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无穷远处Ⅲ型均布荷载与集中荷载作用下应变加强层连接唇形裂纹的解析解*

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摘要: 通过对保角映射及解析延拓的应用,获得了无穷远处Ⅲ型均布荷载和集中荷载作用下,连接在唇形裂纹面上的内埋应变加强层的级数形式应力解析解. 分析了材料匹配、界面连接及几何特征对界面应力的影响,研究发现对于不同的荷载形式,合理的材料匹配、连接及几何特征能够有效地减少应力集中和界面应力.

关 键 词: 唇形裂纹; 内埋应变加强层; 集中荷载; 无穷远处荷载; 本征应变; 界面应力

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引 言

平面弹性断裂问题中,椭圆刻痕与各种异型裂纹(唇裂纹)的应力分析以及位错与裂纹的干涉已经受到了许多学者的关注和研究. 在早期的文献中可以找到均布荷载下同性均质材料中椭圆刻痕附近的应力场的详细解答[1]. Florence 和 Goodier[2] 利用复变函数方法[3] 获得了各向同性材料中圆形孔洞或者卵形孔洞的精确解答. Chao 和 Shen[4] 获得了热应力作用下各向异性体中椭圆夹杂的精确解. Li, Lia 和 Sun[5] 利用复势方法研究了椭圆钝裂纹附近螺型位错的屏蔽效应. Xie 等[6] 分析了螺型位错和刃型位错与唇形裂纹的干涉效应.

在不同形式荷载的情况下,材料内部的椭圆刻痕和各种异形裂纹、孔洞(如唇裂纹)尖端的应力集中对材料本身来说是非常不利的,严重时它将导致材料和结构的失效与破坏,所以,它一直受到广泛的关注和研究. 利用适当几何比例和材料特性的加强涂层连接在裂纹面上是一种有效的削弱应力集中的方法,这种方法已经被广泛运用在许多由于弹性失配产生应力集中的实际问题中,它对保证机构的完整性、安全性是很重要的. Chao 等[7] 研究了圆形孔洞中连接圆环涂层的平面弹性问题. Chen 和 Chao[8] 对无穷远处均匀热流作用下,无穷平面中附有涂

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层的椭圆孔洞进行了应力分析. Chao 等[9]利用不同于标准的 Muskhelishvili 的复势函数方法,获得了连接加强层的椭圆孔洞在无穷远处均布荷载作用下的解析解答. 本文提到了无穷远处Ⅲ型均布荷载和集中荷载作用下,无穷平面中一个连接加强涂层的唇裂纹模型,其中应变涂层是由两条唇形曲线确定的环状面. 本文运用(不同于文献[10])建立在保角映射和解析延拓上的复势方法,将原问题映射成 2 个同心圆并予以求解. 应力场与界面应力的结果都在文中列出并通过数值曲线进行描述.

1 模型描述

如图 1 所示,无穷大基体中被加强的唇形裂纹(参见文献[6])受到远端荷载和集中荷载作用. Ω_1 代表基体,Ω_2 代表非完美连接于裂纹面上的加强层. 加强层的边界是两条唇形曲线 Γ_1,Γ_2;l_1,l_2 和 h_1,h_2 分别为 2 条曲线的半长和半高.

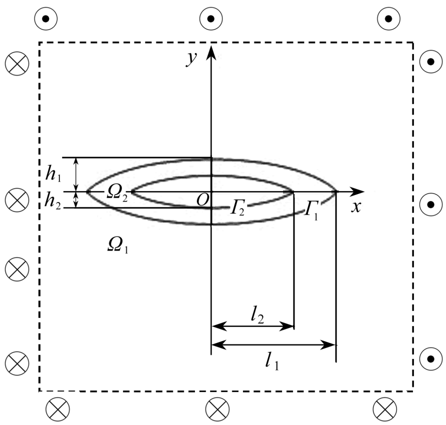


图 1 无穷远处Ⅲ型均布荷载和集中荷载作用下应变加强层连接唇形裂纹面的模型

参考文献[6],引用如下映射函数:

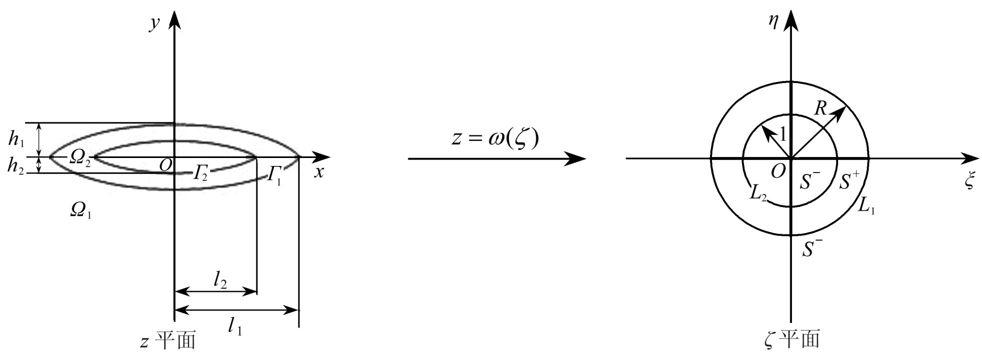
$$z = \omega(\zeta) = \frac{l_2 \rho \mathrm{i}}{2}\left[\zeta + \frac{m}{\zeta} - \frac{\zeta}{\rho^2(\zeta^2 + m)}\right], \quad (1)$$

其中

$$\rho = \frac{a+1}{2},\ m = \frac{a-1}{a+1},$$

$$a = \frac{h_2}{l_2} + \sqrt{\frac{h_2}{l_2} + 1},\ \zeta = r\mathrm{e}^{\mathrm{i}\varphi}.$$

该映射函数将两条物理平面 $z = r\mathrm{e}^{\mathrm{i}\theta}$ 内的唇形曲线,映射成像平面 $\zeta = \delta \mathrm{e}^{\mathrm{i}\varphi}$ 上半径分别为 R 和 1 的同心圆 L_1,L_2(如图 2 所示). 另外,ζ 平面内界面 L_1 上 $\varphi = 0$,$\varphi = \pi/2$,$\varphi = \pi$ 和 $\varphi = 3\pi/2$ 处,分别对应 z 平面内界面 Γ_1 上 $z = \mathrm{i}h_1$,$z = -l_1$,$z = -\mathrm{i}h_1$ 和 $z = l_1$.



z 平面 ζ 平面

图 2 映射模型

2 问题解答

2.1 集中荷载作用在基体上

设 z 平面上的位移

$$w = \mathrm{Re}\, f(z), \quad (2)$$

其中 Re 代表函数实部.

z 平面上的应力表示如下:

直角坐标表示为

$$\tau_{xz} - \mathrm{i}\tau_{yz} = \mu f'(z); \tag{3}$$

极坐标表示为

$$\tau_{rz} - \mathrm{i}\tau_{\theta z} = \mathrm{e}^{\mathrm{i}\theta}\mu f'(z). \tag{4}$$

参考文献[11],基体中的解析函数 $f_1(z)$ 表示如下:

$$f_1(z) = \Gamma z - \frac{Q}{2\pi\mu_1}\ln(z - z_0) + f_{10}(z), \qquad z \in S^-, \tag{5}$$

其中, $\Gamma = (\tau_{xz}^{\infty} - \mathrm{i}\tau_{yz}^{\infty})/\mu_1$, Q 是垂直指向面外的集中力荷载.

