

# 磁场对液体-微伸缩固体介面波传播的影响\*

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**摘要:** 在一个传播理想的非粘性液体半空间,和一个传播理想的微伸缩弹性固体半空间之间,研究介面处纵波的反射和透射.在两个半空间中,满足介面处必需的边界条件下,得到控制方程的适当解,是一组以不同反射和透射波振幅比表示的5个非齐次方程.以水和铝-环氧树脂合成材料介面为实际例子,用 Gauss 消除法的 Fortran 程序求解方程组.考虑存在和不存在外加横向磁场两种情况,在某些入射角范围内,计算振幅比的数值解.最后用图形给出横向磁场对不同反射和透射波振幅比的影响.

**关键词:** 微伸缩; 波的传播; 反射和透射; 磁场

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## 引言

微伸缩连续介质是微形变连续介质的一个特例,除转动外,仅允许在正交方向发生等量的膨胀或收缩.与经典的连续介质微粒相比,微伸缩的连续介质微粒有4个额外的自由度. Eringen<sup>[1-3]</sup>提出了微伸缩弹性体的连续介质理论. Pabst<sup>[4]</sup>从材料科学家和工程师的角度,总结并评论了 Eringen 的微极连续介质理论,引入了固体和液体的运动测度(微极应变和应变率张量),提出了含有微惯性张量的、非经典平衡定律的平衡方程. 在一些文献实例的帮助下,他所展示的微极理论,可以为固体合成材料、多孔介质和悬浮液的建模,提供有用的理论框架.

波的传播现象广泛地用来确定材料的性质,特别用来确定1阶和2阶的弹性常数. Parfitt 和 Eringen<sup>[5]</sup>发现,在一个线性微极弹性固体中,存在4种基本波:纵向的位移波、纵向的微转动波、2组耦合的微转动和横向的位移波. Singh<sup>[6]</sup>在一个线性微伸缩弹性固体中,研究了平面波的传播,发现存在2组和纵向微伸缩波相耦合的纵向位移波. Tomar 和 Garg<sup>[7]</sup>就两个微伸缩固体半空间之间的介面,研究波的反射和透射. Singh 和 Tomar<sup>[8]</sup>在一个覆盖着液体层的微伸缩弹性板中,研究了 Rayleigh-Lamb 波的传播.

众所周知,地壳中存在横向磁场. 组合磁场的研究,在电磁波现象中有着重要的应用价值. Singh<sup>[9]</sup>在一个有磁场存在的液-固介面上,研究了微伸缩弹性波的反射和透射. 本文就传播理想的非粘性液体,和传播理想的微伸缩弹性固体半空间之间的介面,进行纵向波反射和透射的研究.

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# 1 控制方程和求解

根据 Eringen<sup>[2]</sup> 和 Kaliski<sup>[10]</sup>, 无体力和体力矩作用时, 传导性能理想的、线性各向同性微伸缩弹性固体的本构方程和场方程为

$$t_{kl} = \lambda u_{r,r} \delta_{kl} + \mu(u_{k,l} + u_{l,k}) + \kappa(u_{l,k} - \epsilon_{klr} \phi_r) + \lambda_0 \psi \delta_{kl}, \quad (1)$$

$$m_{kl} = \alpha \phi_{r,r} \delta_{kl} + \beta \phi_{k,l} + \gamma \phi_{l,k}, \quad (2)$$

$$\beta_k = \alpha_0 \psi_k, \quad (3)$$

$$\tau_{kl} = \frac{\mu_0}{4\pi} [H_k h_l + H_l h_k - (\mathbf{h} \cdot \mathbf{H}) \delta_{kl}], \quad (4)$$

$$(c_1^2 + c_3^2) \nabla(\nabla \cdot \mathbf{u}) - (c_2^2 + c_3^2) \nabla \times (\nabla \times \mathbf{u}) + c_3^2 (\nabla \times \boldsymbol{\phi}) + \bar{\lambda}_0 \nabla \psi + \frac{\mu_0}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{H} = \ddot{\mathbf{u}}, \quad (5)$$

$$(c_4^2 + c_5^2) \nabla(\nabla \cdot \boldsymbol{\phi}) - c_4^2 \nabla \times (\nabla \times \boldsymbol{\phi}) + \omega_0^2 (\nabla \times \mathbf{u}) - 2\omega_0^2 \boldsymbol{\phi} = \ddot{\boldsymbol{\phi}}, \quad (6)$$

$$c_6^2 \nabla^2 \psi - c_7^2 \psi - c_8^2 (\nabla \cdot \mathbf{u}) = \ddot{\psi}, \quad (7)$$

