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面内平动黏弹性板非线性振动的内-外联合共振*

唐有绮[1], 陈立群[2,3]

(1. 上海应用技术学院 机械工程学院,上海 201418;
2. 上海大学 力学系,上海 200444;
3. 上海市应用数学和力学研究所,上海 200072)

(我刊编委陈立群来稿)

摘要: 研究了同时存在受迫共振和 1:3 内共振时的面内平动黏弹性板的横向非线性振动问题. 板的黏弹性材料用 Kelvin 本构关系描述. 基于系统的运动方程和四边简支的边界条件,对偏微分方程应用直接多尺度法建立了联合共振时的可解性条件. 应用 Routh-Hurvitz 判据对系统幅频响应的稳定性进行了判别. 给出了黏弹性系数、面内平动速度和激励幅值 3 个参数对幅频响应的影响. 最后,应用微分求积数值方法验证了近似解析方法的结论.

关 键 词: 面内平动板; 黏弹性; 受迫振动; 内共振; 多尺度法; 微分求积法
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引 言

运动结构在机械、军事、电子工程以及航空航天等相关领域中有着相当广泛的应用. 磁带、带锯、纸带、动力传送带和航空航天工程中应用的复合材料层合板等都可以简化为运动结构. 运动速度的存在常常导致运动结构产生较大的横向振动. 例如,在热镀锌带钢的生产中,带钢的大幅振动造成镀锌层不均匀并产生噪声,限制了其传输速度的增加,从而影响到产品的加工质量和生产率. 因此,其横向振动及其控制的研究具有相当广阔的应用前景,有着重要的工程意义.

另外,运动结构的研究还具有重要的理论意义. 其研究方向主要有弦线、梁、板和壳等. 目前主要的研究集中在轴向运动弦线和梁,对面内平动板的关注还相当少. 运动板横向振动的研究起源于 1982 年 Ulsoy 和 Mote[1] 的工作. 他们首次把大型带锯的叶片简化为面内运动板来研究,他们采用 Hamilton 原理导出了其横向振动的控制方程,然后分别应用经典的 Ritz 法和 fi-

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作者简介: 唐有绮(1984—),男,上海人,讲师,博士,硕士生导师(通讯作者. Tel:+86-21-60873024; E-mail: tangyouqi2000@163.com).

nite element-Ritz 法分析了板的横向振动，得到了前两阶的固有频率(即最低的横向和最低的扭转频率)，并与实验结果进行了比较。研究结果表明面内运动速度和面内预加张力对固有频率有显著影响。近 20 年内，一些学者运用假设模态法、平均法、有线条法以及数值方法等陆续对面内平动弹性板[2-8]进行了一系列的研究，并且集中于线性阶段。仅有少量的工作涉及到黏弹性板[9-10]，且都是运用数值方法对其自由振动进行分析。对面内平动非线性板[11-13]的研究还不多。

本文研究了存在内–外联合共振的面内平动黏弹性板的横向非线性振动，分别运用近似解析方法(直接多尺度方法)和数值方法(微分求积方法)讨论了系统共振时的幅频响应以及系统各参数对其影响。

1 动力学方程

考虑一边界受有纵向载荷的矩形薄板，板厚与其横向尺寸相比很小，满足 Kirchhoff 基本假设。板以速度 Γ 沿轴线方向作匀速直线运动，其静平衡位形位于 Oxy 平面内。板的单位面积的质量为 ρ，黏弹性系数为 η，弹性模量为 E，长、宽和高分别为 a，b 和 h，Poisson 比为 μ，横向位移为 $w(x, y, t)$。板的黏弹性材料用 Kelvin 本构关系描述，且受到横向分布面载荷 $F = F_0\cos(\Omega t)$，其中 F_0 和 Ω 分别为激励的幅值和频率。

