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使用解析方法获得 FitzHugh-Nagumo 方程新的 peakon 解*

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摘要: 依据对 FitzHugh-Nagumo 方程的研究,通过微分变化法近似分析出 FitzHugh-Nagumo 方程,获得了这个方程的尖峰孤立波(peakon soliton)的解,从而获得了更多形式的 peakon 解,同时也分析了微分变换法(differential transform method, DTM)收敛区域和收敛速度.构建的微分变换法,结合帕德(Padé)逼近,构建一个明确的,完全解析,对 FitzHugh-Nagumo 方程全部有意义的尖波解.其主要思想是限制边界条件而令导数在孤立波不存在峰值,但导数的孤立波在两侧存在.结果表明,微分变换法在参数很小的情况下可以避免摄动的限制.表明这种方法提供了一种强大而有效地获得 FitzHugh-Nagumo 方程新的 peakon 解的数学方法.

关　键　词: FitzHugh-Nagumo 方程; 尖峰解; 微分变换法; Padé 逼近; 收敛区域和速度
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引　　言

考虑下面的 FitzHugh-Nagumo 方程[1-2]:

$$u_t - u_{xx} = u(u - \alpha)(1 - u), \tag{1}$$

其中 α 是任意常数,u_{xx} 是域函数关于空间 x 的二阶导数.方程(1)是一种重要的非线性反应扩散方程,并应用于新型神经传导.广泛应用在遗传学、生物学和热质量转移等研究领域.FitzHugh-Nagumo 方程作为热传导方程,描述温度动力扩散过程.Abbasbandy 等[3]使用同伦分析方法得到 FitzHugh-Nagumo 方程的在波峰处一阶导数连续的孤子解.Li 和 Guo[4]应用首次积分法已取得一系列的 FitzHugh-Nagumo 方程包括特解的新的精确解.Nucci 和 Clarkson[5]推广

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了非经典的对称性约化法,并得到了 FitzHugh-Nagumo 方程的新的 Jacobi 椭圆函数解.Kawahara 和 Tanaka[6] 利用 Hirota 方法发现 FitzHugh-Nagumo 方程新的精确解.

所有之前文献中获得的 FitzHugh-Nagumo 方程的解析解均为连续的.2006 年,Dancer 和 Yan[7] 证明了 FitzHugh-Nagumo 方程在允许的域内有一个尖峰孤立波解的存在性,然而他们没有提供任何具体形式.

在本文中,给 FitzHugh-Nagumo 方程构建了明确的尖峰孤立波解.据笔者所知,这是 Dancer 和 Yan 证明解的存在性之后第一次通过解析渐进的方法获得尖峰孤立波解.在进一步求解之前,引入变换 $\xi = x - ct$, 故方程变形为

$$u'' + cu' - \alpha u + (1 + \alpha)u^2 - u^3 = 0, \tag{2}$$

方程中的“′”表示对 ξ 求导.

1 微分方法简介

微分方程的基本概念和基本运算定义如下[8-17]:因变量 $w(x)$ 的定义域是 Ω,并且取定义域内的 $x = x_0$,这样方程 $w(x)$ 就可以表示成在 $x = x_0$ 处展开的级数形式:

$$W(x) = \frac{1}{k!}\left[\frac{\mathrm{d}^k w(x)}{\mathrm{d}x^k}\right]\bigg|_{x=x_0}, \tag{3}$$

其中, $w(x)$ 是原方程,$W(x)$ 是微分变换方程.

定义微分逆变换为

$$w(x) = \sum_{k=0}^{\infty} W(k)(x - x_0)^k. \tag{4}$$

当 $x_0 = 0$ 时,函数 $w(x)$ 能够表示成为一个无穷级数形式.根据式(2),$w(x)$ 变成

$$w(x) = \sum_{k=0}^{\infty} W(k)x^k = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\mathrm{d}^k w(x)}{\mathrm{d}x^k}\right]\bigg|_{x=0} x^k. \tag{5}$$

目标函数 $w(x)$ 的 M 阶近似为

$$w(x) = \sum_{k=0}^{M} \frac{1}{k!}\left[\frac{\mathrm{d}^k w(x)}{\mathrm{d}x^k}\right]\bigg|_{x=0} x^k = \sum_{k=0}^{M} W(k)x^k. \tag{6}$$

表 1 列出了微分变换法的基本数学运算准则.

