应用数学和力学, 第 24 卷 第 9 期(2003 年 9 月)
Applied Mathematics and Mechanics
应用数学和力学编委会编
重 庆 出 版 社 出 版

文章编号: 1000_0887(2003) 09_0929_11

一类混杂动态系统的能控性(Ⅱ)——含单时滞的情形*

谢广明, 王 龙, 叶庆凯

(北京大学 力学与工程科学系, 北京 100871)

(本刊编委叶庆凯来稿)

摘要: 首次将时滞现象引入到线性切换系统的模型中, 研究含时滞的线性切换系统的能控性及其判定条件• 全部工作由三部分组成• 第Ⅱ部分, 主要研究含单时滞的线性切换系统的能控性及其判定准则• 首先给出周期型系统的单周期能控性和多周期能控性的定义和充要条件, 其次给出非周期系统的能控性的定义和充要条件•

关 键 词: 混杂动态系统; 线性切换系统; 时滞; 能控性; 能控集; 切换序列; 切换路径

中图分类号: TP13; TP273; O317 文献标识码: A

引 言

考虑输入函数含单个时滞的线性切换系统:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{r(t)}\boldsymbol{x}(t) + \boldsymbol{B}_{r(t)}\boldsymbol{u}(t) + \boldsymbol{D}_{r(t)}\boldsymbol{u}(t-\tau), \tag{1}$$

其中, $\boldsymbol{x}(t) \in \mathscr{R}^n$ 为状态向量, $\boldsymbol{u}(t) \in \mathscr{R}^m$ 为输入函数, 分段定常函数 $r(t): \mathbf{R}^+ \to \{1, 2, \cdots, N\}$ 为切换路径, $\{\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{D}_i \mid i = 1, \cdots, N\}$ 为一族系统实现, 或称切换模式• 进一步, $r(t) = i$ 表示在时刻 t, 系统以模式 $(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{D}_i)$ 进行演变• $\tau > 0$ 为固定长度的时间滞后•

本文将研究系统(1)的能控性• 首先研究周期型系统的单周期能控性和多周期能控性, 给出几何形式的充要判据; 然后研究非周期型系统的能控性, 同样给出几何形式的充要判据•

在整个工作的第Ⅰ部分给出如下一些引理, 这些引理将作为讨论能控性的基本工具•

引理 1 给定矩阵 $\boldsymbol{A} \in \mathscr{R}^{n\times n}$, $\boldsymbol{B} \in \mathscr{R}^{n\times m}$, 对任意可逆矩阵 $\boldsymbol{P} \in \mathscr{R}^{m\times m}$, 都有

$$\langle \boldsymbol{A} \mid \boldsymbol{BP} \rangle = \langle \boldsymbol{A} \mid \boldsymbol{B} \rangle• \tag{2}$$

引理 2 给定矩阵 $\boldsymbol{A} \in \mathscr{R}^{n\times n}$, $\boldsymbol{B} \in \mathscr{R}^{n\times m}$, 对任意常数 $h \in \mathbf{R}$, 有

$$\exp(\boldsymbol{A}h)\langle \boldsymbol{A} \mid \boldsymbol{B} \rangle = \langle \boldsymbol{A} \mid \boldsymbol{B} \rangle, \tag{3}$$

$$\langle \boldsymbol{A} \mid \exp(\boldsymbol{A}h)\boldsymbol{B} \rangle = \langle \boldsymbol{A} \mid \boldsymbol{B} \rangle• \tag{4}$$

* 收稿日期: 2002_01_29; 修订日期: 2003_05_28
基金项目: 国家杰出青年科学基金资助项目(69925307, 60274001); 国家重点基础研究与发展计划基金资助项目(2002CB312200); 中国博士后基金资助项目
作者简介: 谢广明(1972—), 男, 北京人, 博士(后), 研究方向为混杂切换系统、广义系统、时滞系统和网络控制系统(E_mail: xiegming@mech.pku.edu.cn)•

引理 3(分离引理) 给定矩阵 $\boldsymbol{A}_1, \boldsymbol{A}_2 \in \mathscr{R}^{n\times n}$, $\boldsymbol{B}_1, \boldsymbol{B}_2 \in \mathscr{R}^{n\times m}$, 有

$$\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle + \langle \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle = \langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1\rangle + \langle \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle \cdot \tag{5}$$

引理 4 给定矩阵 $\boldsymbol{A}_1, \cdots, \boldsymbol{A}_N \in \mathscr{R}^{n\times n}$ 和 $\boldsymbol{B}_1, \cdots, \boldsymbol{B}_N \in \mathscr{R}^{n\times m}$, 对任意 $0 \leqslant t_0 < t_f < +\infty$, 有

$$\left\{\boldsymbol{x} \mid \boldsymbol{x} = \sum_{i=1}^{N}\int_{t_0}^{t_f}\exp(\boldsymbol{A}_i(t_f - s))\boldsymbol{B}_i\boldsymbol{u}(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } \boldsymbol{u}\right\} = \langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N\rangle \cdot \tag{6}$$

特别地

$$\left\{\boldsymbol{x} \mid \boldsymbol{x} = \int_{t_0}^{t_f}\exp(\boldsymbol{A}_1(t_f - s))\boldsymbol{B}_1\boldsymbol{u}(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } \boldsymbol{u}\right\} = \langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1\rangle \cdot \tag{7}$$

引理 5 给定矩阵 $\boldsymbol{A}_{n\times n}$, 对几乎所有 $T > 0$, 对任意线性空间 $\mathscr{W} \subseteq \mathscr{R}$, 成立

$$\langle \boldsymbol{A} \mid \mathscr{W}\rangle = \langle \exp(\boldsymbol{A}T) \mid \mathscr{W}\rangle \tag{8}$$

关于不含时滞的线性切换系统的能控性、稳定性和镇定等问题, 可参考[1~21]•

1 周期型系统的能控性

在本节, 针对输入函数含单时滞的周期型系统, 我们将建立系统能控性的充要判据• 不失一般性, 我们选择切换序列 $\pi = \left\{(1, h_1), \cdots, (N, h_N)\right\}$ 作为系统的周期•

