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# 弹性支承-刚性转子系统同步全周碰摩的分岔响应*

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(我刊编委陈予恕来稿)

**摘要:** 基于航空发动机转子系统的结构特点,将航空发动机转子系统简化为一个非线性弹性支承的刚性转子系统. 根据 Lagrange 方程建立了弹性支承-刚性不对称转子系统同步全周碰摩的运动方程;采用平均法进行求解,得到了关于系统振幅的分岔方程;根据两状态变量约束分岔理论,分别给出了系统在无碰摩和碰摩阶段参数平面的转迁集和分岔图,讨论了转子偏心、阻尼对系统分岔行为的影响;应用 Liapunov 稳定性理论分析了系统碰摩周期解的稳定性和失稳方式,给出了系统参数——转速平面上周期解的稳定范围;该文的研究结果对航空发动机转子系统的设计有一定的理论意义.

**关 键 词:** 弹性支承刚性转子; 碰摩; 两状态变量约束分岔; 稳定性
**中图分类号:** O322; V235.1 **文献标志码:** A
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## 引 言

现代航空发动机普遍采用柔性转子设计,因此大都采用弹性支承[1]. 弹性支承的应用使得转子系统出现了两个低阶临界(支承临界)(见图 1),支承和转子本身的耦合使得转子的弯曲临界有一定的升高,这样就在低阶临界和弯曲临界之间出现了一个较宽阔的、振动较小的转速范围作为转子的工作转速范围. 图 1 给出了弹性支承转子临界转速的弹性线,可以看到在通过低阶临界时,转子本身的弹性变形很小,从而提高了转子的运转寿命和发动机运行的可靠性. 基于这一特点,航空发动机转子可以简化为一个非线性弹性支承的刚性转子系统.

现代航空发动机设计中,为提高系统的性能,旋转与静止部件间的间隙被设计得越来越小,从而导致系统碰摩的可能性越来越大. 碰摩可能会引起叶片断裂、转子失稳等,造成严重的运行事故[2].

近年来,许多学者在转子系统碰摩方面做了大量的研究工作,通过解析、数值和实验方法对系统在碰摩时可能出现的响应进行研究,包括周期运动[3-5]、概周期运动局部碰摩[6-7]、混沌[8-11]、干摩擦反向涡动[12-14]以及碰摩与其它故障的耦合响应[15-16].

在多种碰摩响应中,同步全周碰摩最容易发生在转子的工作转速附近,当它发生时转子和

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静子出现连续的接触,但由于这种接触非常轻微,转子系统的同步全周碰摩在短时间内是可以接受的.但是随着系统参数的改变,同步全周碰摩运动很容易发生失稳,进而导致干摩擦反向涡动、混沌、超谐或者亚谐响应等等对转子系统危害很大的运动形式[17-18].考虑到这一潜在的危险,研究参数对同步全周碰摩运动响应和稳定性的影响是非常重要的.

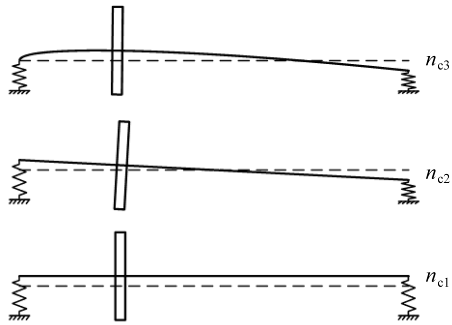



**图 1**　弹性支承转子的振型[1]

Fig. 1　The modes of the elastic-support rotor system[1]

对于转静同步全周碰摩的研究所采用的碰摩模型主要有 3 种:

1) 弹性支承的刚性静子模型(将静子简化为一个弹性支承的刚性环)

针对这种模型,Black[19]研究了同步全周碰摩解的存在条件.Begg[20]采用能量平衡的方法,讨论了柔性悬臂转子-刚性机匣系统的稳定性,指出转静接触的摩擦与系统阻尼的比对同步全周碰摩运动的稳定性影响很大.当它小于某一特定值时,同步全周碰摩运动总是稳定的.Stackley[21]研究了机匣为刚性的转子系统的同步全周碰摩,给出了同步全周碰摩运动稳定的转速范围,这个转速范围可以表示成系统不平衡、转子刚度、阻尼、转静间隙和摩擦因数的表达式.Ehehalt 等[17-18]采用 Liapunov 稳定性理论分析了同步全周碰摩解的稳定性,并讨论了系统参数的影响.

2) 附加刚度模型(将静子简化为附加刚度)

针对这种模型,Yu 等[22],Muszynska[23],Bently 等[24-25]通过实验和解析对单盘转子的同步和反向涡动进行了研究,分析了全周碰摩响应和稳定性,讨论了不平衡量、摩擦因数、阻尼等等系统参数对同步全周碰摩运动向反向涡动转化的影响.Choi[26]采用实验和数值仿真研究了同步全周碰摩运动的边界条件.马建敏等[27-28]通过对单盘转子系统碰摩运动规律的理论分析和计算仿真,得出了转子初次碰摩转速的解析表达式;讨论了阻尼、偏心距和间隙对转子碰摩转速的影响.Jiang 等[29-30]给出了同步全周碰摩运动稳定条件的解析表达式,并讨论了交叉耦合刚度的影响.许和 Zhang 等[31-33]利用摄动法,分别对单盘转子和柔性转子-柔性静子系统的扰动运动进行同步全周碰摩运动的分析,获得同步全周碰摩解的参数存在区,求出该区域内同步全周碰摩解的解析表达式,并对同步全周碰摩解的稳定性进行了讨论.在研究中,他们采用的 Hertz 接触理论推导出的碰摩力模型.刘献栋等[34]分析了转子系统同步全周碰摩的 Hopf 分岔,并求出了系统的 Hopf 分岔解.

3) 弹性转-静耦合模型(将静子简化为一个弹性支承的弹性环)

针对这种模型,Groll 等[35]采用一种由谐波平衡法发展而来的数值方法,研究了碰摩的周期响应及其稳定性.Jiang 等[36]研究了弹性转–静耦合模型同步全周碰摩的稳定性,给出了转子和静子阻尼、摩擦因数以及转静质量比、刚度比对系统稳定性的影响.Shang 等[37]研究了交叉耦合刚度对弹性转-静耦合模型同步全周碰摩运动稳定性的影响.弹性转-静耦合模型也常常用于转子与辅助轴承耦合系统的研究中,Xie 等[38],Cole 等[39-40],Sahinkaya 等[41]等在这方面做了大量的工作.

现有的关于全周碰摩的研究工作大都针对线性支承的转子系统,在现代转子的设计中,越来越多的非线性因素被考虑进去,如滚动轴承的 Hertz 接触力、滑动轴承和挤压油膜阻尼器的

油膜力等等,因此研究非线性支承的转子系统是更加有意义的.

本文考虑一个非线性的弹性支承-刚性不对称转子系统,采用 Lagrange 方程建立系统的运动方程,用平均法进行求解.根据奇异性和两状态变量约束分岔理论,分别给出了系统在无碰摩和碰摩时参数平面的转迁集和分岔图,讨论了转子偏心、阻尼对系统分岔行为的影响;应用 Liapunov 稳定性理论分析了系统周期解的稳定性和失稳方式,给出了参数平面周期解的稳定区域.