将式(3)代入式(5),像平面 ζ 上的应力函数为

$$f_1(\zeta) = \Gamma\frac{l_2\rho\mathrm{i}}{2}\zeta - \frac{Q}{2\pi\mu_1}\ln(\zeta - \zeta_0) + f_{10}(\zeta), \tag{6}$$

其中 $$f_{10}(\zeta) = O\left(\frac{1}{\zeta}\right). \tag{7}$$

复势函数在区域 S^+ 内必须全纯即无奇异值出现,在平面 ζ 上,已经忽略了刚体位移和等势场,在环形区域内 $f_2(\zeta)$ 将被展开成 Laurent 级数:

$$f_2(\zeta) = \sum_{k=0}^{\infty} a_k\zeta^{k+1} + \sum_{k=0}^{\infty} b_k\zeta^{-k-1}. \tag{8}$$

将式(1)代入式(4),应力分量和位移可表示为

$$\tau_{rz} - \mathrm{i}\tau_{\theta z} = \frac{\mu\zeta f'(\zeta)}{|\omega'(\zeta)|\delta}. \tag{9}$$

以 L_1 为圆弧的圆形区域内定义一个辅助函数:

$$f'_*(\zeta) = -\frac{R^2}{\zeta^2}\bar{f}'\left(\frac{R^2}{\zeta}\right). \tag{10}$$

区域 1 和 2 内的辅助函数为

$$f'_{1*}(\zeta) = -\frac{R^2}{\zeta^2}\bar{f}'_1\left(\frac{R^2}{\zeta}\right) = \frac{R^2}{\zeta^2}\bar{\Gamma}\frac{l_2\rho\mathrm{i}}{2} + \frac{Q}{2\pi\mu_1}\left(\frac{1}{\zeta} - \frac{1}{\zeta - \zeta_0^*}\right) + f'_{10*}(\zeta), \tag{11}$$

$$f'_{2*}(\zeta) = -\sum_{k=0}^{\infty}\bar{a}_k(k+1)\frac{R^{2(k+1)}}{\zeta^{k+2}} + \sum_{k=0}^{\infty}\bar{b}_k(k+1)\frac{\zeta^k}{R^{2(k+1)}}, \tag{12}$$

其中 $$\zeta_0^* = \frac{R^2}{\bar{\zeta}_0}.$$

以 L_2 为圆弧的圆形区域内定义另一个辅助函数:

$$f'_{**}(\zeta) = -\frac{1}{\zeta^2}\bar{f}'\left(\frac{1}{\zeta}\right). \tag{13}$$

圆孔区域的辅助函数可表示为

$$f'_{2**}(\zeta) = -\sum_{k=0}^{\infty}\bar{a}_k(k+1)\frac{1}{\zeta^{k+2}} + \sum_{k=0}^{\infty}\bar{b}_k(k+1)\zeta^k. \tag{14}$$

界面 Γ_1 的边界条件为

$$\tau_{rz1}(t) = \tau_{rz2}(t),\ w_2(t) - w_1(t) = w^0, \qquad |t| = R, \tag{15}$$

其中

$$w^0 = z(\varepsilon_{xz}^0 - \mathrm{i}\varepsilon_{yz}^0) + \bar{z}(\varepsilon_{xz}^0 + \mathrm{i}\varepsilon_{yz}^0) = \omega(\zeta)(\varepsilon_{xz}^0 - \mathrm{i}\varepsilon_{yz}^0) + \overline{\omega(\zeta)}(\varepsilon_{xz}^0 + \mathrm{i}\varepsilon_{yz}^0), \tag{16}$$

ε_{xz}^0 和 ε_{yz}^0 是加强层的本征应变.

定义新的应力函数:

$$F'(\zeta)=\zeta f'(\zeta),\ F'_*(\zeta)=\zeta f'_*(\zeta),\ F'_{**}(\zeta)=\zeta f'_{**}(\zeta). \tag{17}$$

