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非线性扰动耦合 Schrödinger 系统激波的近似解法*

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摘要: 研究了一类非线性扰动耦合 Schrödinger 系统. 利用精确解与近似解相关联的特殊技巧,首先讨论了对应典型的耦合系统,利用投射法得到了精确的激波行波解. 再利用近似方法得到了扰动耦合 Schrödinger 系统的行波渐近解.

关 键 词: Schrödinger 系统; 激波; 渐近解
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引 言

激波理论在凝聚态物理、量子物理、光学、流体力学等研究领域中是一个十分关注的对象. 新的研究方法不断地在改进. 许多学者在激波的求解方法上作了许多工作[1-5]. 近来,利用渐近理论来求解激波就是一种较新的方法. 这种方法改变了以往单纯用数值模拟来探讨激波的性态,而是通过解析理论渐近地得到激波的近似表达式. 这种方法的优点在于可以通过渐近表达式进一步用解析运算来对激波性态作更深入的研究[6-7].

近来,许多渐近方法不断地在发展和优化,包括平均法、边界层法、匹配法和多重尺度法等[8-11]. Mo 等也应用渐近方法讨论了一类非线性问题[12-17]. 本文就是利用一种新改进的渐近方法讨论了一类非线性扰动 Schrödinger 耦合系统,并得到了对应激波的近似行波解.

考虑如下非线性扰动 Schrödinger 耦合系统:

$$au - uv - u_{xx} = f_1(u,v), \tag{1}$$

$$v_t - u_x = f_2(u,v), \tag{2}$$

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其中，$u(x,t)$，$v(x,t)$ 为对应系统的物理场函数，a 为正参数，$f_i(i=1,2)$ 为扰动项，它是在相应的变化范围内充分光滑的函数。本系统描述了一类光导纤维等方面的激波传播系统，它的物理背景参见文献[18]。以下提出一个简单的新方法来得到系统(1)和(2)的渐近解。

1 典型的 Schrödinger 耦合系统

首先讨论如下典型的 Schrödinger 耦合系统：

$$au - uv - u_{xx} = 0, \tag{3}$$

$$v_t - u_x = 0. \tag{4}$$

我们引入行波变换 $\xi = x + ct$。这时系统(3)和(4)为

$$au - uv - u_{\xi\xi} = 0, \tag{5}$$

$$cv_\xi - u_\xi = 0. \tag{6}$$

由式(6)有

$$v = \frac{1}{c}u + C, \tag{7}$$

其中 C 为任意常数。这里不妨选取 $C=0$。将式(7)代入式(5)得到

$$cu_{\xi\xi} - acu + u^2 = 0. \tag{8}$$

利用投射理论[7]，我们可得到方程(8)的如下形式的激波解：

$$u(\xi) = k_0 + k_1 v + k_2 v^2 + k_3(v^2 - \sigma^2)^{1/2}, \tag{9}$$

其中，$k_i(i=0,1,\cdots,3)$ 为待定常数，$\sigma > 0$ 和 $\tilde{v}(\xi)$ 满足方程

$$\frac{d\tilde{v}}{d\xi} = \tilde{v}^2 - \sigma^2. \tag{10}$$

显然，方程(10)具有如下激波解：

$$\tilde{v}(\xi) = -\sigma\tanh(\sigma\xi), \qquad \sigma > 0. \tag{11}$$

由式(9)和(10)，我们有

$$u_\xi = -k_1\sigma^2 - 8k_2\sigma^2\tilde{v} + k_1\tilde{v}^2 + 2k_2\tilde{v}^3 + k_3\tilde{v}(\tilde{v}^2 - \sigma^2)^{1/2}, \tag{12}$$

$$\begin{aligned} u_{\xi\xi} = {} & 2k_2\sigma^4 - 8k_1\sigma^2\tilde{v} - 6k_2\sigma^2\tilde{v}^2 + 2k_1\tilde{v}^3 + 6k_2\tilde{v}^4 + \\ & k_3(2\tilde{v}^2 - \sigma^2)(\tilde{v}^2 - \sigma^2)^{1/2}. \end{aligned} \tag{13}$$