1.1 1_周期能控性

定义 1(1_周期能控性) 称系统(1)为 1_周期能控, 若对任意状态 $\boldsymbol{x}_0$, $\boldsymbol{x}_f$ 和初始控制函数 $\boldsymbol{u}_0(t)$, $t \in [t_0 - \tau, t_0]$, 存在一个分段连续函数 $\boldsymbol{u}(t)$ 使得系统状态由 $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$ 演变到 $\boldsymbol{x}(t_N) = \boldsymbol{x}_f$, 其中 $t_N = t_0 + \sum_{m=1}^{N} h_l$ •

对系统(1), 给定初始状态 $\boldsymbol{x}_0$, 和初始控制函数 $\boldsymbol{u}_0(t)$, $t \in [t_0 - \tau, t_0]$, 令 $t_m = t_0 + \sum_{l=1}^{m} h_l (m = 1, \cdots, N)$, 那么, 终止状态 $\boldsymbol{x}_f$ 可以表示为:

$$\boldsymbol{x}_f = \prod_{i=N}^{1}\exp(\boldsymbol{A}_ih_i)\boldsymbol{x}_0 + \sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(\boldsymbol{A}_lh_l)\int_{t_{i-1}}^{t_i}\exp[\boldsymbol{A}_i(t_i - s)](\boldsymbol{B}_i\boldsymbol{u}(s) + \boldsymbol{D}_i(s - \tau))\,\mathrm{d}s + \int_{t_{N-1}}^{t_N}\exp[\boldsymbol{A}_N(t_N - s)](\boldsymbol{B}_N\boldsymbol{u}(s) + \boldsymbol{D}_N\boldsymbol{u}(s - \tau))\,\mathrm{d}s \cdot \tag{9}$$

考虑到

$$\begin{aligned}
&\int_{t_{i-1}}^{t_i}\exp[\boldsymbol{A}_i(t_i - s)](\boldsymbol{B}_i(\boldsymbol{u})s + \boldsymbol{D}_i\boldsymbol{u}(s - \tau))\,\mathrm{d}s = \\
&\quad \int_{t_{i-1}}^{t_i}\exp[\boldsymbol{A}_i(t_i - s)]\boldsymbol{B}_i(\boldsymbol{u})s\,\mathrm{d}s + \int_{t_{i-1}}^{t_i}\exp[\boldsymbol{A}_i(t_i - s)]\boldsymbol{D}_i\boldsymbol{u}(s - \tau)\,\mathrm{d}s = \\
&\quad \int_{t_{i-1}}^{t_i}\exp[\boldsymbol{A}_i(t_i - s)]\boldsymbol{B}_i\boldsymbol{u}(s)\,\mathrm{d}s + \int_{t_{i-1}-\tau}^{t_i-\tau}\exp[\boldsymbol{A}_i(t_i - s)]\exp(-\boldsymbol{A}_i\tau)\boldsymbol{D}_i\boldsymbol{u}(s)\,\mathrm{d}s = \\
&\quad \int_{t_{i-1}-\tau}^{t_{i-1}}\exp[\boldsymbol{A}_i(t_i - s)]\exp(-\boldsymbol{A}_i\tau)\boldsymbol{D}_i\boldsymbol{u}(s)\,\mathrm{d}s + \\
&\quad \int_{t_{i-1}}^{t_i-\tau}\exp[\boldsymbol{A}_i(t_i - s)](\boldsymbol{B}_i + \exp(-\boldsymbol{A}_i\tau)\boldsymbol{D}_i)\boldsymbol{u}(s)\,\mathrm{d}s + \\
&\quad \int_{t_i-\tau}^{t_i}\exp[\boldsymbol{A}_i(t_i - s)]\boldsymbol{B}_i\boldsymbol{u}(s)\,\mathrm{d}s =
\end{aligned}$$

$$\exp(A_i h_i)\int_{t_{i-1}-\tau}^{t_{i-1}}\exp[A_i(t_{i-1}-s)]\exp(-A_i\tau)D_i u(s)\mathrm{d}s+$$
$$\int_{t_{i-1}}^{t_i-\tau}\exp[A_i(t_i-s)](B_i+\exp(-A_i\tau)D_i)u(s)\mathrm{d}s+$$
$$\int_{t_i-\tau}^{t_i}\exp[A_i(t_i-s)]B_i u(s)\mathrm{d}s,$$

于是,(9)可以改写为

$$x_f=\sum_{i=N}^{1}\exp(A_i h_i)\left\{x_0+\int_{t_0-\tau}^{t_0}\exp[A_1(t_0-s)]E_1 u_0(s)\mathrm{d}s\right\}+$$
$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(A_l h_l)\left\{\int_{t_{i-1}}^{t_i-\tau}\exp[A_i(t_i-s)]F_i u(s)\mathrm{d}s+\right.$$
$$\left.\int_{t_i-\tau}^{t_i}(\exp[A_i(t_i-s)]B_i+\exp[A_{i+1}(t_i-s)]E_{i+1})u(s)\mathrm{d}s\right\}+$$
$$\int_{t_{N-1}}^{t_N-\tau}\exp[A_N(t_N-s)](B_N+E_N)u(s)\mathrm{d}s+$$
$$\int_{t_N-\tau}^{t_N}\exp[A_N(t_N-s)]B_N u(s)\mathrm{d}s, \tag{10}$$

其中 $$E_i=\exp(-A_i\tau)D_i,\quad F_i=B_i+E_i\qquad(i=1,\cdots,N)\bullet \tag{11}$$

定义集合

$$\mathscr{C}_1=\left\{x\mid x=\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(A_l h_l)\left\{\int_{t_{i-1}}^{t_i-\tau}\exp[A_i(t_i-s)]F_i u(s)\mathrm{d}s+\right.\right.$$
$$\left.\int_{t_i-\tau}^{t_i}(\exp[A_i(t_i-s)]B_i+\exp[A_{i+1}(t_i-s)]E_{i+1})u(s)\mathrm{d}s\right\}+$$
$$\int_{t_{N-1}}^{t_N-\tau}\exp[A_N(t_N-s)](B_N+E_N)u(s)\mathrm{d}s+$$
$$\left.\int_{t_{N-1}-\tau}^{t_N}\exp[A_N(t_N-s)]B_N u(s)\mathrm{d}s\qquad\forall\ \text{分段连续}\ u\right\}\bullet \tag{12}$$

