

基于高阶变形理论的硬夹芯夹层板 横向载荷条件下的弯曲*

郝加琼, 李明成, 邓宗白

(南京航空航天大学 航空宇航学院, 南京 210016)

摘要: 考虑面板和夹芯的面内刚度及抗弯刚度,基于高阶变形理论考虑各层的横向抗剪刚度,并根据横向剪应变分布情况给出应力函数,基于广义虚位移原理推导了夹层板的基本方程.详细研究了四边简支受横向载荷条件下的夹层板的弯曲,对比计算了面板与芯层厚度变化对计算结果的影响,并与一阶变形理论计算结果进行对比.研究了厚度方向的剪应变分布情况以及中面法线变形后的形态,给出了横向剪应变引起的附加转角在面内的分布情况.

关键词: 高阶变形理论; 夹层板; 硬夹芯; 横向剪切; 广义虚位移原理; 基本方程

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引言

夹层板结构材料具有强度高、刚度大、质量轻等优点,广泛应用于航空、航天、航海领域.波音 787 客机复合材料用量达到 50%,而夹层板是复合材料的重要结构形式之一.随着性能优异的新材料的使用,要求更为精确的理论计算以预测结构的力学性能.夹层板的理论计算模型大致可以分为经典夹层板理论、一阶变形理论、高阶变形理论.经典夹层板理论基于 Kirchhoff 薄板假设,忽略横向剪切变形的影响,用于薄板的计算问题,但是其理论本身存在诸多的不足之处.为了克服经典夹层板理论的局限,Reissner, Mindlin 等研究了一阶剪切变形理论,均假设横向剪切应力在厚度方向上均匀分布.近几年,国内外诸多学者开始将高阶剪切变形理论应用到夹层板结构分析上,引入一个横向剪切应力函数,以克服一阶剪切变形理论的不足.

夹层板的理论研究始于上世纪 40 年代末,Reissner 关于夹层板问题给出的理论计算模型得到了广泛的认同^[1-2],它忽略芯层的面内应力和上下面板的抗弯刚度,并认为横向剪切应力沿厚度方向均匀分布,该理论在软夹芯夹层板的分析计算上具有较好的精度.Hoff 理论模型考虑了上下面板的抗弯刚度^[3],但认为芯层只有抗剪作用,且剪应力沿厚度方向均匀分布.Hoff 理论相对于 Reissner 理论在面板厚度较大时具有更好的计算精度.国内学者在夹层板理论研究上也做出了突出贡献.杜庆华将夹芯和上、下面板独立分析,再用变形一致性条件将其整

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作者简介: 郝加琼(1986—),男,安徽安庆人,工程师,硕士(通讯作者. E-mail: HJQ.2014@163.com).

合^[4],对夹芯只考虑其横向剪应力与厚度方向主应力,且认为横向剪应力沿厚度方向均匀分布.胡海昌对 Reissner 理论做了深入的探讨,为了使之能更有效地应用于工程实际问题,详细解答了反对称变形夹层板的小挠度问题^[5],其理论方法对求解夹层板问题具有非常大的指导意义.周际平、薛大为考虑了夹芯的面内刚度^[6-7],在 Hoff 理论的基础上推导了硬夹芯夹层板的基本方程.胡宁宁等^[8]推导了变厚度硬夹芯夹层板的平衡方程,并分析了硬夹芯夹层板的振动问题.邓宗白等基于 Reissner 理论做了推广^[9-10],考虑夹芯的面内应力,研究了硬夹芯夹层板的弯曲、稳定等问题的解析解.近几年,国外学者就夹层板的高阶变形问题作了很多的探索性研究工作.Reddy 引入抛物线分布的横向应变变形参数^[11-12],获得了对称分布的剪应变分量.Hassis 引入横向翘曲变形以描述层合板的变形^[13-15],其基于 Kirchhoff 和 Mindlin 假设,引入参数描述横向载荷引起的翘曲变形.Ferreira 等将各层独立分析,假定芯层和面板的横向载荷引起的翘曲变形形式不同,用位移一致协调性条件将各层联系起来^[16].Swaminathan 等对比了反对称角度设计夹层板的几种高阶计算模型^[17],并就横向位移、面内应力等做了详细的对比分析.Aydogdu 基于高阶变形理论研究了夹层板的振动、屈曲问题^[18].Berdichevsky 详细分析了夹芯的材料属性对夹层板位移形式的决定性作用,将直法线变形假设分为 3 种情况,并试图将其归并到统一的方程中^[19].Ferreira 等基于高阶剪切变形理论研究了夹层板的屈曲问题^[20].Kheirikhah 等基于高阶变形理论分析了软夹芯夹层板的屈曲问题^[21].Grover 等引入反双曲函数以描述夹层板的变形和屈曲问题^[22].Sahoo 等基于 zigzag 变形假定引入反双曲函数描述夹层板的变形问题^[23].Tounsi 等引入高阶三角函数以描述夹层板的热弹性弯曲问题^[24].高阶变形理论相对于经典夹层板理论和一阶剪切变形理论,能更好地分析夹层板的面内位移和应力分布情况,更有效地评估结构的力学行为,有效地反映横向剪切效应,相比较于三维度的分析更加节约计算资源.