# 1 弹支刚性转子系统碰摩的运动方程与解析计算

考虑如图 2 所示弹支刚性转子系统,其中转子的支承具有 3 次非线性,即支承的回复力可表示为 $F = kr + \alpha r^3$,其中 $r = \sqrt{x^2 + y^2}$ 是转子的径向位移,$k$ 和 $\alpha$ 分别是支承的线性和非线性刚度系数.选取广义坐标 $x, y, \theta_x, \theta_y$,其中 $x, y$ 分别为转子在盘处水平和竖直方向平移运动的位移,$\theta_x, \theta_y$ 分别是转子盘绕水平和竖直方向的偏转角.容易给出系统的动能、势能和 Rayleigh 耗散函数的表达式如下:

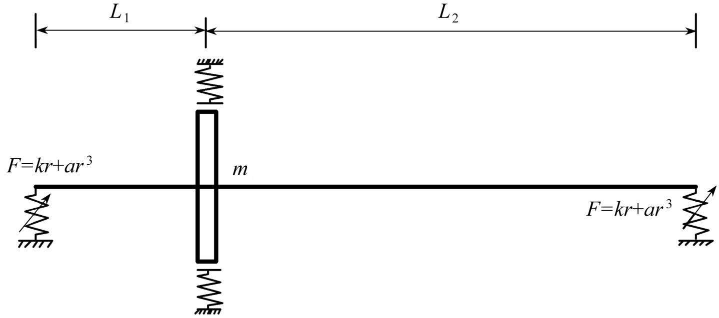



**图 2** 弹性支承刚性转子系统碰摩模型

Fig. 2 The model of rigid-rotor elastic-support rubbing system

$$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\dot{y}^2 + \frac{1}{2} J\dot{\theta}_x^2 + \frac{1}{2} J\dot{\theta}_y^2 + \frac{1}{2} J_p\omega^2 - J_p\omega\dot{\theta}_y\theta_x, \tag{1}$$

$$V = \frac{1}{2} k[(x - L_1\theta_y)^2 + (x + L_2\theta_y)^2 + (y - L_1\theta_x)^2 + (y + L_2\theta_x)^2] + \frac{1}{4}\alpha[(x - L_1\theta_y)^2 + (y - L_1\theta_x)^2]^2 + \frac{1}{4}\alpha[(x + L_2\theta_y)^2 + (y + L_2\theta_x)^2], \tag{2}$$

$$D = \frac{1}{2} c[(\dot{x} - L_1\dot{\theta}_y)^2 + (\dot{x} + L_2\dot{\theta}_y)^2 + (\dot{y} - L_1\dot{\theta}_x)^2 + (\dot{y} + L_2\dot{\theta}_x)^2]. \tag{3}$$

根据 Lagrange 方程可得系统的运动方程为

$$m\ddot{x} + k(2x - L_1\theta_x + L_2\theta_x) + \alpha[(x - L_1\theta_x)^2 + (y - L_1\theta_y)^2](x - L_1\theta_x) + \alpha[(x + L_2\theta_x)^2 + (y + L_2\theta_y)^2](x + L_2\theta_x) + c(2\dot{x} - L_1\dot{\theta}_x + L_2\dot{\theta}_x) + P_x = me\omega^2\cos\omega t, \tag{4a}$$

$$m\ddot{y} + k(2y - L_1\theta_y + L_2\theta_y) + \alpha[(x - L_1\theta_x)^2 + (y - L_1\theta_y)^2](y - L_1\theta_y) + \alpha[(x + L_2\theta_x)^2 + (y + L_2\theta_y)^2](y + L_2\theta_y) + c(2\dot{y} - L_1\dot{\theta}_y + L_2\dot{\theta}_y) + P_y = me\omega^2\sin\omega t, \tag{4b}$$

$$J\ddot{\theta}_x + J_p\omega\dot{\theta}_y + k[(x + L_2\theta_x)L_2 - (x - L_1\theta_x)L_1] - \alpha[(x - L_1\theta_x)^2 + (y - L_1\theta_y)^2](x - L_1\theta_x)L_1 +$$

$$\alpha[(x+L_2\theta_x)^2+(y+L_2\theta_y)^2](x+L_2\theta_x)L_2+$$
$$c[(\dot{x}+L_2\dot{\theta}_x)L_2-(\dot{x}-L_1\dot{\theta}_x)L_1]=0, \tag{4c}$$
$$J\ddot{\theta}_y-J_p\omega\dot{\theta}_x+k[(y+L_2\theta_y)L_2-(y-L_1\theta_y)L_1]-$$
$$\alpha[(x-L_1\theta_x)^2+(y-L_1\theta_y)^2](y-L_1\theta_y)L_1+$$
$$\alpha[(x+L_2\theta_x)^2+(y+L_2\theta_y)^2](y+L_2\theta_y)L_2+$$
$$c[(\dot{y}+L_2\dot{\theta}_y)L_2-(\dot{y}-L_1\dot{\theta}_y)L_1]=0, \tag{4d}$$

其中，$m$ 为转子在盘处的等效质量，$J$ 和 $J_p$ 分别是为转子的等效赤道转动惯量和极转动惯量，$c$ 是支承阻尼，$r_0$ 表示转子和机匣之间的间隙，$e$ 是转子的偏心量，$\mu$ 是Coulomb摩擦因数，$k_c$ 为接触刚度. 这里，我们采用由线性接触力和 Coulomb 摩擦力组合而成的碰摩力模型[29-30]

$$\begin{bmatrix}P_x\\P_y\end{bmatrix}=\begin{cases}0, & r\leqslant r_0,\\ k_c\left(1-\dfrac{r_0}{r}\right)\begin{bmatrix}x-\mu y\\ \mu x+y\end{bmatrix}, & r>r_0.\end{cases} \tag{5}$$

定义无量纲变量

$$q_1=x/r_0,\ q_2=y/r_0,\ q_3=(L_2-L_1)\theta_x/r_0,\ q_4=(L_2-L_1)\theta_y/r_0,\ \tau=\omega t,$$