命题1 系统(1)为1_周期能控当且仅当 $\mathscr{C}_1=\mathscr{R}^n$•

证明 系统(1)为1_周期能控当且仅当对任意 x_0, x_f 和 u_0,存在 $u(t)$ 使得方程(10)有解•这显然等价于 $\mathscr{C}_1=\mathscr{R}^n$• (Q.E.D)

定理1(充分性) 系统(1)为1_周期能控,若

$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(A_l h_l)\langle A_i\mid F_i\rangle+\langle A_N\mid F_N\rangle=\mathscr{R}^n\bullet \tag{13}$$

证明 考虑集合

$$\mathscr{C}_1^*=\left\{x\mid x=\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(A_l h_l)\int_{t_{i-1}}^{t_i-\tau}\exp[A_i(t_i-s)]F_i u(s)\mathrm{d}s+\right.$$
$$\left.\int_{t_{N-1}}^{t_N-\tau}\exp[A_N(t_N-s)]F_N u(s)\mathrm{d}s,\ \forall\ \text{分段连续}\ u\right\}\bullet \tag{14}$$

易证 $\mathscr{C}_1^*\subseteq\mathscr{C}_1$• 实际上,有

$$\mathscr{C}_1^*=\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(A_l h_l)\left\{x\mid x=\int_{t_{i-1}}^{t_i-\tau}\exp[A_i(t_i-s)]F_i u(s)\mathrm{d}s,\ \forall\ \text{分段连续}\ u\right\}+$$

$$\left\{x \mid x = \int_{t_{N-1}}^{t_N - \tau} \exp[A_N(t_N - s)] F_N u(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\}.$$

由引理4，有

$$\mathscr{C}_1^* = \sum_{i=1}^{N-1}\prod_{l=N}^{i+1} \exp(A_l h_l)\langle A_i \mid F_i\rangle + \langle A_N \mid F_N\rangle,$$

因此，$\mathscr{C}_1^* = \mathscr{R}^n$ 蕴含 $\mathscr{C}_1 = \mathscr{R}^n$. (Q. E. D)

注1 对系统(1)，令 $\tau = 0$，我们得到如下的不含时滞的系统

$$\dot{x}(t) = A_{r(t)}x(t) + F_{r(t)}u(t), \tag{15}$$

[1]中给出了系统(15)单周期能控的充要条件，即为(13). 因此，定理1说明若不含时滞的系统能控，则含时滞的系统也必能控.

定理2(充分必要性) 系统(1)为1_周期能控的充要条件为

$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1} \exp(A_l h_l)\langle A_i \mid [B_i, D_i]\rangle + \langle A_N \mid [B_N, D_N]\rangle = \mathscr{R}^n. \tag{16}$$

证明 对(12)，由引理4，有

$$\mathscr{C}_1 = \sum_{i=1}^{N-1}\prod_{l=N}^{i+1} \exp(A_l h_l)\left\{x \mid x = \int_{t_{i-1}}^{t_i - \tau} \exp[A_i(t_i - s)] F_i u(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$

$$\left\{x \mid x = \int_{t_i - \tau}^{t_i} (\exp[A_i(t_i - s)] B_i +\right.$$

$$\left.\exp[A_{i+1}(t_i - s)] E_{i+1}) u(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$

$$\left\{x \mid x = \int_{t_{N-1}}^{t_N - \tau} \exp[A_N(t_N - s)] (B_N + E_N) u(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$

$$\left\{x \mid x = \int_{t_N - \tau}^{t_N} \exp[A_N(t_N - s)] B_N u(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} =$$

$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1} \exp(A_l h_l)(\langle A_i \mid F_i\rangle + \langle A_i \mid B_i + A_{i+1} \mid E_{i+1}\rangle) +$$

$$\langle A_N \mid (B_N + E_N)\rangle + \langle A_N \mid B_N\rangle. \tag{17}$$

下面，我们证明

$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1} \exp(A_l h_l)(\langle A_i \mid F_i\rangle + \langle A_i \mid B_i + A_{i+1} \mid E_{i+1}\rangle) +$$

$$\langle A_N \mid (B_N + E_N)\rangle + \langle A_N \mid B_N\rangle \equiv$$

$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1} \exp(A_l h_l)\langle A_i \mid [B_i, D_i]\rangle\langle A_N \mid [B_N, D_N]\rangle. \tag{18}$$

我们采用数学归纳法. 对 $N = 1$，由引理1和引理2，有

$$\langle A_1 \mid (B_1 + E_1)\rangle + \langle A_1 \mid B_1\rangle = \langle A_1 \mid [B_1, E_1]\begin{bmatrix} I & 0 \\ I & I\end{bmatrix}\rangle =$$

$$\langle A_1 \mid [B_1, E_1]\rangle = \langle A_1 \mid [B_1, D_1]\rangle,$$

因此，$N = 1$ 时(18)成立.

对 $N = 2$，由引理3，有

$$\exp(A_2 h_2)(\langle A_1 \mid B_1 + E_1\rangle + \langle A_1 \mid B_1 + A_2 \mid E_2\rangle) +$$

$$\langle A_2 \mid B_2 + E_2\rangle + \langle A_2 \mid B_2\rangle =$$

$$\exp(A_2h_2)(\langle A_1 \mid B_1 + E_1\rangle + \langle A_1 \mid B_1 + A_2 \mid E_2\rangle + \langle A_2 \mid E_2\rangle) + \langle A_2 \mid B_2\rangle =$$
$$\exp(A_2h_2)(\langle A_1 \mid B_1 + E_1\rangle + \langle A_1 \mid B_1\rangle + \langle A_2 \mid E_2\rangle) + \langle A_2 \mid B_2\rangle =$$
$$\exp(A_2h_2)(\langle A_1 \mid [B_1, E_1]\rangle + \langle A_2 \mid E_2\rangle) + \langle A_2 \mid B_2\rangle =$$
$$\exp(A_2h_2)(\langle A_1 \mid [B_1, D_1]\rangle + \langle A_2 \mid [B_2, D_2]\rangle\cdot$$