其中

$$\begin{cases} c_1^2 = \frac{\lambda + 2\mu}{\rho}, c_2^2 = \frac{\mu}{\rho}, c_3^2 = \frac{\kappa}{\rho}, c_4^2 = \frac{\gamma}{\rho j}, c_5^2 = \frac{\alpha + \beta}{\rho j}, \\ \omega_0^2 = \frac{c_3^2}{j} = \frac{\kappa}{\rho j}, c_6^2 = \frac{2\alpha_0}{\rho j}, c_7^2 = \frac{2\lambda_1}{3\rho j}, c_8^2 = \frac{2\lambda_0}{3\rho j}, \\ \bar{\lambda}_0 = \frac{\lambda_0}{\rho}, \mathbf{h} = \nabla \times (\mathbf{u} \times \mathbf{H}), \mathbf{E} = -\frac{\mu_0}{c_0} (\dot{\mathbf{u}} \times \mathbf{H}), \end{cases} \quad (8)$$

这里,  $\lambda, \mu, \kappa, \gamma, \lambda_0, \lambda_1, \alpha, \beta, \alpha_0$  为材料常数,  $\rho$  为介质密度,  $j$  为微转动惯量,  $\mu_0$  为导磁率,  $c_0$  为光速,  $\mathbf{u}, \boldsymbol{\phi}, \mathbf{E}, \mathbf{h}, \mathbf{H}$  为矢量,  $\psi$  为标量. 方程右边字母上面的点表示对时间的导数. 下文中假设  $\mu_0 = 1$ , 物理上是合理的.

运用 Helmholtz 的矢量表示法, 位移和微转动矢量可表示为

$$\mathbf{u} = \nabla \zeta + \nabla \times \mathbf{U}, \quad \nabla \cdot \mathbf{U} = 0, \quad (9)$$

$$\boldsymbol{\phi} = \nabla \xi + \nabla \times \boldsymbol{\Phi}, \quad \nabla \cdot \boldsymbol{\Phi} = 0. \quad (10)$$

根据式(9)和(10), 方程(5)~(7)可简化为

$$(c_1^2 + c_3^2 + v_0^2) \nabla^2 \zeta + \bar{\lambda}_0 \psi = \ddot{\zeta}, \quad (11)$$

$$c_6^2 \nabla^2 \psi - c_7^2 \psi - c_8^2 \nabla^2 \zeta = \ddot{\psi}, \quad (12)$$

$$(c_2^2 + c_3^2) \nabla^2 \mathbf{U} + c_3^2 (\nabla \times \boldsymbol{\Phi}) = \ddot{\mathbf{U}}, \quad (13)$$

$$c_4^2 \nabla^2 \boldsymbol{\Phi} + \omega_0^2 (\nabla \times \mathbf{U}) - 2\omega_0^2 \boldsymbol{\Phi} = \ddot{\boldsymbol{\Phi}}, \quad (14)$$

$$(c_4^2 + c_5^2) \nabla^2 \xi - 2\omega_0^2 \xi = \ddot{\xi}, \quad (15)$$

这里

$$v_0^2 = \frac{H^2}{4\pi\rho}, \quad H = (\mathbf{H})_y, \quad (16)$$

其中  $v_0$  为 Alfvén 速度. 方程(11)和(12)关系标量  $\zeta$  和  $\psi$  耦合, 方程(13)和(14)关系矢量  $\mathbf{U}$  和  $\boldsymbol{\Phi}$  耦合, 方程(15)是独立的. 方程(13)~(15)与 Parfitt 和 Eringen<sup>[5]</sup> 得到的结果一样, 根据文献[5], 方程(13)和(14)的解对应于两组微转动的横波, 记为 CD I 波和 CD II 波, 它们是耦合的横向位移波, 而方程(15)的解对应于纵向的微转动波(记为 LM 波). 根据 Singh 的文献[6],

方程(11)和(12)的解对应于两组耦合波,称为纵向位移波(LD波)和纵向微伸缩波(LMS波),以上各波均受到横向磁场的影响.在本研究中,方程(11)~(14)的解是必需的,而方程(15)的解是一个特殊波.