将式(9)、(12)及(13)代入方程(8)，得到

$$\begin{aligned} & 2k_2\sigma^4 - ack_0 - 2ck_1\sigma^2\tilde{v} - ack_1v - 8ck_2\sigma^2\tilde{v}^2 - ack_2\tilde{v}^2 + 2ck_1\tilde{v}^3 + 6k_2\tilde{v}^4 + \\ & \quad k_0^2 + k_1^2\tilde{v}^2 + k_2^2\tilde{v}^4 + k_3^2\tilde{v}^2 - k_3^2\sigma^2 + 2k_0k_1\tilde{v} + 2k_0k_2\tilde{v}^2 + 2k_1k_2\tilde{v}^3 + \\ & \quad (-ck_3\sigma^2 - ack_3 + 2k_0k_3 + 2k_1k_3\tilde{v} + 2k_3(c + k_2)\tilde{v}^2)(\tilde{v}^2 - \sigma^2)^{1/2} = 0. \end{aligned}$$

即有

$$\begin{aligned} & (2k_2\sigma^4 - k_3^2\sigma^2 + k_0^2) + 2k_1(c\sigma^2 - k_0)\tilde{v} + (-8ck_2\sigma^2 + \\ & \quad k_1^2 + k_3^2 + 2k_0k_2)\tilde{v}^2 + 2k_1(c + k_2)v^3 + k_2(k_2 + 6)\tilde{v}^4 = 0, \\ & (k_3(-c\sigma^2 - ac + 2k_0) + 2k_1k_3\tilde{v} + 2k_3(c + k_2)\tilde{v}^2)(\tilde{v}^2 - \sigma^2)^{1/2} = 0. \end{aligned}$$

故

$$\begin{aligned} & 2k_2\sigma^4 - k_3^2\sigma^2 + k_0^2 = 0,\ k_1(c\sigma^2 - k_0) = 0, \\ & -8ck_2\sigma^2 + k_1^2 + k_3^2 + 2k_0k_2 = 0,\ k_1(c\sigma^2 - k_0) = 0, \\ & -8ck_2\sigma^2 + k_1^2 + k_3^2 + 2k_0k_2 = 0,\ k_1(c + k_2) = 0, \end{aligned}$$

$$k_2(k_2+6)=0,\ k_3(-c\sigma^2-ac+2k_0)=0,\ k_1k_3=0,\ k_3(c+k_2)=0.$$

由上诸式可得

$$k_0=2\sqrt{3}\,\sigma^2,\ k_1=k_3=0,\ k_2=-6,\ c=\frac{\sqrt{3}}{2}. \tag{14}$$

将式(14)代入式(9)并注意到式(11),可得方程(8)的一个激波解

$$\bar{u}(\xi)=2\sqrt{3}\sigma^2-6\sigma^2\tanh^2(\sigma\xi),$$

再由变换 $\xi=x+(\sqrt{3}/2)t$, 我们得到典型的 Schrödinger 耦合系统(3)和(4)的一组激波的行波精确解:

$$\bar{u}(x,t)=2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right), \tag{15}$$

$$\bar{v}(x,t)=\frac{2\sqrt{3}\sigma^2}{c}-\frac{6\sigma^2}{c}\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right). \tag{16}$$

2 扰动 Schrödinger 耦合系统的近似解

众所周知,非线性耦合系统(1)和(2)一般不能得到初等函数形式的精确解.为了得到扰动 Schrödinger 耦合系统(1)和(2)激波解的近似表示式,引入映射 $H_i(u,v,s)(R^2\times I\to R)$:

$$H_i(u,v,s)=\\ L_i(u,v)-L_i(\tilde{u},\tilde{v})+s[L_i(\tilde{u},\tilde{v})+N_i(u,v)-f_i(u,v)],\qquad i=1,2, \tag{17}$$

其中, $I=[0,1]$, s 为一个人工参数,而算子 $L_i,N_i(i=1,2)$ 为

$$L_1(u,v)=au-u_{\xi\xi},\ L_2(u,v)=cv_\xi-u_\xi,\ N_1(u,v)=-uv,\ N_2(u,v)=0.$$

设

$$u=\sum_{i=0}^{\infty}u_i(x,t)s^i,\ v=\sum_{i=0}^{\infty}v_i(x,t)s^i. \tag{18}$$

由映射(17),将式(18)代入 $H_i(u,v,s)=0(i=1,2)$,按 s 展开非线性项,合并 s 的同次幂项的系数并令其为0,我们可依次得到 $u_i(x,t),v_i(x,t)(i=0,1,2,\cdots)$.

