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不确定离散脉冲系统的鲁棒 H_∞ 滤波问题*

潘圣韬， 孙继涛

(同济大学 数学系,上海 200092)

(戴世强推荐)

摘要: 研究一类离散脉冲不确定系统的鲁棒 H_∞ 滤波器设计问题. 首先,介绍了脉冲线性滤波器与离散脉冲系统的鲁棒 H_∞ 滤波问题. 其次,基于离散 Liapunov 函数方法,给出了用线性矩阵不等式表达的滤波误差系统渐近稳定的充分条件与 H_∞ 脉冲线性滤波器的设计方法. 最后,给出数值例子说明结果的有效性.

关 键 词: 离散; 脉冲; 鲁棒 H_∞ 滤波; 线性矩阵不等式

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引 言

系统中含有脉冲效应的脉冲系统广泛存在于神经网络、通信、医疗、生物等方面. 在过去的一些年里,脉冲系统的稳定性问题得到了越来越多的注意,在连续脉冲系统方面得到了很多好的结果,例如,文献[1-7]与其中的参考文献. 最近,虽然文献[8-9]研究了一般离散脉冲系统的稳定性问题,但对离散脉冲系统远还没有充分的研究.

另一方面,对许多动力系统具有不同性能指标(不同的物理意义)的状态估计问题已经被广泛地研究. 众所周知,不确定存在于许多实际系统之中,因为 H_∞ 滤波问题不仅允许噪声是具有界能量的任意信号,而且允许系统模型有不确定性,因此,在过去的十多年里受到了很大的注意. 过去的一些年里,相当多的兴趣在不确定线性离散系统的鲁棒滤波器问题上,也就是说,对不确定线性离散状态空间模型设计具有某种性能指标的滤波器,例如,文献[10-20]与其中的参考文献. 然而,据作者所知,对线性离散脉冲不确定系统还不存在有用的鲁棒 H_∞ 脉冲线性滤波器的设计方法,本文提供的脉冲线性滤波器的设计是新的.

本文研究一类离散脉冲不确定系统的鲁棒 H_∞ 滤波器设计问题. 本文的结构如下:第1节介绍了离散脉冲线性系统与离散脉冲线性滤波器问题. 在第2节,基于离散 Liapunov 函数方法与分析技巧,给出了滤波误差系统渐近稳定的充分条件,即对离散脉冲系统提供了 H_∞ 脉冲线性滤波器的设计方法,其滤波器增益用可解的线性矩阵不等式表达. 第3节,给出一个数值例子说明本文结果的有效性. 第4节,给出本文的结论.

符号:本文中,“T”表示矩阵的转置; R^n 表示 n 维 Euclid 空间; $R^{n\times m}$ 是 $n\times m$ 实矩阵的集合;

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基金项目: 国家自然科学基金资助项目(60874027)
作者简介: 潘圣韬(1981—),男,江苏宜兴人,硕士;
孙继涛(1963—),教授(联系人. Tel: +86-21-65983240-1307; E-mail: sunjt@sh163.net).

对 $P \in R^{n\times m}$, $P > 0(P \geqslant 0)$ 表示正定(半正定) 矩阵.

1 问题描述

我们考虑以下的离散脉冲系统:

$$\begin{cases} x_{k+1} = Ax_k + B\omega_k, & x_0 = 0,\ k \in \mathbf{Z},\ k \neq \tau_j, & (1) \\ x_{\tau_j+1} = Mx_{\tau_j}, & \tau_j \in \mathbf{Z},\ j \in \mathbf{Z}_+, & (2) \\ y_k = Cx_k + D\omega_k, & & (3) \\ z_k = L_1x_k + L_2\omega_k, & & (4) \end{cases}$$

其中, $x_k \in R^n$ 是状态向量, $y_k \in R^r$ 是测量输出, $z_k \in R^p$ 是被估计的目标信号, $\omega_k \in R^m$ 是噪声输入, $\tau_j \in \Xi = \{\tau_0, \tau_1, \tau_2, \cdots : \tau_0 < \tau_1 < \tau_2 < \cdots\} \subset \mathbf{Z}$ 是脉冲时刻, 并且对于任意的 $j \in \mathbf{Z}_+$, 当 $j \to \infty$ 时 $\tau_j \to \infty$.

系统(1)~(4)中, 当过程噪声与输入噪声不同时(实际上经常这样), 为 ω_{1k} 和 ω_{2k}, 我们可以简单地记 $B = [B_1 \quad 0]$, $D = [0 \quad D_1]$ 并设 $\omega_k = [\omega_{1k}^{\mathrm{T}} \quad \omega_{2k}^{\mathrm{T}}]^{\mathrm{T}}$.