此时,应力边界条件可表示为

应力连续

$$[\mu_2F'_2(t)+\mu_1F'_{1*}(t)]^+=[\mu_1F'_1(t)+\mu_2F'_{2*}(t)]^-; \tag{18}$$

位移跳跃

$$[f_2(t)-f_{1*}(t)]^+-[f_1(t)-f_{2*}(t)]^-=w^0. \tag{19}$$

将应力函数和辅助函数代入以上两个边界条件,并将奇异性主部移至方程右边,

$$[\mu_1F'_{10*}(t)]^+-[\mu_1F'_{10}(t)]^-=-\mu_1\frac{R^2}{t}\bar{\Gamma}\frac{l_2\rho\mathrm{i}}{2}+\mu_1\Gamma\frac{l_2\rho\mathrm{i}}{2}t-\mu_1\frac{Q}{2\pi\mu_1}\left(1-\frac{t}{t-\zeta_0^*}\right)-$$

$$\mu_1\frac{Q}{2\pi\mu_1}\left(\frac{t}{t-\zeta_0}-1+1\right)-\mu_2F'_2(t)+\mu_2F'_{2*}(t)=A(t), \tag{20}$$

$$[-f_{10*}(t)]^+-[f_{10}(t)]^-=-\frac{R^2}{t}\bar{\Gamma}\frac{l_2\rho\mathrm{i}}{2}+\Gamma\frac{l_2\rho\mathrm{i}}{2}t+\frac{Q}{2\pi\mu_1}[\ln t-\ln(t-\zeta_0^*)]-$$

$$\frac{Q}{2\pi\mu_1}\ln(t-\zeta_0)-f_2(t)-f_{2*}(t)+X(t)(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+$$

$$\bar{X}(R^2/t)(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0)=B(t), \tag{21}$$

其中

$$X(t)=\frac{l_2\rho\mathrm{i}}{2}\left\{t+\frac{m}{t}-\frac{1}{2\rho^2}\left(\frac{1}{t+\mathrm{i}\sqrt{m}}+\frac{1}{t-\mathrm{i}\sqrt{m}}\right)\right\}, \tag{22}$$

$$\bar{X}\left(\frac{R^2}{t}\right)=-\frac{l_2\rho\mathrm{i}}{2}\left\{\frac{R^2}{t}+\frac{mt}{R^2}-\frac{R^2}{2\rho^2m}\left(\frac{1}{t+\mathrm{i}R^2/\sqrt{m}}+\frac{1}{t-\mathrm{i}R^2/\sqrt{m}}\right)\right\}. \tag{23}$$

运用 Cauchy 积分和留数定理可以求解以上的边值问题.

$$\mu_1F'_{10*}(\zeta)=\frac{1}{2\pi\mathrm{i}}\oint_{|\zeta|=R}\frac{A(t)}{t-\zeta}\mathrm{d}t+c,\qquad \zeta\in S^+, \tag{24}$$

$$\mu_1F'_{10}(\zeta)=-\frac{1}{2\pi\mathrm{i}}\oint_{|\zeta|=R}\frac{A(t)}{t-\zeta}\mathrm{d}t+c,\qquad \zeta\in S^-, \tag{25}$$

$$-f_{10*}(t)=\frac{1}{2\pi\mathrm{i}}\oint_{|\zeta|=R}\frac{B(t)}{t-\zeta}\mathrm{d}t+d,\qquad \zeta\in S^+, \tag{26}$$

$$f_{10}(\zeta)=\frac{1}{2\pi\mathrm{i}}\oint_{|\zeta|=R}\frac{B(t)}{t-\zeta}\mathrm{d}t+d,\qquad \zeta\in S^-. \tag{27}$$

考虑到 $F'_{10}(\zeta)$ 和 $f_{10}(\zeta)$ 在无穷远为 0,可得 $c=d=0$. 方程(24)~(27)可变为

$$\mu_1F'_{10*}(\zeta)=\mu_1\Gamma\frac{l_2\rho\mathrm{i}}{2}\zeta-\mu_1\frac{Q}{2\pi\mu_1}\left(\frac{\zeta}{\zeta-\zeta_0}-1+1\right)-$$

$$\mu_2\sum_{k=0}^{\infty}a_k(k+1)\zeta^{k+1}+\mu_2\sum_{k=0}^{\infty}\bar{b}_k(k+1)\frac{\zeta^{k+1}}{R^{2(k+1)}},\qquad |\zeta|\leqslant R, \tag{28}$$

$$\mu_1F'_{10}(\zeta)=\mu_1\frac{R^2}{\zeta}\bar{\Gamma}\frac{l_2\rho\mathrm{i}}{2}+\mu_1\frac{Q}{2\pi\mu_1}\left(1-\frac{\zeta}{\zeta-\zeta_0^*}\right)+$$

$$\mu_2\sum_{k=0}^{\infty}\bar{a}_k(k+1)\frac{R^{2(k+1)}}{\zeta^{k+1}}+\mu_2\sum_{k=0}^{\infty}b_k(-k-1)\zeta^{-k-1},\qquad |\zeta|\geqslant R. \tag{29}$$

