

约束层阻尼圆柱壳的自由振动^{*}

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摘要: 给出了被动约束层阻尼圆柱壳(PCLD)的自由振动特性.波传播法被用来求解两端简支的PCLD圆柱壳的振动,而不是用有限元法、传递矩阵法和Rayleigh-Ritz法.基于Sanders薄壳理论,导出了PCLD正交各向异性圆柱壳的控制方程.数值结果表明当前的方法要比目前其它方法有效.讨论了粘弹性层和约束层的厚度,正交各向异性约束层的弹性模量比率和粘弹性层的复剪切模量对频率参数和损失因子的影响.

关键词: 夹心壳; 圆柱壳; 粘弹性材料; 约束层阻尼; 自由振动

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引 言

约束层阻尼(constrained layer damping, CLD)是一种重要的结构阻尼技术,有着高效的阻尼效率,并且提供一种有效的途径来降低振动和声辐射.在航天器舱段、汽车、火箭和潜艇中,CLD有着广泛的应用.被动约束层阻尼(passive constrained layer damping, PCLD)或者主动约束层阻尼(active constrained layer damping, ACLD)的核心层是由粘弹性材料组成的. PCLD与ACLD的差别就在于约束层(constrained layer, CL).如果CL由智能材料组成,例如压电材料、形状记忆合金,它们能够被主动控制,这种CLD属于ACLD.否则,CLD就属于PCLD. CL迫使粘弹性层承受横向剪切变形,通过这种变形使能量耗散^[1].

Mead和Markus^[2]研究了一个带粘弹性层3层夹心梁的强迫响应.在提出的模型中,两个表面层经历相同的横向变形,粘弹性层仅仅通过剪切力同两个表面层相互作用,并且在变形之后,粘弹性层并不改变厚度.在研究带粘弹性层的板壳振动时,这种理论是非常重要的. Rao^[3]分析了不同边界条件下夹心梁的频率和损耗因子. Zheng等^[4]用基于能量的方法和假设模态法研究了带多个PCLD块的梁的自由振动特性.高坚新等^[5]导出了带ACLD的非对称复合板的控制方程,研究了频率和损耗因子.基于能量的方法,Chen和Huang^[6]研究了部分带ACLD的板的自由振动.利用有限元方法, Liu等^[7]研究了带ACLD,旋转的悬臂板的动态特性. Ramesh和Ganesan^[8-9]通过有限元方法研究了带PCLD各向同性圆柱壳的固有频率和损耗因子,分析了剪切参数和粘弹性层与约束层的厚度比率对圆柱壳固有频率和损耗因子的影响.利用PCLD敷设, Oh等^[10]控制了流体载荷作用下的圆柱壳的振动与声辐射.利用结构和流体的有限元模型研究了加强筋、阻尼、流体载荷对圆柱壳的影响.一些研究人员^[11-14]通过有限元方

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法建立带 ACLD 圆柱壳的振动和声辐射模型. 研究人员指出敷设 ACLD 可能是一种不需要高电压, 实现轻质智能结构主动控制的可行方法^[13]. 传递矩阵被用来分析带 PCLD 圆柱壳的振动^[15-16]. 李恩奇等^[17]和向宇等^[18]利用传递矩阵分析部分覆盖 PCLD 圆柱壳的振动, 这种半解析法提供了另外一种研究部分敷设 PCLD 壳振动的方法. 利用 ACLD, Yuan 等^[19]发展了一种方法来控制圆柱壳的周向主要模态. 在模型中, ACLD 部分覆盖在主壳的轴向方向, 在圆周方向完全覆盖, 发展了一种传递矩阵法来分析一个覆盖 ACLD 圆柱壳的动力特性. 基于 Donnell-Mushtari-Vlasov 薄壳理论, Cheng 和 Huang^[20]利用 Rayleigh-Ritz 法研究了部分敷设 PCLD 圆柱壳的响应. Pan^[21]研究了一个有限长带粘弹性层夹心圆柱壳的轴对称振动. 夹心层的粘弹性属性通过复剪切模量表述, 分析了敷设 PCLD 圆柱壳的轴对称振动动力特性. Baz 和 Chen^[22]给出了一种控制全部敷设 ACLD 圆柱壳轴对称振动的解析模型. 王森和方之楚^[23]导出了部分敷设 ACLD 圆柱壳轴对称运动的方程, 研究了频率和损耗因子.

所有以前的研究是通过有限元法、传递矩阵和 Rayleigh-Ritz 法处理敷设 CLD 圆柱壳的振动. 在文献中, 解析法几乎只针对敷设 CLD 圆柱壳的轴对称振动, 从而, 运动方程非常简单. 在本文的研究中, 一个全解析模型提出来研究敷设 PCLD 圆柱壳的非轴对称运动的自由振动. 基于 Sanders 薄壳理论, 导出了 PCLD 圆柱壳的控制方程. 波传播法用来研究两端简支的 PCLD 圆柱壳的自由振动. 讨论了固有频率和损耗因子的特性.

1 PCLD 圆柱壳的控制方程

图 1 所示为一个 PCLD 圆柱壳. PCLD 由线性粘弹性材料 (viscoelastic material, VEM) 层和 CL 组成. 通常, VEM 具有耗散能量的复剪切模量. 在本文中, 符号 h 和 a 表示层合壳体的厚度和半径, 下标或者上标 s, v 和 c 在下面的推导中对应于主壳、粘弹性层和约束层. 圆柱壳的长度为 L . 利用了下面的假设: (a) 主壳和约束层壳满足 Kirchhoff 薄壳假设; (b) 3 层壳的界面的位移和转角满足连续性条件; (c) VEM 仅仅承受剪切应变; (d) 由于 VEM 的密度较低, 仅仅考虑 VEM 的径向惯量.