A, B, M, C, D, L_1, L_2 是含有部分未知参数的维数适当的矩阵. 它们属于以下不确定的多面体:

$$\Omega = \Big\{(A, B, M, C, D, L_1, L_2) \mid (A, B, M, C, D, L_1, L_2) = \sum_{i=1}^{N} \alpha_i (A^{(i)}, B^{(i)}, M^{(i)}, C^{(i)}, D^{(i)}, L_1^{(i)}, L_2^{(i)}), \alpha_i \geqslant 0, \sum_{i=1}^{N} \alpha_i = 1\Big\}, \quad (5)$$

其中, $(A^{(i)}, B^{(i)}, M^{(i)}, C^{(i)}, D^{(i)}, L_1^{(i)}, L_2^{(i)})$, $i = 1, 2, \cdots, N$ 为已知的顶角矩阵.

我们的目的是设计如下形式的滤波器:

$$\begin{cases} \hat{x}_{k+1} = \hat{A}\hat{x}_k + \hat{B}y_k, & \hat{x}_0 = 0,\ k \neq \tau_j, & (6) \\ \hat{x}_{\tau_j+1} = \hat{M}\hat{x}_{\tau_j}, & \tau_j \in \mathbf{Z},\ j \in \mathbf{Z}_+, & (7) \\ \hat{z}_k = \hat{C}\hat{x}_k + \hat{D}y_k, & & (8) \end{cases}$$

其中, 矩阵 $(\hat{A}, \hat{B}, \hat{M}, \hat{C}, \hat{D})$ 待定. 上述结构的滤波器叫脉冲线性滤波器.

2 鲁棒 H_∞ 滤波问题

首先定义 $\xi_k = [x_k^{\mathrm{T}} \quad \hat{x}_k^{\mathrm{T}}]^{\mathrm{T}}$. 由系统(1)~(4)和滤波器(6)~(8)可以得到

$$\begin{cases} \xi_{k+1} = \bar{A}\xi_k + \bar{B}\omega_k, & k \neq \tau_j,\ \xi_0 = 0, & (9) \\ \xi_{\tau_j+1} = \bar{M}\xi_{\tau_j}, & \tau_j \in \mathbf{Z},\ j \in \mathbf{Z}_+, & (10) \\ z_k - \hat{z}_k = \bar{C}\xi_k + \bar{D}\omega_k, & & (11) \end{cases}$$

其中

$$\begin{cases} \bar{A} = \begin{pmatrix} A & 0 \\ \hat{B}C & \hat{A} \end{pmatrix}, \bar{B} = \begin{pmatrix} B \\ \hat{B}D \end{pmatrix}, \bar{M} = \begin{pmatrix} M & 0 \\ 0 & \hat{M} \end{pmatrix}, \\ \bar{C} = (L_1 - \hat{D}C \quad -\hat{C}),\ \bar{D} = L_2 - \hat{D}D. \end{cases} \quad (12)$$

则鲁棒 H_∞ 滤波问题可以描述如下:对不确定离散脉冲系统(1) 至(4) 与先给定的一个噪声水平 $\gamma > 0$, 设计一个脉冲线性滤波器(6) 至(8), 使得滤波误差系统(9) 至(11) 在多面体(5) 上是鲁棒渐近稳定的, 并且在零初始条件与对于所有的非零 $\omega \in l_2[0, +\infty)$, 以下不等式成立:

$$\| z - \hat{z} \|_2 \leqslant \gamma \| \omega \|_2.$$

引理 1 如果存在一个正定矩阵 $\boldsymbol{P} > 0$, 满足以下不等式组:

$$\widetilde{A}^{\mathrm{T}}\mathrm{diag}\{\boldsymbol{P},\boldsymbol{I}\}\widetilde{A} - \mathrm{diag}\{\boldsymbol{P},\gamma^2\boldsymbol{I}\} < 0, \tag{13}$$

$$\widetilde{M}^{\mathrm{T}}\mathrm{diag}\{\boldsymbol{P},\boldsymbol{I}\}\widetilde{M} - \mathrm{diag}\{\boldsymbol{P},\gamma^2\boldsymbol{I}\} < 0, \tag{14}$$

其中

$$\widetilde{A} = \begin{pmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{pmatrix}, \ \widetilde{M} = \begin{pmatrix} \bar{M} & \boldsymbol{0} \\ \bar{C} & \bar{D} \end{pmatrix}.$$

则系统(9)至(11)渐近稳定并且其 H_∞ 模小于 γ.