因此, $N = 2$ 时(18)成立•

假设对 $N-1$, (18)成立, 考虑

$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(A_lh_l)(\langle A_i \mid F_i\rangle + \langle A_i \mid B_i + A_{i+1} \mid E_{i+1}\rangle) +$$
$$\langle A_N \mid (B_N + E_N)\rangle + \langle A_N \mid B_N\rangle =$$
$$\prod_{l=N-1}^{2}\exp(A_lh_l)(\langle A_1 \mid F_1\rangle + \langle A_1 \mid B_1 + A_2 \mid E_2\rangle) +$$
$$\sum_{i=2}^{N-1}\prod_{l=N-1}^{i+1}\exp(A_lh_l)(\langle A_i \mid F_i\rangle +$$
$$\langle A_i \mid B_i + A_{i+1} \mid E_{i+1}\rangle) + \langle A_N \mid (B_N + E_N)\rangle + \langle A_N \mid B_N\rangle =$$
$$\prod_{l=N-1}^{2}\exp(A_lh_l)(\langle A_1 \mid F_1\rangle + \langle A_1 \mid B_1 + A_2 \mid E_2\rangle) +$$
$$\sum_{i=2}^{N-1}\prod_{l=N-1}^{i+1}\exp(A_lh_l)(\langle A_i \mid [B_i, D_i]\rangle + \langle A_N \mid [B_N, D_N]\rangle =$$
$$\prod_{l=N-1}^{2}\exp(A_lh_l)(\langle A_1 \mid F_1\rangle + \langle A_1 \mid B_1 + A_2 \mid E_2\rangle + \langle A_2 \mid E_2\rangle) +$$
$$\sum_{i=2}^{N-1}\prod_{l=N-1}^{i+1}\exp(A_lh_l)(\langle A_i \mid [B_i, D_i]\rangle + \langle A_N \mid [B_N, D_N]\rangle =$$
$$\prod_{l=N-1}^{2}\exp(A_lh_l)\langle A_1 \mid [B_1, D_1]\rangle +$$
$$\sum_{i=2}^{N-1}\prod_{l=N-1}^{i+1}\exp(A_lh_l)(\langle A_i \mid [B_i, D_i]\rangle + \langle A_N \mid [B_N, D_N]\rangle =$$
$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(A_lh_l)(\langle A_i \mid [B_i, D_i]\rangle + \langle A_N \mid [B_N, D_N]\rangle\cdot$$

因此, 对 N, (18) 也成立• 至此可知, 对任意 N, (18) 均成立•

那么, 由命题 1, 定理 2 的结论显然• (Q. E. D)

注 2 若令式(16)中所有 $D_i = 0$, 则其退化为[1]中不含时滞系统单周期能控的充要条件•

注 3 定理 2 表明含时滞系统能控并不能保证不含时滞系统能控, 这从另一角度说明时滞是不可忽略的重要现象•

1.2 多周期能控性

定义 2(m_周期能控性) 系统(1)为 m_周期能控, 若对任意 x_0, x_f 和 $u_0(t)$, $t \in [t_0 - \tau, t_0]$, 存在分段连续函数 $u(t)$ 使得系统由 $x(t_0) = x_0$ 演变到 $x(t_{mN}) = x_f$, 其中

$$t_{mN} = t_0 + m\sum_{l=1}^{N}h_l\cdot$$

注 4 系统(1)称为多周期能控, 若存在正整数 m 使得它为 m_周期能控•

定理 3(充分性) 系统(1)为 m_周期能控, 若

$$\mathscr{C}_1^* + \left(\prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i)\right)\mathscr{C}_1^* + \cdots + \left(\prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i)\right)^{m-1}\mathscr{C}_1^* = \mathscr{R}^n, \tag{19}$$

其中, $\mathscr{C}_1^* = \sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(\boldsymbol{A}_l h_l)\langle \boldsymbol{A}_i \mid \boldsymbol{F}_i\rangle + \langle \boldsymbol{A}_N \mid \boldsymbol{F}_N\rangle$•

证明　证明过程类似于定理 1 的证明, 限于篇幅略去•　　(Q. E. D)

注 5　[3] 中给出了系统(15) 单周期能控的充要条件, 即为(19)•　因此, 不含时滞系统的能控性保证了含时滞系统的能控性•

注 6　对任意 $m \geqslant n$, 系统(1) 为 m_ 周期能控, 若

$$\left\langle \prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i) \,\middle|\, \mathscr{C}_1^* \right\rangle = \mathscr{R}^n \tag{20}$$

定理 4(充分必要性)　系统(1) 为 m_ 周期能控的充分必要条件为

$$\mathscr{C}_1 + \left(\prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i)\right)\mathscr{C}_1 + \cdots + \left(\prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i)\right)^{m-1}\mathscr{C}_1 = \mathscr{R}^n• \tag{21}$$

其中, $\mathscr{C}_1 = \sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(\boldsymbol{A}_l h_l)(\langle \boldsymbol{A}_i \mid [\boldsymbol{B}_i, \boldsymbol{D}_i]\rangle + \langle \boldsymbol{A}_N \mid [\boldsymbol{B}_N, \boldsymbol{D}_N]\rangle)$ •

证明　证明过程类似于定理 1 的证明, 限于篇幅略去•　　(Q. E. D)

注 7　若令式(21) 中所有 $\boldsymbol{D}_i = \boldsymbol{0}$, 则其退化为[3] 中不含时滞系统多周期能控的充要条件•

注 8　对任意 $m \geqslant n$, 系统(1) 为 m_ 周期能控, 当且仅当

$$\left\langle \prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i) \,\middle|\, \mathscr{C}_1 \right\rangle = \mathscr{R}^n \tag{22}$$

注 9　若系统(1) 为多周期能控, 其能控性必可在 n 个周期内实现•

注 10　以上判据均为几何形式, 根据列空间和循环不变子空间的性质, 很容易变换到代数形式, 即验证某些矩阵是否行满秩•

2　非周期型系统的能控性

本节讨论非周期系统的能控性判定问题, 其结果在形式上类似于不含时滞的情形•

定义 3(状态能控性)　对系统(1), 给定状态 $\boldsymbol{x}_0$ 和初始控制函数 $\boldsymbol{u}_0(t)$, $t \in [t_0 - \tau, t_0]$, 称状态 $\boldsymbol{x}_f$ 为$(\boldsymbol{x}_0, \boldsymbol{u}_0)$ _能控, 若存在一个切换序列 $\pi = \{(i_m, h_m)\}_{m=1}^{M}$ 和一个分段连续函数 $\boldsymbol{u}(t)$, $t \in [t_0, t_M]$ 使得系统状态由 $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$ 演变到 $\boldsymbol{x}(t_M) = \boldsymbol{x}_f$, 其中 $t_M = t_0 + \sum_{m=1}^{M} h_l$ •