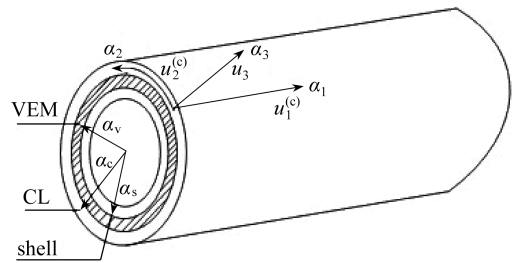


图 1 PCLD 圆柱壳的几何构型

Fig. 1 Geometric configuration of circular cylindrical shell with PCLD

1.1 PCLD 圆柱壳的运动方程

利用 Love 的假设, 壳的位移场能够写为

$$\begin{cases} U_j^{(s)}(\alpha_1, \alpha_2, \alpha_3) = u_j^{(s)}(\alpha_1, \alpha_2) + \left(\alpha_3 + \frac{h_v + h_s}{2} \right) \beta_j^{(s)}(\alpha_1, \alpha_2), \\ U_j^{(v)}(\alpha_1, \alpha_2, \alpha_3) = u_j^{(v)}(\alpha_1, \alpha_2) + \alpha_3 \beta_j^{(v)}(\alpha_1, \alpha_2), \\ U_j^{(c)}(\alpha_1, \alpha_2, \alpha_3) = u_j^{(c)}(\alpha_1, \alpha_2) + \left(\alpha_3 - \frac{h_v + h_c}{2} \right) \beta_j^{(c)}(\alpha_1, \alpha_2), \\ U_3(\alpha_1, \alpha_2) = u_3(\alpha_1, \alpha_2), \end{cases} \quad (1)$$

其中, 下标 j 是 1 或 2, 分别表示在 α_1 和 α_2 方向上的量; u_1, u_2 和 u_3 是中面在 α_1, α_2 和 α_3 方向上的位移; β_1 和 β_2 分别是横向的法线绕曲线坐标轴 α_2 和 α_1 的转角. 图 2 所示为层间位移的分布. 利用上述的 4 个假设, 获得如下的关系:

$$\beta_j^{(c)}(\alpha_1, \alpha_2) = \frac{u_j^{(c)}}{R_j^{(c)}} - \frac{1}{A_j^{(c)}} \frac{\partial u_3}{\partial \alpha_j}, \quad (2)$$

$$\beta_j^{(s)}(\alpha_1, \alpha_2) = \frac{u_j^{(s)}}{R_j^{(s)}} - \frac{1}{A_j^{(s)}} \frac{\partial u_3}{\partial \alpha_j}, \quad (3)$$

$$u_j^{(v)}(\alpha_1, \alpha_2) = \frac{1}{2} \left(u_j^{(c)}(\alpha_1, \alpha_2) - \frac{h_c}{2} \beta_j^{(c)}(\alpha_1, \alpha_2) + u_j^{(s)}(\alpha_1, \alpha_2) + \frac{h_s}{2} \beta_j^{(s)}(\alpha_1, \alpha_2) \right), \quad (4)$$

$$\beta_j^{(v)}(\alpha_1, \alpha_2) = \frac{1}{h_v} \left(u_j^{(c)}(\alpha_1, \alpha_2) - \frac{h_c}{2} \beta_j^{(c)}(\alpha_1, \alpha_2) - u_j^{(s)}(\alpha_1, \alpha_2) - \frac{h_s}{2} \beta_j^{(s)}(\alpha_1, \alpha_2) \right), \quad (5)$$

其中, 标量系数 R_1, R_2, A_1 和 A_2 在曲线坐标 α_1, α_2 和 α_3 下表示为

$$R_1 = \infty, R_2 = a, A_1 = 1, A_2 = a. \quad (6)$$

基于 Sanders 薄壳理论^[24] 的圆柱壳的位移应变关系为

$$\varepsilon_{11}^\circ = \frac{\partial u_1}{\partial \alpha_1}, \varepsilon_{22}^\circ = \frac{1}{a} \frac{\partial u_2}{\partial \alpha_2} + \frac{u_3}{a}, \varepsilon_{12}^\circ = \frac{\partial u_2}{\partial \alpha_1} + \frac{1}{a} \frac{\partial u_1}{\partial \alpha_2}, \quad (7)$$

$$\begin{cases} \kappa_{11} = -\frac{\partial^2 u_3}{\partial \alpha_1^2}, \kappa_{22} = \frac{1}{a^2} \frac{\partial u_2}{\partial \alpha_2} - \frac{1}{a^2} \frac{\partial^2 u_3}{\partial \alpha_2^2}, \\ \kappa_{12} = \frac{3}{2a} \frac{\partial u_2}{\partial \alpha_1} - \frac{2}{a} \frac{\partial^2 u_3}{\partial \alpha_1 \partial \alpha_2} - \frac{1}{2a^2} \frac{\partial u_1}{\partial \alpha_2}, \end{cases} \quad (8)$$

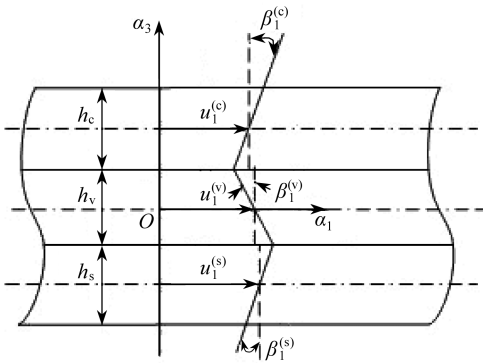


图2 层间位移分布

Fig.2 Displacement distribution among layers

在上面的公式中, 省略了位移量的上标 s 和 c . 主壳和 CL 的应变为

$$\begin{cases} \varepsilon_{11}^{(s)} = \varepsilon_{11}^{\circ(s)} + \left(\alpha_3 + \frac{h_v + h_s}{2} \right) \kappa_{11}^{(s)}, \\ \varepsilon_{22}^{(s)} = \varepsilon_{22}^{\circ(s)} + \left(\alpha_3 + \frac{h_v + h_s}{2} \right) \kappa_{22}^{(s)}, \\ \varepsilon_{12}^{(s)} = \varepsilon_{12}^{\circ(s)} + \left(\alpha_3 + \frac{h_v + h_s}{2} \right) \kappa_{12}^{(s)}, \end{cases} \quad (9)$$

$$\begin{cases} \varepsilon_{11}^{(c)} = \varepsilon_{11}^{\circ(c)} + \left(\alpha_3 - \frac{h_v + h_c}{2} \right) \kappa_{11}^{(c)}, \\ \varepsilon_{22}^{(c)} = \varepsilon_{22}^{\circ(c)} + \left(\alpha_3 - \frac{h_v + h_c}{2} \right) \kappa_{22}^{(c)}, \\ \varepsilon_{12}^{(c)} = \varepsilon_{12}^{\circ(c)} + \left(\alpha_3 - \frac{h_v + h_c}{2} \right) \kappa_{12}^{(c)}, \end{cases} \quad (10)$$

其中, $\varepsilon_{11}, \varepsilon_{22}$ 和 ε_{12} 是壳体上任意一点的应变. VEM 的剪切应变为

$$\varepsilon_{13}^{(v)} = \frac{\partial u_3}{\partial \alpha_1} + \beta_1^{(v)}, \varepsilon_{23}^{(v)} = \frac{1}{a_v} \frac{\partial u_3}{\partial \alpha_2} - \frac{u_2^{(v)}}{a_v} + \beta_2^{(v)}. \quad (11)$$

正交各向异性壳(主壳或 CL)的应力与应变通过如下关系建立联系:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix}, \quad (12)$$