本文基于高阶变形理论研究了硬夹芯夹层板受横向载荷条件下的弯曲问题,引入横向剪应力函数以研究横向剪应力对夹层板弯曲的影响.考虑了上、下面板以及夹芯的面内刚度以及抗弯刚度,并用 Navier 方法求解了四边简支情况下夹层板的弯曲挠度,给出了横向剪应变在厚度方向的分布情况以及中面法线变形情况.

1 基本方程

为了简化分析,做以下假设: 1) 夹层板厚度相对于面内方向的尺寸很小,可以作为薄板问题来考虑,从而忽略其厚度方向的主应力; 2) 变形前的中面在变形后仍然是中面.因为横向剪应力的影响,变形前与中面垂直的法线在变形后不再与之垂直,且不要求其保持为直线.文中坐标系的选取如图 1 所示.

基于前文假设,夹层板的位移场可以用如下方程描述:

$$\begin{cases} u_1(x_1, x_2, x_3) = u_1^0(x_1, x_2) - x_3\theta_1^0(x_1, x_2) + f(x_3)\beta_1^0(x_1, x_2), \\ u_2(x_1, x_2, x_3) = u_2^0(x_1, x_2) - x_3\theta_2^0(x_1, x_2) + f(x_3)\beta_2^0(x_1, x_2), \\ u_3(x_1, x_2, x_3) = u_3^0(x_1, x_2), \end{cases} \quad (1)$$

其中,下标表示坐标轴方向,上标 0 表示位于中面上的广义位移,即 $x_3 = 0$, $\theta_i^0 = \partial u_3^0 / \partial x_i$,与经典夹层板理论一致, β 为横向剪应力引起的中面法线转角, f 为由横向剪应变分布情况确定的未

知函数.

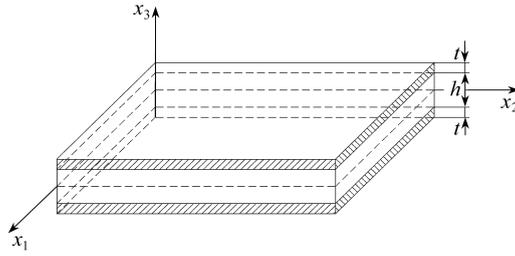


图 1 夹层板坐标系示意图

Fig.1 A sandwich plate with the coordinate system

基于小变形假设,其位移-应变关系为

$$\begin{cases} \varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = \frac{\partial u_1^0}{\partial x_1} - x_3 \frac{\partial \theta_1^0}{\partial x_1} + f(x_3) \frac{\partial \beta_1^0}{\partial x_1}, \\ \varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = \frac{\partial u_2^0}{\partial x_2} - x_3 \frac{\partial \theta_2^0}{\partial x_2} + f(x_3) \frac{\partial \beta_2^0}{\partial x_2}, \\ \varepsilon_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} = \left(\frac{\partial u_1^0}{\partial x_2} + \frac{\partial u_2^0}{\partial x_1} \right) - x_3 \left(\frac{\partial \theta_1^0}{\partial x_2} + \frac{\partial \theta_2^0}{\partial x_1} \right) + f(x_3) \left(\frac{\partial \beta_1^0}{\partial x_2} + \frac{\partial \beta_2^0}{\partial x_1} \right), \\ \varepsilon_{13} = \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} = \frac{\partial f(x_3)}{\partial x_3} \beta_1^0, \\ \varepsilon_{23} = \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} = \frac{\partial f(x_3)}{\partial x_3} \beta_2^0. \end{cases} \quad (2)$$