定义 4(系统能控性)　称系统(1) 为(完全) 能控, 若对任意状态 $\boldsymbol{x}_0$ 和初始控制函数 $\boldsymbol{u}_0(t)$, $t \in [t_0 - \tau, t_0]$, 任意状态 $\boldsymbol{x}_f$ 均为$(\boldsymbol{x}_0, \boldsymbol{u}_0)$ _能控•

下面首先引入能控集的概念, 然后基于能控集给出系统能控性的几何判据•

2.1　切换序列的控集

定义 5(能控集)　对系统(1), 给定状态 $\boldsymbol{x}_0$ 和初始控制函数 $\boldsymbol{u}_0(t)$, $t \in [t_0 - \tau, t_0]$, 给定切换序列 $\pi = \{(i_m, h_m)\}_{m=1}^{M}$, 称所有由 $\boldsymbol{x}_0$ 和 $\boldsymbol{u}_0(t)$ 出发, 经过切换序列 π 后所有可控的状态构成集合为切换序列 π 的$(\boldsymbol{x}_0, \boldsymbol{u}_0)$ _能控集, 记为 $\mathscr{C}(\boldsymbol{x}_0, \boldsymbol{u}_0, \pi)$•

给定状态 $\boldsymbol{x}_0$ 和初始控制函数 $\boldsymbol{u}_0(t)$, $t \in [t_0 - \tau, t_0]$, 经过切换序列 $\pi = \{(i_m, h_m)\}_{m=1}^{M}$ 的状态可表为

$$\boldsymbol{x}_f = \prod_{l=N}^{1}\exp(\boldsymbol{A}_{i_l} h_l)\left\{\boldsymbol{x}_0 + \int_{t_0-\tau}^{t_0}\exp[\boldsymbol{A}_{i_1}(t_0 - s)]\boldsymbol{E}_{i_1}\boldsymbol{u}_0(s)\mathrm{d}s\right\} +$$

$$\sum_{m=1}^{M-1}\prod_{l+M}^{m+1}\exp(\boldsymbol{A}_{i_l}h_l)\left\{\int_{t_{M-1}}^{t_M-\tau}\exp\left[\boldsymbol{A}_{i_m}(t_m-s)\right]\boldsymbol{F}_{i_m}\boldsymbol{u}(s)\mathrm{d}s+\right.$$
$$\left.\int_{t_M-\tau}^{t_M}(\exp\left[\boldsymbol{A}_{i_m}(t_{i_m}-s)\right]\boldsymbol{B}_{i_m}+\exp\left[\boldsymbol{A}_{i_{m+1}}(t_m-s)\right]\boldsymbol{E}_{i_{m+1}})\boldsymbol{u}(s)\mathrm{d}s\right\}+$$
$$\int_{t_{M-1}}^{t_M-\tau}\exp\left[\boldsymbol{A}_{i_M}(t_M-s)\right](\boldsymbol{B}_{i_M}+\boldsymbol{E}_{i_M})\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_M-\tau}^{t_M}\exp\left[\boldsymbol{A}_{i_M}(t_M-s)\right]\boldsymbol{B}_{i_M})\boldsymbol{u}(s)\mathrm{d}s, \tag{23}$$

其中 $$t_m=t_0+\sum_{l=1}^{m}h_l,\ \boldsymbol{E}_{i_m}=\exp(-\boldsymbol{A}_{i_m}h_m)\boldsymbol{D}_{i_m},\ \boldsymbol{F}_{i_m}=\boldsymbol{B}_{i_m}+\boldsymbol{E}_{i_m}\quad(m=1,\cdots,M), \tag{24}$$

于是我们得到如下结论•

定理 5 对系统(1),给定状态 $\boldsymbol{x}_0$ 和初始控制函数 $\boldsymbol{u}_0(t)$, $t\in[t_0-\tau, t_0]$,给定切换序列 $\pi=\{(i_m, h_m)\}_{m=1}^{M}$, 则切换序列 π 的 $(\boldsymbol{x}_0, \boldsymbol{u}_0)$_能控集 $\mathscr{C}(\boldsymbol{x}_0, \boldsymbol{u}_0, \pi)$ 为如下的集合:

$$\mathscr{C}(\boldsymbol{x}_0, \boldsymbol{u}_0, \pi)=\mathscr{T}n(\boldsymbol{x}_0, \boldsymbol{u}_0, \pi)+$$
$$\sum_{m=1}^{M-1}\prod_{l=M}^{m+1}\exp(\boldsymbol{A}_{i_l}h_l)\langle\boldsymbol{A}_{i_m}\mid[\boldsymbol{B}_{i_m}, \boldsymbol{D}_{i_m}]\rangle+\langle\boldsymbol{A}_{i_M}\mid[\boldsymbol{B}_{i_M}, \boldsymbol{D}_{i_M}]\rangle, \tag{25}$$

其中状态

$$\mathscr{T}n(\boldsymbol{x}_0, \boldsymbol{u}_0, \pi)=\prod_{l=N}^{1}\exp(\boldsymbol{A}_{i_l}h_l)\left\{\boldsymbol{x}_0+\int_{t_0-\tau}^{t_0}\exp\left[\boldsymbol{A}_{i_1}(t_0-s)\right]\boldsymbol{E}_{i_1}\boldsymbol{u}_0(s)\mathrm{d}s\right\}• \tag{26}$$

特别地,当 $\boldsymbol{x}_0=\boldsymbol{0}$, $\boldsymbol{u}_0=\boldsymbol{0}$ 时,有

$$\mathscr{C}(\boldsymbol{0}, \boldsymbol{0}, \pi)=\sum_{m=1}^{M-1}\prod_{l=M}^{m+1}\exp(\boldsymbol{A}_{i_l}h_l)\langle\boldsymbol{A}_{i_m}\mid[\boldsymbol{B}_{i_m}, \boldsymbol{D}_{i_m}]\rangle+\langle\boldsymbol{A}_{i_M}\mid[\boldsymbol{B}_{i_M}, \boldsymbol{D}_{i_M}]\rangle• \tag{27}$$