对方程(5)进行无量纲化，有

$$\begin{cases}q_1''+\omega_1^2q_1+\varepsilon a_1q_4+\varepsilon a_2(2q_1'+q_4')+\\ \qquad\varepsilon a_3(q_4q_2^2+2q_1q_2q_3+3q_4q_1^2+2q_1(q_1^2+q_2^2))+\\ \qquad\varepsilon^2a_4(q_1q_3^2+2q_3q_2q_4+3q_4^2q_1)+\varepsilon^2a_5q_4(q_3^2+q_4^2)+P_1=\varepsilon E\cos\tau,\\ q_2''+\omega_1^2q_2+\varepsilon a_1q_3+\varepsilon a_2(2q_2'+q_3')+\\ \qquad\varepsilon a_3(q_1^2q_3+2q_1q_4q_2+3q_2^2q_3+2q_2(q_1^2+q_2^2))+\\ \qquad\varepsilon^2a_4(q_4^2q_2+2q_1q_3q_4+3q_2q_3^2)+\varepsilon^2a_5q_3(q_3^2+q_4^2)+P_2=\varepsilon E\sin\tau,\\ q_3''+\omega_2^2q_3+\varepsilon b_1q_2+\varepsilon b_2q_3'+\varepsilon b_3q_4'+\varepsilon b_4q_2'+\varepsilon b_5q_2(q_1^2+q_2^2)+\\ \qquad\varepsilon^2b_6(q_3q_1^2+2q_1q_4q_2+3q_3q_2^2)+\varepsilon^2b_7(q_2q_4^2+2q_3q_4q_1+3q_3^2q_2)+\\ \qquad\varepsilon^2b_8q_3(q_3^2+q_4^2)=0,\\ q_4''+\omega_2^2q_4+\varepsilon b_1q_1+\varepsilon b_2q_4'-\varepsilon b_3q_3'+\varepsilon b_4q_1'+\varepsilon b_5q_1(q_1^2+q_2^2)+\\ \qquad\varepsilon^2b_6(q_2^2q_4+2q_1q_3q_2+3q_1^2q_4)+\varepsilon^2b_7(q_3^2q_1+2q_2q_3q_4+3q_1q_4^2)+\\ \qquad\varepsilon^2b_8q_4(q_3^2+q_4^2)=0,\end{cases} \tag{6}$$

其中

$$\begin{cases}\omega_1^2=\dfrac{2k}{m\omega^2},\ \varepsilon a_1=\dfrac{k}{m\omega^2},\ \beta=\dfrac{r_0^2\alpha}{\omega^2},\ \zeta=\dfrac{c}{\omega},\ \varepsilon a_2=\dfrac{\zeta}{m},\ \varepsilon a_3=\dfrac{\beta}{m},\\ \varepsilon^2a_4=\dfrac{\beta}{m}\dfrac{L_2^2+L_1^2}{(L_2-L_1)^2},\ \varepsilon^2a_5=\dfrac{\beta}{m}\dfrac{L_2^3-L_1^3}{(L_2-L_1)^3},\ \omega_2^2=\dfrac{k}{J\omega^2}(L_1^2+L_2^2),\\ \varepsilon b_1=\dfrac{k}{J\omega^2}(L_2-L_1)^2,\ \varepsilon b_2=\dfrac{\zeta}{J}(L_1^2+L_2^2),\ \varepsilon b_3=\dfrac{J_p}{J},\\ \varepsilon b_4=\dfrac{\zeta}{J}(L_2-L_1)^2,\ \varepsilon b_5=\dfrac{\beta}{J}(L_2-L_1)^2,\ \varepsilon^2b_6=\dfrac{\beta}{J}(L_1^2+L_2^2),\end{cases} \tag{7a}$$

$$\begin{cases}\varepsilon^2 b_7 = \dfrac{\beta}{J}(L_1^2 + L_2 L_1 + L_2^2),\ \varepsilon^2 b_8 = \dfrac{\beta}{J}\dfrac{L_1^4 + L_2^4}{(L_2 - L_1)^2}, \\ \varepsilon g = \dfrac{k_c}{m\omega^2},\ \varepsilon E = \dfrac{e}{r_0},\ R = \dfrac{r}{r_0},\end{cases} \tag{7b}$$

其中 $\varepsilon$ 为小参数. 无量纲形式的碰摩力可写作

$$\begin{bmatrix}P_1\\P_2\end{bmatrix} = \begin{cases}0, & R \leqslant 1, \\ \varepsilon g\left(1 - \dfrac{1}{R}\right)\begin{bmatrix}q_1 - \mu q_2\\ \mu q_1 + q_2\end{bmatrix}, & R > 1.\end{cases} \tag{8}$$

转子系统的参数见表 1.

**表 1 转子系统的参数**

Table 1 The parameters of the rotor system

| parameters | values |
|---|---|
| equivalent mass at the disc $m$ | 58.361 3 kg |
| equivalent equatorial moment of inertia at the disc $J$ | 4.841 kg·m$^2$ |
| equivalent polar moment of inertia at the disc $J_p$ | 6 kg·m$^2$ |
| distance from the disc to the support on the left $L_1$ | 0.894 2 m |
| distance from the disc to the support on the right $L_2$ | 1.037 7 m |
| stiffness of the support $k$ | $2\times10^6$ N/m |
| nonlinearity of the support $\alpha$ | $3.8\times10^{12}$ N/m$^3$ |
| damping of support $c$ | 261.8 N·s/m |
| clearance of the rotor and the stator $r_0$ | 0.3 mm |
| contact stiffness of the system $k_c$ | $3\times10^6$ N/m |
| frictioncoefficient of the rubbing force $\mu$ | 0.1 |

我们考虑系统在一阶临界转速附近,转子的同步正向涡动的碰摩响应,由于本文所研究的转子是各向同性的,因此转子的轴心轨迹是圆[42],可以设系统的解为

$$\begin{cases}q_1 = A_1\cos(\tau + \theta_1),\ q_2 = A_1\sin(\tau + \theta_1), \\ q_3 = A_2\sin(\tau + \theta_2),\ q_4 = A_2\cos(\tau + \theta_2),\end{cases} \tag{9}$$

其中, $A_1, \theta_1$ 和 $A_2, \theta_2$ 分别是转子平移和偏转运动的振幅和相位. 定义 $1 = \omega_1^2 + \varepsilon\sigma_1, \omega_2^2 = \omega_1^2 + \varepsilon\sigma_2$, 采用平均法[43]求解得

$$\begin{cases}A_1' = \dfrac{\varepsilon}{2}[g\mu - A_1 g\mu + (a_1 A_2 + a_3 A_1^2 A_2)\sin(\theta_1 - \theta_2) - \\ \qquad a_2 A_2\cos(\theta_1 - \theta_2) - 2a_2 A_1 - E\sin\theta_1], \\ A_1\theta_1' = \dfrac{\varepsilon}{2}[2a_3 A_1^3 - \sigma_1 A_1 + gA_1 - g + a_2 A_2\sin(\theta_1 - \theta_2) + \\ \qquad (3a_3 A_1^2 A_2 + A_2 a_1)\cos(\theta_1 - \theta_2) - E\cos\theta_1], \\ A_2' = \dfrac{\varepsilon}{2}[(b_1 A_1 + b_5 A_1^3)\sin(\theta_1 - \theta_2) + \\ \qquad b_2 A_2 + b_4 A_1\cos(\theta_1 - \theta_2)], \\ A_2\theta_2' = \dfrac{\varepsilon}{2}[(b_1 A_1 + b_5 A_1^3)\cos(\theta_1 - \theta_2) - b_3 A_2 - \\ \qquad \sigma_1 A_2 + \sigma_2 A_2 - b_4 A_1\sin(\theta_1 - \theta_2)].\end{cases} \tag{10}$$

令等号右端等于 0, 消去 $\theta_1, \theta_2$, 可以得到系统在碰摩时的分岔方程

$$F_{21}(A_1, A_2, \sigma_1, \eta) = 0,\ F_{22}(A_1, A_2, \sigma_1, \eta) = 0, \qquad A_1 > 1, \tag{11}$$

其中 $\eta$ 表示其他系统参数的集合. 令式(11) 中的 $g = 0$, 可以得到无碰摩状态的分岔方程

$$F_{11}(A_1, A_2, \sigma_1, \eta) = 0,\ F_{12}(A_1, A_2, \sigma_1, \eta) = 0, \qquad A_1 \leqslant 1. \tag{12}$$

# 2 弹支刚性转子系统碰摩的约束分岔

转子系统的碰摩是典型的非光滑动力学问题,在转子没有发生碰摩和碰摩时,对应着不同的约束条件,碰摩力有不同的表达形式,因此,研究系统的约束分岔是非常有意义的事情.