未知函数 f 为 x_3 的函数,为了确定其形式,需要研究横向剪应变在厚度方向上的分布.已知 ε_{13} 沿厚度方向的分布情况为在 $x_3 = 0$ 时 ε_{13} 取得最大值,在 $x_3 = \pm (h/2) \pm t$ 时 $\varepsilon_{13} = 0$,由此可取 $f(x_3) = \sin(\pi x_3 / (h + 2t)) + \pi / (h + 2t)$,为简化表示,记 $g(x_3) = \sin(\pi x_3 / (h + 2t))$, $R = \pi / (h + 2t)$.则 $f(x_3) = g(x_3) + R$.

由应力-应变关系^[25],有

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & G_{13} & 0 \\ 0 & 0 & 0 & 0 & G_{23} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{Bmatrix}, \quad (3)$$

其中, Q_{ij} 取决于各层的材料属性.

由广义位能原理,取微元体 $dv = dx_1 dx_2 dx_3$, 其上应变能可表示为

$$U = \frac{1}{2} \iiint_{x_1 x_2 x_3} (\sigma_{11} \varepsilon_{11} + \sigma_{22} \varepsilon_{22} + \sigma_{12} \varepsilon_{12} + \sigma_{13} \varepsilon_{13} + \sigma_{23} \varepsilon_{23}) dx_1 dx_2 dx_3. \quad (4)$$

由前文假设,忽略厚度方向主应力,则可以将作用在夹层板面板的面力等效到中面上,设各方向面力为 $q_1(x_1, x_2), q_2(x_1, x_2), q_3(x_1, x_2)$, 则外力做功可表示为

$$V = \iint_{x_1 x_2} (q_1 u_1 + q_2 u_2 + q_3 u_3) dx_1 dx_2. \quad (5)$$

由广义虚位移原理 $\delta(U - V) = 0$ 可得夹层板的基本方程:

$$\begin{cases} \frac{\partial N_{11}}{\partial x_1} + \frac{\partial N_{12}}{\partial x_2} + q_1 = 0, \\ \frac{\partial N_{22}}{\partial x_2} + \frac{\partial N_{12}}{\partial x_1} + q_2 = 0, \\ \frac{\partial^2 M_{11}}{\partial x_1^2} + 2 \frac{\partial^2 M_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M_{22}}{\partial x_2^2} + q_3 = 0, \\ \frac{\partial P_{11}}{\partial x_1} + \frac{\partial P_{12}}{\partial x_2} + R \left(\frac{\partial M_{11}}{\partial x_1} + \frac{\partial M_{12}}{\partial x_2} \right) - K_{13} - RQ_{13} = 0, \\ \frac{\partial P_{22}}{\partial x_2} + \frac{\partial P_{12}}{\partial x_1} + R \left(\frac{\partial M_{22}}{\partial x_2} + \frac{\partial M_{12}}{\partial x_1} \right) - K_{23} - RQ_{23} = 0. \end{cases} \quad (6)$$

上式是以力表示的基本方程,其中各力与力矩的表达式为

$$\begin{bmatrix} N_{11} & M_{11} & P_{11} \\ N_{22} & M_{22} & P_{22} \\ N_{12} & M_{12} & P_{12} \end{bmatrix} = \int_{x_3} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \begin{Bmatrix} 1 \\ x_3 \\ g(x_3) \end{Bmatrix}^T dx_3, \quad (7)$$

$$\begin{bmatrix} Q_{13} & K_{13} \\ Q_{23} & K_{23} \end{bmatrix} = \int_{x_3} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \begin{Bmatrix} 1 \\ g'(x_3) \end{Bmatrix}^T dx_3. \quad (8)$$

