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两相饱和介质层与单相介质层应力-位移函数的传递与退化*

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摘要: 根据 Biot 孔隙弹性介质动力控制方程,利用快、慢纵波的解耦,求得满足两相饱和介质位移-应力传播的一阶微分方程组.该方程组及传递函数能退化到单相介质的位移-应力传播微分方程组.利用界面应力-位移连续条件,分析了位移-应力从两相饱和介质向单相介质传播,构建了界面过渡传递矩阵.使原有的 6×6 阶应力-位移传递矩阵过渡为 4×6 阶矩阵,能与单相介质的 4×4 阶应力-位移传递矩阵结合.最后,采用经典的波传播模型对比验算了结果,它们一致吻合.

关 键 词: 两相饱和介质; 单相介质; 多层介质; 应力位移函数; 过渡传递矩阵; 退化

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引 言

多层介质位移-应力波的传播一直是地震工程、土动力学、地球物理、声学等领域的重要研究课题[1-2].多层单相介质位移-应力波传播,上世纪 50 年代已由 Ewing 等与 Brekhovskikh 分别解决,这些成果都反映在 *Elastic Waves in Layered Media*[3] 与 *Waves in Layered Media*[4] 这两本优秀专著中.它包含了计算波多层传播的 Thomson 法[5],及之后 Haskell[6-7] 和 Harkrider[8] 进一步发展的传递矩阵法.上世纪 60,70 年代 Ben-Menahem 和 Singh[9] 结合 Haskell 矩阵与 Hansen 矢量[10] 研究过多层单相、各向同性半空间中位移-应力传播问题;陈运泰[11] 也有过类似研究,这些都推动了地球物理学与土动力学的发展.上世纪 60 年代 Gilbert 和 Backus[12] 提出了求解位移-应力微分方程组,由解矩阵得到传播矩阵的新算法.Kennett 等[13-14] 采用此法,用迭代法计算了层状介质中波的反射参数.Pride 等[15] 讨论了该算法的准确性.之后,许多学者的努力已使这一领域的成果不断完善和发展.此外,由于地层多相性,人们也一直关注着波在多相多层介质中的传播,特别是在多层两相饱和介质中的传播,因为它是模拟含油层的地球物理勘探与饱和土动力反应的理论基础.基于 Biot 等的两相饱和介质动力学方程[16-17],求解波在多层两相介质中的传播,前人取得过一系列成果[18-27].它们都为本文的研究奠定了必要的理论基础.

在本文中,作者首先根据 Biot 两相孔隙介质动力方程,利用快、慢纵波解耦;求得两相饱和介质波传播运动学一阶位移-应力微分方程组;该方程组及传递函数能退化到单相介质波位

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移-应力传播.另外,作者利用界面应力-位移连续条件,构建了界面过渡传递矩阵,分析波从两相饱和介质向单相介质传播;使原有两相介质的 6×6 应力-位移传递矩阵过渡为 4×6 矩阵,能与单相介质 4×4 应力-位移传递矩阵结合.完全有别于原来采用的振幅谱传递函数.这不仅计算简化,物理意义也更明确.最后,作者采用已有的经典的波传播模型对比验算了本文的结果,它们一致地吻合.

1　饱和孔隙弹性介质的动力控制方程与解答

1.1　饱和孔隙弹性介质动力控制方程

1.1.1　饱和孔隙弹性介质的动力控制方程的组成

Biot 饱和孔隙弹性介质的动力控制方程由以下方程组成[16-17]:

① 本构方程

$$\sigma_{ij} = \lambda e\delta_{ij} + 2\mu e_{ij} - \alpha p\delta_{ij}; \tag{1}$$

② 平衡方程

$$\begin{cases} \text{固相: } \sigma_{ij,j} + F_i = \rho\ddot{u}_i + \rho_f\ddot{w}_i, \\ \text{流相: } \zeta = \alpha e + p/M; \end{cases} \tag{2}$$

③ 广义 Darcy 定律

$$q_i = -\kappa(p_{,i} + \rho_f\ddot{u}_i + m'\ddot{w}_{i,i}), \tag{3a}$$

$$m' = \rho_a/\beta_0^2 + \rho_f/\beta_0; \tag{3b}$$

④ 连续方程

$$\dot{\zeta} + q_{i,i} = \chi, \tag{4}$$

式中 λ,μ 是 Lamé 系数, α,M 是 Biot 在饱和多孔介质研究中引入的参量, $\boldsymbol{u}$,$\boldsymbol{w}$ 分别为固相位移和流相相对固相的平均位移矢量($\ddot{\boldsymbol{u}} = \mathrm{d}^2\boldsymbol{u}/\mathrm{d}t^2$,$\ddot{\boldsymbol{w}} = \mathrm{d}^2\boldsymbol{w}/\mathrm{d}t^2$).$\rho$,$\rho_f$ 分别是两相与流相物质密度,$\rho = (1-\beta_0)\rho_s + \beta_0\rho_f$; ρ_s 为固相物质密度, β_0 是孔隙度,式(3b)中 ρ_a 为表观质量密度.式中 λ, μ, α, M 可视为 4 个独立的弹性常数.$e_{ij} = (u_{i,j} + u_{j,i})/2$ 是固相应变张量, $e = e_{ii}$ 是体应变.流体体应变为 $w_{i,i}$.在式(2) 和式(4) 中 ζ 为流体的膨胀度, $\dot{w}_{i,i} = q_{i,i}$, χ 为排水强度.σ_{ij} 为总应力;p 为孔隙压;F_i 为固相体力. $\kappa = k/\eta$,k 为固相渗透率,η 为流体动粘度,δ_{ij} 是 Kronecker-δ 数,i, j = 1,2,3.Biot 动力方程[16-17]是

$$\mu u_{i,jj} + \lambda_c u_{j,ji} + \alpha M w_{j,ji} = -F_i + \rho\ddot{u}_i + \rho_f\ddot{w}_i, \tag{5a}$$

$$\alpha M u_{j,ji} + M w_{j,ji} - \dot{w}_i/\kappa = \rho_f\ddot{u}_i + \ddot{w}_i(\rho_a + \beta_0\rho_f)/\beta_0^2, \tag{5b}$$

其中

$$\lambda_c = \lambda + \alpha^2 M.$$

1.1.2　动力控制方程频域形式

如不考虑体力的影响,即 $F_i = 0$; 对式(5a)和(5b)进行 Fourier 变换可得到两相饱和介质频域 Biot 动力学方程:

$$(\lambda_c + 2\mu)\tilde{u}_{j,ij} + \alpha M\tilde{w}_{j,ij} + \mu\delta_{ikh}\delta_{hlj}\tilde{u}_{i,lk} + \omega^2\rho\tilde{u}_i + \omega^2\rho_f\tilde{w}_i = 0, \tag{6a}$$

$$\alpha M\tilde{u}_{j,ji} + M\tilde{w}_{j,ji} - \mathrm{i}\omega\tilde{w}_i/\kappa = -\omega^2\rho_f\tilde{u}_i - \omega^2\tilde{w}_i(\rho_a + \beta_0\rho_f)/\beta_0^2, \tag{6b}$$

这里 $\tilde{u}$,$\tilde{w}$ 分别是位移 u,w 的 Fourier 谱.δ_{ikh},δ_{hlj} 为置换张量;ω 为频率,i 为虚数单位,k,h,l = 1,2,3.对于式(6b)还可以得到

$$\alpha M\tilde{u}_{j,ij} + M\tilde{w}_{j,ij} + \omega^2\rho_f\tilde{u}_i + \omega^2\gamma\tilde{w}_i = 0, \tag{7}$$

式中

$$\gamma = [-\mathrm{i}\beta_0^2 + \omega\kappa(\rho_a + \beta_0\rho_f)]/(\beta_0^2\omega\kappa) = (\omega\kappa m' - \mathrm{i})/(\omega\kappa). \tag{8}$$

1.2 饱和孔隙介质动力控制方程的解答

1.2.1 动力控制方程的无散场分量解耦

当 $\tilde{u}_i,\tilde{w}_i$ 为 $\tilde{\boldsymbol{u}},\tilde{\boldsymbol{w}}$ 的无散场分量时，由式(7)可得[28-31]

$$\omega^2\rho_{\mathrm{f}}\tilde{u}_i+\omega^2\gamma\tilde{w}_i=0, \tag{9}$$

$$\tilde{w}_i=-(\rho_{\mathrm{f}}/\gamma)\tilde{u}_i, \tag{10}$$

从而对横波有

$$\zeta_z=\tilde{u}_z+\tilde{w}_z=(1-\rho_{\mathrm{f}}/\gamma)\tilde{u}_z. \tag{11}$$

如设 $i=3$ 为 z 轴方向，则 ζ_z 是合成 z 向位移，把式(10)代入式(6a)，可得

$$\mu\tilde{u}_{ij,i}+(\rho-\rho_{\mathrm{f}}^2/\gamma)\omega^2\tilde{u}_j=0, \tag{12}$$

从而有横波速 β：

$$\beta=\sqrt{\mu/(\rho-\rho_{\mathrm{f}}^2/\gamma)},\ K_\beta^2=\omega^2/\beta^2, \tag{13}$$

$K_\beta^{(2)}$ 是两相饱和介质的横波波数.

1.2.2 动力控制方程的无旋场分量解耦

当 $\tilde{u}_i,\tilde{w}_i$ 为 $\tilde{\boldsymbol{u}},\tilde{\boldsymbol{w}}$ 的无旋场分量时[28-31]，可设

$$\tilde{u}_i=\tilde{u}_{1i}+\tilde{u}_{2i},\ \tilde{w}_i=\xi_1\tilde{u}_{1i}+\xi_2\tilde{u}_{2i}, \tag{14a,b}$$

$$\xi_n=(\lambda_{\mathrm{c}}+2\mu-\rho\alpha_n^2)/(\rho_{\mathrm{f}}\alpha_n^2-\alpha M), \tag{15}$$

这里 $n=1,2$. α_1,α_2 分别为快、慢纵波速；而 $\tilde{u}_{1i},\tilde{u}_{2i}$ 分别为快、慢纵波位移.对纵波则有

$$\zeta_z=\tilde{u}_z+\tilde{w}_z=(1+\xi_1)\tilde{u}_{1z}+(1+\xi_2)\tilde{u}_{2z}. \tag{16}$$

