ_i 中有相等的、或者有不大于零的、或者有复数的情形可作类似的讨论• 只有 λ_i 中有纯虚数的情形时，解无物理意义)•

把式(37)代入式(19)、(21)、(25)、(26)、(29)与(30)，由边界条件(35)与(36)，可得确定 $\alpha_i(\xi)\ (i=1,2,3,4)$ 的对偶积分方程组

$$\int_0^\infty \alpha_i(\xi)\xi^8 J_1(\xi r)\,\mathrm{d}\xi=-\Pi_i s \qquad 0\leqslant r\leqslant a, \tag{38}$$

$$\int_0^\infty \alpha_i(\xi)\xi^7 J_1(\xi r)\,\mathrm{d}\xi=0 \qquad r>a, \tag{39}$$

其中 $\Pi_i=\Delta_i/\Delta\ (i=1,2,3,4)$ 是与弹性常数有关的常数

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{vmatrix},\quad \Delta_1=\begin{vmatrix} a_{12} & a_{13} & a_{14}\\ a_{22} & a_{23} & a_{24}\\ a_{42} & a_{43} & a_{44}\end{vmatrix},\quad \Delta_2=-\begin{vmatrix} a_{11} & a_{13} & a_{14}\\ a_{21} & a_{23} & a_{24}\\ a_{41} & a_{43} & a_{44}\end{vmatrix},$$

$$\Delta_3=\begin{vmatrix} a_{11} & a_{12} & a_{14}\\ a_{21} & a_{22} & a_{24}\\ a_{41} & a_{42} & a_{44}\end{vmatrix},\quad \Delta_4=-\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{41} & a_{42} & a_{43}\end{vmatrix},$$

$$a_{1i}=G_1\lambda_i+G_2\lambda_i^3+G_3\lambda_i^5+G_4\lambda_i^7,\quad a_{2i}=K_1\lambda_i+K_2\lambda_i^3+K_3\lambda_i^5+K_4\lambda_i^7,$$

$$a_{3i}=H_1+H_2\lambda_i^2+H_3\lambda_i^4+H_4\lambda_i^6,\quad a_{4i}=L_1+L_2\lambda_i^2+L_3\lambda_i^4+L_4\lambda_i^6•$$

求解对偶积分方程组(38)与(39)，得

$$\alpha_i(\xi)=-\Pi_i s\left[\xi^{-9}(1-\cos a\xi)-\frac{a}{2}\xi^{-8}\sin a\xi\right]• \tag{40}$$

至此，这一问题已得到解决• 把式(40)代入式(37)再代入式(19)～(30)，就得到了所有应力分量与位移分量的解析表达式• 由表示式(26)，容易得到

$$\sigma_{zr}(r,0)=\begin{cases}-s & r<a,\\ -s\left[1-\dfrac{1}{r}(r^2-a^2)^{1/2}-\dfrac{a^2}{2r}(r^2-a^2)^{-1/2}\right] & r>a•\end{cases} \tag{41}$$

从上式(41)可以看到，当 $r-a=\delta\ll 1$ 时，

$$\sigma_{zr}(r,0)\propto\delta^{-1/2}, \tag{42}$$

即应力在裂尖附近呈 1/2 阶奇异性•

由此可得到应力强度因子 K_{II}，应变能 W_{II} 及应变能释放率 G_{II} 如下[12]：

$$K_{\mathrm{II}}=\lim_{r\to a^+}\sqrt{2\pi(r-a)}\,\sigma_{zr}(r,0)=\frac{s\sqrt{\pi a}}{2}, \tag{43}$$

$$W_{\mathrm{II}}=\int_0^a 2\pi r\sigma_{zr}(r,0)\,u_r(r,0)\,\mathrm{d}r=\Pi a^3 s^2, \tag{44}$$

$$G_{\mathrm{II}}=\frac{1}{2\pi a}\frac{\partial W_{\mathrm{II}}}{\partial a}=\frac{3\Pi s^2 a}{2\pi}• \tag{45}$$

这里 $\Pi=\Pi/12\Delta$ 是仅与弹性常数有关的一常数，其中

$$\Pi = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{51} & a_{52} & a_{53} & a_{54} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}, \quad a_{5i} = A_1\lambda_i + A_2\lambda_i^3 + A_3\lambda_i^5 \qquad (i = 1, 2, 3, 4) \cdot$$

3　结论与讨论

准晶弹性理论的基本方程比传统弹性理论要复杂得多, 直接求解非常困难• 包括本文在内的一些工作, 我们通过引入位移函数或应力函数把复杂的原始基本方程简化成一个高阶偏微分方程, 发展了消去变量法这一方法• 用该方法, 求解了立方准晶弹性复杂的边值问题, 得到了圆盘状裂纹的精确解析解• 在一定程度上, 这是经典弹性的方法论在准晶弹性领域的发展•

实验[13]指出, 准晶材料是相当脆弱的, 文[13]也报道了准晶 $Al_{65}Cu_{20}Co_{15}$断裂韧性的测量• 众所周知, 脆性固体的破坏总是与裂纹有关[14], 本文及相关工作试图把经典的 Griffith 断裂理论推广到准晶领域• 目前, 由于缺乏关于准晶圆盘裂纹的实验结果, 本文的解析解为实验研究与数值分析估计提供了理论预测•

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Crack Problem Under Shear Loading in Cubic Quasicrystal

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Abstract: The axisymmetric elasticity problem of cubic quasicrystal is reduced to a single higher_order partial differential equation by introducing a displacement function. Based on the work, the analytic solutions of elastic field of cubic quasicrystal with a penny_shaped crack under the shear loading are found, and the stress intensity factor and strain energy release rate are determined.

Key words: cubic quasicryastal; shear crack; stress intensity factor