为 $\mathscr{R}^n$ 中的线性子空间,我们将其简写为 $\mathscr{C}(\pi)$•

证明 证明过程类似于定理 2 的证明过程,此略• (Q.E.D)

下面我们先给出 $\mathscr{C}(\pi)$ 的一些基本性质•

定义 6(切换序列的积) 给定切换序列 $\pi_1=\{(i_m, h_m)\}_{m=1}^{M}$ 和 $\pi_2=\{(j_m, g_m)\}_{m=1}^{L}$, 如下定义的切换序列

$$\pi_1\wedge\pi_2\overset{\mathrm{def}}{=}\{(i_1, h_1), \cdots, (i_M, h_M), (j_1, g_1), \cdots, (j_L, g_L)\}, \tag{28}$$

称为切换序列 π_1 和 π_2 的积•

显然,容易验证 $(\pi_1\wedge\pi_2)\wedge\pi_3=\pi_1\wedge(\pi_2\wedge\pi_3)$,我们将其简写为 $\pi_1\wedge\pi_2\wedge\pi_3$•

定义 7(切换序列的幂) 给定切换序列 π,如下定义的切换序列

$$\pi^{\wedge n}\overset{\mathrm{def}}{=}\underbrace{\pi\wedge\cdots\wedge\pi}_{n次}• \tag{29}$$

称为切换序列 π 的幂•

定义 8(切换序列的指数矩阵) 给定切换序列 $\pi=\{(i_m, h_m)\}_{m=1}^{M}$, 如下定义的矩阵

$$\exp(\pi)\overset{\mathrm{def}}{=}\prod_{m=M}^{1}\exp(\boldsymbol{A}_{i_m}h_m), \tag{30}$$

称为切换序列 π 的指数矩阵•

定理 6 给定切换序列 π_1 和 π_2, 有

$$\mathscr{C}(\pi_1 \wedge \pi_2) = \exp(\pi_2)\mathscr{C}(\pi_1) + \mathscr{C}(\pi_2) \cdot \tag{31}$$

证明　根据切换序列的积的定义以及能控集的定义容易验证• (Q. E. D)

定理 7　给定切换序列 π, 有

$$\mathscr{C}(\pi^{\wedge n}) = \langle \exp(\pi) \mid \mathscr{C}(\pi) \rangle \tag{32}$$

证明

$$\mathscr{C}(\pi^{\wedge n}) = \exp(\pi)\mathscr{C}(\pi^{\wedge (n-1)}) + \mathscr{C}(\pi) = [\exp(\pi)]^2\mathscr{C}(\pi^{\wedge(n-2)}) + \exp(\pi)\mathscr{C}(\pi) + \mathscr{C}(\pi) = \cdots\cdots = \sum_{i=1}^{n}[\exp(\pi)]^{(i-1)}\mathscr{C}(\pi) = \langle \exp(\pi) \mid \mathscr{C}(\pi)\rangle \cdot$$

(Q. E. D)

推论 1　给定切换序列 π, 有

$$\exp(\pi^{\wedge n})\mathscr{C}(\pi^{\wedge n}) = \mathscr{C}(\pi^{\wedge n}) \cdot \tag{33}$$

证明　由循环不变子空间的性质容易验证• (Q. E. D)

2.2　能控性的几何判据

对系统(1), 我们递归定义如下一个线性子空间序列:

$$\mathscr{W}_1 = \sum_{i=1}^{N}\langle \boldsymbol{A}_i \mid [\boldsymbol{B}_i, \boldsymbol{D}_i]\rangle,\ \mathscr{W}_2 = \sum_{i=1}^{N}\langle \boldsymbol{A}_i \mid \mathscr{W}_1\rangle,\ \ldots,\ \mathscr{W}_n = \sum_{i=1}^{N}\langle \boldsymbol{A}_i \mid \mathscr{W}_{n-1}\rangle \cdot \tag{34}$$

容易验证, 对任意的切换序列 π, 都有 $\mathscr{C}(\pi) \subseteq \mathscr{W}_n$•

定理 8　对系统(1), 必存在某一切换序列 π_b, 使得

$$\mathscr{C}(\pi_b) = \mathscr{W}_n \tag{35}$$

证明　由引理 5, 对每一个切换模式 $(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{D}_i)$, 必存在常数 $h_i > \tau$ 使得对任意的线性空间 $\mathscr{W}$, 都有 $\langle \boldsymbol{A}_i \mid \mathscr{W}\rangle = \langle \exp(\boldsymbol{A}_i h_i) \mid \mathscr{W}\rangle (i = 1, \ldots, N)$•　因此, 子空间序列 $\mathscr{W}_1, \ldots, \mathscr{W}_n$ 可以重新定义为

$$\begin{cases} \mathscr{W}_1 = \sum_{i=1}^{N}\langle \boldsymbol{A}_i \mid [\boldsymbol{B}_i, \boldsymbol{D}_i]\rangle,\ \mathscr{W}_2 = \sum_{i=1}^{N}\langle \exp(\boldsymbol{A}_i h_i) \mid \mathscr{W}_1\rangle,\ \ldots, \\ \mathscr{W}_n = \sum_{i=1}^{N}\langle \exp(\boldsymbol{A}_i h_i) \mid \mathscr{W}_{n-1}\rangle \cdot \end{cases} \tag{36}$$

设 $\dim(\mathscr{W}_n) = d$•　由(36), 必存在 d 个子空间 $\mathscr{X}_1, \ldots, \mathscr{X}_d$, 使得

$$\mathscr{W}_n = \sum_{m=1}^{d}\mathscr{X}_m,$$

并且每个子空间都具有如下形式

$$\prod_{m=1}^{M}\exp(\boldsymbol{A}_{i_m} h_{i_m})\langle \boldsymbol{A}_j \mid [\boldsymbol{B}_j, \boldsymbol{D}_j]\rangle, \tag{37}$$

其中 $M < \infty (i_1, \ldots, i_M, j \in \{1, \ldots, N\})$•

考虑形如(37)的线性子空间, 可以选择切换序列

$$\pi = \{(j, 1), (i_M, h_{i_M}), \ldots, (i_1, h_{i_1})\} \tag{38}$$

使得

$$\prod_{m=1}^{M}\exp(\boldsymbol{A}_{i_m} h_{i_m})\langle \boldsymbol{A}_j \mid [\boldsymbol{B}_j, \boldsymbol{D}_j]\rangle \subseteq \mathscr{C}(\pi) \cdot$$

因此, 我们可以选择切换序列 $\pi_1, \ldots, \pi_d$ 使得对 $m = 1, \ldots, d$, $\mathscr{X}_m \subseteq \mathscr{C}(\pi_m)$•　于是

$$\mathscr{W}_n = \sum_{m=1}^{d}\mathscr{C}(\pi_m) \tag{39}$$

下面我们给出构造切换序列 π_b 的过程.