式(11)和(16)关于合成位移 ζ_z 的表达，会使得对两相饱和介质波传播微分方程组和它的解矩阵及传播矩阵的表达带来方便.将式(14a)和(14b)代入式(6a)与(6b)可得

$$\begin{aligned}&(\lambda_{\mathrm{c}}+2\mu+\alpha M\xi_1)\tilde{u}_{1j,ij}+(\lambda_{\mathrm{c}}+2\mu+\alpha M\xi_2)\tilde{u}_{2j,ij}+\\&\quad(\rho+\rho_{\mathrm{f}}\xi_1)\omega^2\tilde{u}_{1i}+(\rho+\rho_{\mathrm{f}}\xi_2)\omega^2\tilde{u}_{2i}=0,\end{aligned} \tag{17a}$$

$$M(\alpha+\xi_1)\tilde{u}_{1j,ij}+M(\alpha+\xi_2)\tilde{u}_{2j,ij}+(\rho_{\mathrm{f}}+\gamma\xi_1)\omega^2\tilde{u}_{1i}+(\rho_{\mathrm{f}}+\gamma\xi_2)\omega^2\tilde{u}_{2i}=0. \tag{17b}$$

由于在无限两相介质中，只有快、慢两个纵波，由式(17a)和(17b)可得

$$\frac{\lambda_{\mathrm{c}}+2\mu+\alpha M\xi_n}{\rho+\rho_{\mathrm{f}}\xi_n}=\frac{M(\alpha+\xi_n)}{\rho_{\mathrm{f}}+\gamma\xi_n}=\alpha_n^2. \tag{18}$$

从而有

$$\frac{(\gamma\rho-\rho_{\mathrm{f}}^2)}{M(\lambda+2\mu)}\alpha_n^4+\frac{\gamma(\lambda+2\mu)+M(\alpha^2\gamma+\rho-2\alpha\rho_{\mathrm{f}})}{M(\lambda+2\mu)}\alpha_n^2+1=0. \tag{19}$$

记 $K_{\alpha_n}^{(2)}=\omega/\alpha_n$；$K_{\alpha_1}^{(2)},K_{\alpha_2}^{(2)}$ 分别是两相饱和介质的快纵波与慢纵波数.式(19)化为

$$(K_{\alpha_n}^{(2)})^4-\frac{\gamma(\lambda+2\mu)+M(\alpha^2\gamma+\rho-2\alpha\rho_{\mathrm{f}})}{M(\lambda+2\mu)}\omega^2(K_{\alpha_n}^{(2)})^2+\frac{(\gamma\rho-\rho_{\mathrm{f}}^2)}{M(\lambda+2\mu)}\omega^4=0. \tag{20}$$

如设 $\sigma^2=\omega^2(\rho-\rho_{\mathrm{f}}^2/\gamma)/(\lambda+2\mu),\ q_{\mathrm{t}}=\gamma\omega^2/M,\ \varepsilon=M(\alpha-\rho_{\mathrm{f}}/\gamma)^2/(\lambda+2\mu),$ (21)

式(20)可写成

$$(K_{\alpha_n}^{(2)})^4-(\sigma^2+q_{\mathrm{t}}(1+\varepsilon))(K_{\alpha_n}^{(2)})^2+q_{\mathrm{t}}\sigma^2=0. \tag{22}$$

求解式(22)可得

$$(K_{\alpha_n}^{(2)})^2=\frac{1}{2}\left[\sigma^2+q_{\mathrm{t}}(1+\varepsilon)\pm\sqrt{[\sigma^2+q_{\mathrm{t}}(1+\varepsilon)]^2-4q_{\mathrm{t}}\sigma^2}\right]. \tag{23}$$

另有[28-31]

$$(K_{\alpha_1}^{(2)})^2 = \frac{(\rho + \rho_f \xi_1)\omega^2}{\lambda_c + 2\mu + \alpha M \xi_1} = \frac{(\rho_f + \gamma \xi_1)\omega^2}{M(\alpha + \xi_1)}, \tag{24}$$

$$(K_{\alpha_2}^{(2)})^2 = \frac{(\rho + \rho_f \xi_2)\omega^2}{\lambda_c + 2\mu + \alpha M \xi_2} = \frac{(\rho_f + \gamma \xi_2)\omega^2}{M(\alpha + \xi_2)}. \tag{25}$$

2 两相饱和介质与单相介质波传播一阶位移-应力微分方程组及解矩阵退化

采用上标 (n) 表述介质属性,即 $n = 1$ 为单相介质;$n = 2$ 为两相饱和介质;遵照 Gilbert, Backus 方法位移-应力波传播一阶微分方程组的解矩阵和传播矩阵可作如下表述:

2.1 两相饱和介质波传播位移-应力微分方程组与解矩阵

如压缩波(P 波)和偏振剪切波(SV 波)入射在 xOz 平面,介质位移及应力可设(为简洁,表示频域量值的字母上标~省略):

$$\begin{aligned}&[u_z^{(2)},\ \zeta_z^{(2)},\ \bar{\sigma}_{zx}^{(2)},\ \bar{\sigma}_{zz}^{(2)},\ p^{(2)},\ u_x^{(2)}]^{\mathrm T} = \\ &\qquad [f_1^{(2)}, f_2^{(2)}, f_3^{(2)}, f_4^{(2)}, f_5^{(2)}, f_6^{(2)}]^{\mathrm T} \mathrm{e}^{-\mathrm{i}Kx},\end{aligned} \tag{26}$$

其中 $u_x^{(2)}, u_z^{(2)}$ 分别是两相介质 xOz 传播平面上 x 向与 z 向的固相骨架位移,$\bar{\sigma}_{zz}^{(2)}, \bar{\sigma}_{zx}^{(2)}$ 分别是两相介质 Z 面有效正应力与 x 向的有效剪应力($\bar{\sigma}_{zx}^{(2)} = -\sigma_{zx}^{(2)}, \bar{\sigma}_{zz}^{(2)} = -\sigma_{zz}^{(2)} - p^{(2)}$),$p^{(2)}$ 为流相孔隙压力.$f_l^{(2)}(l = 1,2,\cdots,6)$ 代表相关波传播的位移、应力待定函数;e 为自然指数底数.满足两相饱和介质位移-应力波传播的一阶微分方程组是(详见附录 1)

$$\frac{\mathrm d}{\mathrm dz} f_l^{(2)} = \begin{pmatrix} \mathbf 0 & \boldsymbol A_1^{(2)} \\ \boldsymbol A_2^{(2)} & \mathbf 0 \end{pmatrix} f_l^{(2)}, \tag{27}$$

式中

$$\boldsymbol A_1^{(2)} = \begin{pmatrix} \dfrac{-1}{\lambda^{(2)} + 2\mu^{(2)}} & \dfrac{\alpha - 1}{\lambda^{(2)} + 2\mu^{(2)}} & \dfrac{\lambda^{(2)}\mathrm iK}{\lambda^{(2)} + 2\mu^{(2)}} \\ \dfrac{\alpha - 1}{\lambda^{(2)} + 2\mu^{(2)}} & \dfrac{K^2}{\gamma\omega^2} - \dfrac{1}{M} - \dfrac{(1-\alpha)^2}{\lambda^{(2)} + 2\mu^{(2)}} & \left(\dfrac{\lambda^{(2)} + 2\mu^{(2)}\alpha}{\lambda^{(2)} + 2\mu^{(2)}} - \dfrac{\rho_f}{\gamma}\right)\mathrm iK \\ \dfrac{\lambda^{(2)}\mathrm iK}{\lambda^{(2)} + 2\mu^{(2)}} & \left(\dfrac{\lambda^{(2)} + 2\mu^{(2)}\alpha}{\lambda^{(2)} + 2\mu^{(2)}} - \dfrac{\rho_f}{\gamma}\right)\mathrm iK & \left(\rho^{(2)} - \dfrac{\rho_f^2}{\gamma}\right)\omega^2 - \dfrac{4\lambda^{(2)}\mu^{(2)} + 4\mu^{(2)2}}{\lambda^{(2)} + 2\mu^{(2)}}K^2 \end{pmatrix}, \tag{28}$$

$$\boldsymbol A_2^{(2)} = \begin{pmatrix} (\rho^{(2)} + \gamma - 2\rho_f)\omega^2 & (\rho_f - \gamma)\omega^2 & \mathrm iK \\ (\rho_f - \gamma)\omega^2 & \gamma\omega^2 & 0 \\ \mathrm iK & 0 & -\dfrac{1}{\mu^{(2)}} \end{pmatrix}. \tag{29}$$

Lamé 系数 $\lambda^{(2)}, \mu^{(2)}$ 的上标强调两相饱和介质属性,方程组(27)的矩阵特征值是

$$\lambda_{1,2}^2 = K^2 - K_{\alpha_1}^{(2)2},\ \lambda_{3,4}^2 = K^2 - K_{\alpha_2}^{(2)2},\ \lambda_{5,6}^2 = K^2 - K_{\beta}^{(2)2}, \tag{30}$$

其中 K 为视速度波数.设

$$a_1^{(2)} = \begin{cases} \sqrt{K^2 - K_{\alpha_1}^{(2)2}}, & K > K_{\alpha_1}^{(2)}, \\ \mathrm i\sqrt{K_{\alpha_1}^{(2)2} - K^2}, & K < K_{\alpha_1}^{(2)}; \end{cases} \quad a_2^{(2)} = \begin{cases} \sqrt{K^2 - K_{\alpha_2}^{(2)2}}, & K > K_{\alpha_2}^{(2)}, \\ \mathrm i\sqrt{K_{\alpha_2}^{(2)2} - K^2}, & K < K_{\alpha_2}^{(2)}; \end{cases} \tag{31}$$

$$b^{(2)} = \begin{cases} \sqrt{K^2 - K_{\beta}^{(2)2}}, & K > K_{\beta}^{(2)}, \\ \mathrm i\sqrt{K_{\beta}^{(2)2} - K^2}, & K < K_{\beta}^{(2)}. \end{cases} \tag{32}$$

方程(27)解矩阵的特征向量矩阵是

$$\boldsymbol{L}^{(2)}=\begin{pmatrix}\boldsymbol{L}_1^{(2)} & \boldsymbol{L}_1^{(2)}\\ \boldsymbol{L}_2^{(2)} & -\boldsymbol{L}_2^{(2)}\end{pmatrix}, \tag{33}$$

其中

$$\boldsymbol{L}_1^{(2)}=\begin{pmatrix} a_1^{(2)} & a_2^{(2)} & K\\ (1+\xi_1)a_1^{(2)} & (1+\xi_2)a_2^{(2)} & (1+\xi_3)K\\ 2\mu^{(2)}K\mathrm{i}a_1^{(2)} & 2\mu^{(2)}K\mathrm{i}a_2^{(2)} & 2\mu^{(2)}\Omega^{(2)}\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{L}_2^{(2)}=\begin{pmatrix} H_1K_{\alpha_1}^{(2)2}-2\mu^{(2)}K^2 & H_2K_{\alpha_2}^{(2)2}-2\mu^{(2)}K^2 & -2\mu^{(2)}Kb^{(2)}\\ M(\alpha+\xi_1)K_{\alpha_1}^2 & M(\alpha+\xi_2)K_{\alpha_2}^{(2)2} & 0\\ -K\mathrm{i} & -K\mathrm{i} & -b^{(2)}\mathrm{i}\end{pmatrix}, \tag{34}$$

