

# $N$ 个异结构混沌系统的环链耦合同步<sup>\*</sup>

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**摘要:** 提出了一种通过环链耦合实现  $N$  个异结构混沌系统同步的方法. 以 New 系统、Chen 系统、L 系统、Lorenz 系统和 Rössler 系统作为典型的例子, 验证了这种同步控制方法的有效性. 利用 Liapunov 稳定性定理, 构造控制器的具体形式, 并确定了耦合系数的取值范围. 仿真模拟结果表明, 在控制器的作用下, 选择适当的耦合系数值, 可以同时使  $N$  个异结构混沌系统达到完全同步.

**关键词:** 混沌同步; Liapunov 稳定性定理; 环链耦合; 异结构系统; 耦合系数

**中图分类号:** O415.5      **文献标识码:** A

## 引 言

自 1990 年 Pecora 和 Carroll 提出了混沌同步的概念并首先在电子线路中得到实现以来<sup>[1]</sup>, 混沌同步以其在保密通信、物理、生物等领域所显示的巨大应用潜力, 而引起了人们在研究混沌系统的同步问题上的极大兴趣, 从而成为非线性科学中的一个极其重要的研究领域. 近年来, 国内外出现了多种混沌同步的方法, 主要有驱动响应法<sup>[1]</sup>、主动被动法<sup>[2-4]</sup>、自适应法<sup>[5-9]</sup>、变量反馈法<sup>[10-15]</sup>、变量耦合法<sup>[16-23]</sup>等. 按照混沌同步的目标又可以划分为完全同步<sup>[1]</sup>、广义同步<sup>[24-26]</sup>以及投影同步<sup>[27-29]</sup>. 文献[16]研究了两个统一混沌系统的线性耦合, 利用 Liapunov 稳定性定理和线性矩阵不等式(LMI)的方法, 给出了新的稳定性判据. 文献[17]以 Rössler 等系统为例, 研究了两个同结构混沌系统的双向耦合. 但是, 上述文献中的耦合系数大都需要繁杂的数学推导才能确定, 且研究的均是同结构混沌系统的同步问题. 同结构混沌系统具有相同的动力学方程和系统参数, 只是初值稍有不同, 所以实现同结构混沌系统的同步只要抑制由于混沌系统对初值的敏感性引起的轨道分离, 而异结构混沌同步不仅要抑制由于混沌系统对初值的敏感性引起的轨道分离, 而且更要抑制由于混沌系统不同的分岔特性和不同的吸引域引起的轨道分离. 所以同结构混沌系统的同步较容易实现, 但局限性较大, 两个完全相同的系统在实际中很难找到. 同时, 上述方法无法使多个混沌系统同时达到混沌同步.

本文提出了一种通过环链耦合实现多个异结构混沌系统同步的方法. 以 New 系统、Chen 系统、L 系统、Lorenz 系统和 Rössler 系统为例, 利用 Liapunov 稳定性定理, 通过构造适当的控制器并确定了耦合参数的取值范围, 使多个异结构的混沌系统同时达到完全同步. 仿真模拟验

\* 收稿日期: 2007-12-19; 修订日期: 2008-08-25

基金项目: 辽宁省自然科学基金资助项目(20052151); 辽宁省教育厅创新团队计划资助项目

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证了该方法的有效性和可行性. 这种方法把传统的目标系统和响应系统这样一对一的混沌同步法, 拓展为一对多系统的混沌同步, 以较低的代价满足了同步的需要, 具有更好的灵活性和实用性, 为同时进行多方通讯提供了可能.

## 1 同步原理

对于任意的  $N$  个异结构的混沌系统

$$\dot{X} = AX + f(X), \dot{Y} = BY + g(Y), \dot{Z} = CZ + h(Z), \dots \quad (1)$$

这里,  $X = \{x_1, x_2 \dots x_n\}$ ,  $Y = \{y_1, y_2 \dots y_n\}$ , ... 为  $N$  个混沌系统的状态变量,  $A, B, C, \dots \in R^{n \times n}$  为常数矩阵,  $f, g, h, \dots: R^n \rightarrow R^n$  为非线性连续函数.