证明 由(13)和(14)式可得

$$\boldsymbol{P} > \bar{A}^{\mathrm{T}}\boldsymbol{P}\bar{A}, \ \boldsymbol{P} > \bar{M}^{\mathrm{T}}\boldsymbol{P}\bar{M}. \tag{15}$$

设 $\omega_k = \boldsymbol{0}$, 则(9)式与(10)式化为

$$\begin{cases} \xi_{k+1} = \bar{A}\xi_k, & k \neq \tau_j, \ \xi_0 = \boldsymbol{0}, \quad (16) \\ \xi_{\tau_j+1} = \bar{M}\xi_{\tau_j}, & \tau_j \in \mathbf{Z}, \ j \in \mathbf{Z}_+. \quad (17) \end{cases}$$

取 $V_k = \xi_k^{\mathrm{T}}\boldsymbol{P}\xi_k$, 并定义 $\Delta V_k = V_{k+1} - V_k$.

对任何 $k \in (\tau_j, \tau_{j+1})$, 有

$$\Delta V_k = V_{k+1} - V_k = \xi_k^{\mathrm{T}}(\bar{A}^{\mathrm{T}}\boldsymbol{P}\bar{A} - \boldsymbol{P})\xi_k.$$

由(15)式知, ΔV_k 是负定的,因此,存在一个 K - 类函数 $\beta(\cdot)$, 使得

$$\Delta V_k \leqslant -\beta(\| \xi_k \|). \tag{18}$$

另一方面,由(15)式,对 $k \in \{\tau_j\}_{j=1}^{\infty}$, 有

$$\Delta V_{\tau_j} = V_{\tau_j+1} - V_{\tau_j} = \xi_{\tau_j}^{\mathrm{T}}(\bar{M}^{\mathrm{T}}\boldsymbol{P}\bar{M} - \boldsymbol{P})\xi_{\tau_j} \leqslant 0. \tag{19}$$

为方便起见,设 $\tau_0 = 0$,则由(18) 和(19) 式,当 $k \in (\tau_j, \tau_{j+1}]$ 时,有

$$V_0 = V_{\tau_0} \geqslant V_k.$$

由(19)式,当 $k \in \{\tau_j\}_{j=1}^{\infty}$,可以得到 $V_{\tau_j+1} - V_{\tau_j} \leqslant 0$.

因此,对 $k \in \mathbf{Z}$,有 $V_k \leqslant V_{\tau_0} = V_0$. 因为 V_k 是正定的,当 $\xi_0 = \boldsymbol{0}$ 时, $V_0 = 0$,故存在一个 K- 类函数 $\alpha(\cdot)$, 使得

$$V_k \geqslant \alpha(\| \xi_k \|), \quad \forall k \in \mathbf{Z}.$$

另一方面,由 V_k 的定义,故对 $\varepsilon > 0$,存在 $\delta(\varepsilon, k) > 0$,使得 $V_0 < \alpha(\varepsilon)$ 当 $\| \xi_0 \| < \delta$ 时. 综上有

$$\alpha(\| \xi_k \|) \leqslant V_k \leqslant V_0 < \alpha(\varepsilon),$$

对 $k \in \mathbf{Z}$ 成立. 注意到 $\alpha(\cdot)$ 是一个 K- 类函数,故 $\| \xi_k \| < \varepsilon$, 因此,系统(16)和(17)的零解是稳定的.

进一步,由(18)和(19)式,对 $k \in \mathbf{Z}$, 有

$$V_k - V_{\tau_0} = \sum_{i=\tau_0}^{k-1}(V_{i+1} - V_i) \leqslant -\sum_{i=\tau_0}^{k-1}\beta(\| \xi_i \|).$$

另一方面,对 $k \in (\tau_0, \tau_1)$, (16)和(17)式的解满足

$$\sum_{i=\tau_0}^{k-1}(V_{i+1} - V_i) = V_k - V_{\tau_0};$$

对 $k \in (\tau_1, \tau_2)$,

$$\sum_{i=\tau_1+1}^{k-1}(V_{i+1}-V_i)=V_k-V_{\tau_1+1}.$$

由此，有

$$\sum_{i=\tau_m+1}^{k-1}(V_{i+1}-V_i)=V_k-V_{\tau_m+1},$$

对 $k\in(\tau_m,\tau_{m+1})$ 成立.

故对 $k\in(\tau_m,\tau_{m+1})$，有

$$V_k-V_0+\sum_{i=1}^{m}(V_{\tau_i}-V_{\tau_i+1})=\sum_{i=\tau_0}^{k}(V_{i+1}-V_i)\leqslant-\sum_{i=\tau_0}^{k}\beta(\|\xi_i\|).$$

又

$$0\leqslant V_k\leqslant V_0-\sum_{i=\tau_0}^{k}\beta(\|\xi_i\|),$$

因此

$$0\leqslant\sum_{i=\tau_0}^{k}\beta(\|\xi_i\|)\leqslant V_0.$$

两边取极限 $k\to\infty$，得

$$\sum_{i=\tau_0}^{\infty}\beta(\|\xi_i\|)\leqslant V_0.$$

所以

$$\lim_{k\to\infty}\beta(\|\xi_k\|)=0,$$

则 $\lim\limits_{k\to\infty}\xi_k=0$. 因此，系统(16)和(17)的零解是渐近稳定的.