式中 $H_n=\lambda^{(2)}+2\mu^{(2)}+(\alpha-1)M(\alpha+\xi_n)$,$\Omega^{(2)}=K^2-K_\beta^{(2)2}/2$.特征向量矩阵(33)式的逆矩阵为

$$\boldsymbol{L}^{(2)-1}=\frac{1}{2}\begin{pmatrix}\boldsymbol{L}_1^{(2)-1} & \boldsymbol{L}_2^{(2)-1}\\ \boldsymbol{L}_1^{(2)-1} & -\boldsymbol{L}_2^{(2)-1}\end{pmatrix}, \tag{35}$$

其中

$$\boldsymbol{L}_1^{(2)-1}=\begin{pmatrix} \frac{1}{a_1^{(2)}}\left(T_2\frac{2K^2}{K_\beta^{(2)2}}-K_2\right) & \frac{R}{a_1^{(2)}} & \frac{-1}{a_1^{(2)}}\frac{K\mathrm{i}}{\mu^{(2)}K_\beta^{(2)2}}T_2\\ \frac{-1}{a_2^{(2)}}\left(T_1\frac{2K^2}{K_\beta^{(2)2}}-K_1\right) & \frac{-R}{a_2^{(2)}} & \frac{1}{a_2^{(2)}}\frac{K\mathrm{i}}{\mu^{(2)}K_\beta^{(2)2}}T_1\\ \frac{-2K\mathrm{i}}{K_\beta^{(2)2}} & 0 & \frac{-1}{\mu^{(2)}K_\beta^{(2)2}}\end{pmatrix}, \tag{36}$$

$$\boldsymbol{L}_2^{(2)-1}=\frac{1}{\lambda^{(2)}+2\mu^{(2)}}\times$$

$$\begin{pmatrix} -\frac{1}{K_{\alpha_1}^{(2)2}}S_2 & \frac{1}{M}\frac{1}{K_{\alpha_1}^{(2)2}}RH_2 & \frac{2\mu^{(2)}K\mathrm{i}}{K_{\alpha_1}^{(2)2}}S_2\\ \frac{1}{K_{\alpha_2}^{(2)2}}S_1 & \frac{-1}{M}\frac{1}{K_{\alpha_2}^{(2)2}}RH_1 & -\frac{2\mu^{(2)}K\mathrm{i}}{K_{\alpha_2}^{(2)2}}S_1\\ \frac{-K\mathrm{i}}{b^{(2)}}\left(\frac{S_2}{K_{\alpha_1}^{(2)2}}-\frac{S_1}{K_{\alpha_2}^{(2)2}}\right) & \frac{K\mathrm{i}}{b^{(2)}}\frac{R}{M}\left(\frac{H_2}{K_{\alpha_1}^{(2)2}}-\frac{H_1}{K_{\alpha_2}^{(2)2}}\right) & \frac{-2\mu^{(2)}K^2}{b^{(2)}}\left(\frac{S_2}{K_{\alpha_1}^{(2)2}}-\frac{S_1}{K_{\alpha_2}^{(2)2}}\right)-\frac{\lambda^{(2)}+2\mu^{(2)}}{b^{(2)}}\end{pmatrix}, \tag{37}$$

式中

$$R=1/(\xi_1-\xi_2),\ S_n=(\alpha+\xi_n)/(\xi_1-\xi_2),$$
$$T_n=(\rho_{\mathrm{f}}/\gamma+\xi_n)/(\xi_1-\xi_2),\ K_n=(1+\xi_n)/(\xi_1-\xi_2).$$

如剪切波(SH 波)入射在 xOy 平面,波传播的介质位移、应力如下:

$$[u_y^{(2)},\ \bar{\sigma}_{zy}^{(2)}]^{\mathrm{T}}=[f_7^{(2)},f_8^{(2)}]^{\mathrm{T}}\mathrm{e}^{-\mathrm{i}Kx}, \tag{38}$$

这里 $u_y^{(2)}$ 和 $\bar{\sigma}_{zy}^{(2)}$ 分别为 xOy 两相饱和介质传播平面上 y 向位移和 Z 面上 y 向的剪应力.$f_7^{(2)}$ 和 $f_8^{(2)}$ 分别是关于 SH 波传播的待定未知位移-应力函数 $f_7^{(2)}(z)$ 和 $f_8^{(2)}(z)$.同理可得位移-应力解矩阵的特征向量矩阵是

$$\boldsymbol{L}_3^{(2)} = \begin{pmatrix} K_\beta^{(2)} & K_\beta^{(2)} \\ \mu^{(2)} b^{(2)} K_\beta^{(2)} & -\mu^{(2)} b^{(2)} K_\beta^{(2)} \end{pmatrix}, \tag{39}$$

对应的逆矩阵是

$$\boldsymbol{L}_3^{(2)-1} = \frac{1}{2\,\mu^{(2)} K_\beta^{(2)2}} \begin{pmatrix} \mu^{(2)} K_\beta^{(2)} & K_\beta^{(2)}/b^{(2)} \\ \mu^{(2)} K_\beta^{(2)} & -K_\beta^{(2)}/b^{(2)} \end{pmatrix}. \tag{40}$$

2.2　位移-应力微分方程组与解矩阵对单相介质的退化

2.2.1　入射在 xOz 平面的压缩波(P 波)和偏振剪切波(SV 波)

对于单相介质,这时流相相对位移 $w_z^{(2)}$,流相孔隙压力 $p^{(2)}$ 为 0;且 $\alpha = 0$,$\rho_{\mathrm{f}} = 0$;$M = 0$,$\gamma = 0$ ($\rho_{\mathrm{a}} = 0$).如以 $u_x^{(1)}$,$u_z^{(1)}$ 取代 $u_x^{(2)}$,$u_z^{(2)}$,分别代表单相介质 xOz 传播平面上 x 与 z 向位移,$\sigma_{zx}^{(1)}$,$\sigma_{zz}^{(1)}$ 取代 $\bar{\sigma}_{zx}^{(2)}$,$\bar{\sigma}_{zz}^{(2)}$ 分别为 Z 面的剪应力与正应力,$f_i^{(1)}$ 取代 $f_i^{(2)}$ 为相关波传播位移,应力待定函数.$\lambda^{(1)}$,$\mu^{(1)}$ 为单相介质的 Lamé 系数,$\rho^{(1)}$ 为单相介质质量密度;满足波传播的一阶位移-应力微分方程组可由式(27)简单退化得到

$$\frac{\mathrm{d}}{\mathrm{d}z} f_l^{(1)} = \begin{pmatrix} \boldsymbol{0} & \boldsymbol{A}_1^{(1)} \\ \boldsymbol{A}_2^{(1)} & \boldsymbol{0} \end{pmatrix} f_l^{(1)}, \tag{41}$$

$$\begin{cases} \boldsymbol{A}_1^{(1)} = \begin{pmatrix} \dfrac{1}{\lambda^{(1)} + 2\mu^{(1)}} & \dfrac{\lambda^{(1)} K\mathrm{i}}{\lambda^{(1)} + 2\mu^{(1)}} \\ \dfrac{\lambda^{(1)} K\mathrm{i}}{\lambda^{(1)} + 2\mu^{(1)}} & \dfrac{4\mu^{(1)}(\lambda^{(1)} + \mu^{(1)})K^2}{\lambda^{(1)} + 2\mu^{(1)}} - \rho^{(1)}\omega^2 \end{pmatrix}, \\ \boldsymbol{A}_2^{(1)} = \begin{pmatrix} -\rho^{(1)}\omega^2 & \mathrm{i}K \\ \mathrm{i}K & \mu^{(1)} - 1 \end{pmatrix}. \end{cases} \tag{42}$$

这时特征值 $X_{1,2,3,4}$ 是

$$X_{1,2}^2 = K^2 - K_\alpha^{(1)2},\ X_{3,4}^2 = K^2 - K_\beta^{(1)2}, \tag{43}$$

$K_\alpha^{(1)} = \omega/\alpha^{(1)}$,$K_\beta^{(1)} = \omega/\beta^{(1)}$ 分别是单相介质的压缩波数与剪切波数.$\alpha^{(1)}$,$\beta^{(1)}$ 分别为单相介质中传播的纵波速与横波速.方程组的特征向量矩阵

$$\boldsymbol{L}^{(1)} = \begin{pmatrix} \boldsymbol{L}_1^{(1)} & \boldsymbol{L}_1^{(1)} \\ \boldsymbol{L}_2^{(1)} & -\boldsymbol{L}_2^{(1)} \end{pmatrix}, \tag{44}$$

其值退化为

$$\boldsymbol{L}_1^{(1)} = \begin{pmatrix} a^{(1)} & K \\ -2\,\mu^{(1)} K\mathrm{i} a^{(1)} & -2\,\mu^{(1)} \Omega^{(1)} \mathrm{i} \end{pmatrix},\ \boldsymbol{L}_2^{(1)} = \begin{pmatrix} 2\,\mu^{(1)} \Omega^{(1)} & 2\,\mu^{(1)} K b^{(1)} \\ -K\mathrm{i} & -b^{(1)}\mathrm{i} \end{pmatrix}, \tag{45}$$

$\Omega^{(1)} = K^2 - K_\beta^{(1)2}/2$.特征向量矩阵(44)的逆矩阵是

$$\boldsymbol{L}^{(1)-1} = \frac{1}{2} \begin{pmatrix} \boldsymbol{L}_1^{(1)-1} & \boldsymbol{L}_2^{(1)-1} \\ \boldsymbol{L}_1^{(1)-1} & -\boldsymbol{L}_2^{(1)-1} \end{pmatrix}, \tag{46}$$

$$\begin{cases} \boldsymbol{L}_1^{(1)-1} = \dfrac{1}{\mu^{(1)} K_\beta^{(1)2}} \begin{pmatrix} -\dfrac{2\,\mu^{(1)} \Omega^{(1)}}{a^{(1)}} & \dfrac{K\mathrm{i}}{a^{(1)}} \\ 2\,\mu^{(1)} K & -\mathrm{i} \end{pmatrix}, \\ \boldsymbol{L}_2^{(1)-1} = \dfrac{1}{\mu^{(1)} K_\beta^{(1)2}} \begin{pmatrix} -1 & 2\,\mu^{(1)} K\mathrm{i} \\ \dfrac{K}{b^{(1)}} & -\dfrac{2\,\mu^{(1)} \Omega^{(1)} \mathrm{i}}{b^{(1)}} \end{pmatrix}, \end{cases} \tag{47a}$$

$$a^{(1)}=\begin{cases}\sqrt{K^2-K_\alpha^{(1)2}}, & K>K_\alpha^{(1)},\\ \mathrm{i}\sqrt{K_\alpha^{(1)2}-K^2}, & K<K_\alpha^{(1)},\end{cases}\quad b^{(1)}=\begin{cases}\sqrt{K^2-K_\beta^{(1)2}}, & K>K_\beta^{(1)},\\ \mathrm{i}\sqrt{K_\beta^{(1)2}-K^2}, & K<K_\beta^{(1)}.\end{cases} \tag{47b}$$

2.2.2 剪切波(SH 波)入射在 xOy 平面

同理在 SH 波入射的 xOy 平面,满足波传播的一阶微分方程组亦可由式(39)退化.方程解矩阵的特征向量矩阵是

$$\boldsymbol{L}_3^{(1)}=\begin{pmatrix}K_\beta^{(1)} & K_\beta^{(1)}\\ \mu^{(1)}b^{(1)}K_\beta^{(1)} & -\mu^{(1)}b^{(1)}K_\beta^{(1)}\end{pmatrix}; \tag{48}$$

特征向量矩阵的逆矩阵是

$$\boldsymbol{L}_3^{(1)-1}=\frac{1}{2\mu^{(1)}K_\beta^{(1)2}}\begin{pmatrix}\mu^{(1)}K_\beta^{(1)} & K_\beta^{(1)}/b^{(1)}\\ \mu^{(1)}K_\beta^{(1)} & -K_\beta^{(1)}/b^{(1)}\end{pmatrix}. \tag{49}$$

可以验证式(41)~(48)都符合以单相介质波动方程推得的结果.