其中,上标 s 或 c 被省略了。 Q_{ij} 是正交各向异性壳的刚度系数,给定为

$$Q_{11} = \frac{E_1^{(s)}}{1 - \nu_{21}^{(s)} \nu_{12}^{(s)}}, \quad Q_{12} = \frac{E_1^{(s)} \nu_{12}^{(s)}}{1 - \nu_{21}^{(s)} \nu_{12}^{(s)}}, \quad Q_{22} = \frac{E_2^{(s)}}{1 - \nu_{21}^{(s)} \nu_{12}^{(s)}}, \quad Q_{66} = G_{12}^{(s)},$$

其中上标 s 表示为属于主壳的量。如果上标 s 用 c 代替, Q_{ij} 属于 CL 的量。对于 VEM, 剪切应力同剪切应变的关系为

$$\sigma_{13}^{(v)} = G_v \varepsilon_{13}^{(v)}, \quad \sigma_{23}^{(v)} = G_v \varepsilon_{23}^{(v)}, \quad (13)$$

其中, G_v 是 VEM 的复剪切模量。对于这 3 个壳体的应变能表达如下:

$$\begin{aligned} U_{\text{sve}} = & \frac{1}{2} \int_0^l \int_0^{2\pi} \left[\int_{-h_s-(h_v/2)}^{-h_v/2} (\sigma_{11}^{(s)} \varepsilon_{11}^{(s)} + \sigma_{22}^{(s)} \varepsilon_{22}^{(s)} + \sigma_{12}^{(s)} \varepsilon_{12}^{(s)}) a_s d\alpha_3 + \right. \\ & \int_{h_v/2}^{h_c+(h_v/2)} (\sigma_{11}^{(c)} \varepsilon_{11}^{(c)} + \sigma_{22}^{(c)} \varepsilon_{22}^{(c)} + \sigma_{12}^{(c)} \varepsilon_{12}^{(c)}) a_c d\alpha_3 + \\ & \left. \int_{-h_v/2}^{h_v/2} (\sigma_{13}^{(v)} \varepsilon_{13}^{(v)} + \sigma_{23}^{(v)} \varepsilon_{23}^{(v)}) a_v d\alpha_3 \right] d\alpha_1 d\alpha_2. \end{aligned} \quad (14)$$

利用假设(d), 3 个壳体的动能有以下表达式:

$$\begin{aligned} T_{\text{sve}} = & \frac{1}{2} \int_0^l \int_0^{2\pi} [a_s I_1^{(s)} ((\dot{u}_1^{(s)})^2 + (\dot{u}_2^{(s)})^2 + (\dot{u}_3^{(s)})^2) + \\ & a_c I_1^{(c)} ((\dot{u}_1^{(c)})^2 + (\dot{u}_2^{(c)})^2 + (\dot{u}_3^{(c)})^2) + a_v I_1^{(v)} \dot{u}_3^2] d\alpha_1 d\alpha_2, \end{aligned} \quad (15)$$

其中, $I_1^{(s)}$, $I_1^{(c)}$ 和 $I_1^{(v)}$ 分别是主壳、CL 和 VEM 的面密度。它们为

$$I_1^{(s)} = \rho_s h_s, \quad I_1^{(c)} = \rho_c h_c, \quad I_1^{(v)} = \rho_v h_v. \quad (16)$$

由分布的外部面力做的功 W_{sev} 可以表示为

$$\begin{aligned} W_{\text{sev}} = & \int_0^l \int_0^{2\pi} (a_s q_1^{(s)} u_1^{(s)} + a_s q_2^{(s)} u_2^{(s)} + a_s q_3^{(s)} u_3^{(s)} + \\ & a_c q_1^{(c)} u_1^{(c)} + a_c q_2^{(c)} u_2^{(c)} + a_c q_3^{(c)} u_3^{(c)}) d\alpha_1 d\alpha_2, \end{aligned} \quad (17)$$

其中, $q_1^{(s)}$, $q_2^{(s)}$ 和 $q_3^{(s)}$ 表示作用在主壳的 α_1 , α_2 和 α_3 方向上的外部面力。同样, $q_1^{(c)}$, $q_2^{(c)}$ 和 $q_3^{(c)}$ 表示作用在 CL 上的 α_1 , α_2 和 α_3 方向上的外部面力。利用方程(14)、(15)和(17), 能量函数定义为

$$\Pi = \int_0^{T_1} (U_{\text{sve}} - T_{\text{sve}} - W_{\text{sev}}) dt, \quad (18)$$

其中, T_1 是结构运动的周期。对能量函数运用 Hamilton 变分原理, 得到结构的运动方程:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix} \begin{bmatrix} u_1^{(s)} \\ u_2^{(s)} \\ u_3 \\ u_1^{(c)} \\ u_2^{(c)} \end{bmatrix} = \begin{bmatrix} a_s q_1^{(s)} \\ a_s q_2^{(s)} \\ a_s q_3^{(s)} + a_c q_3^{(c)} \\ a_c q_1^{(c)} \\ a_c q_2^{(c)} \end{bmatrix}, \quad (19)$$

其中, 微分算子 L_{ij} 在附录 A 中给出。在纵向和周向, 两个外面的壳体的圆柱面有 4 个运动方

程,还有一个方程表示 PCLD 圆柱壳的横向运动. 由于变量是两个表面壳体的位移,方程(19)在分析离散加筋 CL 或主壳时有很大的优势.

2 PCLD 圆柱壳运动方程求解

对于结构的自由振动,外部力设置为 0. 波传播法用来分析自由振动:

$$\begin{cases} u_1^{(s)} = u_{1n}^{(s)} e^{i(k\alpha_1 + n\alpha_2 + \omega t)}, & u_2^{(s)} = u_{2n}^{(s)} e^{i(k\alpha_1 + n\alpha_2 + \omega t)}, \\ u_3 = u_{3n} e^{i(k\alpha_1 + n\alpha_2 + \omega t)}, & u_1^{(c)} = u_{1n}^{(c)} e^{i(k\alpha_1 + n\alpha_2 + \omega t)}, \\ u_2^{(c)} = u_{2n}^{(c)} e^{i(k\alpha_1 + n\alpha_2 + \omega t)}. \end{cases} \quad (20)$$

将方程(20)代入(19),我们得到了如下的特征方程:

$$\begin{bmatrix} \tilde{L}_{11} & \tilde{L}_{12} & \tilde{L}_{13} & \tilde{L}_{14} & \tilde{L}_{15} \\ \tilde{L}_{21} & \tilde{L}_{22} & \tilde{L}_{23} & \tilde{L}_{24} & \tilde{L}_{25} \\ \tilde{L}_{31} & \tilde{L}_{32} & \tilde{L}_{33} & \tilde{L}_{34} & \tilde{L}_{35} \\ \tilde{L}_{41} & \tilde{L}_{42} & \tilde{L}_{43} & \tilde{L}_{44} & \tilde{L}_{45} \\ \tilde{L}_{51} & \tilde{L}_{52} & \tilde{L}_{53} & \tilde{L}_{54} & \tilde{L}_{55} \end{bmatrix} \begin{bmatrix} u_{1n}^{(s)} \\ u_{2n}^{(s)} \\ u_{3n} \\ u_{1n}^{(c)} \\ u_{2n}^{(c)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (21)$$

其中因子 \tilde{L}_{ij} 在附录 B 中给出. 对于 VEM 的复常剪切模量,特征方程能够利用线性代数的方法求解.