注意 $|\pm \mathrm{i}\sqrt{m}|<1$, $|\pm \mathrm{i}R^2/\sqrt{m}|>R$,

$$f_{10}(\zeta)=\frac{R^2}{\zeta}\bar{\Gamma}\frac{l_2\rho \mathrm{i}}{2}-\frac{Q}{2\pi\mu_1}[\ln\zeta-\ln(\zeta-\zeta_0^*)]+\sum_{k=0}^{\infty}b_k\zeta^{-k-1}+\sum_{k=0}^{\infty}\bar{a}_k\frac{R^{2(k+1)}}{\zeta^{k+1}}-\frac{l_2\rho \mathrm{i}}{2}\left\{\frac{m}{\zeta}-\frac{1}{2\rho^2}\left(\frac{1}{\zeta+\mathrm{i}\sqrt{m}}+\frac{1}{\zeta-\mathrm{i}\sqrt{m}}\right)\right\}(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+\frac{l_2\rho \mathrm{i}}{2}\frac{R^2}{\zeta}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0),\quad |\zeta|\geqslant R, \tag{30}$$

$$f_{10*}(\zeta)=-\Gamma\frac{l_2\rho \mathrm{i}}{2}\zeta+\frac{Q}{2\pi\mu_1}\ln(\zeta-\zeta_0)+\sum_{k=0}^{\infty}a_k\zeta^{k+1}+\sum_{k=0}^{\infty}\bar{b}_k\frac{\zeta^{k+1}}{R^{2(k+1)}}-\frac{l_2\rho \mathrm{i}}{2}\zeta(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+\frac{l_2\rho \mathrm{i}}{2}\left\{\frac{m\zeta}{R^2}-\frac{R^2}{2\rho^2 m}\left(\frac{1}{\zeta+\mathrm{i}R^2/\sqrt{m}}+\frac{1}{\zeta-\mathrm{i}R^2/\sqrt{m}}\right)\right\}\times(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0),\quad |\zeta|\leqslant R. \tag{31}$$

将应力函数(30)和(31)转变为新形式:

$$F'_{10}(\zeta)=-\frac{R^2}{\zeta}\bar{\Gamma}\frac{l_2\rho \mathrm{i}}{2}-\frac{Q}{2\pi\mu_1}\left[1-\frac{\zeta}{\zeta-\zeta_0^*}\right]+\sum_{k=0}^{\infty}b_k(-k-1)\zeta^{-k-1}+\sum_{k=0}^{\infty}\bar{a}_k(-k-1)\frac{R^{2(k+1)}}{\zeta^{k+1}}-\frac{l_2\rho \mathrm{i}}{2}\left\{-\frac{m}{\zeta}-\frac{1}{2\rho^2}\sum_{k=0}^{\infty}\left((-\mathrm{i}\sqrt{m})^k+(\mathrm{i}\sqrt{m})^k\right)\frac{(-k-1)}{\zeta^{k+1}}\right\}(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)-\frac{l_2\rho \mathrm{i}}{2}\frac{R^2}{\zeta}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0),\quad |\zeta|\geqslant R, \tag{32}$$

$$F'_{10*}(\zeta)=-\Gamma\frac{l_2\rho \mathrm{i}}{2}\zeta+\frac{Q}{2\pi\mu_1}\frac{\zeta}{\zeta-\zeta_0}+\sum_{k=0}^{\infty}a_k(k+1)\zeta^{k+1}+\sum_{k=0}^{\infty}\bar{b}_k(k+1)\frac{\zeta^{k+1}}{R^{2(k+1)}}-\frac{l_2\rho \mathrm{i}}{2}\zeta(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+\frac{l_2\rho \mathrm{i}}{2}\left\{\frac{m\zeta}{R^2}-\frac{R^2}{2\rho^2 m}\sum_{k=0}^{\infty}\left((-1)^k-1\right)\frac{k}{(\mathrm{i}R^2/\sqrt{m})^{k+1}}\zeta^k\right\}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0),\quad |\zeta|\leqslant R, \tag{33}$$