对上述  $N$  个系统进行如下的环链耦合, 将第 1 个系统的状态变量与第 2 个系统的状态变量相耦合, 第 2 个系统的状态变量与第 3 个系统的状态变量相耦合 ... 依此类推, 至第  $k$  个系统的状态变量与第 1 个系统的状态变量相耦合, 如此闭环耦合使得  $k$  个系统构成一个闭合的环路. 在此基础上, 对链上的系统依次进行耦合. 将第  $k+1$  个系统的状态变量与第 1 个系统的状态变量相耦合, 第  $k+2$  个系统的状态变量与第  $k+1$  个系统的状态变量相耦合 ... 依此类推, 至第  $N$  个系统的状态变量与第  $N-1$  个系统的状态变量相耦合, 如此环链耦合使得  $N$  个系统相关联. 这样, 方程(1) 经环链耦合后变为

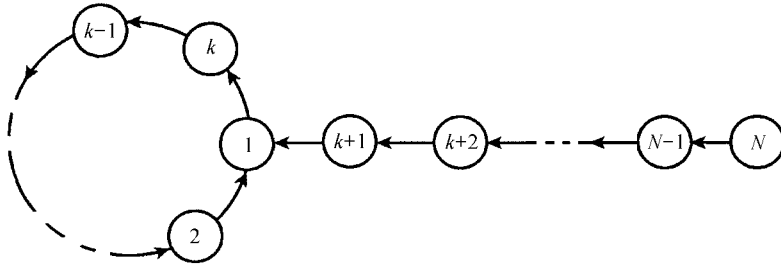


图 1  $N$  个系统的耦合方式

$$\begin{cases} \dot{X} = AX + f(X) + D_1(W - X), \\ \dot{Y} = BY + g(Y) + D_2(X - Y), \\ \dot{Z} = CZ + h(Z) + D_3(Y - Z), \\ \vdots \end{cases} \quad (2)$$

这里,  $D_1, D_2, D_3, \dots$  为耦合系数. 任意选择其中一个系统作为目标系统, 对余下的  $N-1$  个系统上施加控制器. 例如, 选择第一个系统作为目标系统, 式(2) 将变为

$$\begin{cases} \dot{X} = AX + f(X) + D_1(W - X), \\ \dot{Y} = BY + g(Y) + D_2(X - Y) + u_1(t), \\ \dot{Z} = CZ + h(Z) + D_3(Y - Z) + u_2(t), \\ \vdots \end{cases} \quad (3)$$

定义误差系统

$$\begin{bmatrix} e_1 \\ e_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} X - Y \\ Y - Z \\ \vdots \end{bmatrix}, \quad (4)$$

则

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} A - D_2 & 0 & \dots & 0 \\ 0 & B - D_3 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} f(X) - g(Y) + D_1(W - X) + (A - B)Y - u_1(t) \\ g(Y) - h(Z) + D_2(X - Y) + (B - C)Z + u_1(t) - u_2(t) \\ \vdots \end{bmatrix}. \quad (5)$$

构造 Liapunov 函数

$$V = \frac{1}{2} \sum_{i=1}^{N-1} e_i^2. \quad (6)$$

当控制器设计为

$$\begin{cases} u_1(t) = f(X) - g(Y) + D_1(W - X) + (A - B)Y, \\ u_2(t) = g(Y) - h(Z) + D_2(X - Y) + (B - C)Z + u_1(t), \\ \vdots \end{cases} \quad (7)$$

有

$$\dot{V} = \sum_{i=1}^{N-1} e_i \dot{e}_i = (A - D_2)e_1^2 + (B - D_3)e_2^2 + \dots \quad (8)$$

根据 Liapunov 定理, 容易看出, 只要耦合参数的选择满足

$$A - D_2 \leq 0, B - D_3 \leq 0, \dots, \quad (9)$$

就有

$$\dot{V} \leq 0, \quad (10)$$

即实现了 N 个异结构系统的同步.

以 New 系统、Chen 系、L 系统、Lorenz 系统和 Rössler 系统这 5 个异结构的混沌系统为例, 对上述环链耦合同步控制原理加以说明. 其耦合方式如图 2 所示.

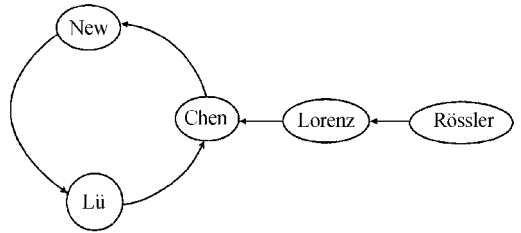


图 2 5 个系统的耦合方式

New 系统

$$\begin{cases} \dot{x}_1 = -x_2 x_3 + a_1 x_1, \\ \dot{x}_2 = x_1 x_3 + b_1 x_2, \\ \dot{x}_3 = (1/3)x_1 x_2 + c_1 x_3, \end{cases} \quad (11)$$

其中,  $a_1, b_1, c_1$  为系统参数, 当  $a_1 = 5, b_1 = -10, c_1 = -3.8$  时, 系统(11) 处于混沌状态, 其混沌吸引子如图 3(a) 所示.