根据 Singh 的文献[6],方程(11)~(14)的通解可以表示为

$$\zeta = [B_1 e^{m_1 z} + B_2 e^{-m_1 z} + B_3 e^{m_2 z} + B_4 e^{-m_2 z}] e^{i(\omega t - kx)}, \tag{17}$$

$$\psi = [a_1 B_1 e^{m_1 z} + a_1 B_2 e^{-m_1 z} + a_2 B_3 e^{m_2 z} + a_2 B_4 e^{-m_2 z}] e^{i(\omega t - kx)}, \tag{18}$$

$$\eta = [B_5 e^{m_3 z} + B_6 e^{-m_3 z} + B_7 e^{m_4 z} + B_8 e^{-m_4 z}] e^{i(\omega t - kx)}, \tag{19}$$

$$\phi_2 = [b_1 B_5 e^{m_3 z} + b_1 B_6 e^{-m_3 z} + b_2 B_7 e^{m_4 z} + b_2 B_8 e^{-m_4 z}] e^{i(\omega t - kx)}, \tag{20}$$

其中

$$\left\{ \begin{aligned} m_1 &= \sqrt{k^2 - \frac{1}{2}[\sqrt{A^2 - 4B} + A]}, m_2 = \sqrt{k^2 + \frac{1}{2}[\sqrt{A^2 - 4B} - A]}, \\ m_3 &= \sqrt{\frac{1}{2}[\sqrt{C^2 - 4D} - C]}, m_4 = \sqrt{-\frac{1}{2}[\sqrt{C^2 - 4D} + C]}, \\ A &= \frac{\omega^2}{V_1^2} + \frac{\omega^2 - c_7^2 - a_0 c_8^2}{c_6^2}, B = \left(\frac{\omega^2}{V_1^2}\right) \left(\frac{\omega^2 - c_7^2}{c_6^2}\right), \\ C &= k^2 \left(\frac{c^2}{c_2^2 + c_3^2} + \frac{c^2}{c_4^2} - 2\right) + \frac{\omega_0^2}{c_4^2} \left(\frac{c_3^2}{c_2^2 + c_3^2} - 2\right), \\ D &= k^4 \left[1 - \frac{c^2}{c_2^2 + c_3^2} - \frac{c^2}{c_4^2} + \frac{c^4}{c_4^2(c_2^2 + c_3^2)}\right] - \\ &\quad k^2 \left[\frac{\omega_0^2}{c_4^2} \left(2 \frac{c^2}{c_2^2 + c_3^2} + \frac{c_3^2}{c_2^2 + c_3^2} - 2\right)\right], \\ a_{1,2} &= \frac{1}{a_0} \left(k^2 - m_{1,2}^2 - \frac{\omega^2}{V_1^2}\right), b_{1,2} = (k^2 - m_{3,4}^2) \left(1 + \frac{c_2^2}{c_3^2}\right) - \frac{\omega^2}{c_3^2}, \\ V_1^2 &= c_1^2 + c_3^2 + v_0^2, a_0 = -\frac{\bar{\lambda}_0}{V_1^2}, \eta = U_y, \phi_2 = \phi_y. \end{aligned} \right. \tag{21}$$

根据 Kaliski 的文献[10],对传导性能理想的液体来说,方程线性组合为

$$\alpha_1^2 \nabla^2 \mathbf{u}^* + \frac{\mu_0^*}{4\pi\rho^*} (\nabla \times \mathbf{h}^*) \times \mathbf{H} = \ddot{\mathbf{u}}^*, \tag{22}$$

其中  $\alpha_1^2 = \lambda^* / \rho^*$  为液体中声速的平方,“\*”表示液体.可压缩非粘性液体的位移矢量为

$$\mathbf{u}^* = \nabla \zeta^*. \tag{23}$$

根据式(23),方程(22)变为

$$\nabla^2 \zeta^* = \frac{1}{\alpha_2^2} \ddot{\zeta}^*, \tag{24}$$

其中

$$\alpha_2^2 = \alpha_1^2 + v_0^{*2}, \tag{25}$$

$v_0$  为液体中的 Alfvén 速度,方程(24)的通解为

$$\zeta^* = [A_0 e^{m_1^* z} + A_1 e^{-m_1^* z}] e^{i(\omega t - kx)}, \tag{26}$$

其中

$$m_1^{*2} = 1 - \frac{c^2}{\alpha_2^2}. \quad (27)$$

取  $\mu = \kappa = \gamma = \lambda_0 = \lambda_1 = \alpha_0 = 0$ , 得到液体介质中力学和电磁学应力张量,并在方程(1)~(4)中,将余下的量标上“\*”。