3 数值结果

在这节中,利用前面的理论,研究真空中 PCLD 圆柱壳的自由振动. 波传播法被用来分析自由振动. 注意力集中于两端简支(SS)的壳体. 在这种边界条件下,圆柱壳纵向方向的波数 k 是 $m\pi/L$ [25]. 对于方程(21),有 3 种不同类型的固有频率. 由于 VEM 的能量耗散,频率都是复数. 频率的实部表示波的传播,虚部表示波的衰减. 一般而言,最低的频率对应于径向运动. 其它频率表示纵向或者周向运动. 方程(21)的特征值 ω^2 能够表示成以下形式:

$$\omega^2 = \omega_0^2(1 + i\eta), \quad (22)$$

其中 η 是对应的损耗因子. ω 的实部给出固有频率. 比率 γ 定义为

$$\gamma = E_1^{(c)}/E_2^{(c)}, \quad (23)$$

其中 $E_1^{(c)}$ 是 CL 纵向方向的弹性模量, $E_2^{(c)}$ 是 CL 周向方向的弹性模量.

为了验证当前方法的正确性,考虑一个 PCLD 圆柱壳. 壳层是各向同性的,几何和材料参数为 [16]

$$L = 100 \text{ mm}, h_s = h_v = h_c = 1/3 \text{ mm}, R_s = 100 \text{ mm},$$

$$R_v = \left(100 + \frac{h_s + h_v}{2}\right) \text{ mm}, R_c = \left(100 + \frac{h_s + 2h_v + h_c}{2}\right) \text{ mm},$$

$$E_s = E_c = 2.1 \times 10^{11} \text{ Pa}, \nu_s = \nu_c = 0.3, \rho_s = \rho_c = 7850 \text{ kg/m}^3, \rho_v = 1340 \text{ kg/m}^3.$$

VEM 的复常剪切模量为

$$G_v = (8.582 + i2.985) \text{ MPa} = 8.582 \times (1 + ig) \text{ MPa}, g = 0.3478.$$

边界条件作简支处理.

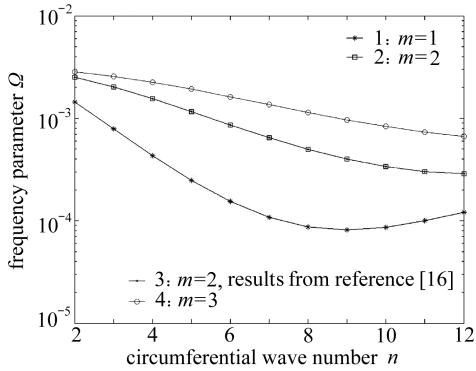


图3 PCLD 圆柱壳的无量纲频率

Fig.3 Dimensionless frequency parameter for PCLD cylindrical shell

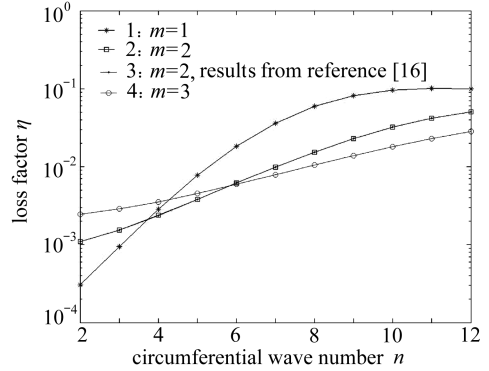
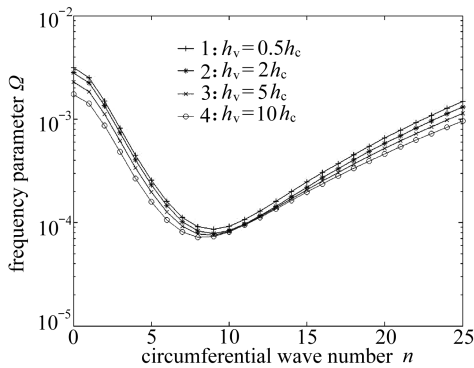
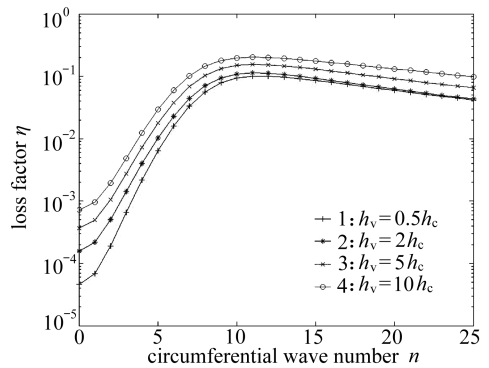


图4 PCLD 圆柱壳的损耗因子

Fig.4 Loss factor for PCLD cylindrical shell

图5 PCLD 圆柱壳的无量纲频率参数随 h_v 的变化关系, $h_c = 1/3$ mm, $m = 1$ Fig.5 Dimensionless frequency parameter for PCLD cylindrical shell with variable h_v , $h_c = 1/3$ mm, $m = 1$ 图6 PCLD 圆柱壳的损耗因子随 h_v 的变化关系, $h_c = 1/3$ mm, $m = 1$ Fig.6 Loss factor for PCLD cylindrical shell with variable h_v , $h_c = 1/3$ mm, $m = 1$

3.1 各向同性 PCLD 圆柱壳的自由振动

利用上面的模型,本文方法的结果与文献[16]的结果作了比较,列于表1中.对于 $n = 0$, PCLD 圆柱壳的振动是轴对称的.可以发现两种结果彼此相同. ω^2 的实部和虚部的相对误差小于1%.对于 CLD 圆柱壳的非轴对称振动,频率参数 Ω 和损耗因子 η 的结果分别显示在图3和图4中.其中, Ω 定义为 $\Omega = \omega_0^2 \rho_s h_s R_s / E_s$. 图3中频率参数曲线1,2,4分别对应于 $m = 1, m = 2$ 和 $m = 3$. 图3中曲线3是由文献[16]给出.能够看到图3中曲线2和3是相合的.图4中损耗因子曲线1,2,4分别对应于 $m = 1, m = 2$ 和 $m = 3$. 图4中曲线3也是由文献[16]给出.图4中曲线2和3相合得很好.因此,本文方法是有效的和可行的.图5和图6显示了 VEM 的厚度对无量纲频率 Ω 和损耗因子 η 的影响.能够看到频率参数随着 VEM 厚度的变化有少量改变.复剪切模量 G_v 的实部和 VEM 的惯量造成了 Ω 的少许变化.由于 G_v 是小量, VEM 的横向惯量对 Ω 起主要作用.随着 VEM 的厚度增加,频率参数 Ω 降低.损耗因子随着 VEM 的厚度增加而明显地增加,尤其在低周向波数,并且由于 VEM 剪切变形,能量耗散更强烈.图7和图8显示了