将得到的 u_i,v_i 代入式(18),可得到系统(1)和(2)的各次近似解.在 $f_i(i=1,2)$ 的假设下,利用不动点原理[6,19]可知,在选择初始近似 $u_0(x,t),v_0(x,t)$ 情况下,这时式(18)就是非线性系统

$$L_i(u,v)+N_i(u,v)=0,\qquad i=1,2, \tag{19}$$

这时, $u=\sum_{i=0}^{\infty}u_i(x,t)s^i$, $v=\sum_{i=0}^{\infty}v_i(x,t)s^i$ 在 $s\in[0,1]$ 上为一致收敛的解.

显然,由映射式(17), $H_i(u,v,1)=0(i=1,2)$ 和方程(19)相同.于是方程(19)的解 u,v 就是 $H_i(u,v,s)=0(i=1,2)$ 当 $s\to 1$ 的解.

比较 $H_i(u,v,s)=0(i=1,2)$ 关于 s 的同次幂的系数.由 $H_i(u,v,s)=0(i=1,2)$ 关于 s 的0次幂的系数,有

$$L_i(u_0,v_0)=L_i(\tilde{u},\tilde{v}),\qquad i=1,2. \tag{20}$$

由式(15)和(16),选择 $\tilde{u},\tilde{v}$ 为系统(3)和(4)的激波解 $\bar{u}(x,t),\bar{v}(x,t)$.于是:

$$\tilde{u}(x,t)=2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right),$$

$$\tilde{v}(x,t)=\frac{2\sqrt{3}\sigma^2}{c}-\frac{6\sigma^2}{c}\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right)$$

为典型系统

$$L_i(u,v)+N_i(u,v)=0,\qquad i=1,2$$

的激波行波解. 这时选择 0 次近似解 $u_0(x,t)$, $v_0(x,t)$ 为

$$u_0(x,t)=2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right),\tag{21}$$

$$v_0(x,t)=\frac{2\sqrt{3}\sigma^2}{c}-\frac{6\sigma^2}{c}\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right).\tag{22}$$

由 $H_i(u,v,s)=0(i=1,2)$ 关于 s 的 1 次幂的系数, 有

$$L_i(u_1,v_1)=L_i(u_0,v_0)+f_i(u_0,v_0),\qquad i=1,2.\tag{23}$$

不难看出, 系统(23)的解 $u_1(x,t)$, $v_1(x,t)$ 为

$$\begin{aligned}u_1(x,t)={}&C_1(t)\exp(\sqrt{a}(x-\eta))+C_2(t)\exp(-\sqrt{a}(x-\eta))+\\&\frac{1}{2\sqrt{a}}\int_0^x\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+\right.\\&\left.6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}+f_1(u_0(\eta,t),v_0(\eta,t))\right][\exp(\sqrt{a}(x-\eta))-\\&\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta,\end{aligned}\tag{24}$$

$$\begin{aligned}v_1(x,t)={}&\frac{\sqrt{a}}{c}\int_0^t[C_1(\tau)\exp(\sqrt{a}(x-\eta))-C_2(\tau)\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\tau-\\&\frac{1}{2c\sqrt{a}}\int_0^t\left[\frac{\partial}{\partial x}\int_{-\infty}^x\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)+\right.\right.\\&\left.6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)_{\eta\eta}+f_1(u_0(\eta,t),v_0(\eta,t))\right]\times\\&\left.[\exp(\sqrt{a}(x-\eta))+\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta\right]\mathrm{d}\tau+\\&\int_0^t f_2(u_0(x,\tau),v_0(x,\tau))\mathrm{d}\tau,\end{aligned}\tag{25}$$

其中 $C_i(t)(i=1,2)$ 为任意函数. u_0, v_0 由式(21)、(22)定义, 下同.