吴志强等[44-46]通过引入适当的变换,将约束含参分岔问题转化为新变量的非约束分岔问题,推导出了约束含参分岔问题转迁集的一般形式. 秦朝红等[47]将多状态变量分岔理论应用到悬索系统中. 李军等[48]将约束分岔理论推广到多状态变量系统中,给出了多状态变量约束系统转迁集的计算方法. 本文将应用两状态变量约束分岔理论研究系统的分岔行为.

为了计算方便,令 $A_1 = X + 1, A_2 = Y, \sigma_1 = \lambda$, 则系统在无碰摩阶段的分岔方程变为

$$F_1 = \begin{cases} F_{11}(X, Y, \lambda, \eta) = 0, \\ F_{12}(X, Y, \lambda, \eta) = 0, \end{cases} \qquad X \leqslant 0. \tag{13}$$

系统在碰摩阶段的分岔方程化为

$$F_2 = \begin{cases} F_{21}(X, Y, \lambda, \eta) = 0, \\ F_{22}(X, Y, \lambda, \eta) = 0, \end{cases} \qquad X > 0. \tag{14}$$

**表 2**　以 $X = 0$ 为约束的两状态变量系统的转迁集

Table 2　Transition sets of the two dimensional system with a constraint $X = 0$

| transition sets | expressions |
|---|---|
| $B_1$ | $H_1(X,Y,\lambda,\eta) = 0,\ H_2(X,Y,\lambda,\eta) = 0,$<br>$H_{1X}(X,Y,\lambda,\eta)H_{2Y}(X,Y,\lambda,\eta) - H_{1Y}(X,Y,\lambda,\eta)H_{2X}(X,Y,\lambda,\eta) = 0$<br>$H_{1Y}(X,Y,\lambda,\eta)H_{2\lambda}(X,Y,\lambda,\eta) - H_{1\lambda}(X,Y,\lambda,\eta)H_{2Y}(X,Y,\lambda,\eta) = 0$ |
| $B_2$ | $H_1(0,Y,\lambda,\eta) = 0, H_2(0,Y,\lambda,\eta) = 0$<br>$H_{1Y}(0,Y,\lambda,\eta)H_{2\lambda}(0,Y,\lambda,\eta) - H_{1\lambda}(0,Y,\lambda,\eta)H_{2Y}(0,Y,\lambda,\eta) = 0$ |
| $\bar{H}_1$ | $H_1(X,Y,\lambda,\eta) = 0, H_2(X,Y,\lambda,\eta) = 0,$<br>$H_{1X}(X,Y,\lambda,\eta)X' + H_{1Y}(X,Y,\lambda,\eta)Y' = 0$<br>$H_{1X}(X,Y,\lambda,\eta)H_{2Y}(X,Y,\lambda,\eta) - H_{1Y}(X,Y,\lambda,\eta)H_{2X}(X,Y,\lambda,\eta) = 0$<br>$H_{1X}(X,Y,\lambda,\eta)f_2 - H_{2X}(X,Y,\lambda,\eta)f_1 = 0$<br>$f_1 = H_{1XX}(X,Y,\lambda,\eta)X'^2 + 2H_{1XY}(X,Y,\lambda,\alpha)X'Y' + H_{1YY}(X,Y,\lambda,\eta)Y'^2$<br>$f_2 = H_{2XX}(X,Y,\lambda,\eta)X'^2 + 2H_{2XY}(X,Y,\lambda,\eta)X'Y' + H_{2YY}(X,Y,\lambda,\eta)Y'^2$ |
| $\bar{H}_2$ | $H_1(0,Y,\lambda,\eta) = 0,\ H_2(0,Y,\lambda,\eta) = 0$<br>$H_{1X}(0,Y,\lambda,\eta)H_{2Y}(0,Y,\lambda,\eta) - H_{1Y}(0,Y,\lambda,\eta)H_{2X}(0,Y,\lambda,\eta) = 0$ |
| $DL_1$ | $H_1(X,Y,\lambda,\eta) = 0,\ H_2(X,Y,\lambda,\eta) = 0$<br>$H_{1X}(X,Y,\lambda,\eta)H_{2Y}(X,Y,\lambda,\eta) - H_{1Y}(X,Y,\lambda,\eta)H_{2X}(X,Y,\lambda,\eta) = 0$<br>$Z_1 \neq Z_2,\ Z = (X,Y)$ |
| $DL_2$ | $H_1(X,Y,\lambda,\eta) = 0,\ H_2(X,Y,\lambda,\eta) = 0$<br>$H_{1X}(X,Y,\lambda,\eta)H_{2Y}(X,Y,\lambda,\eta) - H_{1Y}(X,Y,\lambda,\eta)H_{2X}(X,Y,\lambda,\eta) = 0$<br>$H_1(0,Y,\lambda,\eta) = 0,\ H_2(0,Y,\lambda,\eta) = 0,\ X \neq 0$ |
| $DL_3$ | $H_1(0,Y,\lambda,\eta) = 0,\ H_2(0,Y,\lambda,\eta) = 0,\ Y_1 \neq Y_2$ |

$B$: Bifurcation sets, $H$: Hysteresis, $DL$: Double limit point set

根据文献[48]给出的结果可知,对于两状态变量的系统

$$\begin{cases}\bar{H}_1(X,Y,\lambda,\eta)=0,\\ \bar{H}_2(X,Y,\lambda,\eta)=0.\end{cases} \tag{15}$$

以 $X=0$ 为约束的转迁集如表 2 所示,其中,$B_1$,$H_1$,$DL_1$ 为系统原始的转迁集,$B_2$,$H_2$,$DL_2$,$DL_3$ 为对系统施加约束后新增的转迁集. 由于本文考虑的是工程开折,需要在此理论计算的基础上进一步验证. 这些转迁集两侧的分岔图是否拓扑等价,如果分岔图不等价则该集合是约束转迁集,反之则不是[44-46].