由(13)式与(14)式，存在 η，$0<\eta<1$，使得

$$\begin{pmatrix}P & 0\\ 0 & (1-\eta)\gamma^2 I\end{pmatrix}>\begin{pmatrix}\bar{A} & \bar{B}\\ \bar{C} & \bar{D}\end{pmatrix}^{\mathrm T}\begin{pmatrix}P & 0\\ 0 & I\end{pmatrix}\begin{pmatrix}\bar{A} & \bar{B}\\ \bar{C} & \bar{D}\end{pmatrix},\tag{20}$$

$$\begin{pmatrix}P & 0\\ 0 & (1-\eta)\gamma^2 I\end{pmatrix}>\begin{pmatrix}\bar{M} & 0\\ \bar{C} & \bar{D}\end{pmatrix}^{\mathrm T}\begin{pmatrix}P & 0\\ 0 & I\end{pmatrix}\begin{pmatrix}\bar{M} & 0\\ \bar{C} & \bar{D}\end{pmatrix}.\tag{21}$$

为了证明 $\gamma^2\|\omega\|_2^2$ 是 $\|q\|_2^2(q_k\triangleq z_k-\hat{z}_k)$ 的一个上界，定义

$$J_k=\|q_k\|^2-(1-\eta)\gamma^2\|\omega_k\|^2.$$

于是

$$J_k=[\|q_k\|^2-(1-\eta)\gamma^2\|\omega_k\|^2+\Delta V_k]-\Delta V_k=$$

$$\begin{cases}\begin{pmatrix}\xi_k\\ \omega_k\end{pmatrix}^{\mathrm T}\left[(\bar{C}\ \ \bar{D})^{\mathrm T}(\bar{C}\ \ \bar{D})-\begin{pmatrix}P & 0\\ 0 & (1-\eta)\gamma^2 I\end{pmatrix}+(\bar{A}\ \ \bar{B})^{\mathrm T}P(\bar{A}\ \ \bar{B})\right]\begin{pmatrix}\xi_k\\ \omega_k\end{pmatrix}-\Delta V_k= \\ \quad\begin{pmatrix}\xi_k\\ \omega_k\end{pmatrix}^{\mathrm T}\left[\begin{pmatrix}\bar{A} & \bar{B}\\ \bar{C} & \bar{D}\end{pmatrix}^{\mathrm T}\begin{pmatrix}P & 0\\ 0 & I\end{pmatrix}\begin{pmatrix}\bar{A} & \bar{B}\\ \bar{C} & \bar{D}\end{pmatrix}-\begin{pmatrix}P & 0\\ 0 & (1-\eta)\gamma^2 I\end{pmatrix}\right]\begin{pmatrix}\xi_k\\ \omega_k\end{pmatrix}-\Delta V_k, \qquad k\neq\tau_j, \\ \begin{pmatrix}\xi_k\\ \omega_k\end{pmatrix}^{\mathrm T}\left[(\bar{C}\ \ \bar{D})^{\mathrm T}(\bar{C}\ \ \bar{D})-\begin{pmatrix}P & 0\\ 0 & (1-\eta)\gamma^2 I\end{pmatrix}+(\bar{M}\ \ 0)^{\mathrm T}P(\bar{M}\ \ 0)\right]\begin{pmatrix}\xi_k\\ \omega_k\end{pmatrix}-\Delta V_k= \\ \quad\begin{pmatrix}\xi_k\\ \omega_k\end{pmatrix}^{\mathrm T}\left[\begin{pmatrix}\bar{M} & 0\\ \bar{C} & \bar{D}\end{pmatrix}^{\mathrm T}\begin{pmatrix}P & 0\\ 0 & I\end{pmatrix}\begin{pmatrix}\bar{M} & 0\\ \bar{C} & \bar{D}\end{pmatrix}-\begin{pmatrix}P & 0\\ 0 & (1-\eta)\gamma^2 I\end{pmatrix}\right]\begin{pmatrix}\xi_k\\ \omega_k\end{pmatrix}-\Delta V_k, \qquad k=\tau_j.\end{cases}$$

由(20)与(21)式得 $J_k < -\Delta V_k$, 即

$$\| q_k \|^2 < (1-\eta)\gamma^2 \| \omega_k \|^2 - \Delta V_k.$$

则,有

$$\sum_{k=0}^{n} \| q_k \|^2 < (1-\eta)\gamma^2 \sum_{k=0}^{n} \| \omega_k \|^2 - V_{n+1} \leqslant$$
$$(1-\eta)\gamma^2 \| \omega \|_2^2 - V_{n+1}.$$

注意到以上不等式对所有的 $n > 0$ 成立. 令 $n \to \infty$,由 $\lim\limits_{k\to\infty} \xi_n = 0$ 可以得到

$$\| q \|_2^2 \leqslant (1-\eta)\gamma^2 \| \omega \|_2^2 < \gamma^2 \| \omega \|_2^2.$$

证毕.