Chen 系统

$$\begin{cases} \dot{y}_1 = a_2(y_2 - y_1), \\ \dot{y}_2 = (c_2 - a_2)y_1 - y_1 y_3 + c_2 y_2, \\ \dot{y}_3 = y_1 y_2 - b_2 y_3, \end{cases} \quad (12)$$

其中,  $a_2, b_2, c_2$  为系统参数, 当  $a_2 = 35, b_2 = 3, c_2 = 28$  时, 系统(12) 处于混沌状态, 其混沌吸引子如图 3(b) 所示.

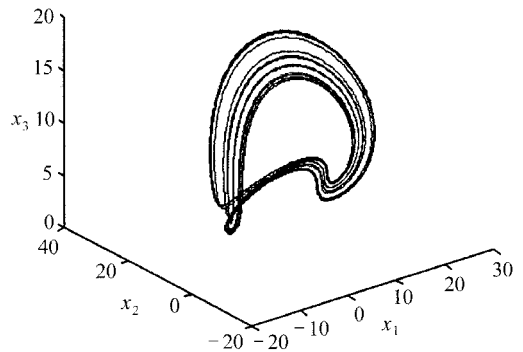


图 3(a) New 系统的混沌吸引子

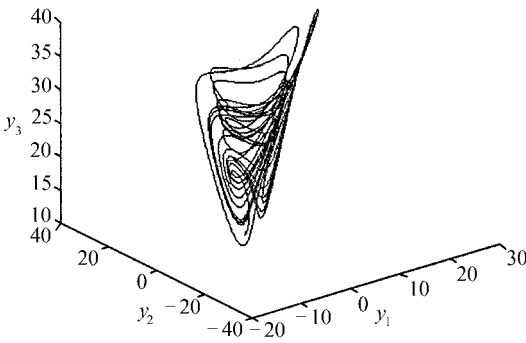


图 3(b) Chen系统的混沌吸引子

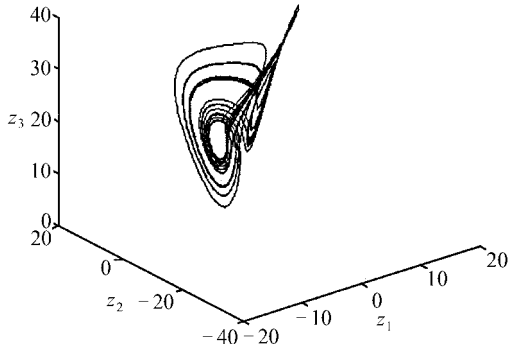


图 3(c) L系统的混沌吸引子

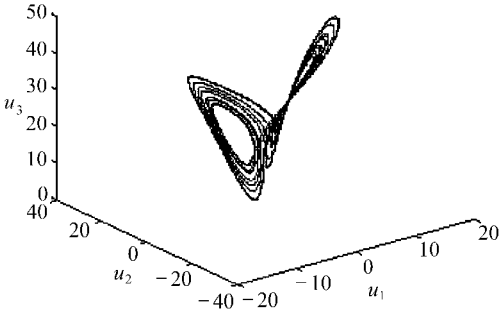


图 3(d) Lorenz系统的混沌吸引子

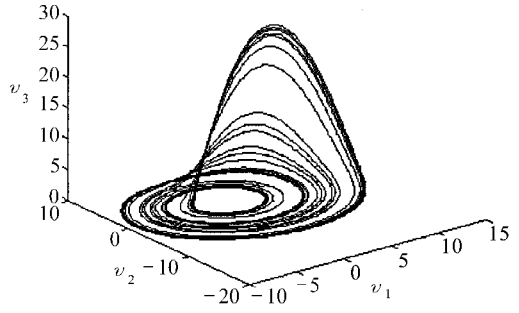


图 3(e) R-ssler系统的混沌吸引子

L 系统

$$\dot{z}_1 = a_3(z_2 - z_1), \quad \dot{z}_2 = -z_1z_3 + c_3z_2, \quad \dot{z}_3 = z_1z_2 - b_3z_3, \tag{13}$$

其中,  $a_3, b_3, c_3$  为系统参数, 当  $a_3 = 36, b_3 = 3, c_3 = 20$  时, 系统(13)处于混沌状态, 其混沌吸引子如图 3(c)所示.