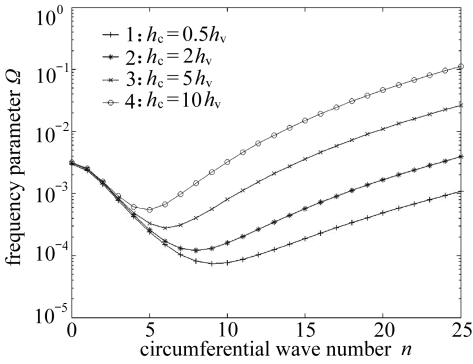


图7 PCLD 圆柱壳的无量纲频率参数随 h_c 的变化关系, $h_v = 1/3$ mm, $m = 1$

Fig. 7 Dimensionless frequency parameter for PCLD cylindrical shell with variable h_c , $h_v = 1/3$ mm, $m = 1$

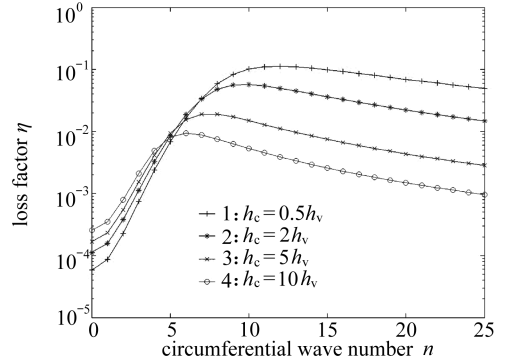


图8 PCLD 圆柱壳的损耗因子随 h_c 的变化关系, $h_v = 1/3$ mm, $m = 1$

Fig. 8 Loss factor for PCLD cylindrical shell with variable h_c , $h_v = 1/3$ mm, $m = 1$

CL 的厚度对频率参数和损耗因子的影响。在高周向波数时, Ω 随着 h_c 的增加而明显增加, 在低周向波数时, 有微小变化。在低周向波数时, 损失因子随着 h_c 的增加而增加。然而, 在高周向波数时, 损耗因子随着 h_c 的增加而明显降低。这是由于在高周向波数时深厚 h_c 圆柱壳的大刚度造成的。因为 CL 使得 VEM 承受较大的剪切变形, 图 7 和图 8 表明随着约束层厚度 h_c 增加, VEM 的阻尼在高频不是非常有效的, 而在低频它却是重要的。

表1 两端简支 PCLD 圆柱壳 ω^2 的比较, $n = 0$

Table 1 Comparison of ω^2 for PCLD cylindrical shell with SS, $n = 0$

longitudinal half wave number m	results from reference[16]	present solution
1	2.443 8 E9+i2.057 0 E5	2.427 7 E9+i2.069 3 E5
2	2.470 3 E9+i1.968 3 E6	2.453 9 E9+i1.969 3 E6
3	2.498 9 E9+i5.330 5 E6	2.482 5 E9+i5.330 4 E6
4	2.556 6 E9+i1.013 4 E7	2.540 2 E9+i1.013 2 E7
5	2.665 1 E9+i1.633 8 E7	2.648 6 E9+i1.633 8 E7
6	2.851 1 E9+i2.393 5 E7	2.834 7 E9+i2.393 4 E7
7	3.147 0 E9+i3.291 9 E7	3.130 6 E9+i3.291 8 E7
8	3.591 0 E9+i4.328 9 E7	3.574 5 E9+i4.328 7 E7
9	4.227 0 E9+i5.505 4 E7	4.210 6 E9+i5.504 1 E7
10	5.105 0 E9+i6.830 0 E7	5.088 5 E9+i6.817 9 E7

3.2 正交各向异性 CL 圆柱壳的自由振动

主壳和 VEM 的几何和材料参数保持不变。但是, CL 由正交各向异性材料组成。CL 的几何参数同前面的模型中数据一样。 γ 对 PCLD 圆柱壳的频率参数和损失因子的影响如图 9 和图 10 所示。当 γ 从 1.2 增加到 20 时, 图 9 表明 Ω 是降低的, 周向波数的增加轻微改变了 Ω 。这是由于当 γ 增加时, CL 的周向刚度下降较大, 并且同纵向刚度相比较, 它对 Ω 的影响减少, 微弱的周向刚度放松了它对 Ω 的贡献。当 γ 变大时, 在低周向波数时, 图 10 中的损失因子出现波动和变大。图 11 至图 14 显示了 g 对频率参数和损耗因子的影响(图 11 和图 13 中的 4 条曲线全

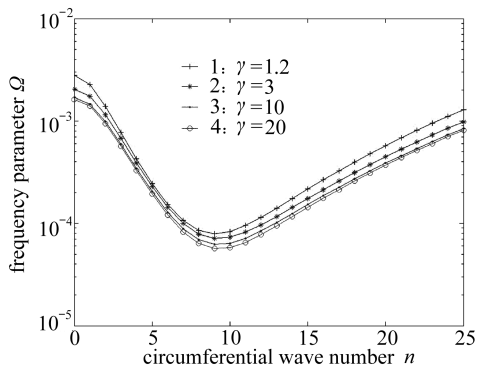


图9 PCLD 圆柱壳的频率参数随弹性模量比率 γ 的变化关系, $m = 1$

Fig. 9 Frequency parameter for PCLD cylindrical shell with variable elastic modulus ratio γ , $m = 1$

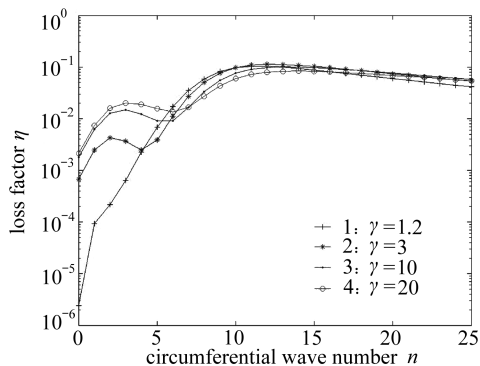


图10 PCLD 圆柱壳的损耗因子随弹性模量比率 γ 的变化关系, $m = 1$

Fig. 10 Loss factor for PCLD cylindrical shell with variable elastic modulus ratio γ , $m = 1$