引理 2 考虑系统(9)~(11),条件(13)与(14)成立的充要条件是存在矩阵 $\boldsymbol{P}, \boldsymbol{F}, \boldsymbol{G}(\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}})$, 使得以下不等式组成立:

$$\begin{pmatrix} -\operatorname{diag}\{\boldsymbol{P}, \gamma^2 \boldsymbol{I}\} + \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{F} + \boldsymbol{F}^{\mathrm{T}}\tilde{\boldsymbol{A}} & -\boldsymbol{F}^{\mathrm{T}} + \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{G} \\ -\boldsymbol{F} + \boldsymbol{G}^{\mathrm{T}}\tilde{\boldsymbol{A}} & \operatorname{diag}\{\boldsymbol{P}, \boldsymbol{I}\} - (\boldsymbol{G} + \boldsymbol{G}^{\mathrm{T}}) \end{pmatrix} < 0, \tag{22}$$

$$\begin{pmatrix} -\operatorname{diag}\{\boldsymbol{P}, \gamma^2 \boldsymbol{I}\} + \tilde{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{F} + \boldsymbol{F}^{\mathrm{T}}\tilde{\boldsymbol{M}} & -\boldsymbol{F}^{\mathrm{T}} + \tilde{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{G} \\ -\boldsymbol{F} + \boldsymbol{G}^{\mathrm{T}}\tilde{\boldsymbol{M}} & \operatorname{diag}\{\boldsymbol{P}, \boldsymbol{I}\} - (\boldsymbol{G} + \boldsymbol{G}^{\mathrm{T}}) \end{pmatrix} < 0. \tag{23}$$

证明 这个证明是直接的. 如果对于某个 $\boldsymbol{P} > 0$,(13)与(14)式成立,令 $\boldsymbol{F} = 0, \boldsymbol{G}^{\mathrm{T}} = \boldsymbol{G} = \operatorname{diag}\{\boldsymbol{P}, \boldsymbol{I}\}$, 并运用 Schur 补知,(22)与(23)式成立. 另一方面,如果对于某些 $\boldsymbol{P}, \boldsymbol{F}, \boldsymbol{G}$,(22)与(23)式成立,多次用 $\boldsymbol{\Gamma}^{\mathrm{T}}$ 和 $\boldsymbol{\Gamma}(\boldsymbol{\Gamma}^{\mathrm{T}} = [\boldsymbol{I} \quad \tilde{\boldsymbol{A}}^{\mathrm{T}}])$ 分别左乘和右乘(22)和(23)两个不等式,可以得到(13)与(14)式. 证毕.

引理 3 给定滤波器(6)至(8),如果存在矩阵 $\boldsymbol{P}^{(i)}, \boldsymbol{F}, \boldsymbol{G}(\boldsymbol{P}^{(i)} = \boldsymbol{P}^{(i)\mathrm{T}})$ 满足以下不等式组:

$$\begin{pmatrix} -\operatorname{diag}\{\boldsymbol{P}^{(i)}, \gamma^2 \boldsymbol{I}\} + \tilde{\boldsymbol{A}}^{(i)\mathrm{T}}\boldsymbol{F} + \boldsymbol{F}^{\mathrm{T}}\tilde{\boldsymbol{A}}^{(i)} & -\boldsymbol{F}^{\mathrm{T}} + \tilde{\boldsymbol{A}}^{(i)\mathrm{T}}\boldsymbol{G} \\ -\boldsymbol{F} + \boldsymbol{G}^{\mathrm{T}}\tilde{\boldsymbol{A}}^{(i)} & \operatorname{diag}\{\boldsymbol{P}^{(i)}, \boldsymbol{I}\} - (\boldsymbol{G} + \boldsymbol{G}^{\mathrm{T}}) \end{pmatrix} < 0, \tag{24}$$