3 半空间多层两相饱和介质应力-位移传递函数对单相介质的退化

3.1 波从两相饱和多层介质向单相介质的传递矩阵

3.1.1 介质的应力-位移函数传递矩阵

如两相饱和多层或单相介质(见图 1),第 m 层位移-应力函数为 $\boldsymbol{f}^{(n)}(z)_m$,第 $m-1$ 层为 $\boldsymbol{f}^{(n)}(z)_{m-1}$;则由传递矩阵 $\boldsymbol{P}^{(n)}(z_m,z_{m-1})$ 有

$$\boldsymbol{f}^{(n)}(z_m)=\boldsymbol{P}^{(n)}(z_m,z_{m-1})\boldsymbol{f}^{(n)}(z_{m-1}). \tag{50}$$

介质传递矩阵可表达为[13-14]

$$\begin{aligned}\boldsymbol{P}^{(n)}(z_i,z_{i-1})&=\boldsymbol{L}_{(i)}^{(n)}\boldsymbol{\Lambda}_{(i)}^{(n)}\boldsymbol{L}_{(i)}^{(n)-1}=\frac{1}{2}\begin{pmatrix}\boldsymbol{L}_{1(i)}^{(n)} & \boldsymbol{L}_{1(i)}^{(n)}\\ \boldsymbol{L}_{2(i)}^{(n)} & -\boldsymbol{L}_{2(i)}^{(n)}\end{pmatrix}\begin{pmatrix}\boldsymbol{\Lambda}_{1(i)}^{(n)} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{\Lambda}_{2(i)}^{(n)}\end{pmatrix}\begin{pmatrix}\boldsymbol{L}_{1(i)}^{(n)-1} & \boldsymbol{L}_{2(i)}^{(n)-1}\\ \boldsymbol{L}_{1(i)}^{(n)-1} & -\boldsymbol{L}_{2(i)}^{(n)-1}\end{pmatrix}=\\ &\frac{1}{2}\begin{pmatrix}\boldsymbol{L}_{1(i)}^{(n)}(\boldsymbol{\Lambda}_{1(i)}^{(n)}+\boldsymbol{\Lambda}_{2(i)}^{(n)})\boldsymbol{L}_{1(i)}^{(n)-1} & \boldsymbol{L}_{1(i)}^{(n)}(\boldsymbol{\Lambda}_{1(i)}^{(n)}-\boldsymbol{\Lambda}_{2(i)}^{(n)})\boldsymbol{L}_{2(i)}^{(n)-1}\\ \boldsymbol{L}_{2(i)}^{(n)}(\boldsymbol{\Lambda}_{1(i)}^{(n)}-\boldsymbol{\Lambda}_{2(i)}^{(n)})\boldsymbol{L}_{1(i)}^{(n)-1} & \boldsymbol{L}_{2(i)}^{(n)}(\boldsymbol{\Lambda}_{1(i)}^{(n)}+\boldsymbol{\Lambda}_{2(i)}^{(n)})\boldsymbol{L}_{2(i)}^{(n)-1}\end{pmatrix}=\\ &\begin{pmatrix}\boldsymbol{L}_{1(i)}^{(n)}\boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(n)}\boldsymbol{L}_{1(i)}^{(n)-1} & \boldsymbol{L}_{1(i)}^{(n)}\boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(n)}\boldsymbol{L}_{2(i)}^{(n)-1}\\ \boldsymbol{L}_{2(i)}^{(n)}\boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(n)}\boldsymbol{L}_{1(i)}^{(n)-1} & \boldsymbol{L}_{2(i)}^{(n)}\boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(n)}\boldsymbol{L}_{2(i)}^{(n)-1}\end{pmatrix},\end{aligned} \tag{51}$$

式中

$$\boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(n)}=(\boldsymbol{\Lambda}_{1(i)}^{(n)}+\boldsymbol{\Lambda}_{2(i)}^{(n)})/2,\ \boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(n)}=(\boldsymbol{\Lambda}_{1(i)}^{(n)}-\boldsymbol{\Lambda}_{2(i)}^{(n)})/2.$$

因而两相饱和多层弹性介质的 P-SV 波传递矩阵是

$$\boldsymbol{P}^{(2)}(z_i,z_{i-1})=\begin{pmatrix}\boldsymbol{L}_{1(i)}^{(2)}\boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(2)}\boldsymbol{L}_{1(i)}^{(2)-1} & \boldsymbol{L}_{1(i)}^{(2)}\boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(2)}\boldsymbol{L}_{2(i)}^{(2)-1}\\ \boldsymbol{L}_{2(i)}^{(2)}\boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(2)}\boldsymbol{L}_{1(i)}^{(2)-1} & \boldsymbol{L}_{2(i)}^{(2)}\boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(2)}\boldsymbol{L}_{2(i)}^{(2)-1}\end{pmatrix}, \tag{52}$$

$$\boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(2)}=\mathrm{diag}[\mathrm{ch}\,a_{1(i)}^{(2)}d_i,\ \mathrm{ch}\,a_{2(i)}^{(2)}d_i,\ \mathrm{ch}\,b_{(i)}^{(2)}d_i],$$

$$\boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(2)}=\mathrm{diag}[\mathrm{sh}\,a_{1(i)}^{(2)}d_i,\ \mathrm{sh}\,a_{2(i)}^{(2)}d_i,\ \mathrm{sh}\,b_{(i)}^{(2)}d_i],$$

d_i 为介质层厚度.两相饱和多层弹性介质的 SH 波传递矩阵

$$\bar{\boldsymbol{P}}^{(2)}(z_i,z_{i-1})=\begin{pmatrix}\mathrm{ch}\,b_{(i)}^{(2)}d_i & \dfrac{\mathrm{sh}\,b_{(i)}^{(2)}d_i}{\mu_{(i)}^{(2)}b_{(i)}^{(2)}}\\ \mu_{(i)}^{(2)}b_{(i)}^{(2)}\mathrm{sh}\,b_{(i)}^{(2)}d_i & \mathrm{ch}\,b_{(i)}^{(2)}d_i\end{pmatrix}, \tag{53}$$

它们也能退化得到单相介质传递矩阵:单相多层弹性介质的 P-SV 波传递矩阵是

$$\boldsymbol{P}^{(1)}(z_i, z_{i-1}) = \begin{pmatrix} \boldsymbol{L}_{1(i)}^{(1)} \boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(1)} \boldsymbol{L}_{1(i)}^{(1)-1} & \boldsymbol{L}_{1(i)}^{(1)} \boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(1)} \boldsymbol{L}_{2(i)}^{(1)-1} \\ \boldsymbol{L}_{2(i)}^{(1)} \boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(1)} \boldsymbol{L}_{1(i)}^{(1)-1} & \boldsymbol{L}_{2(i)}^{(1)} \boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(1)} \boldsymbol{L}_{2(i)}^{(1)-1} \end{pmatrix}, \tag{54}$$

$$\boldsymbol{\Lambda}_{\mathrm{ch}(i)}^{(1)} = \mathrm{diag}[\mathrm{ch}\, a_{(i)}^{(1)} d_i,\ \mathrm{ch}\, b_{(i)}^{(1)} d_i],\ \boldsymbol{\Lambda}_{\mathrm{sh}(i)}^{(1)} = \mathrm{diag}[\mathrm{sh}\, a_{(i)}^{(1)} d_i,\ \mathrm{sh}\, b_{(i)}^{(1)} d_i]. \tag{55}$$

单相多层弹性介质的 SH 波传递矩阵是

$$\bar{\boldsymbol{P}}^{(1)}(z_i, z_{i-1}) = \begin{pmatrix} \mathrm{ch}\, b_{(i)}^{(1)} d_i & \dfrac{\mathrm{sh}\, b_{(i)}^{(1)} d_i}{\mu_{(i)}^{(1)} b_{(i)}^{(1)}} \\ \mu_{(i)}^{(1)} b_{(i)}^{(1)} \mathrm{sh}\, b_{(i)}^{(1)} d_i & \mathrm{ch}\, b_{(i)}^{(1)} d_i \end{pmatrix}. \tag{56}$$

同样也可以验证,式(54)~(56)也都符合以单相介质波动方程推得的结果.

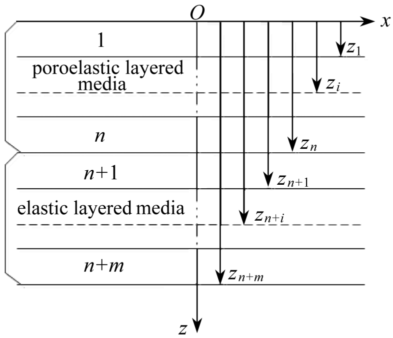


图 1　直角坐标系下多层弹性介质半空间模型

Fig.1　The half-space model of a multilayer elastic media in Cartesian coordinate system

3.1.2　介质界面过渡传递矩阵

1) 两相饱和介质对单相介质界面应力-位移连续与连续条件

两相饱和介质与单相介质界面处于 m_1 层,应力-位移连续条件是

① $u_z^{(2)}|_{z=z_{m_1}^-} = u_z^{(1)}|_{z=z_{m_1}^+}$　(界面两侧固相骨架法向位移连续),　(57)

② $u_x^{(2)}|_{z=z_{m_1}^-} = u_x^{(1)}|_{z=z_{m_1}^+}$　(界面两侧固相骨架切向位移连续),　(58)

③ $u_y^{(2)}|_{z=z_{m_1}^-} = u_y^{(1)}|_{z=z_{m_1}^+}$　(界面两侧固相骨架 y 向位移连续),　(59)

④ 界面处, $w_z^{(2)}|_{z=z_{m_1}^-} = 0$　(两相饱和介质流相介质不流入单相介质),　(60)

⑤ $-(\bar{\sigma}_{zz}^{(2)} + p^{(2)})|_{z=z_{m_1}^-} = \sigma_{zz}^{(1)}|_{z=z_{m_1}^+}$　(界面两侧正应力连续),　(61)

⑥ $-\bar{\sigma}_{zx}^{(2)}|_{z=z_{m_1}^-} = \sigma_{zx}^{(1)}|_{z=z_{m_1}^+}$　(界面两侧 x 向剪应力连续),　(62)

⑦ $\bar{\sigma}_{zy}^{(2)}|_{z=z_{m_1}^-} = \sigma_{zy}^{(1)}|_{z=z_{m_1}^+}$　(界面两侧 y 向剪应力连续),　(63)

⑧ $\bar{\sigma}_{zz}^{(2)}|_{z=0} = 0$　(自由表面正应力为 0),　(64)

⑨ $\bar{\sigma}_{zx}^{(2)}|_{z=0} = 0$, $\sigma_{zy}^{(2)}|_{z=0} = 0$　(自由表面剪应力为 0),　(65)

⑩ $p^{(2)}|_{z=0} = 0$　(在自由表面处不考虑大气压力影响).　(66)

上述①~④为位移连续条件,⑤~⑦为应力连续条件,⑧~⑩为自由表面应力边界条件.相应的位移连续条件已包含波的反射.