联立方程(29)和方程(32),得

$$\sum_{k=0}^{\infty}\left[\left(\frac{\mu_1+\mu_2}{\mu_1}\right)\bar{a}_k(k+1)R^{2(k+1)}-\left(\frac{\mu_2-\mu_1}{\mu_1}\right)b_k(k+1)\right]\frac{1}{\zeta^{k+1}}+\frac{R^2}{\zeta}\bar{\Gamma}l_2\rho \mathrm{i}+\frac{Q}{\pi\mu_1}\left(1-\frac{\zeta}{\zeta-\zeta_0^*}\right)+\frac{l_2\rho \mathrm{i}}{2}\left\{-\frac{m}{\zeta}-\frac{1}{2\rho^2}\sum_{k=0}^{\infty}\left((-\mathrm{i}\sqrt{m})^k+(\mathrm{i}\sqrt{m})^k\right)\frac{(-k-1)}{\zeta^{k+1}}\right\}(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+\frac{l_2\rho \mathrm{i}}{2}\frac{R^2}{\zeta}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0)=0,\quad |\zeta|\geqslant R. \tag{34}$$

联立方程(28)和方程(33),得

$$\sum_{k=0}^{\infty}\left[\left(\frac{\mu_1+\mu_2}{\mu_1}\right)a_k(k+1)-\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\frac{\bar{b}_k(k+1)}{R^{2(k+1)}}\right]\zeta^{k+1}-\Gamma l_2\rho \mathrm{i}\zeta+\frac{Q}{\pi\mu_1}\left(\frac{\zeta}{\zeta-\zeta_0}\right)-\frac{l_2\rho \mathrm{i}}{2}\zeta(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+$$

$$\frac{l_2\rho\mathrm{i}}{2}\left\{\frac{m\zeta}{R^2}-\frac{R^2}{2\rho^2 m}\sum_{k=0}^{\infty}((-1)^k-1)\frac{k}{(\mathrm{i}R^2/\sqrt{m})^{k+1}}\zeta^k\right\}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0)=0,\quad |\zeta|\leqslant R. \tag{35}$$

基于只有找到 a_k 和 b_k 的系数关系才能将其解出,考虑边界 Γ_2 的边界条件

$$\tau_{rz2}(t)=0,\qquad |t|=1. \tag{36}$$

自由边界条件可转化为

$$[\mu_2 F_2'(t)]^+-[\mu_2 F_{2**}'(t)]^-=0. \tag{37}$$

将应力函数和辅助函数代入以上边界条件,并将奇异性主部移至方程右边,

$$0^+-0^-=-\mu_2 F_2'(t)+\mu_2 F_{2**}'(t). \tag{38}$$

用以上边值问题方法,可得

$$0=\mu_2\sum_{k=0}^{\infty}a_k(k+1)\zeta^{k+1}-\mu_2\sum_{k=0}^{\infty}\bar{b}_k(k+1)\zeta^{k+1},\qquad |\zeta|\leqslant 1, \tag{39}$$

$$0=-\mu_2\sum_{k=0}^{\infty}\bar{a}_k(k+1)\frac{1}{\zeta^{k+1}}-\mu_2\sum_{k=0}^{\infty}b_k(-k-1)\zeta^{-k-1},\qquad |\zeta|\geqslant 1. \tag{40}$$