Lorenz 系统

$$\dot{w}_1 = a_4(w_2 - w_1), \quad \dot{w}_2 = w_1(c_4 - w_3) - w_2, \quad \dot{w}_3 = w_1w_2 - b_4w_3, \tag{14}$$

其中,  $a_4, b_4, c_4$  为系统参数, 当  $a_4 = 10, b_4 = 8/3, c_4 = 28$  时, 系统(14)处于混沌状态, 其混沌吸引子如图 3(d)所示.

R<sup>L</sup>-ssler 系统

$$\dot{v}_1 = -v_2 - v_3, \quad \dot{v}_2 = v_1 + a_5v_2, \quad \dot{v}_3 = b_5 + (v_1 - c_5)v_3, \tag{15}$$

其中,  $a_5, b_5, c_5$  为系统参数, 当  $a_5 = b_5 = 0.2, c_5 = 5.7$  时, 系统(15)处于混沌状态, 其混沌吸引子如图 3(e)所示.

任选 Chen 系统作为目标系统, 在 New 系统、L 系统、Lorenz 系统和 R<sup>L</sup>-ssler 系统上加入耦合项以及控制器  $U = [u_1(t), u_2(t), u_3(t), \dots, u_{12}(t)]^T$ , 方程(11)~(15)变为下列形式:

$$\begin{cases} \dot{x}_1 = -x_2x_3 + a_1x_1 + d_{11}(z_1 - x_1) + u_1(t), \\ \dot{x}_2 = x_1x_3 + b_1x_2 + d_{12}(z_2 - x_2) + u_2(t), \\ \dot{x}_3 = (1/3)x_1x_2 + c_1x_3 + d_{13}(z_3 - x_3) + u_3(t), \end{cases} \tag{16}$$

$$\begin{cases} \dot{y}_1 = a_2(y_2 - y_1) + d_{21}(x_1 - y_1), \\ \dot{y}_2 = (c_2 - a_2)y_1 - y_1y_3 + c_2y_2 + d_{22}(x_2 - y_2), \\ \dot{y}_3 = y_1y_2 - b_2y_3 + d_{23}(x_3 - y_3), \end{cases} \tag{17}$$

$$\begin{cases} \dot{z}_1 = a_3(z_2 - z_1) + d_{31}(y_1 - z_1) + u_4(t), \\ \dot{z}_2 = -z_1 z_3 + c_3 z_2 + d_{32}(y_2 - z_2) + u_5(t), \\ \dot{z}_3 = z_1 z_2 - b_3 z_3 + d_{33}(y_3 - z_3) + u_6(t), \\ \dot{w}_1 = a_4(w_2 - w_1) + d_{41}(y_1 - w_1) + u_7(t), \\ \dot{w}_2 = w_1(c_4 - w_3) - w_2 + d_{42}(y_2 - w_2) + u_8(t), \\ \dot{w}_3 = w_1 w_2 - b_4 w_3 + d_{43}(y_3 - w_3) + u_9(t) \end{cases} \quad (18)$$

$$\begin{cases} \dot{w}_1 = a_4(w_2 - w_1) + d_{41}(y_1 - w_1) + u_7(t), \\ \dot{w}_2 = w_1(c_4 - w_3) - w_2 + d_{42}(y_2 - w_2) + u_8(t), \\ \dot{w}_3 = w_1 w_2 - b_4 w_3 + d_{43}(y_3 - w_3) + u_9(t) \end{cases} \quad (19)$$

和

$$\begin{cases} \dot{v}_1 = -v_2 - v_3 + d_{51}(w_1 - v_1) + u_{10}(t), \\ \dot{v}_2 = v_1 + a_5 v_2 + d_{52}(w_2 - v_2) + u_{11}(t), \\ \dot{v}_3 = b_5 + (v_1 - c_5)v_3 + d_{53}(w_3 - v_3) + u_{12}(t). \end{cases} \quad (20)$$

构造 5 个系统的误差系统为

$$\begin{pmatrix} e_1, e_2, e_3, e_4 \end{pmatrix}^T = \begin{pmatrix} y - x, x - z, y - w, w - v \end{pmatrix}^T, \quad (21)$$

其中

$$\begin{aligned} e_1 &= (e_{11}, e_{12}, e_{13})^T = (y_1 - x_1, y_2 - x_2, y_3 - x_3)^T, \\ e_2 &= (e_{21}, e_{22}, e_{23})^T = (x_1 - z_1, x_2 - z_2, x_3 - z_3)^T, \\ e_3 &= (e_{31}, e_{32}, e_{33})^T = (y_1 - w_1, y_2 - w_2, y_3 - w_3)^T, \\ e_4 &= (e_{41}, e_{42}, e_{43})^T = (w_1 - v_1, w_2 - v_2, w_3 - v_3)^T. \end{aligned}$$