首先，若 $\mathscr{C}(\pi_1^{\wedge n})=\mathscr{W}_n$，则我们可以令 $\pi_b=\pi_1^{\wedge n}$. 若否，则必存在某个切换序列 $k\in\{2,\cdots,d\}$，(不失一般性，令 $k=2$) 使得

$$\mathscr{C}(\pi_2)\not\subseteq\mathscr{C}(\pi_1^{\wedge n}),$$

既然 $\mathscr{C}(\pi_2\wedge\pi_1^{\wedge n})=\exp(\pi_1^{\wedge n})\mathscr{C}(\pi_2)+\mathscr{C}(\pi_1^{\wedge n})$,

由(33)，可得

$$\mathscr{C}(\pi_2\wedge\pi_1^{\wedge n})=\exp(\pi_1^{\wedge n})(\mathscr{C}(\pi_2)+\mathscr{C}(\pi_1^{\wedge n})),$$

于是 $\dim(\mathscr{C}(\pi_2\wedge\pi_2^{\wedge n}))=\dim(\mathscr{C}(\pi_2)+\mathscr{C}(\pi_2^{\wedge n}))\geqslant$

$$\dim(\mathscr{C}(\pi_1^{\wedge n}))+1=2.$$

由此，我们按如下过程构造切换序列:

$$\pi=\pi_1,\quad \pi_2=\pi_2\wedge\pi_1^{\wedge n},$$

$$\cdots\cdots,\quad \pi_d=\pi_d\wedge(\pi_{d-1})^{\wedge n}.$$

且令 $\pi_b=\pi_d$,

容易验证

$$\dim(\mathscr{C}(\pi_b))\geqslant d,$$

因此，$\mathscr{C}(\pi_b)=\mathscr{W}_n$. (Q. E. D)

推论 2(充分必要性) 系统(1)能控的充分必要条件为

$$\mathscr{W}_n=\mathscr{R}^n. \tag{40}$$

证明 由定理 8，必存在切换序列 $\pi_b=\{(i_m,h_m)\}_{m=1}^{M}$ 使得 $\mathscr{C}(\pi_b)=\mathscr{W}_n=\mathscr{R}^n$. 于是，对任意初始状态 $\boldsymbol{x}_0$ 和初始函数 $\boldsymbol{u}_0$，以及任意状态 $\boldsymbol{x}_f$，考虑状态

$$\boldsymbol{x}_f-\prod_{l=N}^{1}\exp(\boldsymbol{A}_{i_l}h_l)\left\{\boldsymbol{x}_0+\int_{t_0-\tau}^{t_0}\exp\left[\boldsymbol{A}_{i_1}(t_0-s)\right]\boldsymbol{E}_{i_1}\boldsymbol{u}_0(s)\mathrm{d}s\right\}\in\mathscr{C}(\pi_b).$$

必存在输入函数 $\boldsymbol{u}(t)$，使得

$$\begin{aligned}
&\boldsymbol{x}_f-\prod_{l=N}^{1}\exp(\boldsymbol{A}_{i_l}h_l)\left\{\boldsymbol{x}_0+\int_{t_0-\tau}^{t_0}\exp\left[\boldsymbol{A}_{i_1}(t_0-s)\right]\boldsymbol{E}_{i_1}\boldsymbol{u}_0(s)\mathrm{d}s\right\}=\\
&\sum_{m=1}^{M-1}\prod_{l=M}^{m+1}\exp(\boldsymbol{A}_{i_l}h_l)\left\{\int_{t_{m-1}}^{t_m-\tau}\exp\left[\boldsymbol{A}_{i_m}(t_m-s)\right]\boldsymbol{F}_{i_m}\boldsymbol{u}(s)\mathrm{d}s+\right.\\
&\left.\int_{t_m-\tau}^{t_m}(\exp\left[\boldsymbol{A}_{i_m}(t_{i_m}-s)\right]\boldsymbol{B}_{i_m}+\exp\left[\boldsymbol{A}_{i_{m+1}}(t_m-s)\right]\boldsymbol{E}_{i_{m+1}})\boldsymbol{u}(s)\mathrm{d}s\right\}+\\
&\int_{t_{M-1}}^{t_M-\tau}(\exp\left[\boldsymbol{A}_{i_M}(t_M-s)\right](\boldsymbol{B}_{i_M}+\boldsymbol{E}_{i_M})\boldsymbol{u}(s)\mathrm{d}s+\\
&\int_{t_M-\tau}^{t_M}(\exp\left[\boldsymbol{A}_{i_M}(t_M-s)\right]\boldsymbol{B}_{i_M}\boldsymbol{u}(s)\mathrm{d}s,
\end{aligned}$$

其中

$$t_m=t_0+\sum_{l=1}^{m}h_l,\quad \boldsymbol{E}_{i_m}=\exp(-\boldsymbol{A}_{i_m}h_m)\boldsymbol{D}_{i_m},\quad \boldsymbol{F}_{i_m}=\boldsymbol{B}_{i_m}+\boldsymbol{E}_{i_m}\qquad(m=1,\cdots,M).$$

亦即

$$\boldsymbol{x}_f=\prod_{l=N}^{1}\exp(\boldsymbol{A}_{i_l}h_l)\left\{\boldsymbol{x}_0+\int_{t_0-\tau}^{t_0}\exp\left[\boldsymbol{A}_{i_1}(t_0-s)\right]\boldsymbol{E}_{i_1}\boldsymbol{u}_0(s)\mathrm{d}s\right\}+$$

$$\sum_{m=1}^{M-1}\prod_{l=M}^{m+1}\exp(A_{i_l}h_l)\left\{\int_{t_{m-1}}^{t_m-\tau}\exp\left[A_{i_m}(t_m-s)\right]F_{i_m}u(s)\,ds+\right.$$
$$\left.\int_{t_m-\tau}^{t_m}\left(\exp\left[A_{i_m}(t_{i_m}-s)\right]B_{i_m}+\exp\left[A_{i_{m+1}}(t_m-s)\right]E_{i_{m+1}}\right)u(s)\,ds\right\}+$$
$$\int_{t_{M-1}}^{t_M-\tau}\left(\exp\left[A_{i_M}(t_M-s)\right]\right)(B_{i_M}+E_{i_M})u(s)\,ds+$$
$$\int_{t_M-\tau}^{t_M}\left(\exp\left[A_{i_M}(t_M-s)\right]\right)B_{i_M}u(s)\,ds•$$