$$\begin{pmatrix} -\operatorname{diag}\{\boldsymbol{P}^{(i)}, \gamma^2 \boldsymbol{I}\} + \tilde{\boldsymbol{M}}^{(i)\mathrm{T}}\boldsymbol{F} + \boldsymbol{F}^{\mathrm{T}}\tilde{\boldsymbol{M}}^{(i)} & -\boldsymbol{F}^{\mathrm{T}} + \tilde{\boldsymbol{M}}^{(i)\mathrm{T}}\boldsymbol{G} \\ -\boldsymbol{F} + \boldsymbol{G}^{\mathrm{T}}\tilde{\boldsymbol{M}}^{(i)} & \operatorname{diag}\{\boldsymbol{P}^{(i)}, \boldsymbol{I}\} - (\boldsymbol{G} + \boldsymbol{G}^{\mathrm{T}}) \end{pmatrix} < 0, \tag{25}$$

则系统(9)至(11)渐近稳定并且其 H_∞ 模小于 γ;其中 $i = 1, 2, \cdots, N$,

$$\tilde{\boldsymbol{A}}^{(i)} = \begin{pmatrix} \bar{\boldsymbol{A}}^{(i)} & \bar{\boldsymbol{B}}^{(i)} \\ \bar{\boldsymbol{C}}^{(i)} & \bar{\boldsymbol{D}}^{(i)} \end{pmatrix}, \quad \tilde{\boldsymbol{M}}^{(i)} = \begin{pmatrix} \bar{\boldsymbol{M}}^{(i)} & \boldsymbol{0} \\ \bar{\boldsymbol{C}}^{(i)} & \bar{\boldsymbol{D}}^{(i)} \end{pmatrix},$$

$\bar{\boldsymbol{A}}^{(i)}, \bar{\boldsymbol{B}}^{(i)}, \bar{\boldsymbol{C}}^{(i)}, \bar{\boldsymbol{D}}^{(i)}, \bar{\boldsymbol{M}}^{(i)}$ 是(12)式中多面体 Ω 的第 i 个顶角矩阵.

证明 考虑到

$$\begin{pmatrix} \bar{\boldsymbol{A}} & \bar{\boldsymbol{B}} \\ \bar{\boldsymbol{C}} & \bar{\boldsymbol{D}} \end{pmatrix} = \sum \alpha_i \begin{pmatrix} \bar{\boldsymbol{A}}^{(i)} & \bar{\boldsymbol{B}}^{(i)} \\ \bar{\boldsymbol{C}}^{(i)} & \bar{\boldsymbol{D}}^{(i)} \end{pmatrix},$$

$$\begin{pmatrix} \bar{\boldsymbol{M}} & \boldsymbol{0} \\ \bar{\boldsymbol{C}} & \bar{\boldsymbol{D}} \end{pmatrix} = \sum \alpha_i \begin{pmatrix} \bar{\boldsymbol{M}}^{(i)} & \boldsymbol{0} \\ \bar{\boldsymbol{C}}^{(i)} & \bar{\boldsymbol{D}}^{(i)} \end{pmatrix},$$

故(22)和(23)式成立. 由引理 2,(13)与(14)式也成立,再由引理 1,引理 3 成立. 证毕.

在(24)和(25)式中令

$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{\Lambda\Phi} & \boldsymbol{0} \\ \boldsymbol{0} & \varepsilon\boldsymbol{\Psi} \end{pmatrix}, \quad \boldsymbol{G} = \begin{pmatrix} \boldsymbol{\Phi} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Psi} \end{pmatrix}, \tag{26}$$

其中，$\boldsymbol{\Phi} \in R^{n\times n}$，$\boldsymbol{\Psi} \in R^{p\times p}$，$\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1 \boldsymbol{I}_n, \lambda_2 \boldsymbol{I}_n\}$，$\varepsilon$，$\lambda_1$ 和 λ_2 是实数.

由引理 3，下列结果给出鲁棒 H_∞ 滤波问题的解.

定理 1 考虑在多面体(5)上的系统(1)至(4)，如果对给定的 $\lambda_1, \lambda_2, \varepsilon$，满足以下线性矩阵不等式组(LMIs)的解 $(\hat{\boldsymbol{P}}_{11}^{(i)}, \hat{\boldsymbol{P}}_{12}^{(i)}, \hat{\boldsymbol{P}}_{22}^{(i)}, \boldsymbol{R}, \boldsymbol{W}, \boldsymbol{S}_A, \boldsymbol{S}_M, \boldsymbol{S}_B, \boldsymbol{S}_C, \boldsymbol{S}_D, \boldsymbol{T}, \boldsymbol{\Psi})$ 存在