表 1 微分变换方法的运算

Table 1 The operation for differential transform method

original function	transformed function
$w(x) = g(x) + h(x)$	$W(k) = G(k) + H(k)$
$w(x) = \alpha g(x)$	$W(k) = \alpha G(k)$
$w(x) = \dfrac{\partial g(x)}{\partial x}$	$W(k) = (k+1)G(k)$
$w(x) = g(x)h(x)$	$W(k) = \sum_{r=0}^{k} G(r)H(k-r)$
$w(x) = \dfrac{\partial^m g(x)}{\partial x^m}$	$W(k) = (k+1)(k+2)\cdots(k+m)G(k+m)$

2 构建 peakon soliton 解

孤立波波峰的一阶导数不存在.在这种特殊情况下,相应的边界条件为

$$u(0)=1,\ u(+\infty)=0. \tag{7}$$

需要强调一下,此时边界条件 $u'(0)=0$ 是不存在的.但导数在孤立波波峰右侧是存在的,即存在 $u'_+(0), u''_+(0), \cdots, u_+^{(n)}(0)$.转换后的方程(2)形式如下:

$$(k+1)(k+2)U(k+2)+c(k+1)U(k+1)-\alpha U(k)+$$
$$(1+\alpha)\sum_{j=0}^{k}U(j)U(k-j)-\sum_{i=0}^{k}\left(U(k-i)\sum_{j=0}^{i}U(j)U(i-j)\right)=0. \tag{8}$$

转化的初始条件为

$$U(0)=1,\ U(1)=-1. \tag{9}$$

根据表 1 中的原函数和变换后函数之间的关系,得到下面的表达式,为简单起见,令 $c=1$,

$$\begin{cases} U(2)=\dfrac{1}{2},\ U(3)=-\dfrac{1}{3}+\dfrac{1}{6}\alpha,\ U(4)=\dfrac{7}{24}-\dfrac{1}{6}\alpha, \\ U(5)=-\dfrac{9}{40}+\dfrac{13}{120}\alpha-\dfrac{1}{120}\alpha^2,\ U(6)=\dfrac{19}{120}-\dfrac{31}{360}\alpha+\dfrac{13}{720}\alpha^2. \end{cases} \tag{10}$$

现在得到了方程的封闭形式:

$$u(\xi)=1-\xi+\frac{1}{2}\xi^2+\left(-\frac{1}{3}+\frac{1}{6}\alpha\right)\xi^3+\left(\frac{7}{24}-\frac{1}{6}\alpha\right)\xi^4+$$
$$\left(-\frac{9}{40}+\frac{13}{120}\alpha-\frac{1}{120}\alpha^2\right)\xi^5+\left(\frac{19}{120}-\frac{31}{360}\alpha+\frac{13}{720}\alpha^2\right)\xi^6+\cdots. \tag{11}$$

由于该 peakon 孤子的对称性,可以很容易地得到 FitzHugh-Nagumo 方程的解.

3 主要结果

为了提高收敛域,结合帕德(Padé)逼近和微分变换法,即微分变换-帕德(differential transform-Padé, D-P)逼近法.