采用广义 Hamilton 原理可以得到无量纲化的动力学方程[14]

$$\begin{aligned} &w_{,tt} + 2\gamma w_{,xt} + (\gamma^2 - 1)w_{,xx} + \zeta(w_{,xxxx} + 2\xi^2 w_{,xxyy} + \xi^4 w_{,yyyy}) = \\ &\quad \varepsilon F_1\cos\ \omega t + 6\varepsilon\zeta[(w_{,xx} + \mu\xi^2 w_{,yy})w_{,x}^2 + \xi^2(\mu w_{,xx} + \xi^2 w_{,yy})w_{,y}^2 + \\ &\quad 2\xi^2(1-\mu)w_{,x}w_{,y}w_{,xy}] - \varepsilon\eta[w_{,xxxxt} + 2\xi^2 w_{,xxyyt} + \xi^4 w_{,yyyyt} + \\ &\quad \gamma(w_{,xxxxx} + 2\xi^2 w_{,xxxyy} + \xi^4 w_{,xyyyy})] + O(\varepsilon^2) \end{aligned} \tag{1}$$

和四边简支(SSSS)的无量纲化边界条件

$$w\Big|_{x=0}^{x=1} = 0,\ w_{,xx}\Big|_{x=0}^{x=1} = 0;\ w\Big|_{y=0}^{y=1} = 0,\ w_{,yy}\Big|_{y=0}^{y=1} = 0, \tag{2}$$

其中无量纲化参数为

$$\begin{cases} w \leftrightarrow \dfrac{w}{\sqrt{\varepsilon}h},\ t \leftrightarrow \dfrac{t}{a}\sqrt{\dfrac{N_{x0}}{\rho}},\ x \leftrightarrow \dfrac{x}{a},\ y \leftrightarrow \dfrac{y}{b},\ \zeta = \dfrac{D}{N_{x0}a^2},\ \gamma = \Gamma\sqrt{\dfrac{\rho}{N_{x0}}},\ \xi = \dfrac{a}{b}, \\ \eta \leftrightarrow \dfrac{D\eta}{\varepsilon E a^3\sqrt{\rho N_{x0}}},\ \omega = a\Omega\sqrt{\dfrac{\rho}{N_{x0}}},\ F_1 = \dfrac{\rho a^2 F_0}{\varepsilon^{1.5} h N_{x0}}. \end{cases} \tag{3}$$

ε 是一个无量纲的参数，没有具体的物理意义，仅仅表征板的横向位移、黏弹性系数和激励幅值均为小量。

2 直接多尺度分析

本文应用直接多尺度方法，对面内平动黏弹性板在 1∶3 内共振和主参数共振的情况进行研究。设其一阶近似解为

$$w(x,y,t;\varepsilon) = w_0(x,y,T_0,T_1) + \varepsilon w_1(x,y,T_0,T_1) + O(\varepsilon^2), \tag{4}$$

这里 $T_0 = t$ 和 $T_1 = \varepsilon t$ 分别为系统的快时间和慢时间尺度。将式(4)代入方程(1)，令等式两边 ε 同次幂的系数相等，得到

$$\varepsilon^0: w_{0,T_0T_0} + 2\gamma w_{0,xT_0} + (\gamma^2 - 1)w_{0,xx} + \zeta(w_{0,xxxx} + 2\xi^2 w_{0,xxyy} + \xi^4 w_{0,yyyy}) = 0, \tag{5}$$

$$\begin{aligned}\varepsilon^1: & w_{1,T_0T_0} + 2\gamma w_{1,xT_0} + (\gamma^2 - 1)w_{1,xx} + \zeta(w_{1,xxxx} + 2\xi^2 w_{1,xxyy} + \xi^4 w_{1,yyyy}) = \\ & -\{\eta[w_{0,xxxxT_0} + 2\xi^2 w_{0,xxyyT_0} + \xi^4 w_{0,yyyyT_0} + \\ & \gamma(w_{0,xxxxx} + 2\xi^2 w_{0,xxxyy} + \xi^4 w_{0,xyyyy})] + 2(w_{0,T_0T_1} + \gamma w_{0,xT_1})\} + \\ & 6\zeta[(w_{0,xx} + \xi^2\mu w_{0,yy})w_{0,x}^2 + \xi^2(\mu w_{0,xx} + \xi^2 w_{0,yy})w_{0,y}^2 + \\ & 2\xi^2(1-\mu)w_{0,x}w_{0,y}w_{0,xy}] + F_1\cos\omega T_0,\end{aligned} \tag{6}$$

其中式(5)为面内平动板的线性自由振动方程,其固有频率和模态已由文献[13]给出.