$$\begin{pmatrix} \lambda_1(\boldsymbol{A}^{(i)\mathrm{T}}\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}^{(i)}) - \hat{\boldsymbol{P}}_{11}^{(i)} & \cdots & \cdots & \cdots & \cdots & \cdots \\ \lambda_1 \boldsymbol{W}^{\mathrm{T}}\boldsymbol{A}^{(i)} + \lambda_2(\boldsymbol{S}_B\boldsymbol{C}^{(i)} + \boldsymbol{S}_A) - \hat{\boldsymbol{P}}_{12}^{(i)\mathrm{T}} & -\lambda_2(\boldsymbol{S}_A + \boldsymbol{S}_A^{\mathrm{T}}) - \hat{\boldsymbol{P}}_{22}^{(i)} & \cdots & \cdots & \cdots & \cdots \\ \varepsilon(\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{L}_1^{(i)} - \boldsymbol{S}_D\boldsymbol{C}^{(i)} - \boldsymbol{S}_C) + \lambda_1\boldsymbol{B}^{(i)\mathrm{T}}\boldsymbol{R} & \varepsilon\boldsymbol{S}_C + \lambda_1\boldsymbol{B}^{(i)\mathrm{T}}\boldsymbol{W} + \lambda_2\boldsymbol{D}^{(i)\mathrm{T}}\boldsymbol{S}_B^{\mathrm{T}} & \boldsymbol{\Pi}_1 & \cdots & \cdots & \cdots \\ -\lambda_1\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}^{(i)} & -\lambda_1\boldsymbol{W} - \lambda_2\boldsymbol{T}^{\mathrm{T}} & \boldsymbol{R}^{\mathrm{T}}\boldsymbol{B}^{(i)} & \hat{\boldsymbol{P}}_{11}^{(i)} - (\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}) & \cdots & \cdots \\ \boldsymbol{W}^{\mathrm{T}}\boldsymbol{A}^{(i)} + \boldsymbol{S}_B\boldsymbol{C}^{(i)} + \boldsymbol{S}_A & \lambda_2\boldsymbol{T}^{\mathrm{T}} - \boldsymbol{S}_A & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{B}^{(i)} + \boldsymbol{S}_B\boldsymbol{D}^{(i)} & \hat{\boldsymbol{P}}_{12}^{(i)\mathrm{T}} - (\boldsymbol{W}^{\mathrm{T}} + \boldsymbol{T}) & \boldsymbol{\Pi}_2 & \cdots \\ \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{L}_1^{(i)} - \boldsymbol{S}_D\boldsymbol{C}^{(i)} - \boldsymbol{S}_C & \boldsymbol{S}_C & -\varepsilon\boldsymbol{\Psi} + \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{L}_2^{(i)} - \boldsymbol{S}_D\boldsymbol{D}^{(i)} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Pi}_3 \end{pmatrix} < 0, \tag{27}$$

$$\begin{pmatrix} \lambda_1(\boldsymbol{M}^{(i)\mathrm{T}}\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{M}^{(i)}) - \hat{\boldsymbol{P}}_{11}^{(i)} & \cdots & \cdots & \cdots & \cdots & \cdots \\ \lambda_1 \boldsymbol{W}^{\mathrm{T}}\boldsymbol{M}^{(i)} + \lambda_2\boldsymbol{S}_M - \hat{\boldsymbol{P}}_{12}^{(i)\mathrm{T}} & -\lambda_2(\boldsymbol{S}_M + \boldsymbol{S}_M^{\mathrm{T}}) - \hat{\boldsymbol{P}}_{22}^{(i)} & \cdots & \cdots & \cdots & \cdots \\ \varepsilon(\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{L}_i^{(i)} - \boldsymbol{S}_D\boldsymbol{C}^{(i)} - \boldsymbol{S}_C) & \varepsilon\boldsymbol{S}_C & \boldsymbol{\Pi}_1 & \cdots & \cdots & \cdots \\ -\lambda_1\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{M}^{(i)} & -\lambda_1\boldsymbol{W} - \lambda_2\boldsymbol{T}^{\mathrm{T}} & \boldsymbol{0} & \hat{\boldsymbol{P}}_{11}^{(i)} - (\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}) & \cdots & \cdots \\ \boldsymbol{W}^{\mathrm{T}}\boldsymbol{M}^{(i)} + \boldsymbol{S}_M & \lambda_2\boldsymbol{T}^{\mathrm{T}} - \boldsymbol{S}_M & \boldsymbol{0} & \hat{\boldsymbol{P}}_{12}^{(i)} - (\boldsymbol{W}^{\mathrm{T}} + \boldsymbol{T}) & \boldsymbol{\Pi}_2 & \cdots \\ \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{L}_1^{(i)} - \boldsymbol{S}_D\boldsymbol{C}^{(i)} - \boldsymbol{S}_C & \boldsymbol{S}_C & -\varepsilon\boldsymbol{\Psi} + \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{L}_2^{(i)} - \boldsymbol{S}_D\boldsymbol{D}^{(i)} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Pi}_3 \end{pmatrix} < 0, \tag{28}$$