6 阶近似为

$$u_6(\xi)=1-\xi+\frac{1}{2}\xi^2+\left(-\frac{1}{3}+\frac{1}{6}\alpha\right)\xi^3+\left(\frac{7}{24}-\frac{1}{6}\alpha\right)\xi^4+$$
$$\left(-\frac{9}{40}+\frac{13}{120}\alpha-\frac{1}{120}\alpha^2\right)\xi^5+\left(\frac{19}{120}-\frac{31}{360}\alpha+\frac{13}{720}\alpha^2\right)\xi^6; \tag{12}$$

10 阶近似为

$$u_{10}(\xi)=1-\xi+\frac{1}{2}\xi^2+\left(-\frac{1}{3}+\frac{1}{6}\alpha\right)\xi^3+\left(\frac{7}{24}-\frac{1}{6}\alpha\right)\xi^4+$$
$$\left(-\frac{9}{40}+\frac{13}{120}\alpha-\frac{1}{120}\alpha^2\right)\xi^5+\left(\frac{19}{120}-\frac{31}{360}\alpha+\frac{13}{720}\alpha^2\right)\xi^6+$$
$$\left(-\frac{571}{5\,040}+\frac{7}{90}\alpha-\frac{29}{1\,680}\alpha^2+\frac{1}{5\,040}\alpha^3\right)\xi^7+$$
$$\left(\frac{479}{5\,760}-\frac{1\,307}{20\,160}\alpha+\frac{97}{6\,720}\alpha^2-\frac{23}{20\,160}\alpha^3\right)\xi^8+$$
$$\left(\frac{2\,669}{51\,840}\alpha+\frac{319}{181\,440}\alpha^3-\frac{1\,693}{120\,960}\alpha^2-\frac{1}{362\,880}\alpha^4-\frac{1\,381}{22\,680}\right)\xi^9+$$
$$\left(-\frac{1\,037}{25\,200}\alpha-\frac{3\,221}{1\,814\,400}\alpha^3+\frac{49\,307}{3\,628\,800}\alpha^2+\frac{1}{20\,736}\alpha^4+\frac{160\,873}{3\,628\,800}\right)\xi^{10}. \tag{13}$$

利用上述结果,可以对 6 阶近似结果进行帕德逼近,并且得到[1,4]阶微分变换-帕德近似解(D-P approximation):

$$u_{6[1,4]}(\xi)=\left\{-3+4\alpha+\frac{\alpha^2-13\alpha+7}{5}\xi\right\}\Big/\left\{-3+4\alpha+\frac{\alpha^2+7\alpha-8}{5}\xi+\frac{2\alpha^2-6\alpha-2}{10}\xi^2+\frac{16\alpha-17\alpha^2-9}{30}\xi^3+\frac{11-12\alpha-20\alpha^2-4\alpha^3}{120}\xi^4\right\}. \tag{14}$$

再对 10 阶近似结果进行帕德逼近,并且得到[1,4] 阶微分变换-帕德近似解(D-P approximation):

$$u_{10[1,4]}(\xi)=\left\{-3+4\alpha+\frac{\alpha^2-13\alpha+7}{5}\xi\right\}\Big/\left\{-3+4\alpha+\frac{\alpha^2+7\alpha-8}{5}\xi+\frac{2\alpha^2-6\alpha-2}{10}\xi^2+\frac{16\alpha-17\alpha^2-9}{30}\xi^3+\frac{11-12\alpha-20\alpha^2-4\alpha^3}{120}\xi^4\right\}. \tag{15}$$

3.1 算例 1: $\alpha=0.5$

当 $\alpha=0.5$ 时,可以在 $\xi>0$ 的范围内得到 6 阶渐进级数解为

$$u_6(\xi)=1-\xi+0.5\xi^2-0.25\xi^3+0.2\xi^4-0.172\ 9\xi^5+0.12\xi^6 \tag{16}$$

和 10 阶渐进级数解为

$$u_{10}(\xi)=1-\xi+\frac{1}{2}\xi^2-0.25\xi^3+0.2\xi^4-0.172\ 9\xi^5+0.12\xi^6-0.078\ 7\xi^7+0.054\ 21\xi^8-0.038\ 4\xi^9+0.027\xi^{10}. \tag{17}$$

用帕德逼近计算 $u_6(\xi)$, 在正半轴上得到[1,4]阶 D-P 近似值为

$$u_+(\xi)=\frac{1-0.15\xi}{1.0+0.85\xi+0.35\xi^2+0.175\xi^3+0.004\ 166\ 666\ 517\xi^4}. \tag{18}$$