这里考虑1∶3 内共振,引入相应的解谐参数 σ_1,同时考虑分布面载荷的激励频率 ω 在第1个基谐波频率附近,引入相应的解谐参数 σ_2, 其共振关系为

$$\omega_{s'l'} = 3\omega_{sl} + \varepsilon\sigma_1,\ \omega = \omega_{sl} + \varepsilon\sigma_2. \tag{7}$$

方程(5)的通解可以用已经分离变量的复数形式表示为

$$w_0(x,y,T_0,T_1) = \psi_{sl}(x,y)A_{sl}(T_1)e^{i\omega_{sl}T_0} + \psi_{s'l'}(x,y)A_{s'l'}(T_1)e^{i\omega_{s'l'}T_0} + cc. \tag{8}$$

将式(7)和(8)代入式(6)并消去长久项,可得系统的可解性条件为

$$\begin{cases}A_{sl,T_1} + \eta m_1 A_{sl} + g_{11}A_{sl}^2\bar{A}_{sl} + g_{12}A_{sl}A_{s'l'}\bar{A}_{s'l'} + h_1 A_{s'l'}\bar{A}_{sl}^2 e^{i\sigma_1 T_1} + nF_1 e^{i\sigma_2 T_1} = 0, \\ A_{s'l',T_1} + \eta m_2 A_{s'l'} + g_{22}A_{s'l'}^2\bar{A}_{s'l'} + g_{21}A_{s'l'}A_{sl}\bar{A}_{sl} + h_2 A_{sl}^3 e^{-i\sigma_1 T_1} = 0.\end{cases} \tag{9}$$

对于预先设定的一系列物理参数,方程(9)中推出的常系数,经数值计算均表明 $m_i(i=1,2)$ 是正实数,$g_i(i=11,12,21,22)$ 是负虚数,$h_i(i=1,2)$ 和 n 是复数.因为它们的表达式太复杂,在本文里不给出它们的具体表达式.引入极坐标形式解

$$A_{sl} = \alpha_{sl}(T_1)e^{i\beta_{sl}(T_1)},\ A_{s'l'} = \alpha_{s'l'}(T_1)e^{i\beta_{s'l'}(T_1)}, \tag{10}$$

式中, α_i 和$\beta_i(i=sl,s'l')$ 分别为对应阶模态的幅值和相角,它们均为 T_1 的实函数.将式(10)代入式(9)并分离实部和虚部,得到

$$\begin{cases}\dot{\alpha}_{sl} = -\eta m_1\alpha_{sl} + (h_1^{\mathrm I}\sin\theta_2 - h_1^{\mathrm R}\cos\theta_2)\alpha_{sl}^2\alpha_{s'l'} - F_1(n^{\mathrm R}\cos\theta_1 - n^{\mathrm I}\sin\theta_1), \\ \alpha_{sl}\dot{\beta}_{sl} = -(g_{11}^{\mathrm I}\alpha_{sl}^2 + g_{12}^{\mathrm I}\alpha_{s'l'}^2)\alpha_{sl} - (h_1^{\mathrm I}\cos\theta_2 + h_1^{\mathrm R}\sin\theta_2)\alpha_{sl}^2\alpha_{s'l'} - \\ \qquad F_1(n^{\mathrm I}\cos\theta_1 + n^{\mathrm R}\sin\theta_1), \\ \dot{\alpha}_{s'l'} = -\eta m_2\alpha_{s'l'} - (h_2^{\mathrm I}\sin\theta_2 + h_2^{\mathrm R}\cos\theta_2)\alpha_{sl}^3, \\ \alpha_{s'l'}\dot{\beta}_{s'l'} = -(g_{21}^{\mathrm I}\alpha_{sl}^2 + g_{22}^{\mathrm I}\alpha_{s'l'}^2)\alpha_{s'l'} - (h_2^{\mathrm I}\cos\theta_2 - h_2^{\mathrm R}\sin\theta_2)\alpha_{sl}^3,\end{cases} \tag{11}$$