由 $H_i(u,v,s)=0(i=1,2)$ 关于 s 的 2 次幂的系数, 有

$$L_i(u_2,v_2)=L_i(u_0,v_0)+F_{i2}(x,t),\qquad i=1,2,\tag{26}$$

其中

$$\begin{aligned}F_{12}(x,t)={}&\left[\frac{\partial}{\partial s}\left(-\left(\sum_{i=0}^{\infty}u_i(x,t)s^i\right)\left(\sum_{i=0}^{\infty}v_i(x,t)s^i\right)\right)+\right.\\&\left.f_1\left(\sum_{i=0}^{\infty}u_i(x,t)s^i,\sum_{i=0}^{\infty}v_i(x,t)s^i\right)\right]_{s=0},\end{aligned}$$

$$F_{22}(x,t)=\left[\frac{\partial}{\partial s}f_1\left(\sum_{i=0}^{\infty}u_i(x,t)s^i,\sum_{i=0}^{\infty}v_i(x,t)s^i\right)\right]_{s=0}.$$

方程(26)的解 $u_2(x,t)$, $v_2(x,t)$ 为

$$u_2(x,t)=\frac{1}{2\sqrt{a}}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}+F_{21}(\eta,t)\right][\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta, \tag{27}$$

$$v_2(x,t)=\frac{1}{2\sqrt{a}}\int_0^t\left[\frac{\partial}{\partial x}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)_{\eta\eta}+F_{21}(\eta,\tau)\right][\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta\right]\mathrm{d}\tau+\int_0^t F_{22}(u_0(x,\tau),v_0(x,\tau))\mathrm{d}\tau. \tag{28}$$

用同样的方法,我们还能得到 $u_n(x,t),v_n(x,t)(n=3,4,\cdots)$.

于是,由式(18),我们便得到了扰动 Schrödinger 耦合系统(1)和(2)激波解的 n 次近似 $u_{n\,\mathrm{app}}(x,t),v_{n\,\mathrm{app}}(x,t),n=1,2,\cdots,$ 为

$$u_{n\,\mathrm{app}}(x,t)=2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right)+C_1(t)\exp(\sqrt{a}(x-\eta))+C_2(t)\exp(-\sqrt{a}(x-\eta))+\frac{1}{2\sqrt{a}}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}+f_1(u_0(\eta,t),v_0(\eta,t))\right][\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta+\frac{1}{2\sqrt{a}}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}+\sum_{i=2}^{n}F_{i1}(\eta,t)\right][\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta,$$

$$v_{n\,\mathrm{app}}(x,t)=\frac{2\sqrt{3}\sigma^2}{c}-\frac{6\sigma^2}{c}\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right)+\frac{\sqrt{a}}{c}\int_0^t[C_1(\tau)\exp(\sqrt{a}(x-\eta))-C_2(\tau)\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\tau+\frac{1}{2c\sqrt{a}}\int_0^t\left[\frac{\partial}{\partial x}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)_{\eta\eta}+f_1(u_0(\eta,\tau),v_0(\eta,\tau))\right]\times[\exp(\sqrt{a}(x-\eta))+\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta\right]\mathrm{d}\tau+\int_0^t f_2(u_0(x,\tau),v_0(x,\tau))\mathrm{d}\tau-\frac{1}{2\sqrt{a}}\int_0^t\left[\frac{\partial}{\partial x}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)+\right.\right.$$

$$6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)_{\eta\eta}+\sum_{i=2}^{n}F_{i1}(\eta,\tau)\Bigg][\exp(\sqrt{a}(x-\eta))-$$
$$\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta+$$
$$\frac{\sqrt{a}}{2}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}+\right.$$
$$\left.\sum_{i=2}^{n}F_{i1}(\eta,t)\right][\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta+$$
$$\int_0^t\sum_{i=2}^{n}F_{i2}(u_0(x,\tau),v_0(x,\tau))\mathrm{d}\tau,$$