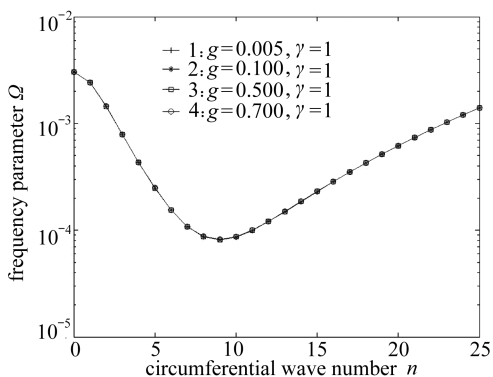


图11 PCLD 圆柱壳的频率参数随变量 g 的变化关系, $m = 1$, $\gamma = 1$

Fig. 11 Frequency parameter of PCLD cylindrical shell with variable g , $m = 1$, $\gamma = 1$

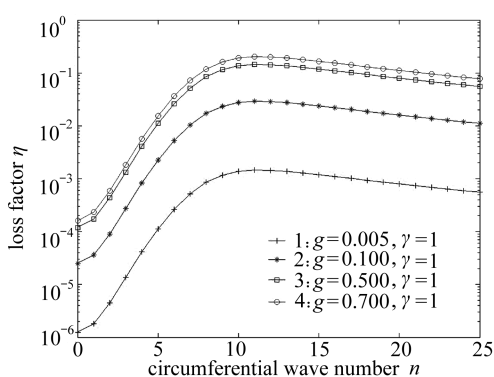


图12 PCLD 圆柱壳的损耗因子随变量 g 的变化关系, $m = 1$, $\gamma = 1$

Fig. 12 Loss factor of PCLD cylindrical shell with variable g , $m = 1$, $\gamma = 1$

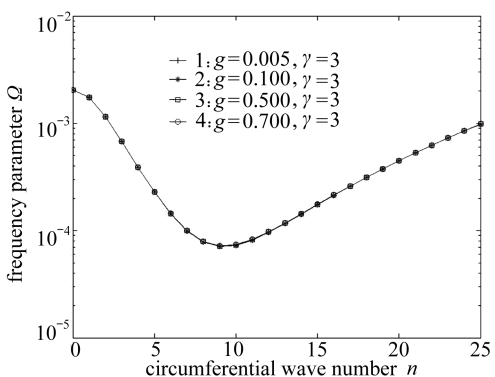


图13 PCLD 圆柱壳的频率参数随变量 g 的变化关系, $m = 1$, $\gamma = 3$

Fig. 13 Frequency parameter of PCLD cylindrical shell with variable g , $m = 1$, $\gamma = 3$

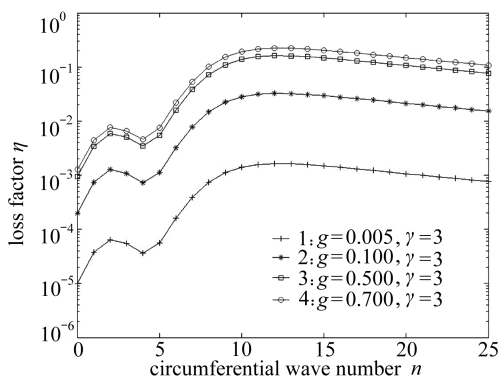


图14 PCLD 圆柱壳的损耗因子随变量 g 的变化关系, $m = 1$, $\gamma = 3$

Fig. 14 Loss factor of PCLD cylindrical shell with variable g , $m = 1$, $\gamma = 3$

部重合)。频率参数 Ω 几乎不受 g 的影响, 损失因子随 g 的增加而增加, 图 12 与图 14 的差异在于低周向波数时图 14 中的损失因子有小幅波动, 并且同图 12 中的结果相比较时, 它有较大的值。由于结构周向刚度的减少, 较大的 γ 造成较大的损失因子。

4 结 论

基于 Sanders 薄壳理论, 导出了 PCLD 圆柱壳的 5 个运动方程。波传播法被用来求解 PCLD 圆柱壳的自由振动。通过同文献[16]的结果作比较, 验证了本文提出的方法。VEM 的厚度对频率参数 Ω 的影响较小, 对损耗因子的影响很大。在高周向波数时, 随着 CL 的厚度增加, Ω 明显增加。在低周向波数时, 随着 CL 的厚度增加, 损失因子也增加。然而, 在高周向波数时, 损失因子随着 CL 的厚度增加而明显降低。当正交各向异性 CL 的弹性模量比率变大时, Ω 减少, 损耗因子在低频时出现波动和变大。 Ω 不受 VEM 的剪切模量比率 g 的影响。损耗因子随 g 的增加而增加。 g 一定时, 由于结构的周向刚度减少, 较大的 γ 造成较大的损失因子。利用波传播法, 能分析其它边界条件下的 PCLD 圆柱壳的动力特性。本文的方法既简单又直接。基于提出的模型, 进一步的研究能够进行, 例如强迫振动, 声辐射。