其中新相位角 $\theta_1 = \sigma_2 T_1 - \beta_{sl}, \theta_2 = \sigma_1 T_1 - 3\beta_{sl} + \beta_{s'l'}$, 上标 R 和 I 分别表示相应参数的实部和虚部.对于系统的稳态响应,其幅值和新的相角应该为常数.由式(11)导出

$$-\eta m_1\alpha_{sl} + (h_1^{\mathrm I}\sin\theta_2 - h_1^{\mathrm R}\cos\theta_2)\alpha_{sl}^2\alpha_{s'l'} - F_1(n^{\mathrm R}\cos\theta_1 - n^{\mathrm I}\sin\theta_1) = 0, \tag{12}$$

$$\begin{aligned}&\sigma_2 + (g_{11}^{\mathrm I}\alpha_{sl}^2 + g_{12}^{\mathrm I}\alpha_{s'l'}^2) + (h_1^{\mathrm I}\cos\theta_2 + h_1^{\mathrm R}\sin\theta_2)\alpha_{sl}\alpha_{s'l'} + \\ &\quad (n^{\mathrm I}\cos\theta_1 + n^{\mathrm R}\sin\theta_1)F_1/\alpha_{sl} = 0,\end{aligned} \tag{13}$$

$$-\eta m_2\alpha_{s'l'} - (h_2^{\mathrm I}\sin\theta_2 + h_2^{\mathrm R}\cos\theta_2)\alpha_{sl}^3 = 0, \tag{14}$$

$$\begin{aligned}&\sigma_1 + (3g_{11}^{\mathrm I} - g_{21}^{\mathrm I})\alpha_{sl}^2 + (3g_{12}^{\mathrm I} - g_{22}^{\mathrm I})\alpha_{s'l'}^2 + 3(n^{\mathrm I}\cos\theta_1 + n^{\mathrm R}\sin\theta_1)F_1/\alpha_{sl} + \\ &\quad [\cos\theta_2(3h_1^{\mathrm I}\alpha_{s'l'}^2 - h_2^{\mathrm I}\alpha_{sl}^2) + \sin\theta_2(3h_1^{\mathrm R}\alpha_{s'l'}^2 + h_2^{\mathrm R}\alpha_{sl}^2)]\alpha_{sl}/\alpha_{s'l'} = 0.\end{aligned} \tag{15}$$

因为 h_1 和 h_2 仅仅与面内平动黏弹性板的固有参数有关,而与 η 和 F_1 无关,固由式(12)和(14)

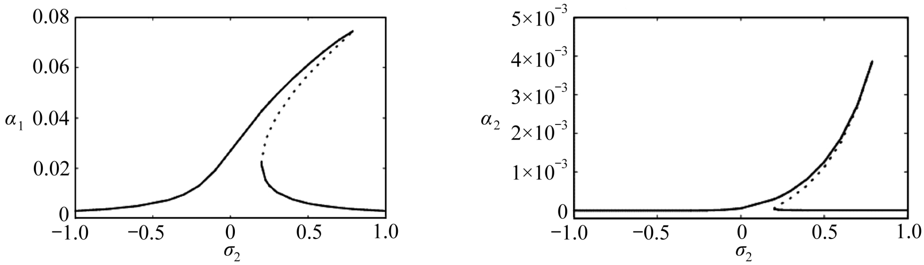
可以得到

$$h_1^{\mathrm{R}}/h_2^{\mathrm{R}} = -h_1^{\mathrm{I}}/h_2^{\mathrm{I}} = C. \tag{16}$$

联立式(12)~(16),从中消去新相位角 θ_1 和 θ_2,得到

$$\begin{cases} \alpha_{s'l'}^2[\eta^2 m_2^2 + (3\sigma_2 - \sigma_1 + g_{21}^{\mathrm{I}}\alpha_{sl}^2 + g_{22}^{\mathrm{I}}\alpha_{s'l'}^2)^2] = \alpha_{sl}^6\,|h_2|^2, \\ \eta^2\,(Cm_2\alpha_{s'l'}^2 - m_1\alpha_{sl}^2)^2 + [C\alpha_{s'l'}^2(3\sigma_2 - \sigma_1 + g_{21}^{\mathrm{I}}\alpha_{sl}^2 + g_{22}^{\mathrm{I}}\alpha_{s'l'}^2) + \\ \qquad \alpha_{sl}^2(\sigma_2 + g_{11}^{\mathrm{I}}\alpha_{sl}^2 + g_{12}^{\mathrm{I}}\alpha_{s'l'}^2)]^2 = \alpha_{sl}^2\,|n|^2F_1^2. \end{cases} \tag{17}$$

对于稳态响应,将解得稳态响应的振幅和相位角回代 Jacobi 矩阵的特征方程,根据 Routh-Hurwitz 判据可得到系统幅频响应的稳定性.