以上方程的解为

$$a_k=\bar{b}_k. \tag{41}$$

考虑到此系数关系和方程(34)和(35),并注意在区域 Ω_1 中 Laurent 级数展开:

$$\frac{Q}{\pi\mu_1}\left(1-\frac{\zeta}{\zeta-\zeta_0^*}\right)=-\frac{Q}{\pi\mu_1}\sum_{k=0}^{\infty}\left(\frac{\zeta_0^*}{\zeta}\right)^{k+1}$$

和在区域 Ω_2 中 Taylor 级数展开:

$$\frac{Q}{\pi\mu_1}\left(\frac{\zeta}{\zeta-\zeta_0}\right)=-\frac{Q}{\pi\mu_1}\sum_{k=0}^{\infty}\left(\frac{\zeta}{\zeta_0}\right)^{k+1}.$$

应力函数级数形式中的系数可求得如下:

$$b_0=\frac{-R^2\bar{\Gamma}l_2\rho\mathrm{i}+\dfrac{Q\zeta_0^*}{\pi\mu_1}-\dfrac{l_2\rho\mathrm{i}}{2}\left\{-m+\dfrac{1}{\rho^2}\right\}(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)-\dfrac{l_2\rho\mathrm{i}}{2}R^2(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0)}{\left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)R^2-\left(\dfrac{\mu_2-\mu_1}{\mu_1}\right)}, \tag{42}$$

$$a_0=\frac{R^2\Gamma l_2\rho\mathrm{i}+\dfrac{Q\bar{\zeta}_0^*}{\pi\mu_1}+\dfrac{l_2\rho\mathrm{i}}{2}\left\{-m+\dfrac{1}{\rho^2}\right\}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0)+\dfrac{l_2\rho\mathrm{i}}{2}R^2(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)}{\left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)R^2-\left(\dfrac{\mu_2-\mu_1}{\mu_1}\right)}, \tag{43}$$

$$b_k=\frac{\dfrac{Q\zeta_0^{*(k+1)}}{\pi\mu_1}-\dfrac{l_2\rho\mathrm{i}}{2}\left\{-\dfrac{1}{2\rho^2}((-\mathrm{i}\sqrt{m})^k+(\mathrm{i}\sqrt{m})^k)(-k-1)\right\}(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)}{\left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)(k+1)R^{2(k+1)}-\left(\dfrac{\mu_2-\mu_1}{\mu_1}\right)(k+1)},\quad k\geqslant 1, \tag{44}$$

$$a_k=\frac{\dfrac{Q\bar{\zeta}_0^{*(k+1)}}{\pi\mu_1}+\dfrac{l_2\rho\mathrm{i}}{2}\left\{-\dfrac{1}{2\rho^2}((\mathrm{i}\sqrt{m})^k+(-\mathrm{i}\sqrt{m})^k)(-k-1)\right\}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0)}{\left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)(k+1)R^{2(k+1)}-\left(\dfrac{\mu_2-\mu_1}{\mu_1}\right)(k+1)},\quad k\geqslant 1. \tag{45}$$

将以上 4 个系数代入方程(6)和(8)可得应力函数。

2.2 集中荷载作用在增强层中

由文献[11-12],解析函数 $f_1(z)$ 和 $f_2(z)$ 可表示为

$$f_1(z)=\Gamma z-\frac{Q}{2\pi\mu_1}\ln(z)+f_{10}(z),\qquad z\in S^-, \tag{46}$$

$$f_2(z)=-\frac{Q}{2\pi\mu_2}\ln(z-z_0)+f_{20}(z),\qquad z\in S^+. \tag{47}$$

将方程(3)代入方程(46)和(47),可得像平面 ζ 中的应力函数:

$$f_1(\zeta)=\Gamma\frac{l_2\rho\mathrm{i}}{2}\zeta-\frac{Q}{2\pi\mu_1}(\ln(\zeta-\zeta_1)+\ln(\zeta-\zeta_2)+\ln(\zeta-\zeta_3)+\ln(\zeta-\zeta_4)-\ln(\zeta-\zeta_5)-\ln(\zeta-\zeta_6)-\ln(\zeta))+f_{10}(\zeta), \tag{48}$$