将式(2)代入式(3)再代入式(7)、(8)得到以位移表示的力与力矩,将其代入式(6)即可得到以广义位移 $u_1, u_2, u_3, \beta_1^0, \beta_2^0$ 表示的基本方程。

2 边界条件与方程的解

对于受横向载荷的四边简支夹层板,其边界条件为

$$x_1 = 0, a: u_1^0 = 0, u_3^0 = 0, \frac{\partial u_3^0}{\partial x_2} = 0, \beta_2^0 = 0;$$

$$x_2 = 0, b: u_2^0 = 0, u_3^0 = 0, \frac{\partial u_3^0}{\partial x_1} = 0, \beta_1^0 = 0.$$

对方程的求解选用 Navier 方法^[26].假设四边简支夹层板的位移为

$$u_1^0 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{1mn} \cos(\lambda_1 x_1) \sin(\lambda_2 x_2), \quad (9)$$

$$u_2^0 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{2mn} \sin(\lambda_1 x_1) \cos(\lambda_2 x_2), \quad (10)$$

$$u_3^0 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{3mn} \sin(\lambda_1 x_1) \sin(\lambda_2 x_2), \quad (11)$$

$$\beta_1^0 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} X_{mn} \cos(\lambda_1 x_1) \sin(\lambda_2 x_2), \quad (12)$$

$$\beta_2^0 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_{mn} \sin(\lambda_1 x_1) \cos(\lambda_2 x_2), \tag{13}$$

其中

$$\lambda_1 = \frac{m\pi}{a}, \lambda_2 = \frac{n\pi}{b}.$$

综合前文,可以得到以 5 个未知量表示的 5 个方程.其中式(6)前两式可解得 u_1, u_2 , 后 3 式可解得 u_3 与法线转角 β_1, β_2 .

3 高阶理论与一阶理论对比

以下对比分析高阶变形理论与一阶变形理论挠度计算结果之间的差异.为了研究上、下面板的抗弯刚度对计算结果的影响,对比计算面板厚度变化时夹层板的最大挠度计算结果.其中 $E_f = 70 \text{ GPa}$, $E_c = 7 \text{ GPa}$, 夹芯厚度为 10 mm, 边长 $a = b = 100 \text{ mm}$, 四边简支, 横向均布载荷 $q_3 = 1 \text{ MPa}$.

表 1 面板厚度变化时,高阶变形理论与一阶变形理论计算结果的对比

Table 1 Comparison of the center deflection calculation results between the high-order deformation theory and the 1st-order deformation theory with different surface layer thickness

h/t	u_{3H}	u_{3F}	$(u_{3H} - u_{3F})/u_{3H}$
1	3.36E-06	2.63E-06	21.82%
2	1.10E-05	9.25E-06	15.81%
4	2.95E-05	2.59E-05	12.18%
5	3.91E-05	3.47E-05	11.34%
10	8.44E-05	7.67E-05	9.11%
15	1.24E-04	1.14E-04	7.89%
20	1.58E-04	1.47E-04	7.02%
30	2.15E-04	2.02E-04	5.82%
50	2.95E-04	2.82E-04	4.37%
100	4.04E-04	3.93E-04	2.74%

表 1 中 u_{3H} 为高阶变形理论计算结果, u_{3F} 为一阶变形理论计算结果^[9].

由上表计算数据可知,当面板厚度相对于夹芯厚度较小时,一阶变形理论能较好地分析夹层板的弯曲情形.随着面板厚度增大,计算差异增大.这主要是因为一阶变形理论将横向剪切应力简化为沿厚度方向均匀分布,且假设面板上的面内应力沿厚度方向也均匀分布,导致法线转角的计算误差,这种误差在厚度尺寸相对较大时表现得尤为明显.因此,当 h/t 比值小于 20 时,应该考虑采用高阶变形理论分析夹层板问题.

4 高阶变形理论

对于夹层板结构,横向剪切是其主要破坏因素之一,所以有必要研究其横向剪切应力沿厚度方向的分布情形.而一阶变形理论均假设其横向剪切应力沿厚度方向均匀分布,所以有必要采用高阶变形理论对其进行进一步的研究分析.