为了减少问题的复杂性,假设本问题为一平面应变状态:

$$\begin{cases} u_1 = u_1(x, z, t), u_2 = 0, u_3 = u_3(x, z, t), \\ u_1^* = u_1^*(x, z, t), u_2^* = 0, u_3^* = u_3^*(x, z, t), \\ \phi_1 = 0, \phi_2 = \phi_2(x, z, t), \phi_3 = 0, \end{cases} \quad (28)$$

其中

$$u_1 = \zeta_{,x} - \eta_{,z}, u_3 = \zeta_{,z} + \eta_{,x}, u_1^* = \zeta_{,x}^*, u_3^* = \zeta_{,z}^*. \quad (29)$$

## 2 反射和透射

设  $z = 0$  为两个半空间之间的平面边界,  $z > 0$  的区域为传导性能理想的可压缩非粘性液体,  $z < 0$  为各向同性微伸缩弹性导体,不考虑重力场对液体的影响.  $x$  轴与表面波的传播方向相一致,取入射波的传播平面为  $x$ - $z$  平面. 由于入射介质为可压缩的非粘性液体,它无法忍受横波,因此入射波选择纵波,基于同样的理由反射波也选择纵波. 透射介质则存在 4 种波: 2 组耦合的 LD 波和 LMS 波, 2 组耦合的 CD I 波和 CD II 波. 对入射纵波来说,  $c = \alpha_2 \csc \theta$ , 其中  $\theta$  为入射角.

在二维运动情况下,当初始磁场矢量方向与接触面平行时,介面  $z = 0$  处的边界条件与 Kaliski 和 Nowacki<sup>[11]</sup> 的一致: 法向应力和法向位移连续,切向应力、切向力矩以及跨越介面  $z = 0$  的一阶矩矢量为 0, 即

$$t_{33} + \tau_{33} = t_{33}^* + \tau_{33}^*, E_1 = E_1^* \text{ (即 } u_3 = u_3^*), t_{31} + \tau_{31} = 0, m_{32} = 0, \beta_3 = 0. \quad (30)$$

借助于方程(1)~(4)、(28)和(29),满足边界条件(30)的解(17)~(20)和(26),为如下 5 个非齐次方程:

$$\sum d_{ij} Z_j = e_i \quad (i, j = 1, 2, \dots, 5), \quad (31)$$

其中

$$\begin{cases} d_{11} = (\lambda^* + \rho^* v_0^{*2})(m_1^{*2} + k^2), \\ d_{12} = -(\lambda + \rho v_0^2)(m_1^2 + k^2) - (2\mu + \kappa)m_1^2 - \lambda_0 a_1, \\ d_{13} = -(\lambda + \rho v_0^2)(m_2^2 + k^2) - (2\mu + \kappa)m_2^2 - \lambda_0 a_2, d_{14} = i(2\mu + \kappa)km_3, \\ d_{15} = i(2\mu + \kappa)km_4, d_{21} = m_1^*, d_{22} = m_1, d_{23} = m_2, d_{24} = d_{25} = -ik, \\ d_{31} = 0, d_{32} = i(2\mu + \kappa)km_1, d_{33} = i(2\mu + \kappa)km_2, \\ d_{34} = \mu(k^2 + m_3^2) + \kappa(m_3^2 + b_1), d_{35} = \mu(k^2 + m_4^2) + \kappa(m_4^2 + b_2), \\ d_{41} = d_{42} = d_{43} = 0, d_{44} = -m_3 b_1, d_{45} = -m_4 b_2, \\ d_{51} = d_{54} = d_{55} = 0, d_{52} = -m_1 a_1, d_{53} = -m_2 a_2, \\ e_1 = -d_{11}, e_2 = d_{21}, e_3 = -d_{31}, e_4 = d_{41}, e_5 = -d_{51}, \end{cases} \quad (32)$$

以及反射的纵向波振幅比、折射的 LD 波、LMS 波、CD I 波、CD II 波的振幅比分别为

$$Z_1 = \frac{A_1}{A_0}, Z_2 = \frac{B_1}{A_0}, Z_3 = \frac{B_3}{A_0}, Z_4 = \frac{B_5}{A_0}, Z_5 = \frac{B_7}{A_0}. \quad (33)$$