则鲁棒 H_∞ 滤波问题中具有形式(6)至(8)的脉冲线性滤波器存在．其中 $i = 1,2,\cdots,N$，

$$\boldsymbol{\Pi}_1 = -\gamma^2\boldsymbol{I} + \varepsilon(\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{L}_2^{(i)} + \boldsymbol{L}_2^{(i)\mathrm{T}}\boldsymbol{\Psi} - \boldsymbol{S}_D\boldsymbol{D}^{(i)} - \boldsymbol{D}^{(i)\mathrm{T}}\boldsymbol{S}_D^{\mathrm{T}}),$$

$$\boldsymbol{\Pi}_2 = \hat{\boldsymbol{P}}_{22}^{(i)} + (\boldsymbol{T} + \boldsymbol{T}^{\mathrm{T}}),\ \boldsymbol{\Pi}_3 = \boldsymbol{I} - (\boldsymbol{\Psi} + \boldsymbol{\Psi}^{\mathrm{T}}).$$

这种情况下，一个合适的 H_∞ 滤波器参数由以下的等式给出：

$$\hat{\boldsymbol{A}} = \boldsymbol{T}^{-1}\boldsymbol{S}_A,\ \hat{\boldsymbol{B}} = \boldsymbol{T}^{-1}\boldsymbol{S}_B,\ \hat{\boldsymbol{C}} = \boldsymbol{\Psi}^{-\mathrm{T}}\boldsymbol{S}_C,\ \hat{\boldsymbol{D}} = \boldsymbol{\Psi}^{-\mathrm{T}}\boldsymbol{S}_D,\ \hat{\boldsymbol{M}} = \boldsymbol{T}^{-1}\boldsymbol{S}_M.$$

注 1 给定 ε，λ_1 和 λ_2 后，(27)与(28)式是线性矩阵不等式，因此，可以用文献[21]给出的 LMI 工具求解.

3 数值例子

考虑以下离散脉冲系统:

$$\begin{cases} x_{k+1} = \begin{pmatrix} 1 & 0.01 \\ -0.05 & -0.1 \end{pmatrix} x_k + \begin{pmatrix} 1 \\ 0.1 \end{pmatrix} \omega_k, & x_0 = \mathbf{0},\ k \in \mathbf{Z},\ k \neq \tau_j, \\ x_{\tau_j+1} = \begin{pmatrix} 1 & 0 \\ 0 & 0.8 \end{pmatrix} x_{\tau_j}, & \tau_j \in \mathbf{Z},\ j \in \mathbf{Z}_+, \\ y_k = (1 \quad 0)\, x_k + \omega_k, \\ z_k = (1 \quad 0)\, x_k. \end{cases}$$

令 $\gamma = 0.5$,当取 $\lambda_1 = 0.2, \lambda_2 = -0.3, \varepsilon = 0.1$ 的时候,我们通过计算软件得到问题的一个解,其滤波器的参数为

$$\hat{A} = \begin{pmatrix} 0.2955 & 0.1235 \\ 1.1128 & 0.1294 \end{pmatrix},\ \hat{B} = \begin{pmatrix} 1.2788 \\ -0.0368 \end{pmatrix},\ \hat{M} = \begin{pmatrix} 0.4198 & -0.1499 \\ -1.1959 & 0.0066 \end{pmatrix},$$
$$\hat{C} = (0.0440 \quad -0.0319),\ \hat{D} = 0.9551.$$

4 结　论

本文研究了一类离散脉冲不确定系统的 H_∞ 滤波器设计问题. 基于离散 Liapunov 函数与分析技巧,给出了 H_∞ 脉冲线性滤波器的设计方法. 数值例子说明了本文结果的有效性.

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Robust H_∞ Filtering for Discrete-Time Impulsive Systems With Uncertainty

PAN Sheng-tao, SUN Ji-tao

(*Department of Mathematics*, *Tongji University*, *Shanghai* 200092, *P. R. China*)

Abstract: Robust filter design for linear discrete-time impulsive systems with uncertainty under H_∞ performance is investigated. First, an impulsive linear filter and robust H_∞ filtering problem were introduced for a discrete-time impulsive systems. Then, sufficient condition of asymptotical stability and H_∞ performance for the filtering error system was provided by discrete-time Liapunov function method, and the filter gains can be obtained by solving a set of linear matrix inequalities (LMIs). Finally, a numerical example was also included to illustrate the effectiveness of the proposed result.

Key words: discrete-time; impulses; robust H_∞ filtering; linear matrix inequalities