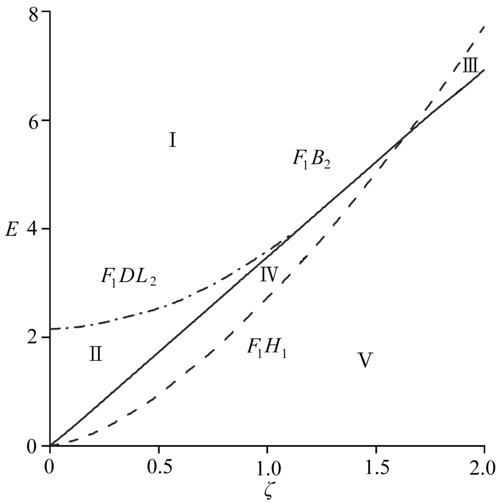



**图 3** $F_1$ 的转迁集

Fig. 3 Transition sets of $F_1$

图 3 和图 4 分别是系统在未碰摩阶段的转迁集和分岔图(其中图 4 中的圆点表示分岔图与约束平面的交点),可以看到,区域Ⅰ,Ⅱ,Ⅲ的分岔图和约束平面有 2 个交点,表明对应图 3 中的参数域Ⅰ,Ⅱ,Ⅲ系统可能会出现碰摩;在区域Ⅳ和Ⅴ分岔图和约束平面没有交点,说明对应该参数域系统不可能发生碰摩;区域Ⅰ,Ⅱ,Ⅳ的分岔图都出现了对应一个 $\lambda$ 值系统有多解的情况,说明系统随着 $\lambda$ 的变化可能出现跳跃现象.

图 5 和图 6 分别是系统在碰摩阶段的转迁集和分岔图(其中图 6 中的圆点表示分岔图与约束平面的交点),可以看到,区域Ⅰ和Ⅱ的分岔图和约束平面有 2 个交点,表明对应图 5 中的参数域Ⅰ和Ⅱ转子和机匣之间可能会出现一次接触碰摩;而区域Ⅲ的分岔图为空,说明对应该参数域系统不可能发生碰摩.

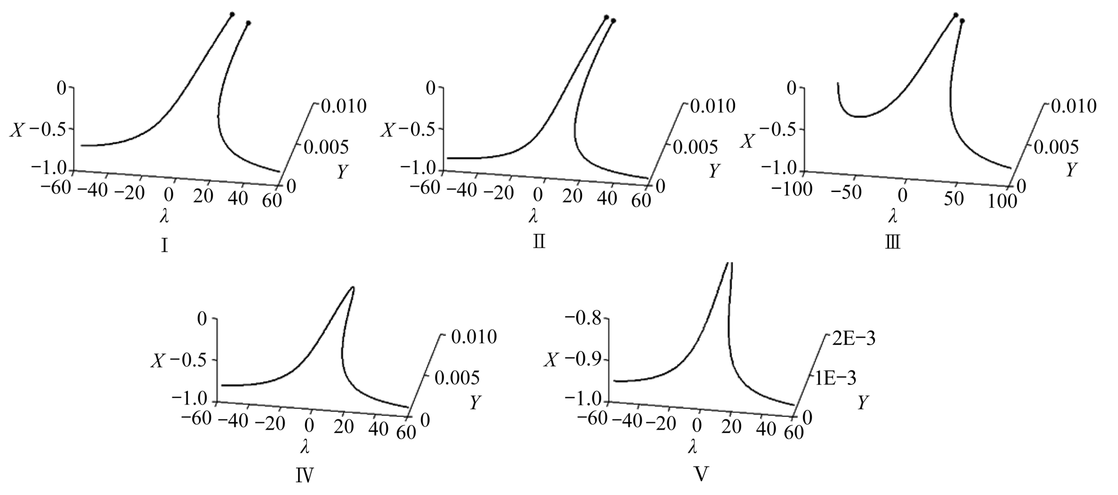



**图 4** $F_1$ 的分岔图

Fig. 4 Bifurcation diagrams of $F_1$

为了更清楚地描述参数变化对转子系统动力学行为的影响,我们将前面的分析结合起来,给出了转子系统碰摩的转迁集和对应不同保持域的幅频特性曲线(见图 7 至图 9),其中图 8 是图 7 中区域①和②的局部放大,可以看到系统的转迁集将参数平面分为 8 个保持域,对应着每个保持域的分岔图具有不同的拓扑结构;升速时,在区域Ⅰ-Ⅳ和Ⅶ、Ⅷ对应的参数范围内,

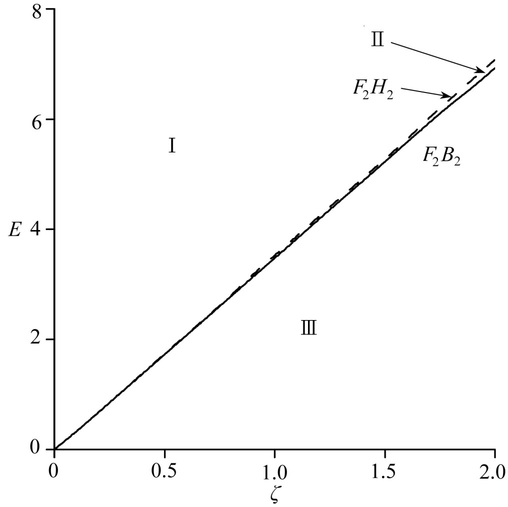



**图 5**　$F_2$ 的转迁集

Fig. 5　Transition sets of $F_2$

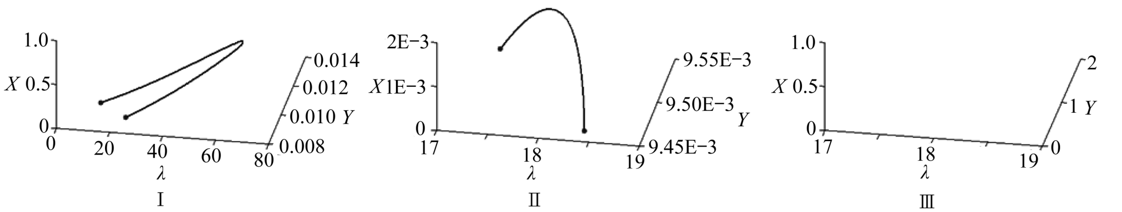



**图 6**　$F_6$ 的分岔图

Fig. 6　Bifurcation diagrams of $F_2$

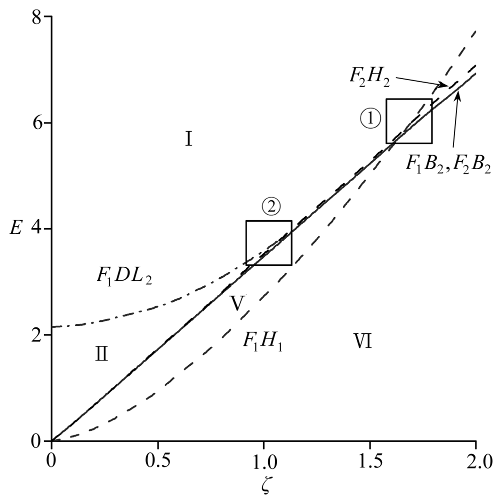
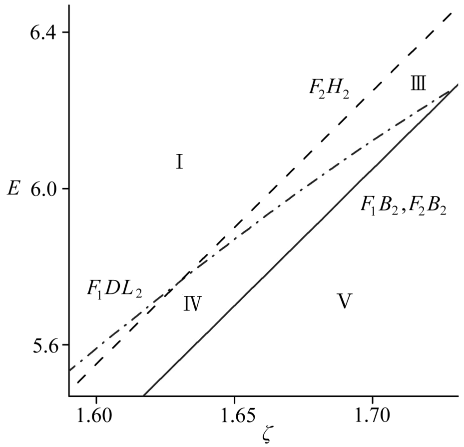