### 3 数值结果和讨论

根据 Gauthier 的文献[12], 传导性能理想的微伸缩弹性固体相关的物理常量为(注  $1 \text{ dyn} = 10^{-5} \text{ N}$ )

$$\begin{aligned} \lambda &= 7.59 \times 10^{11} \text{ dyn} \cdot \text{cm}^{-2}, \mu = 1.89 \times 10^{11} \text{ dyn} \cdot \text{cm}^{-2}, \\ \kappa &= 0.0149 \times 10^{11} \text{ dyn} \cdot \text{cm}^{-2}, \gamma = 0.0268 \times 10^{11} \text{ dyn}, \lambda_0 = 0.062 \times 10^{11} \text{ dyn}, \\ \lambda_1 &= 0.063 \times 10^{11} \text{ dyn}, \alpha_0 = 0.95 \times 10^{11} \text{ dyn}, \rho = 2.19 \text{ g} \cdot \text{cm}^{-3}, j = 0.0196 \text{ cm}^2, \\ \omega^2/\omega_0^2 &= 10. \end{aligned}$$

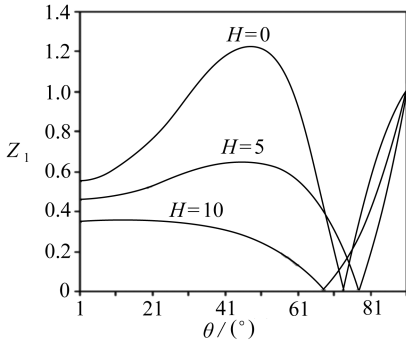


图 1 反射纵波振幅比  $Z_1$  随着入射角  $\theta$  的变化

Fig.1 Variations of the amplitude ratios of reflected longitudinal wave against the angle of incidence

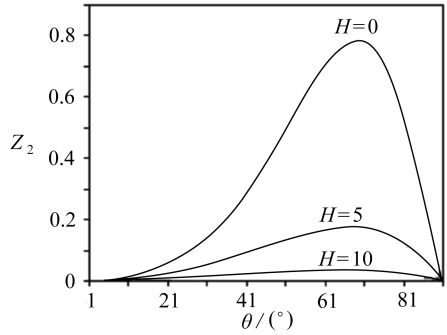


图 2 透射 LD 波振幅比  $Z_2$  随着入射角  $\theta$  的变化

Fig.2 Variations of the amplitude ratios of reflected LD wave against the angle of incidence

作为液体半空间的水, 相关的参数为

$$\rho^* = 1 \text{ g} \cdot \text{cm}^{-3}, \lambda^* = 2.04 \times 10^{11} \text{ dyn} \cdot \text{cm}^{-2}.$$

利用 Gauss 消去法的 Fortran 程序, 当  $H = 0, H = 5 \times 10^5 \text{ Oe}$  和  $H = 10 \times 10^5 \text{ Oe}$  (注  $1 \text{ Oe} = 1000/(4\pi) \text{ A/m}$ ), 入射角满足  $0 < \theta < 90^\circ$  时, 由解方程组(31), 计算不同反射波和透射波的振幅比. 图 1 ~ 5 给出了不同反射波和透射波的振幅比, 随纵波入射角  $\theta$  的变化.

图 1 给出了反射纵波的振幅比. 当  $H = 0$  时, 反射纵波的振幅比先增加, 在  $\theta = 47^\circ$  时达到最大值 1.233, 然后急剧下降, 在  $\theta = 73^\circ$  时达到最小值  $0.4672 \times 10^{-2}$ , 接着在  $\theta = 90^\circ$  附近时的值为 1. 随着磁场参数  $H$  值的增加, 每个入射角的反射纵波的振幅比在减少,  $70^\circ < \theta < 90^\circ$  除外. 当  $H = 0, H = 5 \times 10^5 \text{ Oe}$  和  $H = 10 \times 10^5 \text{ Oe}$  时, 反射纵波振幅比达到不同的最大值和最小值.

图 2 给出了透射 LD 波的振幅比. 当  $H = 0$  时, 透射耦合 LD 波的振幅比, 在  $\theta = 0^\circ$  附近时的值为 0, 在  $\theta = 69^\circ$  附近时达到最大值, 然后在  $\theta = 90^\circ$  附近减少到 0.  $H = 5 \times 10^5 \text{ Oe}$  和  $H = 10 \times 10^5 \text{ Oe}$  时, 透射 LD 波的振幅比和  $H = 0$  时相似, 但最大值不一样. 随着外加磁场的加强, 每个入射角对应的振幅比在减少.