2) 两相饱和介质对单相介质的界面应力-位移过渡的传递矩阵

对于单相介质第 $m_2 - 1$ 层底面的位移-应力关系对界面 m_1 层,由传递矩阵有

$$(\boldsymbol{f}_l^{(1)})_{m_2-1} = \boldsymbol{P}^{(1)}(z_{m_2-1}, z_{m_1})(\boldsymbol{f}_l^{(1)})_{m_1^+}. \tag{67}$$

另外,通过传递矩阵建立多层两相饱和介质界面 m_1 层的位移、应力与自由表面的位移、应力关系是

$$(\boldsymbol{f}_l^{(2)})_{m_{\bar{1}}} = \boldsymbol{P}^{(2)}(z_{m_1}, z_0)(\boldsymbol{f}_l^{(2)})_0. \tag{68}$$

利用两相饱和介质与单相介质在界面处的连续条件可得

$$(\boldsymbol{f}_l^{(1)})_{m_2-1} = \boldsymbol{P}^{(1)}(z_{m_2-1}, z_{m_1})\boldsymbol{J}_{12}\boldsymbol{P}^{(2)}(z_{m_1}, z_0)(\boldsymbol{f}_l^{(1)})_0, \tag{69}$$

其中矩阵 $\boldsymbol{J}_{12}$ 为

$$\boldsymbol{J}_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{70}$$

定义矩阵 $\boldsymbol{J}_{12}$ 为界面过渡传递矩阵，是 4×6 矩阵，它联系两相传递矩阵和单相传递矩阵.原有的两相介质的 6×6 应力-位移传递矩阵通过过渡矩阵，能与单相介质的 4×4 应力-位移传递矩阵结合.

对于 SH 波从两相到单相的传播，由于不管在两相和单相介质中都只有 2 个未知函数(单相介质中是 $u_y^{(1)}$ 和 $\boldsymbol{\sigma}_{zy}^{(1)}$，两相饱和介质中是 $u_y^{(2)}$ 和 $\boldsymbol{\sigma}_{zy}^{(2)}$)，所以两相和单相介质中的传递矩阵都是 2×2 矩阵，故 SH 波从两相传到单相不需要过渡传递矩阵联系.

4 波函数退化与传递的数值分析比较

在以下多层半空间波传播模型(图 1)的参数计算中，取 3 层两相饱和介质和 3 层单相介质.各层的物理参数见表 1 与表 2.

表 1 3 层两相饱和弹性层物理参数[27]

Table 1 The physical parameters of three layers of two-phase saturated media[27]

layer	1	2	3
thickness d /m	0.25	0.35	0.40
ρ_f /(kg/m^3)	1 000	1 000	1 000
ρ_s /(kg/m^3)	2 220	2 216	2 221
n	0.24	0.32	0.35
λ /GPa	8.933 3	3.666 7	2.933 3
μ /GPa	9.7	6.2	5.5
α	0.579 2	0.786 9	0.819 7
M /GPa	8.448 8	6.320 7	5.745 2
κ /(m^2/s)	10^{-9}	4×10^{-9}	2×10^{-9}
m /(kg/m^3)	8 750	5 938	5 634

表 2 3 层单相弹性层物理参数

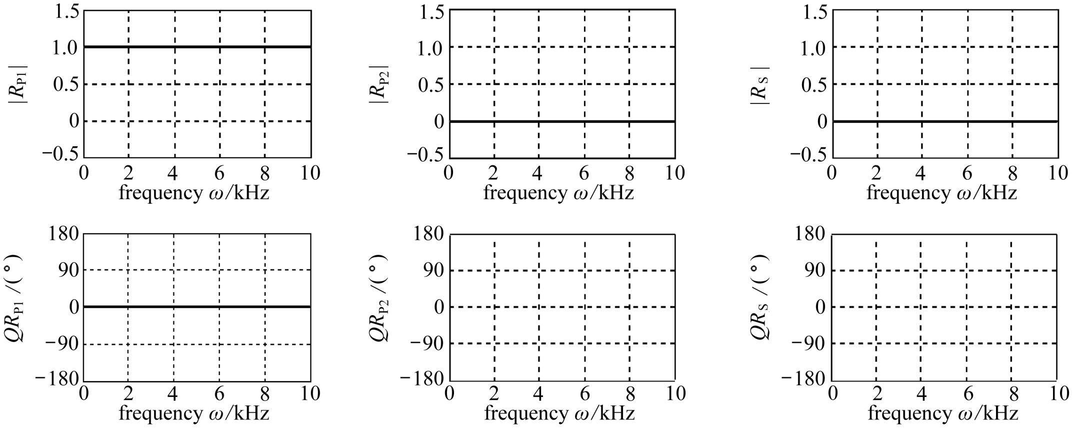
Table 2 The physical parameters of three layers of single-phase media

layer	thickness d /m	ρ /(kg/m^3)	λ /GPa	μ /GPa
4	0.25	1 720	6.2	9.2
5	0.35	1 650	5.1	8.4
6	0.40	1 700	5.0	8.1

1) 多层两相饱和介质的反射和透射系数计算

如 P1 波从入射层入射，计算中入射层采用表 1 第 1 列参数，透射层采用表 1 第 2 列参数.图 2 为垂直入射波在两相饱和介质自由界面传播的反射系数及相位.图 3 是斜入射波在两相饱和介质自由界面传播的反射系数及相位.图 4 是斜入射波在两相饱和介质层界面的透射系数及相位.这些计算与 Deresiewicz 等[20]的结果吻合.

对于波从底层入射的多层半空间两相饱和介质,其在自由界面传播的的反射系数计算结果可见图5,这里没有透射波.

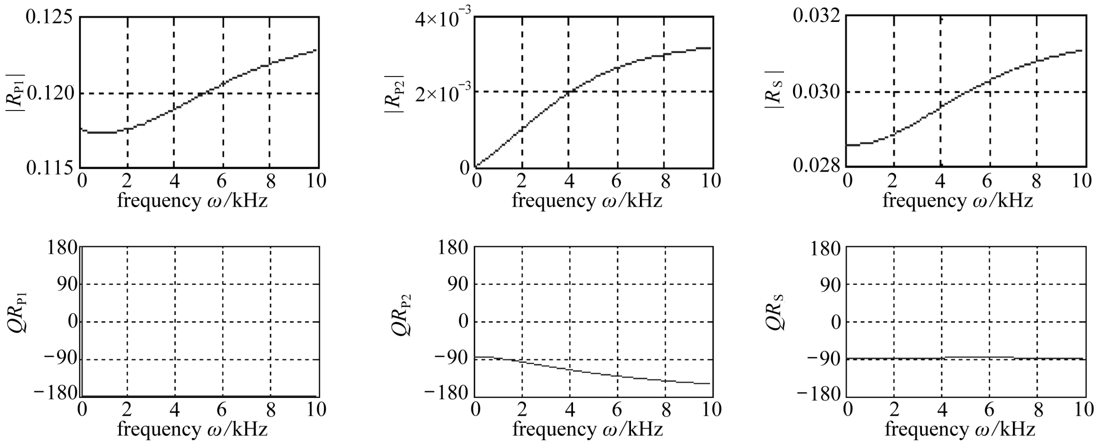


（a）P1 波　（b）P2 波　（c）S 波

（a）P1 wave　（b）P2 wave　（c）S wave

图 2　垂直入射波在两相饱和弹性介质自由界面传播的反射系数绝对值及其相位

Fig.2　Absolute reflection coefficients and phase values at the free surface of poroelastic saturated media induced by normal-incidence waves

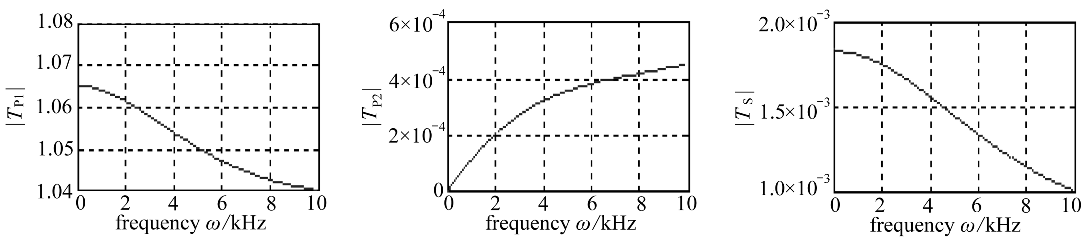


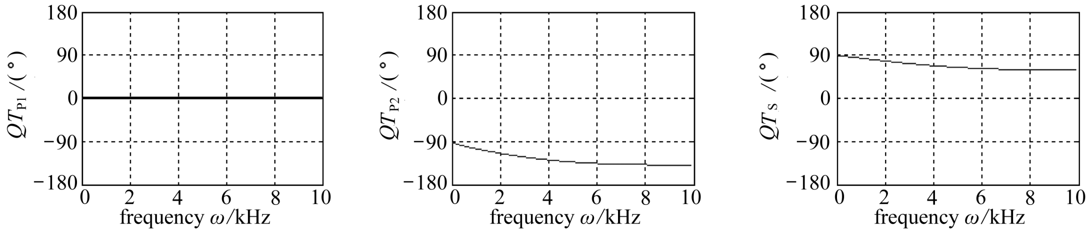
（a）P1 波　（b）P2 波　（c）S 波

（a）P1 wave　（b）P2 wave　（c）S wave

图 3　斜入射波在两相饱和弹性介质自由界面传播的反射系数绝对值及其相位

Fig.3　Absolute reflection coefficients and phase values at the free surface of poroelastic saturated media induced by oblique-incidence waves