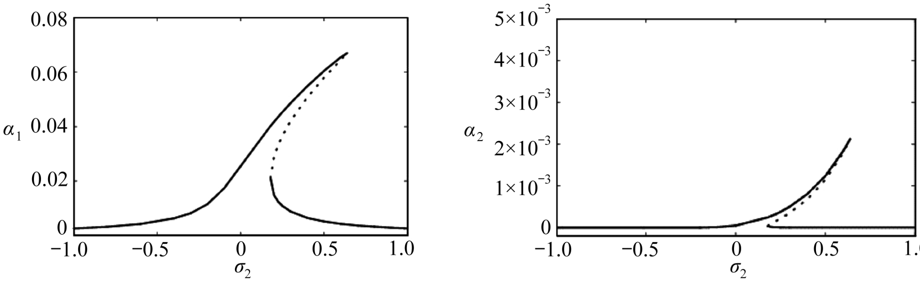


(a) 第 1 阶模态 (b) 第 2 阶模态

(a) The first mode (b) The second mode

图 1 幅频响应曲线及其稳定性($\eta = 0.000\,1,\ \gamma = 3.924\,3,\ F_1 = 0.35$)

Fig. 1 Frequency response curves and stability for $\eta = 0.000\,1,\ \gamma = 3.924\,3,\ F_1 = 0.35$



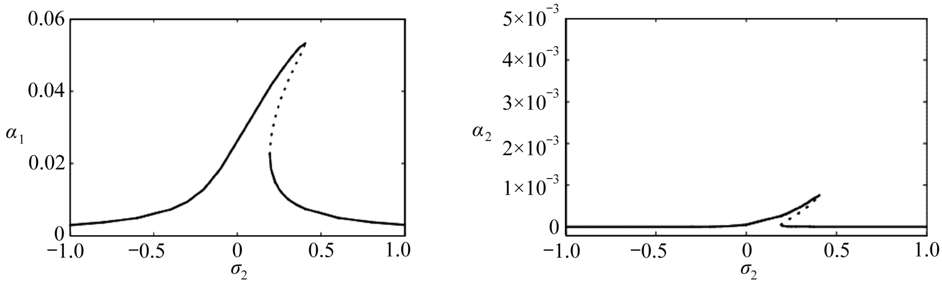
(a) 第 1 阶模态 (b) 第 2 阶模态

(a) The first mode (b) The second mode

图 2 幅频响应曲线及其稳定性($\eta = 0.000\,1,\ \gamma = 3.924\,3,\ F_1 = 0.3$)

Fig. 2 Frequency response curves and stability for $\eta = 0.000\,1,\ \gamma = 3.924\,3,\ F_1 = 0.3$

考虑一面内平动黏弹性板[15],$E = 2.10 \times 10^{11}$ Pa,$\rho = 7\,850$ kg/m^3,$a = 1.5$ m,$b = 1.5$ m,$h = 0.02$ m 和 $P_0 = 68\,376$ N,解得 $\zeta = 1$ 和 $\xi = 1$. 当 $\gamma = 3.924\,3$ 时,前两阶的固有频率分别为 $\omega_1 = 13.959\,66$ 和 $\omega_2 = 43.803\,6$,对应的内共振解谐参数为 $\sigma_1 = 1.924\,62$;当 $\gamma = 3.4$ 时,$\omega_1 = 15.486\,2$ 和 $\omega_2 = 45.321\,8$,$\sigma_1 = -1.136\,8$. 3 组图给出了板主参数受迫共振和 1∶3 内共振时前两阶模态的幅频响应曲线以及稳定性. 图 1 对应的参数为 $\eta = 0.000\,1$, $\gamma = 3.924\,3$ 和 $F_1 = 0.35$;图 2 对应的参数为 $\eta = 0.000\,1$,$\gamma = 3.924\,3$ 和 $F_1 = 0.3$;图 3 对应的参数为 $\eta = 0.000\,15$,$\gamma = 3.924\,3$ 和 $F_1 = 0.35$;图 4 对应的参数为 $\eta = 0.000\,1$,$\gamma = 3.4$ 和 $F_1 = 0.35$. 其中实线表示稳定解,点线表示非稳定解.