根据对称性,可以得到负半轴的形式:

$$u_-(\xi)=\frac{1+0.15\xi}{1.0-0.85\xi+0.35\xi^2-0.175\xi^3+0.004\ 166\ 666\ 517\xi^4}. \tag{19}$$

3.2 算例 2: $\alpha=0.6$

当 $\alpha=0.6$ 时,可以得到 6 阶微分变换解为

$$u_6(\xi)=1-\xi+0.5\xi^2-0.23\xi^3+0.19\xi^4-0.163\xi^5+0.113\xi^6 \tag{20}$$

和 10 阶微分变换解为

$$u_{10}(\xi)=1-\xi+0.5\xi^2-0.23\xi^3+0.19\xi^4-0.163\xi^5+0.113\xi^6-0.073\xi^7+0.05\xi^8-0.035\xi^9+0.024\xi^{10}. \tag{21}$$

对 $u_6(\xi)$ 和 $u_{10}(\xi)$ 应用帕德逼近,得到[1,4]阶 D-P 近似值:

$$u_+(\xi)=\frac{1+0.147\xi}{1.0+1.147\xi+0.65\xi^2+0.3\xi^3+0.06\xi^4} \tag{22}$$

和

$$u_-(\xi)=\frac{1-0.147\xi}{1.0-1.147\xi+0.65\xi^2-0.3\xi^3+0.06\xi^4}. \tag{23}$$

3.3 算例 3：$\alpha = 0.7$

当 $\alpha = 0.7$ 时，可以得到 6 阶微分变换解为

$$u_6(\xi) = 1 - \xi + 0.5\xi^2 - 0.2\xi^3 + 0.175\xi^4 - 0.153\,25\xi^5 + 0.1\xi^6 \tag{24}$$

和 10 阶微分变换解为

$$u_{10}(\xi) = 1 - \xi + 0.5\xi^2 - 0.2\xi^3 + 0.175\xi^4 - 0.153\,25\xi^5 + 0.1\xi^6 - 0.067\,2\xi^7 + 0.044\,46\xi^8 - 0.031\,1\xi^9 + 0.02\xi^{10}. \tag{25}$$

对 $u_6(\xi)$ 和 $u_{10}(\xi)$ 应用帕德逼近，得到［1,4］阶 D-P 近似值：

$$u_+(\xi) = \frac{1 + 1.6\xi}{1 + 2.6\xi + 2.1\xi^2 + \xi^3 + 0.357\xi^4} \tag{26}$$

和

$$u_-(\xi) = \frac{1 - 1.6\xi}{1 - 2.6\xi + 2.1\xi^2 - \xi^3 + 0.357\xi^4}. \tag{27}$$

3.4 算例 4：$\alpha = 0.8$

当 $\alpha = 0.8$ 时，可以得到 6 阶微分变换解为

$$u_6(\xi) = 1 - \xi + 0.5\xi^2 - 0.2\xi^3 + 0.158\xi^4 - 0.144\xi^5 + 0.1\xi^6 \tag{28}$$

和 10 阶微分变换解为

$$u_{10}(\xi) = 1 - \xi + 0.5\xi^2 - 0.2\xi^3 + 0.158\xi^4 - 0.144\xi^5 + 0.1\xi^6 - 0.062\xi^7 + 0.04\xi^8 - 0.027\,76\xi^9 + 0.019\,2\xi^{10}. \tag{29}$$