4.1 中面各点的挠度

设夹层板上、下面板厚度 $t = 1 \text{ mm}$, 夹芯厚度 $h = 10 \text{ mm}$, 面板弹性模量 $E_f = 70 \text{ GPa}$, 夹芯

弹性模量 $E_c = 7 \text{ GPa}$, 面板和夹芯 Poisson(泊松)比 $\nu_f = \nu_c = 0.3$, 夹层板边长 $a = b = 500 \text{ mm}$, 四边简支, 受横向均布载荷 $q_3 = 1 \text{ MPa}$. 使用 Navier 方法能快速的求得其各点的位移和转角, 沿 x_1, x_2 方向各取 51 个计算节点, 得到位移变形图(图 2).

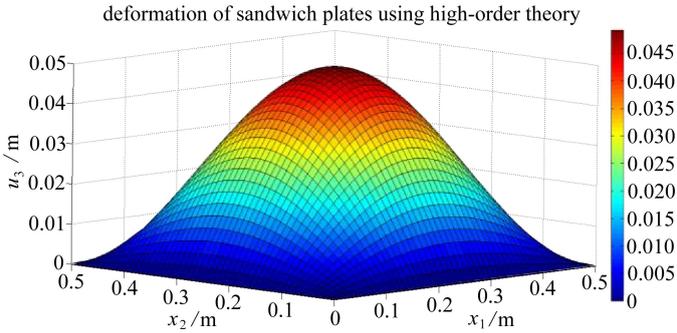


图 2 四边简支受横向载荷夹层板变形图

Fig.2 Deformation of the simply supported sandwich plate under transverse loading

4.2 中面各点的法线转角

由式(2)可知, 夹层板各处的横向剪应变与转角 β 相关. 由此可求解出夹层板中面上的法线转角 β 的分布情况, 以得到整体的横向剪应力分布情况.

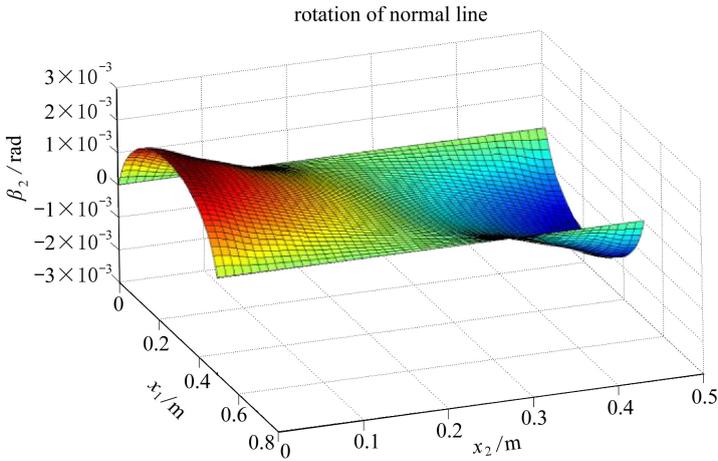


图 3 四边简支受横向载荷夹层板中面法线转角

Fig.3 Normal line rotation of the simply supported sandwich plate under transverse loading

图 3 为转角 β_2 在夹层板中面内的分布图, 由图可知最大横向剪应力 σ_{23} 位于 $x_1 = a/2, x_2 = 0, b$ 处. 由结构的对称性可知最大剪应力 σ_{13} 位于 $x_2 = b/2, x_1 = 0, a$ 处. 对于更复杂的情况, 同样可以用该法快速地得到中面法线转角在面内的分布情况, 为夹层板的强度分析计算提供有效的定位手段.

4.3 厚度方向应变分布

在厚度方向上, 横向剪应变的分布呈现出规整的半正弦分布形式, 如图 4 所示. 由图 4 可知, 横向剪应变在中面处取得最大值, 在上、下面板的外表面处为 0. 对于四边简支受均布横向载荷的夹层板, 越靠近中点, 其横向剪应变越小, 且在板的内面中点处取得 0 值.