（a）第 1 阶模态

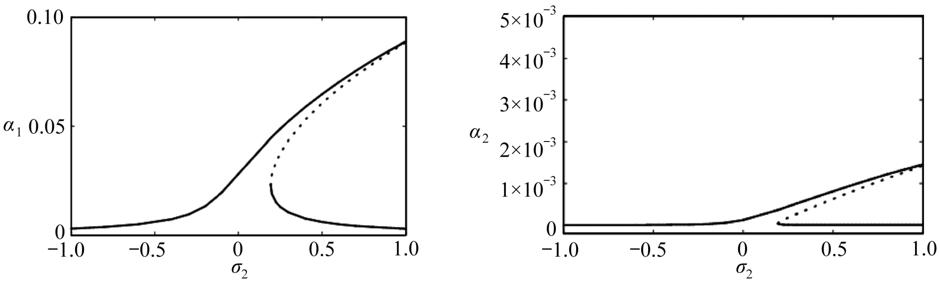
（a）The first mode

（b）第 2 阶模态

（b）The second mode

图 3　幅频响应曲线及其稳定性（$\eta = 0.000\,15$，$\gamma = 3.924\,3$，$F_1 = 0.35$）

Fig. 3　Frequency response curves and stability for $\eta = 0.000\,15$, $\gamma = 3.924\,3$, $F_1 = 0.35$



（a）第 1 阶模态

（a）The first mode

（b）第 2 阶模态

（b）The second mode

图 4　幅频响应曲线及其稳定性（$\eta = 0.000\,1$，$\gamma = 3.4$，$F_1 = 0.35$）

Fig. 4　Frequency response curves and stability for $\eta = 0.000\,1$, $\gamma = 3.4$, $F_1 = 0.35$

3　数 值 验 证

为了检验直接多尺度方法的正确性，我们使用微分求积法对其进行数值验证。这里，我们采用与 Tang 和 Chen[13-14] 相同的非均匀网点布置。其相应的微分求积近似离散为

$$
\begin{aligned}
\ddot{\phi}_i =& -2\gamma\sum_{k=2}^{N_x-1}A_{ik}^{(1)}\dot{\phi}_k-\eta\left(\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(4)}\dot{\phi}_k-2m^2\pi^2\xi^2\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\dot{\phi}_k+m^4\pi^4\xi^4\dot{\phi}_i\right)-\\
&(\gamma^2-1)\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\phi_k-\zeta\left(\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(4)}\phi_k-2m^2\pi^2\xi^2\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\phi_k+m^4\pi^4\xi^4\phi_i\right)+\\
&\frac{3}{2}\zeta\left[3\left(\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\phi_k-\mu m^2\pi^2\xi^2\phi_i\right)\left(\sum_{k=2}^{N_x-1}A_{ik}^{(1)}\phi_k\right)^2+\right.\\
&m^2\pi^2\xi^2\left(\mu\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(2)}\phi_k-m^2\pi^2\xi^2\phi_i\right)\phi_i^2+\\
&\left.2m^2\pi^2\xi^2(1-\mu)\left(\sum_{k=2}^{N_x-1}A_{ik}^{(1)}\phi_k\right)^2\phi_i\right]-\\
&\eta\gamma\left(\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(5)}\phi_k-2m^2\pi^2\xi^2\sum_{k=2}^{N_x-1}\tilde{A}_{ik}^{(3)}\phi_k+\right.
\end{aligned}
$$

$$m^4\pi^4\xi^4\sum_{k=2}^{N_x-1}A_{ik}^{(1)}\phi_k\Big) + \frac{4}{\pi}F_1\cos\omega t. \tag{18}$$

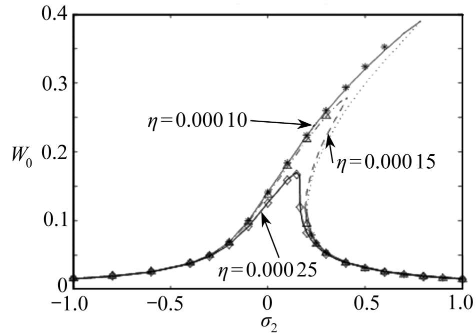


图 5　解析结果和数值结果的比较(不同黏弹性系数)