若选取控制器  $U = [u_1(t), u_2(t), u_3(t), \dots, u_{12}(t)]^T$  的形式为

$$\begin{cases} u_1(t) = m_1(t) - y_2 e_{13} - y_3 e_{12} - (a_1 + a_2)y_1 + y_2 y_3 + e_{12} e_{13} + a_2 y_2, \\ u_2(t) = m_2(t) + y_1 e_{13} + y_3 e_{11} + (c_2 - a_2)y_1 + (c_2 - b_1)y_2 - \\ \quad 2y_1 y_3 - e_{11} e_{13}, \\ u_3(t) = m_3(t) + (1/3)y_1 e_{12} + (1/3)y_2 e_{11} + (2/3)y_1 y_2 - \\ \quad (1/3)e_{11} e_{12} - (b_2 + c_1)y_3, \\ u_4(t) = m_4(t) + (a_3 - a_2)y_1 - (a_3 - a_2)y_2, \\ u_5(t) = m_5(t) + (c_2 - a_2)y_1 - (c_3 - c_2)y_2 - \\ \quad y_1(e_{13} + e_{23}) - (y_3 - e_{13} - e_{23})(e_{11} + e_{21}), \\ u_6(t) = m_6(t) - (b_2 - b_3)y_3 + y_1(e_{12} + e_{22}) + \\ \quad (y_2 - e_{12} - e_{22})(e_{11} + e_{21}), \\ u_7(t) = m_7(t) + (a_2 - a_4)(y_2 - y_1), \\ u_8(t) = m_8(t) + (c_2 - a_2 - c_4)y_1 + (c_2 + 1)y_2 - y_1 e_{33} - y_3 e_{31} + e_{31} e_{33}, \\ u_9(t) = m_9(t) + (b_4 - b_2)y_3 + y_1 e_{32} + y_2 e_{31} - e_{31} e_{32}, \\ u_{10}(t) = m_{10}(t) + (a_4 + 1)w_2 - a_4 w_1 + w_3 + (a_2 - a_4)(y_2 - y_1), \\ u_{11}(t) = m_{11}(t) + (c_4 - 1)w_1 - (1 + a_5)w_2 - w_1 w_3 + (c_2 - a_2 - c_4)y_1 + \\ \quad (c_2 + 1)y_2 - y_1 e_{33} - y_3 e_{31} + e_{31} e_{33}, \\ u_{12}(t) = m_{12}(t) + w_1 w_2 - w_1 w_3 + (c_5 - b_4)w_3 - b_5 + w_1 e_{43} + w_3 e_{41} - \\ \quad e_{41} e_{43} + (b_4 - b_2)y_3 + y_1 e_{32} + y_2 e_{31} - e_{31} e_{32} \end{cases} \quad (22)$$

可以得到

$$\begin{cases} \dot{e}_1 = -(d_{21} - a_1)e_{11} + d_{11}e_{21} - m_1(t), \\ \dot{e}_2 = -(d_{22} - b_1)e_{12} + d_{12}e_{22} - m_2(t), \\ \dot{e}_3 = -(d_{23} - c_1)e_{13} + d_{13}e_{22} - m_3(t), \\ \dot{e}_{21} = -(d_{31} + a_3)e_{21} + (a_2 - a_1 - a_3 - d_{31})e_{11} + a_3e_{12} + a_3e_{22} - m_4(t), \\ \dot{e}_{22} = -(d_{32} - c_3)e_{22} + (c_3 - d_{32} - b_1 - c_2)e_{12} - m_5(t), \\ \dot{e}_{23} = -(d_{33} + d_3)e_{23} + (b_2 - c_1 - b_3 - d_{33})e_{13} - m_6(t), \\ \dot{e}_{31} = -(d_{41} + a_4)e_{31} - d_{21}e_{11} + a_4e_{32} - m_7(t), \\ \dot{e}_{32} = -(d_{42} + 1)e_{32} + c_4e_{31} - d_{22}e_{12} - m_8(t), \\ \dot{e}_{33} = -(d_{43} + b_4)e_{33} - d_{23}e_{13} - m_9(t), \\ \dot{e}_{41} = -d_{51}e_{41} - e_{42} - e_{43} + d_{41}e_{31} + m_7(t) - m_{10}(t), \\ \dot{e}_{42} = -(d_{52} - a_5)e_{42} + e_{41} + d_{42}e_{32} + m_8(t) - m_{11}(t), \\ \dot{e}_{43} = -(d_{53} + c_5)e_{43} + d_{43}e_{33} + m_9(t) - m_{12}(t), \end{cases} \quad (24)$$

$$\begin{cases} \dot{e}_{41} = -d_{51}e_{41} - e_{42} - e_{43} + d_{41}e_{31} + m_7(t) - m_{10}(t), \\ \dot{e}_{42} = -(d_{52} - a_5)e_{42} + e_{41} + d_{42}e_{32} + m_8(t) - m_{11}(t), \\ \dot{e}_{43} = -(d_{53} + c_5)e_{43} + d_{43}e_{33} + m_9(t) - m_{12}(t), \end{cases} \quad (25)$$