其中

$$F_{i1}(x,t)=\frac{1}{(i-1)!}\left[\frac{\partial^{i-1}}{\partial s^{i-1}}\left(-\sum_{i=0}^{\infty}u_i(x,t)s^i\right)\left(\sum_{i=0}^{\infty}v_i(x,t)s^i\right)+\right.$$
$$\left.f_1\left(\sum_{i=0}^{\infty}u_i(x,t)s^i,\sum_{i=0}^{\infty}v_i(x,t)s^i\right)\right]_{s=0},$$
$$F_{i2}(x,t)=\frac{1}{(i-1)!}\left[\frac{\partial^{i-1}}{\partial s^{i-1}}f_2\left(\sum_{i=0}^{\infty}u_i(x,t)s^i,\sum_{i=0}^{\infty}v_i(x,t)s^i\right)\right]_{s=0}.$$

3 举 例

为简单起见,考虑一个特殊的扰动 Schrödinger 耦合系统,其扰动项为 $f_1(u,v)=\varepsilon\sin u$, $f_2(u,v)=\varepsilon\cos v$,其中 ε 为小参数。这时系统(1)和(2)为

$$au-uv-u_{xx}=\varepsilon\sin u, \tag{29}$$
$$v_t-u_x=\varepsilon\cos v. \tag{30}$$

由映射式(17),选取 0 次近似 $u_0(x,t)$, $v_0(x,t)$ 为

$$u_0(x,t)=2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right), \tag{31}$$
$$v_0(x,t)=\frac{2\sqrt{3}\sigma^2}{c}-\frac{6\sigma^2}{c}\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right). \tag{32}$$

由式(23)~(25)和式(26)~(28),有

$$u_1(x,t)=C_1(t)\exp(\sqrt{a}(x-\eta))+C_2(t)\exp(-\sqrt{a}(x-\eta))+$$
$$\frac{1}{2\sqrt{a}}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}+\right.$$
$$\left.\varepsilon\sin\left(2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)\right][\exp(\sqrt{a}(x-\eta))-$$
$$\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta, \tag{33}$$

$$v_1(x,t)=\frac{\sqrt{a}}{c}\int_0^t[C_1(\tau)\exp(\sqrt{a}(x-\eta))-C_2(\tau)\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\tau-$$
$$\frac{1}{2c\sqrt{a}}\int_0^t\left[\frac{\partial}{\partial x}\int_{-\infty}^{x}\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)+\right.\right.$$
$$6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)_{\eta\eta}+$$

$$\varepsilon\sin\left(2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)\Big]\times$$

$$[\exp(\sqrt{a}(x-\eta))+\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta\Big]\mathrm{d}\tau+$$

$$\varepsilon\int_0^t\cos\left(\frac{2\sqrt{3}\sigma^2}{c}-\frac{6\sigma^2}{c}\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}\tau\right)\right)\mathrm{d}\tau, \tag{34}$$

$$u_2(x,t)=\frac{1}{2\sqrt{a}}\int_{-\infty}^x\Big[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+$$

$$6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}-u_0(\eta,t)u_1(\eta,t)-$$

$$u_1(\eta,t)u_0(\eta,t)+\varepsilon u_1(\eta,t)\Big]\cos\Big[2\sqrt{3}\sigma^2-$$

$$6\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\Big][\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta, \tag{35}$$

$$v_2(x,t)=\frac{1}{2\sqrt{a}}\int_0^t\Big[\frac{\partial}{\partial x}\int_{-\infty}^x\Big[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)+$$

$$6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)_{\eta\eta}-$$

$$2u_0(\eta,t)v_1(\eta,t)+\cos(u_0(\eta,\tau))\Big]\times$$

$$[\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta\Big]\mathrm{d}\tau-$$

$$\varepsilon\int_0^t v_1(x,\tau)\sin(v_0(x,\tau))\mathrm{d}\tau. \tag{36}$$

由式(31)~(36),这时我们便得到扰动 Schrödinger 耦合系统(29)~(30)如下的 2 次近似激波行波解 $u_{2\,\mathrm{app}}(x,t)$, $v_{2\,\mathrm{app}}(x,t)$, $n=1,2,\cdots$, 为

$$u_{2\,\mathrm{app}}(x,t)=2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right)+$$

$$C_1(t)\exp(\sqrt{a}(x-\eta))+C_2(t)\exp(-\sqrt{a}(x-\eta))+$$

$$\frac{1}{2\sqrt{a}}\int_{-\infty}^x\Big[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}+$$

$$\varepsilon\sin\left(2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)\Big][\exp(\sqrt{a}(x-\eta))-$$