图 3 给出了  $H = 0, H = 5 \times 10^5 \text{ Oe}$  和  $H = 10 \times 10^5 \text{ Oe}$  时, 透射 LMS 波振幅比随着入射角

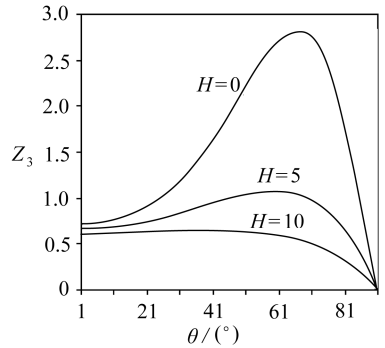


图 3 透射 LMS 波振幅比  $Z_3$  随着入射角  $\theta$  的变化

Fig.3 Variations of the amplitude ratios of reflected LMS wave against the angle of incidence

的变化.当  $H = 0$  时,透射 LMS 波的振幅比首先急速增加,在  $\theta = 67^\circ$  时达到最大值 2.809,然后在  $\theta = 90^\circ$  附近取得最小值 0.当  $H = 5 \times 10^5$  Oe 和  $H = 10 \times 10^5$  Oe 时,透射 LMS 波的振幅比和  $H = 0$  时相似,但是最大值不一样.随着磁场的加强,每个入射角所对应的透射 LMS 波振幅比在减少.

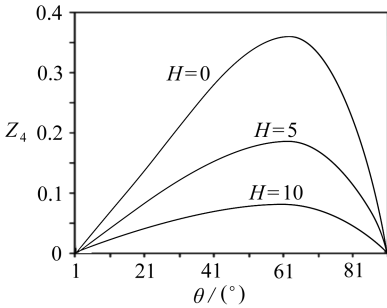


图4 透射 CD I 波振幅比  $Z_4$  随着入射角  $\theta$  的变化

Fig.4 Variations of the amplitude ratios of reflected CD I wave against the angle of incidence

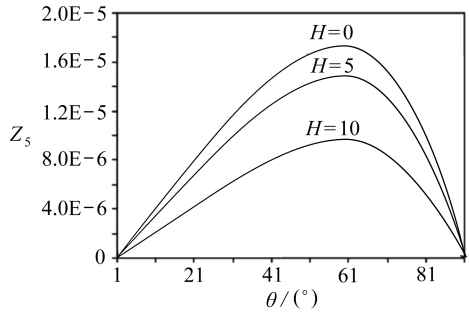


图5 透射 CD II 波振幅比  $Z_5$  随着入射角  $\theta$  的变化

Fig.5 Variations of the amplitude ratios of reflected CD II wave against the angle of incidence

图4和图5分别给出了透射 CD I 波和 CD II 波的振幅比.当  $H = 0$  时, CD I 波和 CD II 波的振幅比,彼此相似,并分别在  $\theta = 60^\circ$  和  $\theta = 57^\circ$  处达到最大值.当  $H = 5 \times 10^5$  Oe 和  $H = 10 \times 10^5$  Oe 时,透射 CD I 波和 CD II 波的振幅比和  $H = 0$  时相似,但最大值各不相同.随着外加磁场的加强,每个入射角所对应的这些透射波的振幅比都在减少.

## 4 结 论

从数值结果可归结如下:反射波和透射波的振幅比,依赖于入射角和外加磁场.外加磁场加强时,反射波和透射波的振幅比在减少.因此,波在液体和固体半空间之间介面处传播的研究中,外加磁场被认为是一个重要的因素.

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## Influence of Magnetic Field on Wave Propagation at Liquid-Microstretch Solid Interface

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**Abstract:** The reflection and refraction of longitudinal wave at an interface between perfectly conducting non-viscous liquid half-space and a perfectly conducting microstretch elastic solid half-space was studied. The appropriate solutions of the governing equations were obtained in both the half-spaces which satisfy the required boundary conditions at the interface to obtain a system of five non-homogeneous equations in the amplitude ratios of various reflected and transmitted waves. The system of equations was solved by Fortran program of Gauss elimination method for a particular example of an interface between water and aluminum-epoxy composite. The numerical values of amplitude ratios were computed for a certain range of the angle of incidence both in presence and absence of impressed transverse magnetic field. The effects of the presence of transverse magnetic field on the amplitude ratios of various reflected and transmitted waves were shown graphically.

**Key words:** microstretch; wave propagation; reflection and transmission; magnetic field