其中,  $M = [m_1(t), m_2(t), m_3(t), \dots, m_{12}(t)]^T$  是误差信号的函数, 以上误差系统是一个由控制函数  $M$  控制的一个线性系统.

不妨取控制函数

$$M = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} e, \quad (26)$$

其中,  $A_{12}$  中各元素为 0,

$$A_{11} = \begin{pmatrix} 0 & 0 & 0 & d_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{13} \\ a_2 - a_1 - a_3 - d_{31} & a_3 & 0 & 0 & a_3 & 0 \\ 0 & c_3 - b_1 - c_2 - d_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & b_2 - c_1 - b_3 - d_{33} & 0 & 0 & 0 \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} -d_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & -d_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & -d_{23} & 0 & 0 & 0 \\ -d_{21} & a_3 & 0 & 0 & 0 & 0 \\ 0 & -d_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & -d_{23} & 0 & 0 & 0 \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} 0 & a_4 & 0 & 0 & 0 & 0 \\ c_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ d_{41} & a_4 & 0 & 0 & -1 & -1 \\ c_4 & d_{42} & 0 & 1 & 0 & 0 \\ 0 & 0 & d_{43} & 0 & 0 & 0 \end{pmatrix}.$$

构造如下形式正定的 Liapunov 函数

$$V = \frac{1}{2}(e_{11}^2 + e_{12}^2 + e_{13}^2 + e_{21}^2 + e_{22}^2 + e_{23}^2 + e_{31}^2 + e_{32}^2 + e_{33}^2 + e_{41}^2 + e_{42}^2 + e_{43}^2).$$

对其求导

$$\begin{aligned} \dot{V} = & e_{11}\dot{e}_{11} + e_{12}\dot{e}_{12} + e_{13}\dot{e}_{13} + e_{21}\dot{e}_{21} + e_{22}\dot{e}_{22} + e_{23}\dot{e}_{23} + \\ & e_{31}\dot{e}_{31} + e_{32}\dot{e}_{32} + e_{33}\dot{e}_{33} + e_{41}\dot{e}_{41} + e_{42}\dot{e}_{42} + e_{43}\dot{e}_{43} = \\ & - (d_{21} - a_1)e_{11}^2 - (d_{22} - b_1)e_{12}^2 - (d_{23} - c_1)e_{13}^2 - (d_{31} + a_3)e_{21}^2 - \\ & (d_{32} - c_3)e_{22}^2 - (d_{33} + b_3)e_{23}^2 - (d_{41} + a_4)e_{31}^2 - (d_{42} + 1)e_{32}^2 - \\ & (d_{43} + b_4)e_{33}^2 - d_{51}e_{41}^2 - (d_{52} - a_5)e_{42}^2 - (d_{53} + c_5)e_{43}^2. \end{aligned} \quad (27)$$

根据 Liapunov 稳定性定理, 容易看出, 只要耦合参数的选择满足

$$\begin{cases} d_{21} - a_1 \geq 0, & d_{22} - b_1 \geq 0, & d_{23} - c_1 \geq 0, \\ d_{31} + a_3 \geq 0, & d_{32} - c_3 \geq 0, & d_{33} + b_3 \geq 0, \\ d_{41} + a_4 \geq 0, & d_{42} + 1 \geq 0, & d_{43} + b_4 \geq 0, \\ d_{51} \geq 0, & d_{52} - a_5 \geq 0, & d_{53} + c_5 \geq 0, \end{cases} \quad (28)$$

则满足  $\dot{V}$  是负半定的, 即  $\dot{V} \leq 0$ , 那么, 随着时间趋近无穷, 系统的误差变量  $e_1, e_2, e_3, \dots, e_{N-1}$  均收敛于 0, 即实现了 New 系统、Chen 系统、L 系统、Lorenz 系统和  $R^{L-ssler}$  系统的异结构同步。