**图 7**　系统的转迁集

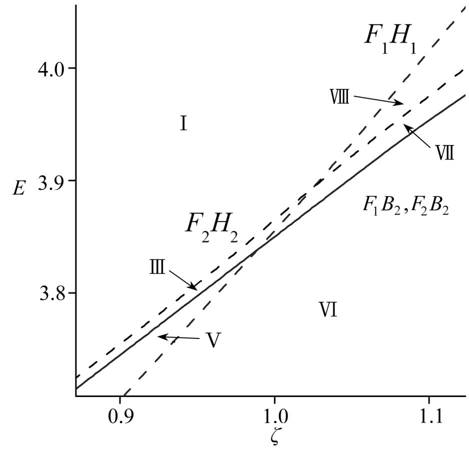
Fig. 7　Transition sets of the system

系统会发生碰摩；降速时，碰摩发生在区域Ⅰ，Ⅲ，Ⅶ，Ⅷ。在Ⅰ-Ⅴ和Ⅷ区域，随着转速的变化会有跳跃现象发生，其中在转子降速时，Ⅰ-Ⅴ区域跳跃发生在系统振幅小于 1 的时候，而对应区域Ⅷ，跳跃发生在振幅为 1 即转子和机匣的接触点处。




（a）①的局部放大

（a）Local enlargements of ①




（b）②的局部放大

（b）Local enlargements of ②

**图8　系统的转迁集的局部放大**

Fig. 8　Local enlargements of transition sets

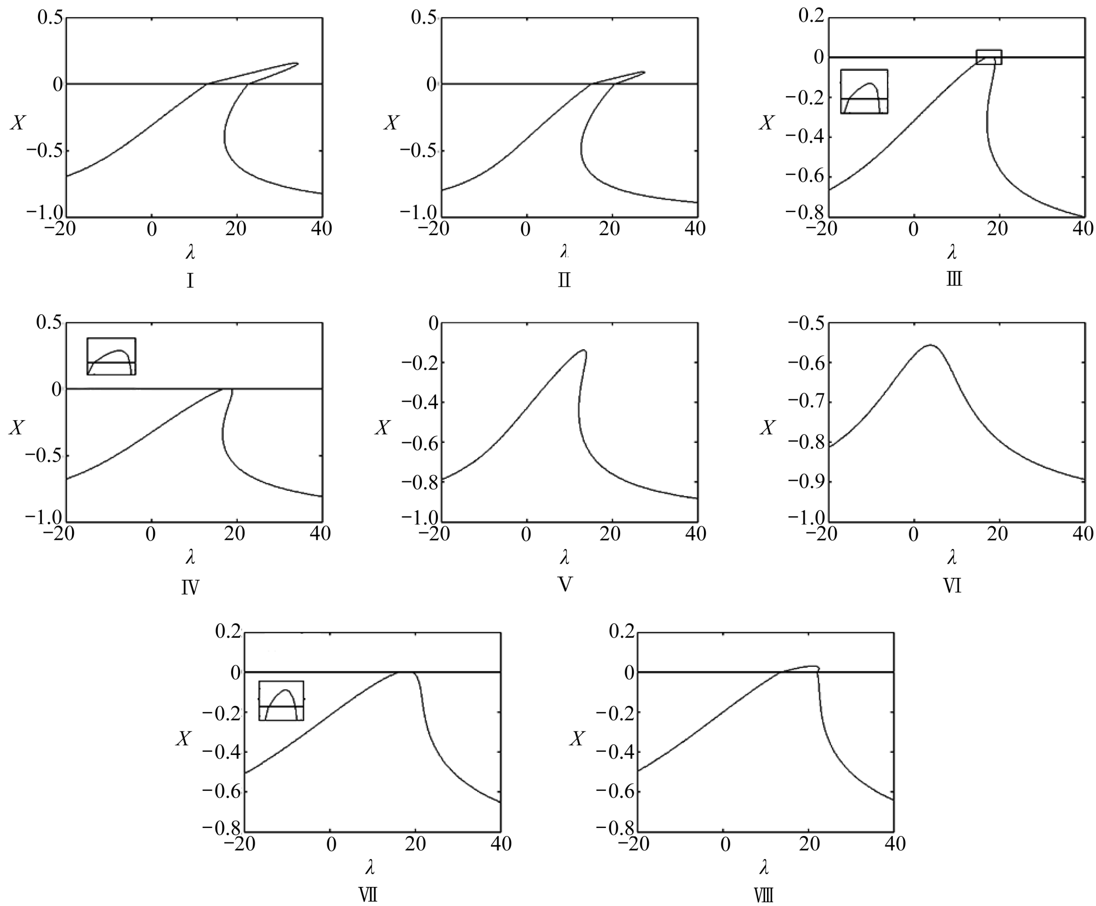



**图9　转子系统碰摩的幅频特性曲线**

Fig. 9　The frequency-amplitude curves of the rubbing system

## 3 同步全周碰摩解的稳定性

在上一节,我们讨论了同步全周碰摩的约束分岔. 在实际系统中,碰摩非常容易导致复杂的动力学行为,只有稳定的周期解才会真实的存在,本节将对同步全周碰摩的稳定性进行讨论. 为了判断碰摩运动周期解的稳定性,我们做如下变换:

$$\begin{cases} q_1 = A_1(\tau)\cos(\tau + \theta_1(\tau)), \\ q_2 = A_1(\tau)\sin(\tau + \theta_1(\tau)), \\ q_3 = A_2(\tau)\sin(\tau + \theta_2(\tau)), \\ q_4 = A_2(\tau)\cos(\tau + \theta_2(\tau)). \end{cases} \tag{16}$$

将式(16)代入到式(4)中,并令

$$A_1 = x_1,\ A_1' = x_2,\ A_2 = x_3,\ A_2' = x_4,\ \theta_1 = x_5,\ \theta_1' = x_6,\ \theta_2 = x_7,\ \theta_2' = x_8,$$