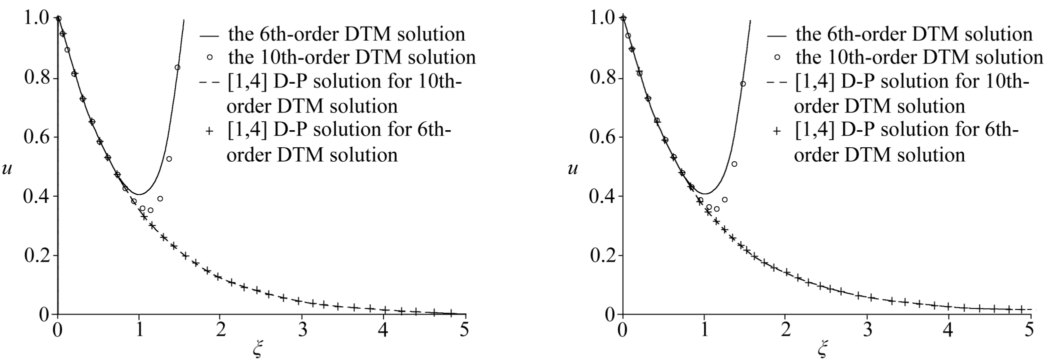
对 $u_6(\xi)$ 和 $u_{10}(\xi)$ 应用帕德逼近，得到［1,4］阶 D-P 近似值：

$$u_+(\xi) = \frac{1 - 2.76\xi}{1 - 1.76\xi - 2.26\xi^2 - 1.18\xi^3 - 0.56\xi^4}, \tag{30}$$

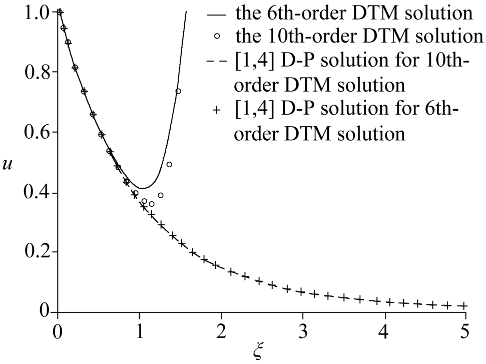
$$u_-(\xi) = \frac{1 + 2.76\xi}{1 + 1.76\xi - 2.26\xi^2 + 1.18\xi^3 - 0.56\xi^4}. \tag{31}$$

通过以上的计算，得出结论，$0.4 < \alpha < 0.9$ 时存在尖峰孤立波解。对 6 阶近似值和 10 阶近似的解析渐进解，使用微分变换-帕德逼近法，得到了 6 阶近似值的［1,4］阶 D-P 近似值 $u_{6[1,4]}$ 和 10 阶近似值的［1,4］阶 D-P 近似值 $u_{10[1,4]}$。

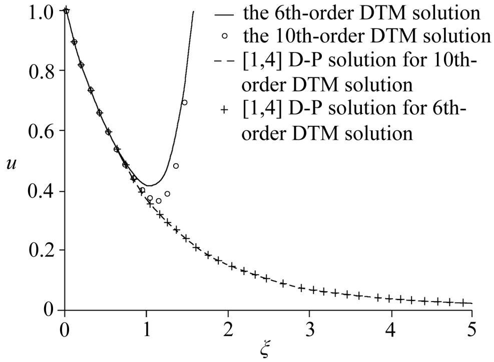
图 1 显示了当 $\xi \geqslant 0$ 时（$\alpha = 0.5, \alpha = 0.6, \alpha = 0.7, \alpha = 0.8$），6 阶近似值 u_6 和 10 阶近似值 u_{10} 的比较。从图形可以看出，要想提高精度，可以计算更多阶的近似解。显然，u_{10} 收敛域比 u_6 收敛域大。由本文提出的微分变换-帕德近似法计算得出 $u_{6[1,4]}$ 和 $u_{10[1,4]}$ 一致，得到了一个有尖