$$\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta+$$

$$\frac{1}{2\sqrt{a}}\int_{-\infty}^x\Big[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)+6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\right)_{\eta\eta}-$$

$$u_0(\eta,t)u_1(\eta,t)+\varepsilon u_1(\eta,t)\Big]\cos\Big(2\sqrt{3}\sigma^2-$$

$$6\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}t\right)\Big)[\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta,$$

$$v_{2\,\mathrm{app}}(x,t)=\frac{2\sqrt{3}\sigma^2}{c}-\frac{6\sigma^2}{c}\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}t\right)+$$

$$\frac{\sqrt{a}}{c}\int_0^t[C_1(\tau)\exp(\sqrt{a}(x-\eta))-C_2(\tau)\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\tau-$$

$$\frac{1}{2c\sqrt{a}}\int_0^t\left[\frac{\partial}{\partial x}\int_{-\infty}^x\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)+\right.\right.$$

$$6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)_{\eta\eta}+\varepsilon\left[\sin\left(2\sqrt{3}\sigma^2-6\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)\right]\times$$

$$\left.\left.[\exp(\sqrt{a}(x-\eta))+\exp(-\sqrt{a}(x-\eta))]\right]\mathrm{d}\eta\right]\mathrm{d}\tau+$$

$$\varepsilon\int_0^t\cos\left(\frac{2\sqrt{3}\sigma^2}{c}-\frac{6\sigma^2}{c}\tanh^2\sigma\left(x+\frac{\sqrt{3}}{2}\tau\right)\right)\mathrm{d}\tau+$$

$$\frac{1}{2\sqrt{a}}\int_0^t\left[\frac{\partial}{\partial x}\int_{-\infty}^x\left[2\sqrt{3}a\sigma^2-6a\sigma^2\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)+\right.\right.$$

$$6\sigma^2\left(\tanh^2\sigma\left(\eta+\frac{\sqrt{3}}{2}\tau\right)\right)_{\eta\eta}-u_0(\eta,t)v_1(\eta,t)+\cos(u_0(\eta,\tau))\Big]\times$$

$$\left.[\exp(\sqrt{a}(x-\eta))-\exp(-\sqrt{a}(x-\eta))]\mathrm{d}\eta\right]\mathrm{d}\tau-$$

$$\varepsilon\int_0^t v_1(x,\tau)\sin(v_0(x,\tau))\mathrm{d}\tau,$$

其中，$u_i(x,t)$，$v_i(x,t)$ $(i=0,1)$ 分别由式(31)～(34)表示．

继续地，利用同样的方法能够得到扰动 Schrödinger 耦合系统(29)和(30)的更高次近似的激波行波解．

我们还能证明扰动 Schrödinger 耦合系统(29)和(30)的激波解有如下的估计式[20]：

$$u(x,t)=u_{2\,\mathrm{app}}(x,t)+O(\varepsilon^3),\ v(x,t)=v_{2\,\mathrm{app}}(x,t)+O(\varepsilon^3),\qquad 0<\varepsilon<<1.$$

因此，利用本文提出的渐近方法得到的激波近似解具有较好的精确度．

4 结束语

激波理论出自于一类复杂的自然现象．因此，我们需要简化它为基本模型．利用近似方法去求解这类模型是激波理论的重要方面．本文就是利用映射理论并指出了一个简单而有效的方法得到了扰动 Schrödinger 耦合系统激波渐近解．

由渐近方法求解模型的近似解，不同于单纯的模拟得到的数值近似解，由于渐近解是具有解析形式的结构．因此，它还可以进行微分、积分等解析运算，从而能进一步地了解相应激波解的更深层的性态．

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Approximate Solving Method of Shock for Nonlinear Disturbed Coupled Schrödinger System

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Abstract: A class of the nonlinear disturbed coupled Schrödinger system was studied. Using the specific technique to relate the exact and approximate solutions, firstly, the corresponding typical coupled system was considered. The exact shock travelling solution was obtained by using the mapping method. Then, the travelling asymptotic solutions of the disturbed coupled Schrödinger system was found by using an approximate method.

Key words: Schrödinger system; solitary wave; asymptotic solution