## 2 模拟结果

为了验证上述结论, 利用 Matlab 编程进行仿真模拟. 数值模拟时, New 系统、Chen 系统、L 系统、Lorenz 系统和  $R^{L-ssler}$  系统的初值分别取为  $x_1(0) = 1, x_2(0) = 6, x_3(0) = 8, y_1(0) = 10, y_2(0) = 20, y_3(0) = 30, z_1(0) = -5.8, z_2(0) = 8, z_3(0) = 10, w_1(0) = 11, w_2(0) = 15, w_3(0) = 26, v_1(0) = 4.8, v_2(0) = 0.8, v_3(0) = 0.2$ , 根据不等式(28), 对于不等式中未涉及到的耦合系数  $d_{11}, d_{12}$  和  $d_{13}$ , 我们取  $d_{11} = d_{12} = d_{13} = 0$ , 也取  $d_{22} = d_{23} = d_{31} = d_{33} = 0, d_{41} = d_{42} = d_{43} = d_{53} = 0$ , 这样既简化了控制器的设计, 又降低了成本, 以较低的代价满足了同步的需要, 取  $d_{21} = 6, d_{32} = 21, d_{51} = d_{52} = 1$ . 图 4(a)、图 4(b)、图 4(c) 分别给出了 3 个系统的变量随时间演化的显示结果. 图 5(a) 给出了 New 系统和 Chen 系统状态变量的误差信号随时间的演化的结果, 即  $e_1 = (e_{11}, e_{12}, e_{13})^T = (y_1 - x_1, y_2 - x_2, y_3 - x_3)^T$ , 图 5(b) 给出了新系统和 L 系统状态变量的误差信号随时间演化的结果, 即  $e_2 = (e_{21}, e_{22}, e_{23})^T = (x_1 - z_1, x_2 - z_2, x_3 - z_3)^T$ . 图 5(c) 给出了 Lorenz 系统和 Chen 系统状态变量的误差信号随时间的演化的结果, 即  $e_3 = (e_{31}, e_{32}, e_{33})^T = (y_1 - w_1, y_2 - w_2, y_3 - w_3)^T$ , 图 5(d) 给出了  $R^{L-ssler}$  系统和 Lorenz 系统状态变量的误差信号随时间演化的结果, 即  $e_4 = (e_{41}, e_{42}, e_{43})^T = (w_1 - v_1, w_2 - v_2, w_3 - v_3)^T$ .

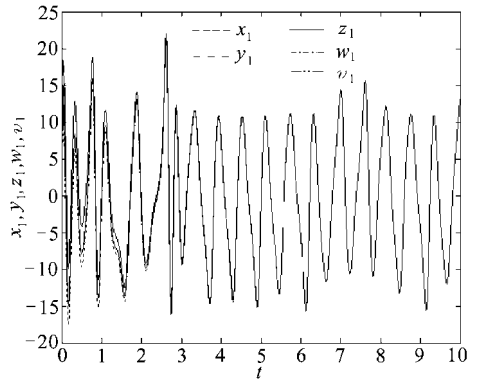
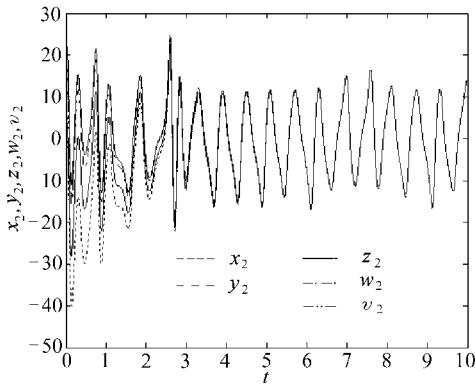
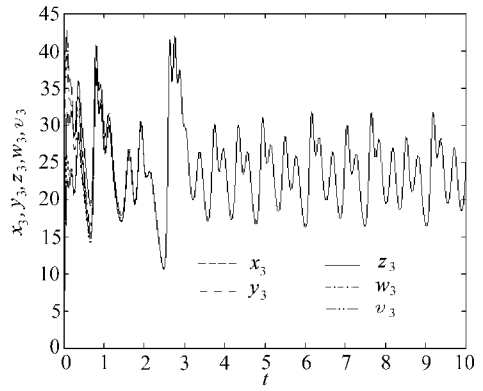
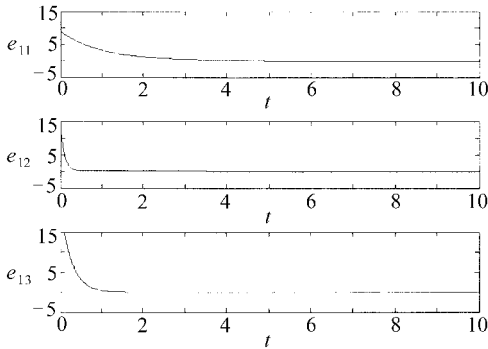
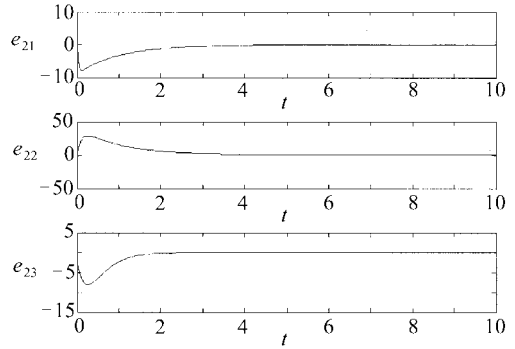
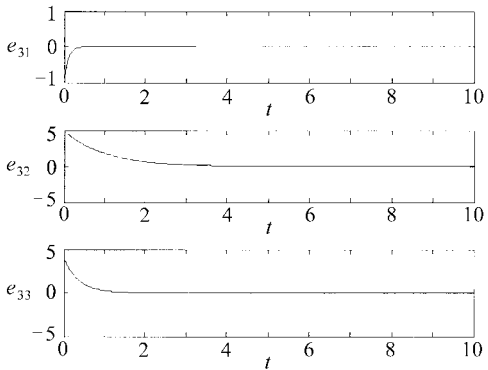
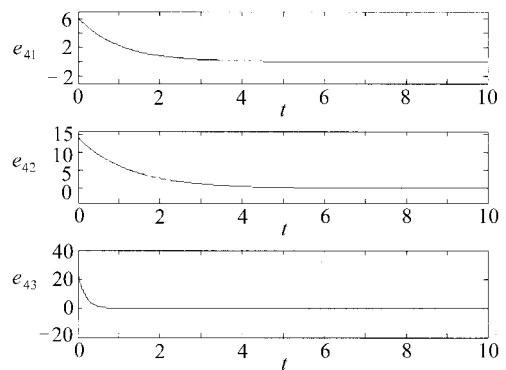