（a）$\alpha = 0.5$　　　　（b）$\alpha = 0.6$



(c) α = 0.7

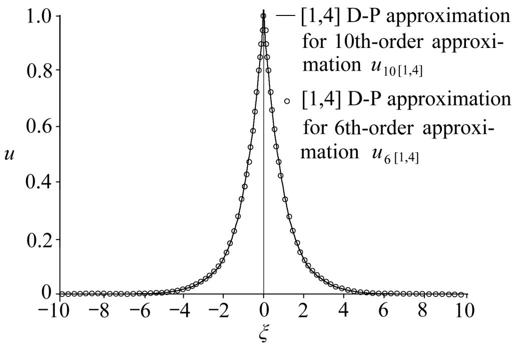


(d) α = 0.8

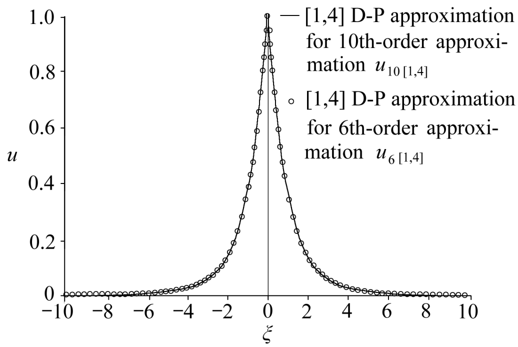
图 1 比较微分变换法的解和微分变换-帕德近似法的解

Fig.1 Comparison of the differential transform solution with the differential transform-Padé approximation solution

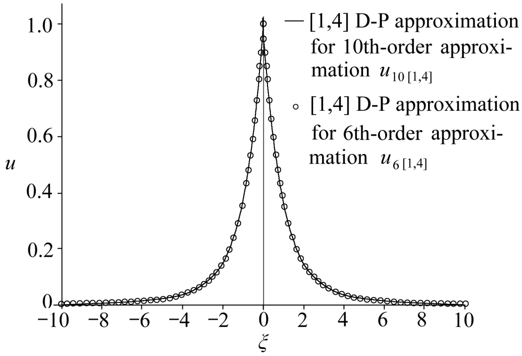
峰孤立波解,并且该解析解收敛.收敛级数是所考虑问题的一种情况,同时也检查了相应的控制方程的残差,发现这确实是真的收敛.此外,还证明了微分变换对不连续地解决孤立波的有效性[18-19].该解析解可以很好地被微分变换-帕德近似法表达.微分变换-帕德近似法在很大程度上提高了收敛域和速率.由于 peakon 的对称性,可以很容易地获得 $\xi \leqslant 0$ 的波剖面.成功地得到了明确的解析表达式,如图 2(a)、(b)、(c)和(d)所示.



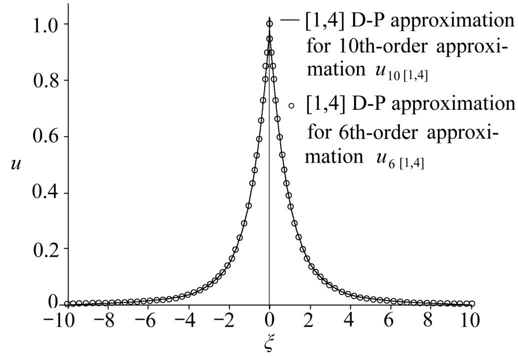
(a) α = 0.5



(b) α = 0.6



(c) α = 0.7



(d) α = 0.8

图 2 $u(\xi)$ 的近似解析

Fig.2 The analytical approximation of $u(\xi)$

图 3 显示了当 $\xi \geqslant 0$ 时($\alpha = 0.5, \alpha = 0.6, \alpha = 0.7, \alpha = 0.8$),$u$ 的剖面随空间变量 ξ 的变换.更

清晰地,在子图上将区间[3,4] 放大.从图中,可以看到,随着 α 增大,波陡在下降.