附 录

A 微分算子 L_{ij} 表达为

$$\begin{aligned}
 L_{11} &= -a_s A_{11}^{(s)} \frac{\partial^2}{\partial \alpha_1^2} - \left(\frac{A_{66}^{(s)}}{a_s} + \frac{D_{66}^{(s)}}{4a_s^3} \right) \frac{\partial^2}{\partial \alpha_2^2} + \frac{a_v G_v}{h_v} + a_s I_1^{(s)} \frac{\partial^2}{\partial t^2}, \\
 L_{12} &= L_{21} = - \left(A_{12}^{(s)} + A_{66}^{(s)} - \frac{3D_{66}^{(s)}}{4a_s^2} \right) \frac{\partial^2}{\partial \alpha_1 \partial \alpha_2}, \\
 L_{13} &= -L_{31} = - \left(A_{12}^{(s)} + a_v G_v \left(\frac{h_c + h_s + 2h_v}{2h_v} \right) \right) \frac{\partial}{\partial \alpha_1} - \frac{D_{66}^{(s)}}{a_s^2} \frac{\partial^3}{\partial \alpha_2^2 \partial \alpha_1}, \\
 L_{14} &= L_{41} = - \frac{a_v G_v}{h_v}, \quad L_{15} = L_{51} = 0, \\
 L_{22} &= - \left(a_s A_{66}^{(s)} + \frac{9D_{66}^{(s)}}{4a_s} \right) \frac{\partial^2}{\partial \alpha_1^2} - \left(\frac{A_{22}^{(s)}}{a_s} + \frac{D_{22}^{(s)}}{a_s^3} \right) \frac{\partial^2}{\partial \alpha_2^2} + \gamma_3 G_v + a_s I_1^{(s)} \frac{\partial^2}{\partial t^2}, \\
 L_{23} &= -L_{32} = \left(\frac{3D_{66}^{(s)}}{a_s} + \frac{D_{12}^{(s)}}{a_s} \right) \frac{\partial^3}{\partial \alpha_1^2 \partial \alpha_2} - \left(\frac{A_{22}^{(s)}}{a_s} + \gamma_1 G_v \right) \frac{\partial}{\partial \alpha_2} + \frac{D_{22}^{(s)}}{a_s^3} \frac{\partial^3}{\partial \alpha_2^3}, \\
 L_{24} &= L_{42} = 0, \quad L_{25} = L_{52} = -\gamma_2 G_v, \\
 L_{33} &= (a_s D_{11}^{(s)} + a_c D_{11}^{(c)}) \frac{\partial^4}{\partial \alpha_1^4} + \left(\frac{2D_{12}^{(s)}}{a_s} + \frac{4D_{66}^{(s)}}{a_s} + \frac{2D_{12}^{(c)}}{a_c} + \frac{4D_{66}^{(c)}}{a_c} \right) \frac{\partial^4}{\partial \alpha_1^2 \partial \alpha_2^2} - \\
 &\quad \frac{a_v G_v (h_c + h_s + 2h_v)^2}{h_v} \frac{\partial^2}{\partial \alpha_1^2} - a_v G_v \gamma_4 \frac{\partial^2}{\partial \alpha_2^2} + \left(\frac{D_{22}^{(s)}}{a_s^3} + \frac{D_{22}^{(c)}}{a_c^3} \right) \frac{\partial^4}{\partial \alpha_2^4} + \\
 &\quad \left(\frac{A_{22}^{(s)}}{a_s} + \frac{A_{22}^{(c)}}{a_c} \right) + (a_s I_1^{(s)} + a_c I_1^{(c)} + a_v I_1^{(v)}) \frac{\partial^2}{\partial t^2}, \\
 L_{34} &= -L_{43} = \left(A_{12}^{(c)} - a_v G_v \left(\frac{h_c + h_s + 2h_v}{2h_v} \right) \right) \frac{\partial}{\partial \alpha_1} + \frac{D_{66}^{(c)}}{a_c^2} \frac{\partial^3}{\partial \alpha_2^2 \partial \alpha_1}, \\
 L_{35} &= -L_{53} = - \left(\frac{3D_{66}^{(c)}}{a_c} + \frac{D_{12}^{(c)}}{a_c} \right) \frac{\partial^3}{\partial \alpha_1^2 \partial \alpha_2} + \left(\frac{A_{22}^{(c)}}{a_c} - a_v G_v \gamma_5 \right) \frac{\partial}{\partial \alpha_2} - \frac{D_{22}^{(c)}}{a_c^3} \frac{\partial^3}{\partial \alpha_2^3},
 \end{aligned}$$

$$\begin{aligned}
L_{44} &= -a_c A_{11}^{(c)} \frac{\partial^2}{\partial \alpha_1^2} - \left(\frac{A_{66}^{(c)}}{a_c} + \frac{D_{66}^{(c)}}{4a_c^3} \right) \frac{\partial^2}{\partial \alpha_2^2} + \frac{a_v G_v}{h_v} + a_c I_1^{(c)} \frac{\partial^2}{\partial t^2}, \\
L_{45} &= L_{54} = - \left(A_{12}^{(c)} + A_{66}^{(c)} - \frac{3D_{66}^{(c)}}{4a_c^2} \right) \frac{\partial^2}{\partial \alpha_1 \partial \alpha_2}, \\
L_{55} &= - \left(a_c A_{66}^{(c)} + \frac{9D_{66}^{(c)}}{4a_c} \right) \frac{\partial^2}{\partial \alpha_1^2} - \left(\frac{A_{22}^{(c)}}{a_c} + \frac{D_{22}^{(c)}}{a_c^3} \right) \frac{\partial^2}{\partial \alpha_2^2} + a_v G_v \gamma_6 + a_c I_1^{(c)} \frac{\partial^2}{\partial t^2},
\end{aligned}$$

其中系数 $\gamma_1, \gamma_2, \dots, \gamma_6$ 表达为

$$\begin{aligned}
\gamma_1 &= \left(\frac{h_v}{2} + a_v \right) \left(1 + \frac{h_s}{2a_s} \right) \left[\frac{1}{a_v} - \frac{1}{2a_v} \left(\frac{h_c}{2a_c} - \frac{h_s}{2a_s} \right) + \frac{1}{h_v} \left(\frac{h_c}{2a_c} + \frac{h_s}{2a_s} \right) \right], \\
\gamma_2 &= \left(\frac{h_v}{2} + a_v \right) \left(1 + \frac{h_s}{2a_s} \right) \left(\frac{1}{h_v} - \frac{1}{2a_v} \right) \left(1 - \frac{h_c}{2a_c} \right), \\
\gamma_3 &= \left(\frac{h_v}{2} + a_v \right) \left(1 + \frac{h_s}{2a_s} \right) \left(\frac{1}{2a_v} + \frac{1}{h_v} \right) \left(1 + \frac{h_s}{2a_s} \right), \\
\gamma_4 &= \left[\frac{1}{a_v} - \frac{1}{2a_v} \left(\frac{h_c}{2a_c} - \frac{h_s}{2a_s} \right) + \frac{1}{h_v} \left(\frac{h_c}{2a_c} + \frac{h_s}{2a_s} \right) \right] \times \\
&\quad \left[\frac{h_v}{a_v} - \frac{h_v}{2a_v} \left(\frac{h_c}{2a_c} - \frac{h_s}{2a_s} \right) + \left(\frac{h_c}{2a_c} + \frac{h_s}{2a_s} \right) \right], \\
\gamma_5 &= \left[\frac{1}{a_v} - \frac{1}{2a_v} \left(\frac{h_c}{2a_c} - \frac{h_s}{2a_s} \right) + \frac{1}{h_v} \left(\frac{h_c}{2a_c} + \frac{h_s}{2a_s} \right) \right] \left(1 - \frac{h_v}{2a_v} \right) \left(1 - \frac{h_c}{2a_c} \right), \\
\gamma_6 &= \left(1 - \frac{h_v}{2a_v} \right) \left(1 - \frac{h_c}{2a_c} \right) \left(\frac{1}{h_v} - \frac{1}{2a_v} \right) \left(1 - \frac{h_c}{2a_c} \right).
\end{aligned}$$