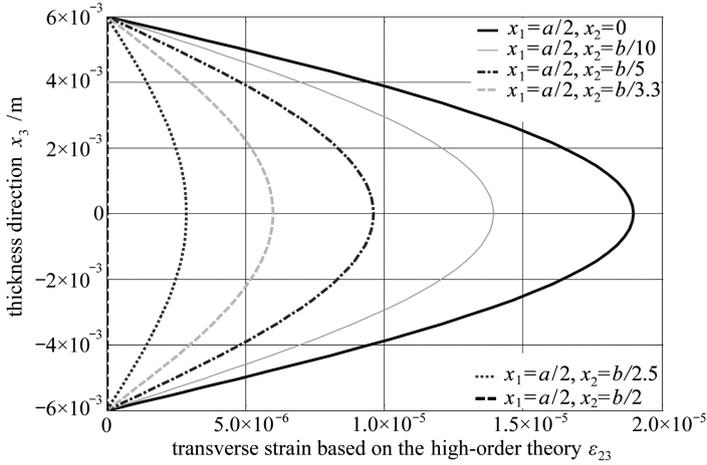


图4 横向剪应变沿厚度方向分布情况

Fig.4 Transverse strain along the thickness direction

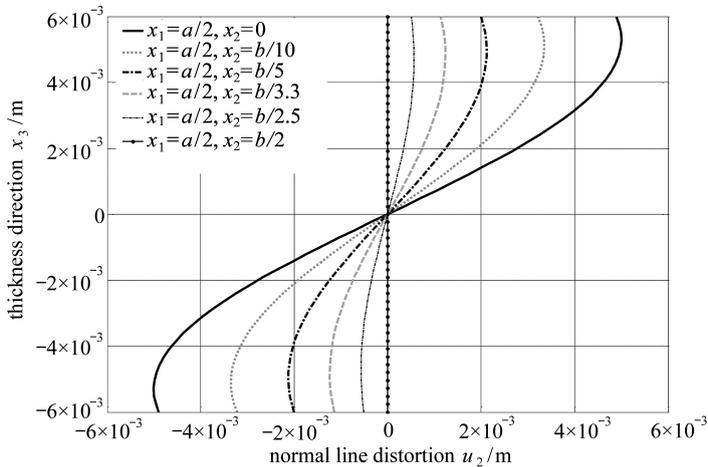


图5 中面法线变形图

Fig.5 The center-plane normal line distortion

4.4 中面法线变形

考虑到横向剪切应力对中面法线变形的影响,变形后将不再保持为直线.由式(1)可知,求解各点处的面内位移,减除中面上对应点的位移,即可得到各法线的变形情况.如图5所示.

图5中纵坐标为夹层板厚度方向,横坐标为法向上各点在 x_2 方向的位移.由图5可知,对于四边简支受横向均布载荷的夹层板,其边界处的法线转角位移最大,而板的面内中点处中面法线转角位移为0.

5 结 论

1) 由于考虑了面内方向的应力在厚度方向的分布情况,提高了夹层板问题的分析精度,扩展了理论计算的适用范围.

2) 研究了横向剪切应力的面内分布情况,对于夹层板的横向载荷破坏能快速准确地找到危险点,并且考虑了横向剪切应力在厚度方向的分布情况,提高了研究精度.

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Bending of Sandwich Plates With Hard Cores Under Transverse Loading Based on the High-Order Deformation Theory

HAO Jia-qiong, LI Ming-cheng, DENG Zong-bai

(*College of Aerospace Engineering, Nanjing University of Aeronautics
and Astronautics, Nanjing 210016, P.R.China*)

Abstract: Based on the high-order deformation theory, the in-plane stiffness and bending stiffness of both surface layer and core layer of the sandwich plate were considered to derive the transverse shearing stiffnesses of all the layers. The transverse stress function was given according to the transverse strain distribution, and the differential equations for the sandwich plate were deduced with the generalized principle of virtual displacement. The bending deformation of simply supported rectangular sandwich plates with different core-to-surface thickness ratios were detailedly studied under transverse loading, and the calculation results were compared with those from the 1st-order deformation theory to give a bigger relative deformation difference at a smaller thickness ratio. The distribution of transverse strain along the thickness direction makes a half sine curve, and the center-plane normal line distortion culminates at the surface height.

Key words: high-order deformation theory; sandwich plate; hard core; transverse shear; generalized principle of virtual displacement; basic equation