Fig. 5　Comparison of analytical and numerical results(different viscosity coefficients)

对于一维简化模型,选取网点数 $N_x = 9$. W_0 是 w_0 的最大值. 粗实线、点划线和细实线表示稳定的解析解;点线和虚线表示不稳定的解析解;“ * ”“△”和“◇”表示稳定的数值解. 图 5 ~ 图 7 分别给出了不同黏弹性系数、激励幅值和面内平动速度下解析结果和数值结果的比较. 由图可见,随着黏弹性系数的减小、激励幅值的增大以及面内平动速度的增大,共振幅值增大. 并且两种方法所得结果吻合得非常好.

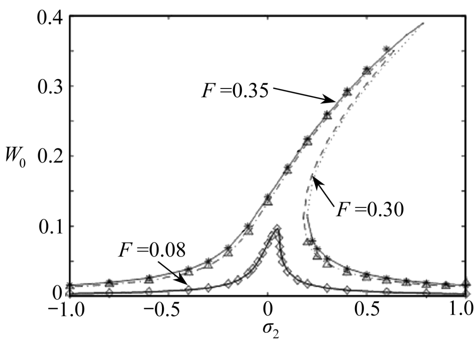


图 6　解析结果和数值结果的比较(不同激励幅值)

Fig. 6　Comparison of analytical and numerical results(different excitation amplitudes)

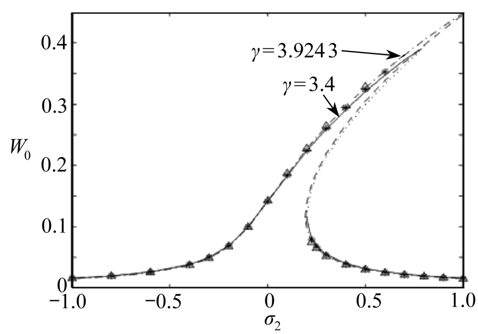


图 7　解析结果和数值结果的比较(不同面内平动速度)

Fig. 7　Comparison of analytical and numerical results(different in-plane translating speeds)

4　结　论

当考虑主参数受迫共振和 1:3 内共振且有平面张力时,基于黏弹性板的运动方程和四边简支的边界条件,由直接多尺度法建立的可解性条件得到了系统的幅频响应曲线. 利用 Routh-Hurvitz 判据求得了幅频响应的稳定性问题. 最后考察了系统的参数对幅频响应曲线的影响. 得到如下结论:1) 增大黏弹性系数使共振幅值减小;2) 增大激励幅值使共振幅值增大;3) 增大面内速度使共振幅值增大;4) 利用微分求积法对直接多尺度方法进行数值验证,结果吻合得非常好.

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Internal-External Combination Resonance of Nonlinear Vibration of in-Plane Translating Viscoelastic Plates

TANG You-qi[1], CHEN Li-qun[2,3]

(1. *School of Mechanical Engineering*, *Shanghai Institute of Technology*, *Shanghai* 201418, *P. R. China*;

2. *Department of Mechanics*, *Shanghai University*, *Shanghai* 200444, *P. R. China*;

3. *Shanghai Institute of Applied Mathematics and Mechanics*, *Shanghai* 200072, *P. R. China*)

Abstract: Nonlinear vibrations of in-plane translating viscoelastic plates were investigated on the steady-state responses in external and internal resonances. The plate's material obeyed the Kelvin model in which the material time derivative was used. Based on the governing equation and boundary conditions for four edges simple supports, the method of multiple scales was applied to establish the solvability conditions in the primary resonance and the 1∶3 internal resonance. The Routh-Hurvitz criterion was used to determine the stabilities of the steady-state responses. The effects of the viscosity coefficient, the in-plane translating speed, and the excitation amplitude on the steady-state responses were examined. The differential quadrature scheme was developed for the plate model to solve the nonlinear governing equations numerically. The numerical calculations confirm the approximate analytical results regarding the solutions of the steady-state responses.

Key words: in-plane translating plate; viscoelastic; forced vibration; internal resonance; method of multiple scales; differential quadrature scheme