图 4(a)  $x_1, y_1, z_1, w_1$  和  $v_1$  随时间的演化

从图 4 和图 5 可以看出, 在时间演化序列不到 2 s 时, 3 个系统的状态变量均达到同步, 而且在以后的时间里始终保持稳定. 同时, 相应的误差变量趋于 0, 随着时间演化, 误差曲线一直保持平稳, 无跳跃和抖动现象.

图 4(b)  $x_2, y_2, z_2, w_2$  和  $v_2$  随时间的演化图 4(c)  $x_3, y_3, z_3, w_3$  和  $v_3$  随时间的演化图 5(a)  $e_{11}, e_{12}, e_{13}$  随时间的演化图 5(b)  $e_{21}, e_{22}, e_{23}$  随时间的演化图 5(c)  $e_{31}, e_{32}, e_{33}$  随时间的演化图 5(d)  $e_{41}, e_{42}, e_{43}$  随时间的演化

### 3 结 论

本文研究了环链耦合实现多个异结构混沌系统同步的问题. 利用 Liapunov 稳定性定理, 构造控制器的具体形式, 确定了耦合系数的取值范围. 通过任意  $N$  个异结构的混沌系统状态变量之间的环链耦合, 最终使  $N$  个异结构的混沌系统达到了完全同步. 在实施保密通讯时, 实现异结构混沌系统的同步, 克服了同结构系统同步保密通讯系统结构的单一性, 混沌掩盖信号的单调性和重复性的缺点, 这无疑增加了破解系统的难度, 因此提高了保密通讯的性能, 在实际生产或生活中具有良好的应用价值. 而且, 同时实现  $N$  个异结构混沌系统达到同步, 为



同时进行多方通讯提供了可能和更为广阔的发展空间.

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## Synchronization of $N$ Different Coupled Chaotic Systems With Ring and Chain Connections

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**Abstract:** The synchronization of  $N$  different coupled chaotic systems with ring and chain connections is investigated. The New system, the Chen system, the L system, the Lorenz system and the Rössler system were taken as examples to verify the effectiveness of the method. Based on Liapunov stability theory, the form of controller was designed and the area of coupling coefficients were determined. Artificial simulations indicate that the global synchronization of the  $N$  different chaotic systems can be realized by choosing appropriate coupling coefficients under the function of controller.

**Key words:** chaos synchronization; Liapunov stability theory; coupling with ring and chain connection; different chaotic systems; coupling coefficient