材料参数定义为

$$\begin{aligned}
A_{11}^{(s)} &= h_s Q_{11}^{(s)}, A_{12}^{(s)} = h_s Q_{12}^{(s)}, A_{22}^{(s)} = h_s Q_{22}^{(s)}, A_{66}^{(s)} = h_s Q_{66}^{(s)}, \\
D_{11}^{(s)} &= \frac{h_s^3}{12} Q_{11}^{(s)}, D_{12}^{(s)} = \frac{h_s^3}{12} Q_{12}^{(s)}, D_{22}^{(s)} = \frac{h_s^3}{12} Q_{22}^{(s)}, D_{66}^{(s)} = \frac{h_s^3}{12} Q_{66}^{(s)}, \\
A_{11}^{(c)} &= h_c Q_{11}^{(c)}, A_{12}^{(c)} = h_c Q_{12}^{(c)}, A_{22}^{(c)} = h_c Q_{22}^{(c)}, A_{66}^{(c)} = h_c Q_{66}^{(c)}, \\
D_{11}^{(c)} &= \frac{h_c^3}{12} Q_{11}^{(c)}, D_{12}^{(c)} = \frac{h_c^3}{12} Q_{12}^{(c)}, D_{22}^{(c)} = \frac{h_c^3}{12} Q_{22}^{(c)}, D_{66}^{(c)} = \frac{h_c^3}{12} Q_{66}^{(c)}.
\end{aligned}$$

B 因子 \tilde{L}_{ij} 为

$$\begin{aligned}
\tilde{L}_{11} &= a_s A_{11}^{(s)} k^2 + \left(\frac{A_{66}^{(s)}}{a_s} + \frac{D_{66}^{(s)}}{4a_s^3} \right) n^2 + \frac{a_v G_v}{h_v} - a_s \omega^2 I_1^{(s)}, \\
\tilde{L}_{12} &= \tilde{L}_{21} = \left(A_{12}^{(s)} + A_{66}^{(s)} - \frac{3D_{66}^{(s)}}{4a_s^2} \right) kn, \\
\tilde{L}_{13} &= -\tilde{L}_{31} = ikn^2 \frac{D_{66}^{(s)}}{a_s^2} - ik \left(A_{12}^{(s)} + a_v G_v \left(\frac{h_c + h_s + 2h_v}{2h_v} \right) \right), \\
\tilde{L}_{14} &= \tilde{L}_{41} = -\frac{a_v G_v}{h_v}, \tilde{L}_{15} = \tilde{L}_{51} = 0, \\
\tilde{L}_{22} &= k^2 \left(a_s A_{66}^{(s)} + \frac{9D_{66}^{(s)}}{4a_s} \right) + n^2 \left(\frac{A_{22}^{(s)}}{a_s} + \frac{D_{22}^{(s)}}{a_s^3} \right) + \gamma_3 G_v - a_s \omega^2 I_1^{(s)}, \\
\tilde{L}_{23} &= -\tilde{L}_{32} = -ink^2 \left(\frac{3D_{66}^{(s)}}{a_s} + \frac{D_{12}^{(s)}}{a_s} \right) - in \left(\frac{A_{22}^{(s)}}{a_s} + \gamma_1 G_v \right) - in^3 \frac{D_{22}^{(s)}}{a_s^3}, \\
\tilde{L}_{24} &= \tilde{L}_{42} = 0, \tilde{L}_{25} = \tilde{L}_{52} = -\gamma_2 G_v, \\
\tilde{L}_{33} &= (a_s D_{11}^{(s)} + a_c D_{11}^{(c)}) k^4 + \left(\frac{2D_{12}^{(s)}}{a_s} + \frac{4D_{66}^{(s)}}{a_s} + \frac{2D_{12}^{(c)}}{a_c} + \frac{4D_{66}^{(c)}}{a_c} \right) n^2 k^2 +
\end{aligned}$$

$$\begin{aligned}
& k^2 \frac{a_v G_v}{h_v} \left(\frac{h_c + h_s + 2h_v}{2} \right)^2 + n^2 a_v G_v \gamma_4 + \left(\frac{D_{22}^{(s)}}{a_s^3} + \frac{D_{22}^{(c)}}{a_c^3} \right) n^4 + \\
& \frac{A_{22}^{(s)}}{a_s} + \frac{A_{22}^{(c)}}{a_c} - (a_s I_1^{(s)} + a_c I_1^{(c)} + a_v I_1^{(v)}) \omega^2, \\
\tilde{L}_{34} = -\tilde{L}_{43} &= ik \left(A_{12}^{(c)} - a_v G_v \left(\frac{h_c + h_s + 2h_v}{2h_v} \right) \right) - ikn^2 \frac{D_{66}^{(c)}}{a_c^2}, \\
\tilde{L}_{35} = -\tilde{L}_{53} &= ink^2 \left(\frac{3D_{66}^{(c)}}{a_c} + \frac{D_{12}^{(c)}}{a_c} \right) + in \left(\frac{A_{22}^{(c)}}{a_c} - a_v G_v \gamma_5 \right) + in^3 \frac{D_{22}^{(c)}}{a_c^3}, \\
\tilde{L}_{44} &= a_c A_{11}^{(c)} k^2 + \left(\frac{A_{66}^{(c)}}{a_c} + \frac{D_{66}^{(c)}}{4a_c^3} \right) n^2 + \frac{a_v G_v}{h_v} - a_c I_1^{(c)} \omega^2, \\
\tilde{L}_{45} = \tilde{L}_{54} &= nk \left(A_{12}^{(c)} + A_{66}^{(c)} - \frac{3D_{66}^{(c)}}{4a_c^2} \right), \\
\tilde{L}_{55} &= k^2 \left(a_c A_{66}^{(c)} + \frac{9D_{66}^{(c)}}{4a_c} \right) + n^2 \left(\frac{A_{22}^{(c)}}{a_c} + \frac{D_{22}^{(c)}}{a_c^3} \right) + a_v G_v \gamma_8 - a_c I_1^{(c)} \omega^2.
\end{aligned}$$

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Free Vibration of Circular Cylindrical Shell With Constrained Layer Damping

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Abstract: The free vibration characteristic of circular cylindrical shell with passive constrained layer damping (PCLD) was presented. Wave propagation approach rather than finite element method, transfer matrix method and Rayleigh-Ritz method were used to solve the vibration of PCLD circular cylindrical shell with simply supported boundary condition at two ends. The governing equations of motion for the orthotropic cylindrical shell with PCLD were derived on the base of Sanders' thin shell theory. Numerical results show that the present method is more effective in comparison with other methods. The effect of the thickness of viscoelastic core and constrained layer, the elastic modulus ratio of orthotropic constrained layer and complex shear modulus of viscoelastic core on frequency parameter and loss factor is discussed.

Key words: sandwich shell; cylindrical shell; viscoelastic material; constrained layer damping; free vibration