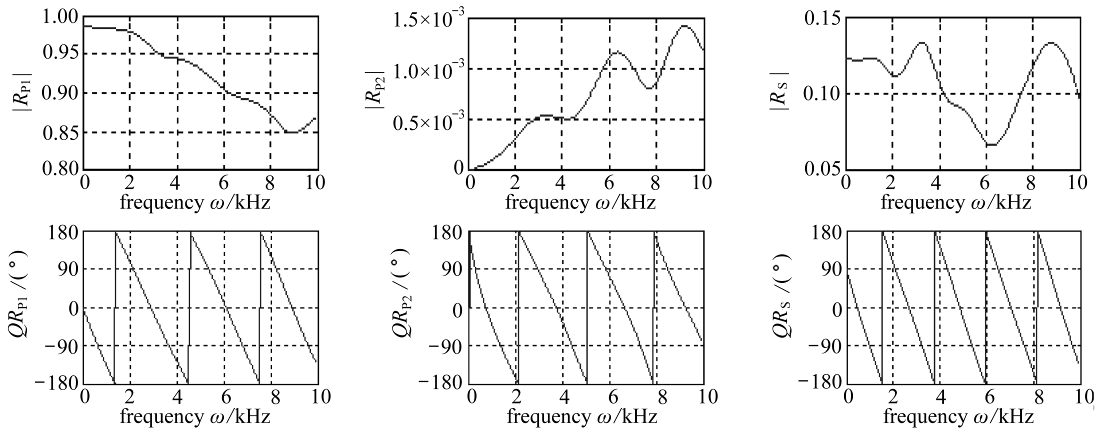


(a) P1 波 (b) P2 波 (c) S 波

(a) P1 wave (b) P2 wave (c) S wave

图 4 斜入射波在两相饱和介质层界面传播的透射系数绝对值及其相位

Fig.4 Absolute transmission coefficients and phase values at the interface of poroelastic saturated media induced by oblique-incidence waves

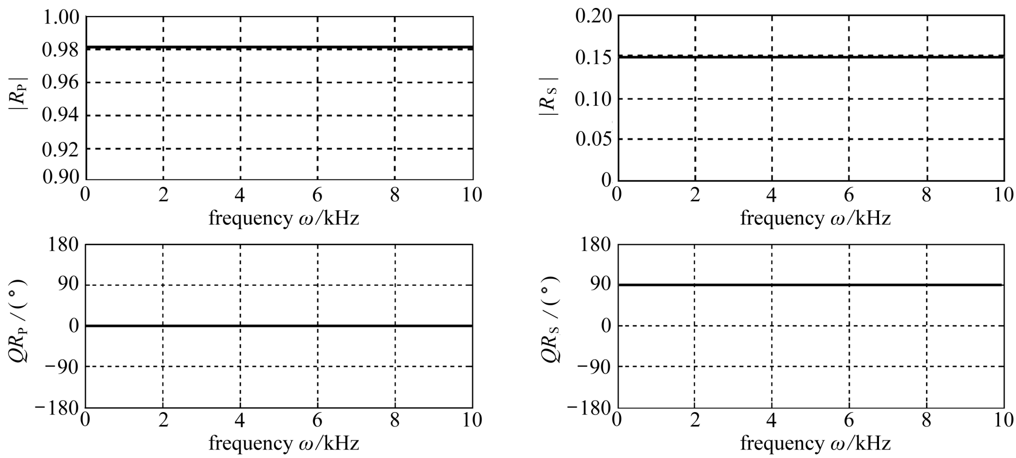


(a) P1 波 (b) P2 波 (c) S 波

(a) P1 wave (b) P2 wave (c) S wave

图 5 底层入射波在多层两相饱和介质自由界面传播的反射系数绝对值及其相位

Fig.5 Absolute reflection coefficients and phase values at the free surface of poroelastic multi-layered saturated media induced by incident waves at the bottom



(a) P 波 (b) S 波

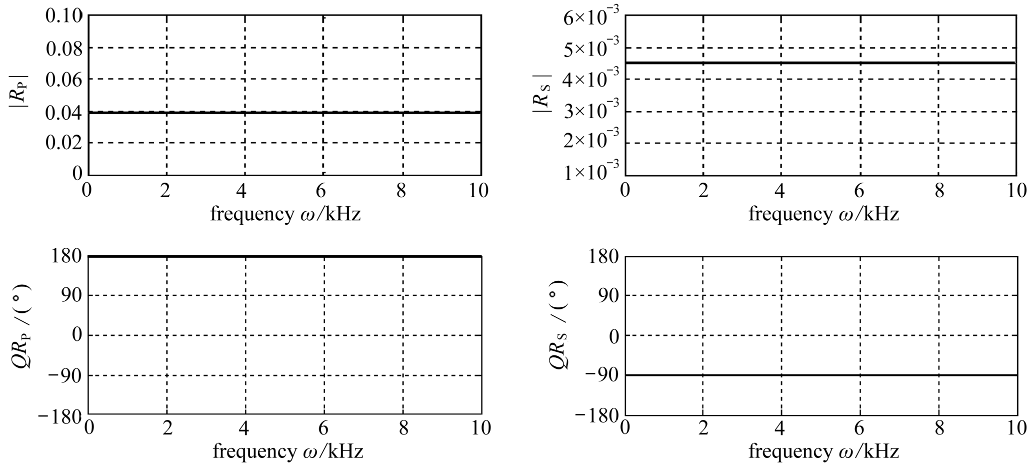
(a) P wave (b) S wave

图 6 斜入射波在单相介质自由界面传播的反射系数绝对值及其相位

Fig.6 Absolute reflection coefficients and phase values at the free surface of elastic media induced by oblique-incidence waves

2）退化成多层单相介质波传播的反射系数

取 $w_z^{(2)}=0, p^{(2)}=0; \alpha=0, \rho_f=0; M=\infty, \gamma=\infty$；两相饱和介质传递矩阵能够退化为单相介质传递矩阵.如果波从顶层入射，其参数采用表 2 第 4 层的结果，而透射层采用表 2 第 5 层的结果.斜入射波在自由界面反射系数和相位的计算结果可如图 6 所示.而在层界面的反射系数和相位的计算结果如图 7 所示.图 8 则为层界面的透射系数和相位的计算结果.



（a）P 波

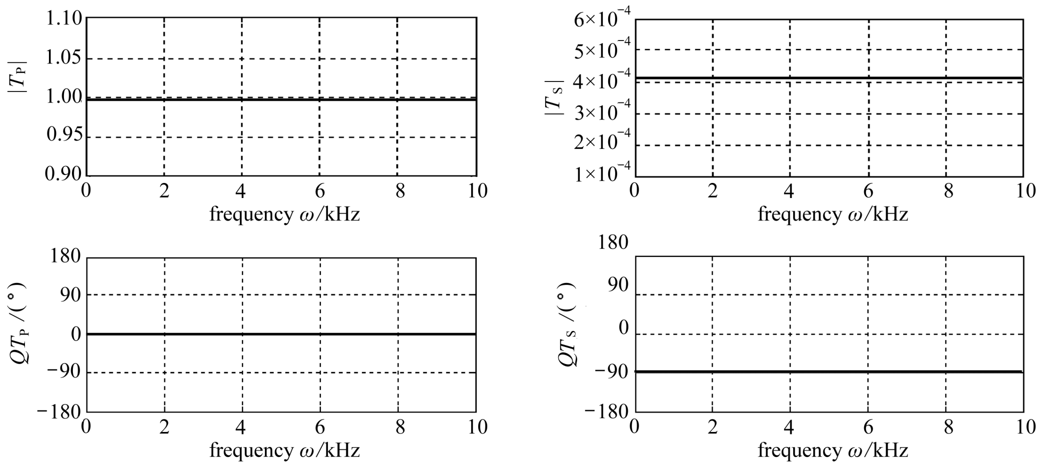
（a）P wave

（b）S 波

（b）S wave

图 7　斜入射波在单相介质层界面传播的反射系数绝对值及其相位

Fig.7　Absolute phase values and the reflection coefficients at the interface of elastic media induced by oblique-incidence waves



（a）P 波

（a）P wave

（b）S 波

（b）S wave

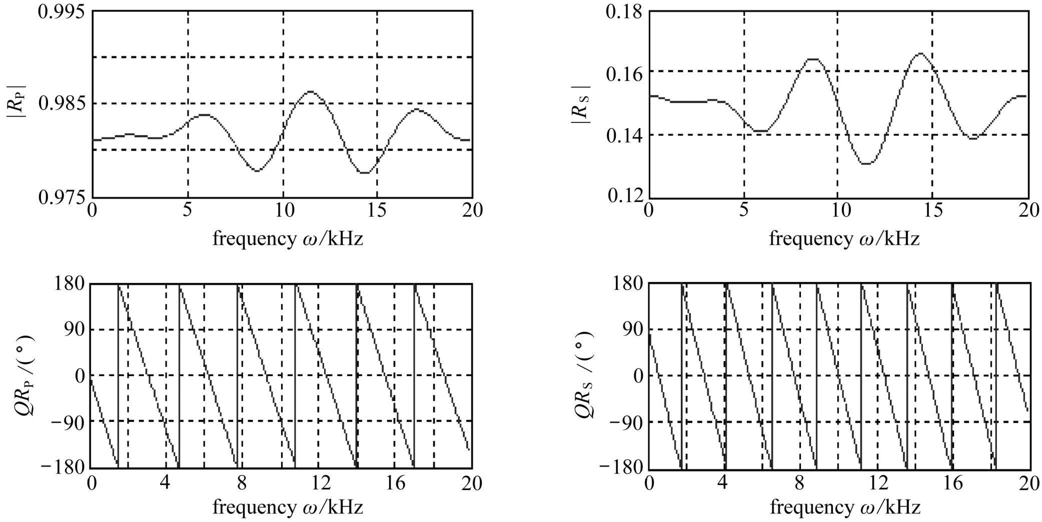
图 8　斜入射波在单相介质层界面传播的透射系数绝对值及其相位

Fig.8　Absolute transmission coefficients and phase values at the interface of elastic media induced by oblique-incidence waves

图 9 与图 10 是从底层入射的多层单相介质自由界面波传播的反射系数绝对值与相位.图 9 是斜入射的，图 10 是垂直入射的.

值得注意的是：图 6～10 的结果分别都能和 Ben-Menahem，Singh[9] 以及陈运泰[11] 在多层

地层震源机制中计算的结果吻合.