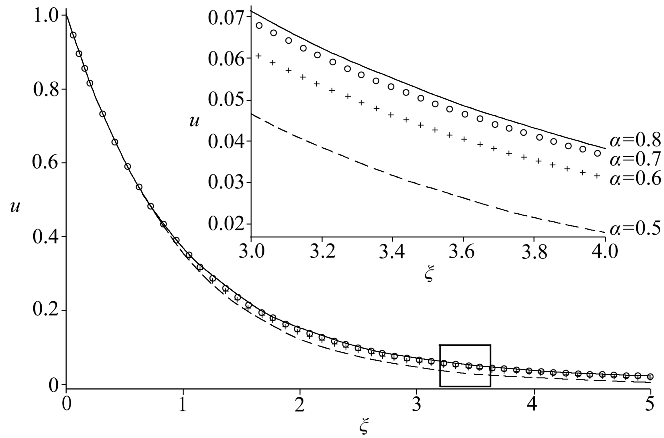


图 3 通过微分变换-帕德近似法得到的不同 α 所对应的 u (为了便于比较在子图上将区间[3,4]放大)

Fig.3 The u profile obtained from differential transform-Padé approximation method at different values of α (the profile section interval [3,4] is zoomed in a sub-figure for easy comparison)

表 2 方程(1)中 $\alpha = 0.5$, $\alpha = 0.6$, $\alpha = 0.7$, $\alpha = 0.8$ 的取值情况

Table 2 Numerical values when $\alpha = 0.5$, $\alpha = 0.6$, $\alpha = 0.7$, $\alpha = 0.8$ in eq.(1)

ξ	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$
0.5	0.602 749 024 50	0.604 002 454 00	0.605 190 641 30	0.605 557 583 80
1.0	0.357 267 950 90	0.362 958 534 10	0.367 666 048 50	0.369 722 008 20
1.5	0.210 929 194 20	0.221 423 142 80	0.228 906 097 80	0.231 833 766 30
2.0	0.125 748 503 20	0.139 354 468 00	0.147 821 213 40	0.150 840 971 70
2.5	0.076 130 055 77	0.091 029 984 94	0.992 536 430 80	0.101 986 981 50
3.0	0.046 758 767 57	0.061 704 589 36	0.369 128 658 77	0.071 465 734 69
3.5	0.028 979 520 88	0.043 274 773 28	0.049 743 679 03	0.051 694 542 07
4.0	0.017 964 072 24	0.031 282 311 96	0.036 827 793 78	0.038 443 080 73
4.5	0.010 991 624 57	0.023 221 715 85	0.027 946 070 78	0.029 283 238 54
5.0	0.064 970 225 99	0.017 642 568 39	0.021 664 322 70	0.022 775 462 98

表 2 显示了使用微分变换-帕德近似法计算方程(1)获得的不同 α 值的近似解.值得注意的是,表 2 只使用了 10 阶近似解(加上[1,4]阶的微分变换-帕德近似法).当然,精度可以通过计算更多阶近似解来提高.

4 结　　论

在本文中,成功提出的微分变换法,结合帕德逼近,构建了一个明确的,完全解析,对 FitzHugh-Nagumo 方程全部有意义的尖波解.其主要思想是限制边界条件而令导数在孤立波不存在峰值,但导数的孤立波在两侧存在.结果表明,微分变换法在参数很小的情况下可以避免摄动的限制.这表明这种方法提供了一种强大而有效地获得 FitzHugh-Nagumo 方程新的 peakon 解的数学方法.

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Finding New Types of Peakon Solutions for FitzHugh-Nagumo Equation by an Analytical Technique

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Abstract: The FitzHugh-Nagumo equation was studied with an approximate analytical method: the differential transform method. Peakon soliton solutions to this equation were presented. As a result, more new types of peakon solutions were obtained. The convergence region and rate of the differential transform method were also analyzed. The differential transform method was successfully combined with the Padé approximation technique, to construct an explicit, totally analytical and uniformly valid peakon soliton solution to FitzHugh-Nagumo equation. The main idea was to limit the boundary conditions while let the derivative at the crest of the solitary wave not exist but the solitary waves of the derivative exist at both sides. The obtained results show that the differential transform method can avoid the limitation of perturbation under conditions of very small parameters. The present method provides a powerful and effective mathematical tool to obtain new types of precise peakon solutions for FitzHugh-Nagumo equation.

Key words: FitzHugh-Nagumo equation; peakon solution; differential transform method; Padé approximation; convergence region and rate

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