则系统化为

$$\begin{cases} x_1' = x_2, \\ x_2' = -2\varepsilon a_2 x_2 - 2\varepsilon a_3 x_1^3 - \varepsilon g x_1 + \varepsilon E\cos x_5 - 3\varepsilon a_3 x_1^2 x_3 \cos(x_5 - x_7) - \\ \qquad \varepsilon a_2 x_3 x_8 \sin(x_5 - x_7) + \varepsilon g + \varepsilon x_1 \sigma_1 + 2x_1 x_6 + x_1 x_6^2 - \\ \qquad \varepsilon a_1 x_3 \cos(x_5 - x_7) - \varepsilon a_2 x_4 \cos(x_5 - x_7) - \varepsilon a_2 x_3 \sin(x_5 - x_7), \\ x_3' = x_4, \\ x_4' = \varepsilon x_3 \sigma_1 - \varepsilon x_3 \sigma_2 + \varepsilon b_3 x_3 + 2x_3 x_8 + x_3 x_8^2 - \varepsilon b_5 x_1^3 \cos(x_5 - x_7) + \\ \qquad \varepsilon b_4 x_1 x_6 \sin(x_5 - x_7) - \varepsilon b_2 x_4 + \varepsilon b_3 x_3 x_8 - \varepsilon b_1 x_1 \cos(x_5 - x_7) - \\ \qquad \varepsilon b_4 x_2 \cos(x_5 - x_7) + \varepsilon b_4 x_1 \sin(x_5 - x_7), \\ x_5' = x_6, \\ x_6' = \dfrac{1}{x_1}[\varepsilon g\mu - 2x_2 - 2\varepsilon a_2 x_1 - \varepsilon E\sin x_5 - \varepsilon g\mu x_1 - 2x_2 x_6 - \\ \qquad \varepsilon a_2 x_3 \cos(x_5 - x_7) - 2\varepsilon a_2 x_1 x_6 + \varepsilon a_3 x_1^2 x_3 \sin(x_5 - x_7) + \\ \qquad \varepsilon a_1 x_3 \sin(x_5 - x_7) + \varepsilon a_2 x_4 \sin(x_5 - x_7) - \varepsilon a_2 x_3 x_8 \cos(x_5 - x_7)], \\ x_7' = x_8, \\ x_8' = \dfrac{1}{x_3}[-2x_4 - \varepsilon b_5 x_1^3 \sin(x_5 - x_7) - \varepsilon b_2 x_3 x_8 - \varepsilon b_1 x_1 \sin(x_5 - x_7) - \\ \qquad \varepsilon b_4 x_2 \sin(x_5 - x_7) - \varepsilon b_2 x_3 - \varepsilon b_4 x_1 \cos(x_5 - x_7) - \\ \qquad \varepsilon b_4 x_1 x_6 \cos(x_5 - x_7) - 2x_4 x_8 - \varepsilon b_3 x_4]. \end{cases} \tag{17}$$

显然,式(17)的平衡点为 $(X_{10}, 0, X_{30}, 0, X_{50}, 0, X_{70}, 0)$,其中 $X_{10}, X_{30}, X_{50}, X_{70}$ 是系统的稳态响应的振幅. 对式(17)做线性变换,令

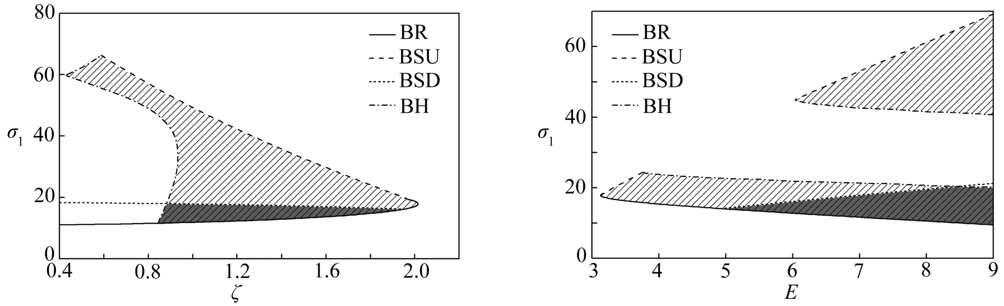
$$x_1 = X_1 + X_{10},\ x_2 = X_2,\ x_3 = X_3 + X_{30},\ x_4 = X_4,$$
$$x_5 = X_5 + X_{50},\ x_6 = X_6,\ x_7 = X_7 + X_{70},\ x_8 = X_8,$$

有

$$X_1' = X_2,\ X_2' = f_1,\ X_3' = X_4,\ X_4' = f_2,\ X_5' = X_6,\ X_6' = f_3,\ X_7' = X_8,\ X_8' = f_4, \tag{18}$$

其中，$f_1$，$f_2$，$f_3$，$f_4$ 的表达式见附录. 这样周期解的稳定性问题转化为平衡点的稳定性问题，容易求得式(18)的 Jacobi 矩阵的特征方程为

$$\lambda^8 + \delta_1\lambda^7 + \delta_2\lambda^6 + \delta_3\lambda^5 + \delta_4\lambda^4 + \delta_5\lambda^3 + \delta_6\lambda^2 + \delta_7\lambda + \delta_8 = 0. \tag{19}$$




(a) $\zeta$-$\sigma_1$　　(b) $E$-$\sigma_1$

**图 10**　碰摩周期解的稳定域和分岔边界 ($E = 7$, $\zeta = 0.9$)

Fig. 10　The stability regions of the steady-state rubbing motion and the bifurcation boundary ($E = 7$, $\zeta = 0.9$)

**表 3**　参数对系统运动演化规律的影响

Table 3　The influence of system parameters on the evolution of system's motion

| working condition | range of parameters | evolution of motion |
|---|---|---|
| run-up ($E = 7$) | $\zeta \geqslant 2.01$ | no rubbing |
| | $1.97 \leqslant \zeta < 2.01$ | no rubbing→SFARM→no rubbing |
| | $0.93 \leqslant \zeta < 1.97$ | no rubbing→SFARM→jump→no rubbing |
| | $0.84 \leqslant \zeta < 0.93$ | no rubbing→SFARM→QPRM→SFARM→jump→no rubbing |
| | $0.58 \leqslant \zeta < 0.84$ | no rubbing→QPRM→SFARM→jump→no rubbing |
| | $0.43 \leqslant \zeta < 0.58$ | no rubbing→QPRM→SFARM→QPRM |
| | $\zeta < 0.43$ | no rubbing→QPRM |
| run-down ($E = 7$) | $\zeta \geqslant 1.95$ | no rubbing |
| | $0.88 \leqslant \zeta < 1.95$ | no rubbing→jump→SFARM→no rubbing |
| | $0.84 \leqslant \zeta < 0.88$ | no rubbing→jump→QPRM→SFARM→no rubbing |
| | $\zeta < 0.84$ | no rubbing→jump→QPRM→no rubbing |
| run-up ($\zeta = 0.9$) | $E \leqslant 3.12$ | no rubbing |
| | $3.12 < E \leqslant 3.18$ | no rubbing→SFARM→no rubbing |
| | $3.18 < E \leqslant 3.76$ | no rubbing→SFARM→jump→no rubbing |
| | $3.76 < E \leqslant 6.03$ | no rubbing→SFARM→QPRM |
| | $E > 6.03$ | no rubbing→SFARM→QPRM→SFARM→jump→no rubbing |
| run-down ($\zeta = 0.9$) | $E \leqslant 4.93$ | no rubbing |
| | $4.93 < E \leqslant 8.46$ | no rubbing→jump→SFARM→no rubbing |
| | $E > 8.46$ | no rubbing→QPRM→SFARM→no rubbing |

根据 Liapunov 稳定性理论，当且仅当式(19)的解的实部均小于 0 时周期解是稳定的. 图 10 分别给出了升速和降速时 $E$-$\sigma_1$ 平面和 $\zeta$-$\sigma_1$ 平面上碰摩周期解的稳定边界和稳定域. 其中 BR 表示碰摩的发生的边界，BH 表示 Hopf 分岔边界，BSU 表示升速时静态分岔边界，BSD 表示升速时静态分岔边界，斜线填充区域表示升速时碰摩周期运动的稳定范围，阴影填充区域表示降速时碰摩周期运动的稳定范围. 升速时，通过 Hopf 分岔，系统的运动形式由周期运动变为概周期运动；通过静态分岔，系统的振幅发生跳跃，脱离碰摩. 降速时，通过 Hopf 分岔，系统的运动形

式由概周期运动回到周期运动形式，通过静态分岔，系统的振幅跳跃后突然增大，碰摩周期运动出现. 表 3 给出了图 10 所示不平衡和阻尼对系统运动演化规律的影响，可以看到对应不同的参数，系统动力学行为随转速的变化呈现出的不同的演化方式，其中 SFARM 表示同步全周碰摩运动，QPRM 表示概周期局部碰摩运动.