根据定义可知系统必为完全能控•　(Q. E. D)

3　结　论

本文首次将时滞引入到线性切换系统中, 研究含单时滞的线性切换系统的能控性判定问题• 运用第I部分所给出的基本工具, 首先给出周期型系统单周期能控性的充要条件, 接着给出多周期能控性的充要条件, 最后给出非周期型系统能控性的充要条件• 在第II部分中将进一步讨论含多时滞情形线性切换系统的能控性的判定问题•

[参　考　文　献]

[1] Liberzon A S, Morse A S. Basic problems in stability and design of switched systems[J]. IEEE Contr Syst Mag, 1999, **19**(5): 59—70.

[2] Ezzine J. Haddad A H. Controllability and observability of hybrid system[J]. Int J Control, 1989, **49**(6): 2045—2055.

[3] SUN Zhen_dong, ZHENG Da_zhong. On reachability and stabilization of switched linear systems[J]. IEEE Trans Automat Contr, 2001, **46**(2): 291—295.

[4] 谢广明, 郑大钟. On the controllability and reachability of a class of hybrid dynamical systems[A]. 见: 秦化淑 编. 19届中国控制会议[C]. 香港: 香港工程师协会, 2000, 114—117.

[5] XIE Guang_ming, WANG Long. Necessary and sufficient conditions for controllability of switched linear systems[A]. In: American Automatic Control Counci Ed. Proceedings of the American Control Conference 2002[C]. USA: IEEE Service Center, 2002, 1897—1902.

[6] XU Xu_ping, Antsaklis P J. On the reachability of a class of second_order switched systems[A]. In: American Automatic Control Counci Ed. Proceedings of the American Control Conference 1999[C]. USA: IEEE Service Center, 1999, 2955—2959.

[7] Ishii H, Francis B A. Stabilization with control networks[J]. Automatica, 2002, **38**(10): 1745—1751.

[8] Ishii H, Francis B A. Stabilizing a linear system by switching control with dwell time[A]. In: American Automatic Control Counci Ed. Proceedings of the American Control Conference 2001[C]. USA: IEEE Service Center, 2001, 1876—1881.

[9] Morse A S. Supervisory control of families of linear set_point controllers_Part1: Exact matching[J]. IEEE Trans Automat Contr, 1996, **41**(7): 1413—1431.

[10] Liberzon D, Hespanha J P, Morse A S. stability of switched systems: a Lie_algebraic condition[J]. Systems and Control Letters, 1999, **37**(3): 117—122.

[11] Hespanha J P, Morse A S. Stability of switched systems with average dwell_time[A]. In: IEEE Control Systems Society Ed. Proceedings of the 38th Conference on Decesion and Control[C]. USA: IEEE Customer Service, 1999, 2655—2660.

[12] Narendra K S, Balakrishnan J. A common Lyapunov function for stable LTI systems with commuting A_matrices[J]. IEEE Trans Automat Contr, 1994, **39**(12): 2469—2471.

[13] Narendra K S, Balakrishnan J. Adaptive control using multiple models[J]. IEEE Trans Automat Contr, 1997, **42**(1): 171—187.

[14] Petterson S, Lennartson B. Stability and robustness for hybrid systems[A]. In: IEEE Control Systems Society Ed. Proceedings of the 35th Conference on Decesion and Control[C]. USA: IEEE Customer Service, 1996, 1202—1207.

[15] YE Hong, Michel A N, HOU Ling. Stability theory for hybrid dynamical systems[J]. IEEE Trans Automat Contr, 1998, **43**(4): 461—474.

[16] HU Bo, XU Xu_ping, Antsaklis P J, et al. Robust stabilizing control laws for a class of second_order switched systems[J]. Systems and Control Letters, 1999, **38**(2): 197—207.

[17] Branicky M S. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems [J]. IEEE Trans Automat Contr, 1998, **43**(4): 475—482.

[18] Shorten R N, Narendra K S. On the stability and existence of common Lyapunov functions for stable linear switching systems[A]. In: IEEE Control Systems Society Ed. Proceedings of the 37th Conference on Decesion and Control[C]. USA: IEEE Customer Service, 1998, 3723—3724.

[19] Johansson M, Rantzer A. Computation of piecewise quadratic Lyapunov funtions for hybrid systems [J]. IEEE Trans Automat Contr, 1998, **43**(4): 555—559.

[20] Wicks M A, Peleties P, DeCarlo R A. Construction of piecewise Lyapunov funtions for stabilizing switched systems[A]. In: IEEE Control Systems Society Ed. Proceedings of the 33th Conference on Decesion and Control[C]. USA: IEEE Customer Service, 1994, 3492—3497.

[21] Peleties P, DeCarlo R A. Asymptotic stability of m_switched systems using Lyapunov_like functions [A]. In: American Automatic Control Counci Ed. Proceedings of American Control Conference 1991 [C]. USA: IEEE Service Center, 1991, 1679—1684.

Controllability of a Class of Hybrid Dynamic Systems(Ⅱ) —Single Time_Delay Case

XIE Guang_ming, WANG Long, YE Qing_kai

(Department of Mechanics and Engineering Science, Peking University, Beijing 100871, P.R.China)

Abstract: The controllability for switched linear systems with time_delay in controls is first investigated. The whole work contains three parts. This is the second part. The definition and determination of controllability of switched linear systems with single time_delay in control functions is mainly investigated. The sufficient and necessary conditions for the 1_periodic, multiple_periodic controllability of periodic_type systems and controllability of aperiodic systems are presented, respectively.

Key words: hybrid dynamic system; switched linear system; time_delay; controllability; controllable set; switching sequence; switching path