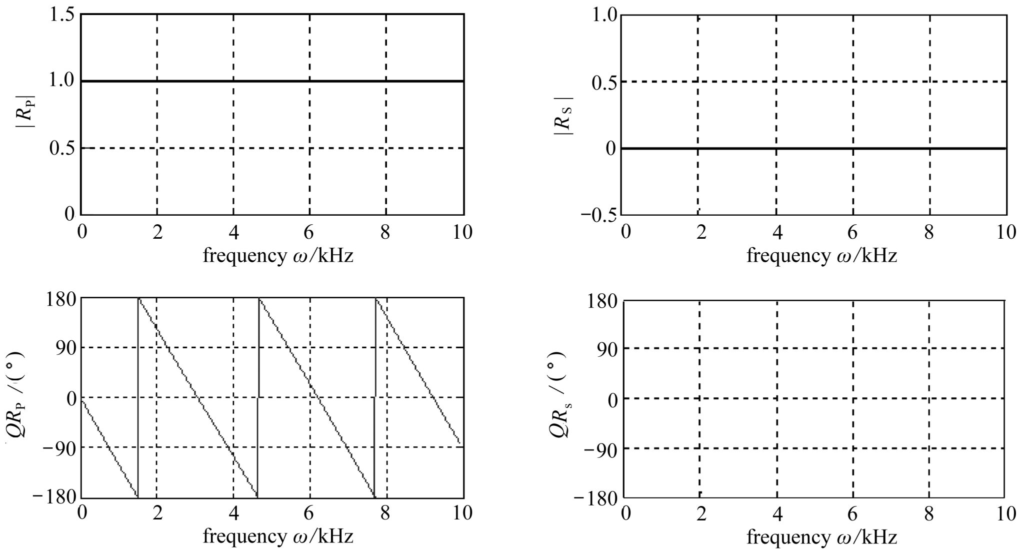


（a）P 波　　　　（b）S 波

（a）P wave　　　　（b）S wave

图 9　斜入射波在单相多层介质中传播的反射系数绝对值及其相位

Fig.9　Absolute reflection coefficients and phase values of elastic multi-layered media induced by oblique-incidence waves



（a）P 波　　　　（b）S 波

（a）P wave　　　　（b）S wave

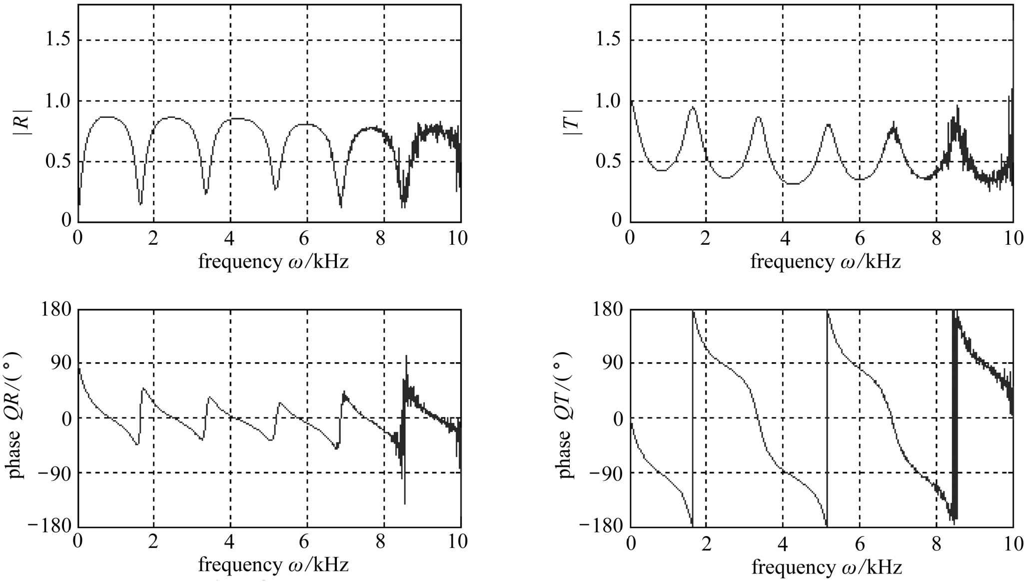
图 10　垂直入射波在单相多层介质中传播的反射系数绝对值及其相位

Fig.10　Absolute reflection coefficients and phase values of elastic multi-layered media induced by normal-incidence waves

3）上表面与下表面都为流相介质层的两相饱和介质

图 11 为当上表面与下表面都是流相介质层时，垂直入射波在两相饱和多层介质中传播的反射系数与透射系数及其相位.从图中可以看到，此结果与 Jocker 等[27]在 2004 年为海上石油勘探模型所做的计算结果完全一致.图 12 表示计算中传播参数关于土层厚度与频率的稳定

性，图中实线为不同渗透系数下数值计算稳定与不稳定的临界线，虚线为相对应的慢波影响曲线[27]．



（a）反射系数　　（b）透射系数

（a）Reflection coefficients　　（b）Transmission coefficients

图 11　垂直入射波在顶底面皆流相的两相饱和多层介质中传播的反射与透射系数绝对值及其相位

Fig.11　Absolute reflection and transmission coefficients and phase values of the fluid-poroelastic-fluid multi-layered staturated media induced by normal-incidence waves

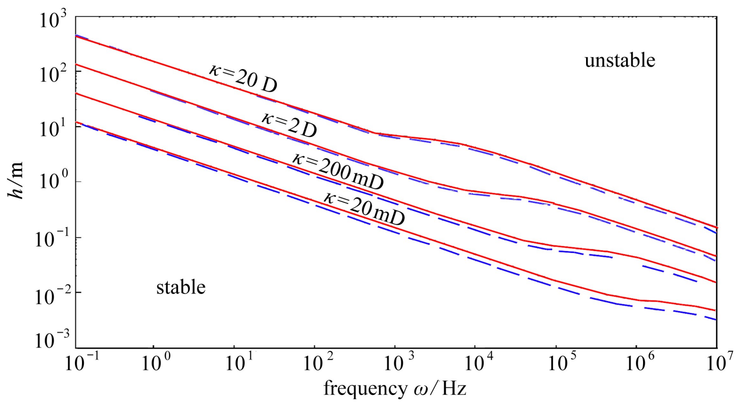
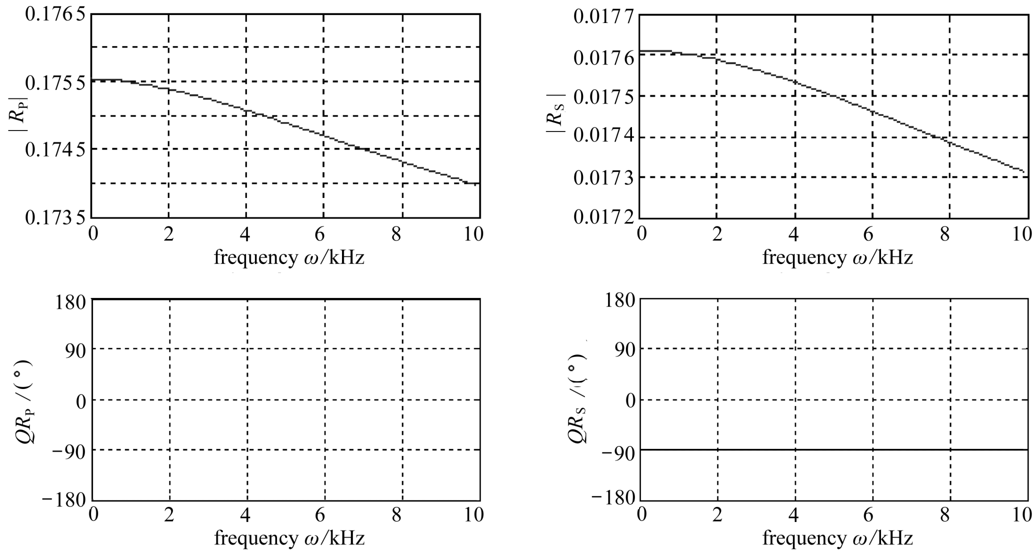


图 12　Jocker[27] 的稳定性分析图

Fig.12　Numerically stable analysis of Jocker[27]

4）单相弹性介质与两相饱和介质层界面反射与透射

计算中透射的两相饱和介质层采用表 1 第 1 层的参数，单相入射层采用表 2 第 4 层的参数．计算结果如图 13 与图 14 所示．图 13 是斜入射波从单相介质向两相饱和介质传播的反射系数及其相位．图 14 是斜入射波从单相介质向两相饱和介质传播的透射系数及其相位．结果也能与弹性介质纯力学分析方法结果互为引证．限于篇幅，此处从略．

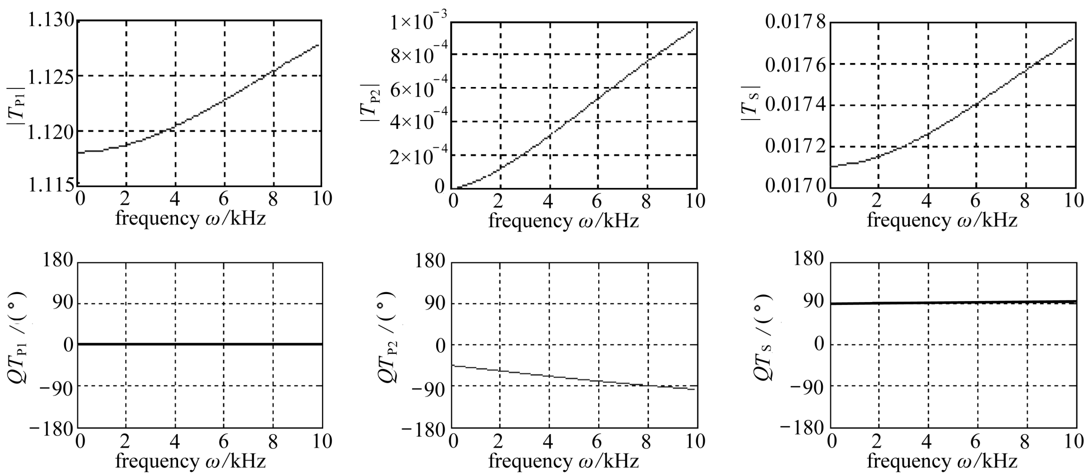


(a) P 波　　(b) S 波

(a) P wave　　(b) S wave

图 13　斜入射波从单相介质向两相饱和介质传播的反射系数绝对值及其相位

Fig.13　Absolute reflection coefficients and phase values of wave propagation from single-phase satarated media to two-phase media induced by oblique-incidence waves



(a) P1 波　　(b) P2 波　　(c) S 波

(a) P1 wave　　(b) P2 wave　　(c) S wave

图 14　斜入射波从单相介质向两相饱和介质传播的透射系数绝对值及其相位

Fig.14　Absolute transmission coefficients and phase values of wave propagation from single-phase media to two-phase saturated media induced by oblique-incidence waves

5　结　　论

1）本文通过 Biot 两相饱和孔隙弹性介质的动力控制方程，利用快纵波、慢纵波解耦，求得满足波传播运动学的一阶位移-应力微分方程组.该方程组及其解答退化后都能满足单相介质的波位移-应力传播微分方程组.