# 4 结　论

本文基于航空发动机转子系统的特点，将航空发动机转子系统简化为一个弹性支承的刚性转子系统. 应用 Lagrange 方程建立了弹支刚性转子系统碰摩的动力学方程，通过平均法和 Liapunov 稳定性理论对系统的运动方程进行研究，取得了以下新的成果，为转子系统的非线性设计和故障诊断提供了一定的理论参考.

将两状态变量的约束分岔理论首次应用到转子系统碰摩的分岔研究中，分别给出了转子无碰摩和碰摩阶段的转迁集和分岔图，并将两者相结合给出了转子系统碰摩参数域内完整的转迁集和分岔形式，讨论了转子不平衡和支承阻尼对系统碰摩和跳跃现象的影响.

给出了系统参数（不平衡和阻尼）——转速平面的静态分岔和 Hopf 分岔边界曲线和碰摩周期解的稳定区域，讨论了参数对系统动力学行为随转速的变化的演化方式的影响.

## 附　录

$$\begin{aligned} f_1 = &-2\varepsilon a_2 X_2 - 2\varepsilon a_3 (X_1 + X_{10})^3 - \varepsilon g(X_1 + X_{10}) + \varepsilon E\cos(X_5 + X_{50}) + \\ &2(X_1 + X_{10})X_6 - 3\varepsilon a_3 (X_1 + X_{10})^2 (X_3 + X_{30})\cos(X_5 + X_{50} - X_7 - X_{70}) - \\ &\varepsilon a_2 X_4 \cos(X_5 + X_{50} - X_7 - X_{70}) - \varepsilon a_2 (X_3 + X_{30}) X_8 \sin(X_5 + X_{50} - X_7 - X_{70}) + \varepsilon g + \\ &\varepsilon(X_1 + X_{10})\sigma_1 + (X_1 + X_{10}) X_6^2 - \varepsilon a_1 (X_3 + X_{30})\cos(X_5 + X_{50} - X_7 - X_{70}) - \\ &\varepsilon a_2 (X_3 + X_{30})\sin(X_5 + X_{50} - X_7 - X_{70}), \end{aligned} \tag{A1}$$

$$\begin{aligned} f_2 = &\varepsilon(X_3 + X_{30})\sigma_1 - \varepsilon(X_3 + X_{30})\sigma_2 + \varepsilon b_3 (X_3 + X_{30}) + \\ &2(X_3 + X_{30})X_8 + (X_3 + X_{30})X_8^2 - \varepsilon b_5 (X_1 + X_{10})^3 \cos(X_5 + X_{50} - X_7 - X_{70}) + \\ &\varepsilon b_4 (X_1 + X_{10}) X_6 \sin(X_5 + X_{50} - X_7 - X_{70}) - \varepsilon b_2 X_4 + \varepsilon b_3 (X_3 + X_{30}) X_8 - \\ &\varepsilon b_1 (X_1 + X_{10})\cos(X_5 + X_{50} - X_7 - X_{70}) - \\ &\varepsilon b_4 X_2 \cos(X_5 + X_{50} - X_7 - X_{70}) + \varepsilon b_4 (X_1 + X_{10})\sin(X_5 + X_{50} - X_7 - X_{70}), \end{aligned} \tag{A2}$$

$$\begin{aligned} f_3 = &\frac{1}{X_1 + X_{10}}[-2X_2 + \varepsilon a_3 (X_1 + X_{10})^2 (X_3 + X_{30})\sin(X_5 + X_{50} - X_7 - X_{70}) - \\ &2X_2 X_6 - \varepsilon a_2 (X_3 + X_{30}) X_8 \cos(X_5 + X_{50} - X_7 - X_{70}) - \\ &2\varepsilon a_2 (X_1 + X_{10}) X_6 - \varepsilon g\mu(X_1 + X_{10}) + \varepsilon a_1 (X_3 + X_{30})\sin(X_5 + X_{50} - X_7 - X_{70}) - \\ &\varepsilon E\sin(X_5 + X_{50}) + \varepsilon g\mu - 2\varepsilon a_2 (X_1 + X_{10}) + \\ &\varepsilon a_2 X_4 \sin(X_5 + X_{50} - X_7 - X_{70}) - \varepsilon a_2 (X_3 + X_{30})\cos(X_5 + X_{50} - X_7 - X_{70})], \end{aligned} \tag{A3}$$

$$\begin{aligned} f_4 = &\frac{1}{X_3 + X_{30}}[-2X_4 - \varepsilon b_5 (X_1 + X_{10})^3 \sin(X_5 + X_{50} - X_7 - X_{70}) - \\ &\varepsilon b_2 (X_3 + X_{30}) X_8 - \varepsilon b_1 (X_1 + X_{10})\sin(X_5 + X_{50} - X_7 - X_{70}) - \\ &\varepsilon b_4 X_2 \sin(X_5 + X_{50} - X_7 - X_{70}) - 2X_4 X_8 - \varepsilon b_4 (X_1 + X_{10})\cos(X_5 + X_{50} - X_7 - X_{70}) - \\ &\varepsilon b_4 (X_1 + X_{10}) X_6 \cos(X_5 + X_{50} - X_7 - X_{70}) - \varepsilon b_2 (X_3 + X_{30}) - \varepsilon b_3 X_4]. \end{aligned} \tag{A4}$$

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# Bifurcation on the Synchronous Full Annular Rub of a Rigid-Rotor Elastic-Support System

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**Abstract**: Aero engine rotor system was simplified to be an unsymmetrical-rigid-rotor with nonlinear-elastic-support based on its characteristics. Governing equations of the rubbing system obtained from Lagrange equation were solved by averaging method to find the bifurcation equations. Then, according to the two-dimensional constraint bifurcation theory, transition sets and bifurcation diagrams of the system with and without rubbing were given to study the influence of system's eccentric and damping on the bifurcation behaviors, respectively. Finally, according to Liapunov stability theory, the stability region of steady-state rubbing solution and the boundary of static bifurcation and Hopf bifurcation were determined to discuss the influence of system parameters on the evolution of system's motion. The research results may provide some references for the design of aero rotor systems.

**Key words**: unsymmetrical-rigid-rotor elastic-support system; rubbing; two dimensional constraint bifurcation theory; stability