2）本文构建的两相饱和介质与单相介质界面过渡传递矩阵.使原有的两相介质的 6×6 位

移-应力传递矩阵转化为 4×6 矩阵,能与单相介质的 4×4 位移-应力传递矩阵结合.

附　录

入射波在两相饱和介质的 xOz 平面,固相骨架位移、固流合成位移,介质有效应力及孔隙压力见式(26),其本构关系是

$$\sigma_{xx}^{(2)} = \lambda^{(2)}\left(\frac{\partial u_x^{(2)}}{\partial x} + \frac{\partial u_z^{(2)}}{\partial z}\right) + 2\mu^{(2)}\frac{\partial u_x^{(2)}}{\partial x} - \alpha p^{(2)}, \tag{A1a}$$

$$\sigma_{zx}^{(2)} = \mu^{(2)}\left(\frac{\partial u_x^{(2)}}{\partial z} + \frac{\partial u_z^{(2)}}{\partial x}\right), \tag{A1b}$$

$$\sigma_{zz}^{(2)} = \lambda^{(2)}\left(\frac{\partial u_x^{(2)}}{\partial x} + \frac{\partial u_z^{(2)}}{\partial z}\right) + 2\mu^{(2)}\frac{\partial u_z^{(2)}}{\partial z} - \alpha p^{(2)}. \tag{A1c}$$

连续条件是

$$\theta = \alpha u_{j,ji} + p^{(2)}/M, \tag{A2a}$$

$$p^{(2)} = -\alpha M u_{j,ji}^{(2)} - M w_{j,ji}^{(2)} = -\alpha M\left(\frac{\partial u_x^{(2)}}{\partial x} + \frac{\partial u_z^{(2)}}{\partial z}\right) - M\left(\frac{\partial w_x^{(2)}}{\partial x} + \frac{\partial w_z^{(2)}}{\partial z}\right). \tag{A2b}$$

动力平衡方程有以下等式:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zx}}{\partial z} = -\omega^2(\rho^{(2)} u_x^{(2)} + \rho_f w_x^{(2)}), \tag{A3a}$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = -\omega^2(\rho^{(2)} u_z^{(2)} + \rho_f w_z^{(2)}). \tag{A3b}$$

由 Darcy 定律可得

$$\mathrm{i}\omega w_x^{(2)} = \kappa(\rho_f\omega^2 u_x^{(2)} + m'\omega^2 w_x^{(2)} - \partial p^{(2)}/\partial x), \tag{A4a}$$

$$\mathrm{i}\omega w_z^{(2)} = \kappa(\rho_f\omega^2 u_z^{(2)} + m'\omega^2 w_z^{(2)} - \partial p^{(2)}/\partial z). \tag{A4b}$$

这样,就能得到

$$\sigma_{xx}^{(2)} = \lambda^{(2)}(-\mathrm{i}Kf_6^{(2)} + f_1^{'(2)}) - 2\mu^{(2)}\mathrm{i}Kf_6^{(2)} - \alpha f_5^{(2)}, \tag{A5a}$$

$$f_6^{'(2)} = \mathrm{i}Kf_1^{(2)} - f_3^{(2)}/\mu^{(2)}, \tag{A5b}$$

$$f_1^{'} = \frac{\lambda^{(2)}\mathrm{i}K}{\lambda^{(2)} + 2\mu^{(2)}}f_6^{(2)} + \frac{\alpha - 1}{\lambda^{(2)} + 2\mu^{(2)}}f_5^{(2)} - \frac{1}{\lambda^{(2)} + 2\mu^{(2)}}f_4^{(2)}. \tag{A5c}$$

再由式(A2)和式(A3)可得

$$f_5^{(2)} = -\alpha M(-\mathrm{i}Kf_6^{(2)} + f_1^{'(2)}) - M(f_2^{'(2)} - f_1^{'(2)} + \partial w_x/\partial x), \tag{A6}$$

$$\lambda^{(2)}(-K^2 f_6^{(2)} - \mathrm{i}Kf_1^{'(2)}) - 2\mu^{(2)}K^2 f_6^{(2)} + \mathrm{i}K\alpha f_5^{(2)} - f_3^{'(2)} = -\omega^2(\rho^{(2)}f_6^{(2)} + \rho_f w_x^{(2)}), \tag{A7a}$$

$$\mathrm{i}Kf_3^{(2)} - f_4^{'(2)} - f_5^{'(2)} = -\omega^2(\rho^{(2)}f_1^{(2)} + \rho_f f_2^{(2)} - \rho_f f_1^{(2)}). \tag{A7b}$$

另外由式(A4)得出

$$\mathrm{i}\omega w_x^{(2)} = -\kappa(-\mathrm{i}Kf_5^{(2)} - \rho_f\omega^2 f_6^{(2)} - m'\omega^2 w_x^{(2)}), \tag{A8a}$$

$$\mathrm{i}\omega f_3^{(2)} = -\kappa(f_5^{'(2)} - \rho_f\omega^2 f_1^{(2)} - m'\omega^2 f_2^{(2)}). \tag{A8b}$$

由于 $\gamma = (\kappa m'\omega - \mathrm{i})/(\kappa\omega)$, 由式(A8a)可得

$$w_x^{(2)} = -(\mathrm{i}K/\gamma\omega^2)f_5^{(2)} - (\rho_f/\gamma)f_6^{(2)}. \tag{A9}$$

将式(A9)代入到式(A6)与式(A7a)并结合式(A5c),可有

$$f_2^{'} = \left(\frac{\lambda^{(2)} + 2\mu^{(2)}\alpha}{\lambda^{(2)} + 2\mu^{(2)}} - \frac{\rho_f}{\gamma}\right)\mathrm{i}Kf_6^{(2)} + \left[\frac{MK^2 - \gamma\omega^2}{\gamma\omega^2 M} - \frac{(1-\alpha)^2}{\lambda^{(2)} + 2\mu^{(2)}}\right]f_5^{(2)} + \frac{\alpha - 1}{\lambda^{(2)} + 2\mu^{(2)}}f_4^{(2)}, \tag{A10a}$$

$$f_3^{'} = \left[\left(\rho^{(2)} - \frac{\rho_f^2}{\gamma}\right)\omega^2 - \frac{4\lambda^{(2)}\mu^{(2)} + 4\mu^{(2)2}}{\lambda^{(2)} + 2\mu^{(2)}}K^2\right]f_6^{(2)} +$$

$$\left(\frac{\lambda^{(2)}+2\mu^{(2)}\alpha}{\lambda^{(2)}+2\mu^{(2)}}-\frac{\rho_{\mathrm{f}}}{\gamma}\right)\mathrm{i}Kf_5^{(2)}+\frac{\lambda^{(2)}\mathrm{i}K}{\lambda^{(2)}+2\mu^{(2)}}f_4^{(2)}. \tag{A10b}$$

再由式(A8b)解得

$$f_5'=(\rho_{\mathrm{f}}-\gamma)\omega^2 f_1^{(2)}+\gamma\omega^2 f_2^{(2)}. \tag{A11}$$

将式(A11)代入式(A7b),得

$$f_4'=(\rho^{(2)}+\gamma-2\rho_{\mathrm{f}})\omega^2 f_1^{(2)}+(\rho_{\mathrm{f}}-\gamma)\omega^2 f_2^{(2)}+\mathrm{i}Kf_3^{(2)}. \tag{A12}$$

至此已得一阶微分方程的矩阵形式(27)。

对 xOy 平面 SH 波,y 向固相骨架位移及有效应力已示于式(38)。同样由本构关系有

$$\sigma_{yx}^{(2)}=\mu^{(2)}\partial u_y/\partial x, \tag{A13a}$$

$$\sigma_{yz}^{(2)}=\mu^{(2)}\partial u_y/\partial z. \tag{A13b}$$

由动力平衡条件可得

$$\frac{\partial\sigma_{yx}^{(2)}}{\partial x}+\frac{\partial\sigma_{yz}^{(2)}}{\partial z}=-\omega^2(\rho^{(2)}u_y^{(2)}+\rho_{\mathrm{f}}w_y^{(2)}), \tag{A14}$$

这里 $w_y^{(2)}$ 是流相介质 y 向位移,可设 $w_y^{(2)}=f_9^{(2)}\mathrm{e}^{-\mathrm{i}Kx}$。由(A2a),(A2b)式及 Darcy 定律有

$$\mathrm{i}\omega w_y^{(2)}=\kappa\omega^2(\rho_{\mathrm{f}}u_y^{(2)}+m'w_y^{(2)}). \tag{A15}$$

而由式(A13b)和式(A14)及式(A15)可得

$$f_7'^{(2)}=f_8^{(2)}/\mu^{(2)}, \tag{A16}$$

$$f_8'^{(2)}=(\mu^{(2)}K^2-\rho^{(2)}\omega^2)f_7^{(2)}-\rho_{\mathrm{f}}\omega^2 f_9^{(2)}, \tag{A17}$$

$$\mathrm{i}\omega f_9^{(2)}=\kappa\omega^2(\rho_{\mathrm{f}}f_7^{(2)}+m'f_9^{(2)}). \tag{A18}$$

由式(A18)可得中介函数 $f_9^{(2)}\mathrm{e}^{-\mathrm{i}Kx}$:

$$f_9^{(2)}=[-\rho_{\mathrm{f}}\kappa\omega/(-\mathrm{i}+m'\kappa\omega)]f_7^{(2)}=-\rho_{\mathrm{f}}/\gamma f_7^{(2)}. \tag{A19}$$

将式(A19)代入式(A17)得

$$f_8'^{(2)}=[\mu^{(2)}K^2-(\rho^{(2)}-\rho_{\mathrm{f}}^2/\gamma)\omega^2]f_7^{(2)}. \tag{A20}$$

至此,可将式(A16)和式(A20)写成一阶微分方程的矩阵形式。

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Degeneration and Transfer of the Displacement-Stress Functions From Poroelastic Layered Media to Elastic Layered Media

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Abstract: Based on Biot's dynamic governing equations, through decoupling of the fast and slow dilational waves, the first-order differential simultaneous equations for the displacement-stress propagation were obtained, which satisfy the kinetics of wave propagation in the multi-layer poroelastic saturated media. Both the simultaneous equations and the transfer funtions could be degenerated to those for the multilayer single-phase media. With the displacement-stress continuity conditions at the interface between the poroelastic and single-phase media, the interfacial transitional transfer matrix was established by analysis of the propagation of displacement-stress from the poroelastic medium to the single-phase medium. The 4×6 transfer matrix was derived from the 6×6 transitional transfer matrix of the multilayer poroelastic medium and could be combined with the 4×4 transfer matrix of the single-phase medium. Finally, the degenerated results from presented method were compared with those from the previous classical wave-propagation models to get good consistency between them. The presented method has merits of simpler calculation and clearer physical sense compared with the classical ones.

Key words: two-phase saturated media; single-phase media; multilayered media; stress-displacement function; transitional transfer matrix; degeneration

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