其中

$$\zeta_1=\frac{-1+\sqrt{1-4\rho^2m}}{2\rho},\ \zeta_2=\frac{-1-\sqrt{1-4\rho^2m}}{2\rho},\ \zeta_3=\frac{1+\sqrt{1-4\rho^2m}}{2\rho},$$

$$\zeta_4=\frac{1-\sqrt{1-4\rho^2m}}{2\rho},\ \zeta_5=\mathrm{i}\sqrt{m},\ \zeta_6=-\mathrm{i}\sqrt{m},\qquad |\zeta_j|<1\ (j=1,2,\cdots,6).$$

$$f_2(\zeta)=-\frac{Q}{2\pi\mu_2}\ln(\zeta-\zeta_0)+\sum_{k=0}^{\infty}a_k\zeta^{k+1}+\sum_{k=0}^{\infty}b_k\zeta^{-k-1}. \tag{49}$$

以 L_1 为圆弧的圆形区域内定义一个辅助函数:

$$f'_*(\zeta)=-\frac{R^2}{\zeta^2}\bar{f}'\left(\frac{R^2}{\zeta}\right). \tag{50}$$

区域 1 和 2 内的辅助函数为

$$f'_{1*}(\zeta)=-\frac{R^2}{\zeta^2}\bar{f}'_1\left(\frac{R^2}{\zeta}\right)=\frac{R^2}{\zeta^2}\bar{\Gamma}\frac{l_2\rho\mathrm{i}}{2}+\frac{Q}{2\pi\mu_1}\left(\frac{1}{\zeta}-\frac{1}{\zeta-\zeta_1^*}-\frac{1}{\zeta-\zeta_2^*}-\frac{1}{\zeta-\zeta_3^*}-\frac{1}{\zeta-\zeta_4^*}+\frac{1}{\zeta-\zeta_5^*}+\frac{1}{\zeta-\zeta_6^*}\right)+f'_{10*}(\zeta), \tag{51}$$

其中 $\zeta_j^*=\dfrac{R^2}{\bar{\zeta}_j}$,

$$f'_{2*}(\zeta)=\frac{Q}{2\pi\mu_2}\left(\frac{1}{\zeta}-\frac{1}{\zeta-\zeta_2^*}\right)-\sum_{k=0}^{\infty}\bar{a}_k(k+1)\frac{R^{2(k+1)}}{\zeta^{k+2}}+\sum_{k=0}^{\infty}\bar{b}_k(k+1)\frac{\zeta^k}{R^{2(k+1)}}. \tag{52}$$

以 L_2 为圆弧的圆形区域内定义另一个辅助函数:

$$f'_{**}(\zeta)=-\frac{1}{\zeta^2}\bar{f}'\left(\frac{1}{\zeta}\right). \tag{53}$$

圆孔区域的辅助函数可表示为

$$f'_{2**}(\zeta)=\frac{Q}{2\pi\mu_1}\left(\frac{1}{\zeta}-\frac{1}{\zeta-\zeta_0^{**}}\right)-\sum_{k=0}^{\infty}\bar{a}_k(k+1)\frac{1}{\zeta^{k+2}}+\sum_{k=0}^{\infty}\bar{b}_k(k+1)\zeta^k, \tag{54}$$

其中 $\zeta_0^{**}=\dfrac{1}{\bar{\zeta}_0}$。

界面 Γ_1 的边界条件为

$$\tau_{rz1}(t)=\tau_{rz2}(t),\ w_2(t)-w_1(t)=w^0,\qquad |t|=R. \tag{55}$$

定义新的应力函数:

$$F'(\zeta)=\zeta f'(\zeta),\ F'_*(\zeta)=\zeta f'_*(\zeta),\ F'_{**}(\zeta)=\zeta f'_{**}(\zeta). \tag{56}$$

此时,应力条件可表示为

应力连续

$$[\mu_2 F'_2(t)+\mu_1 F'_{1*}(t)]^+=[\mu_1 F'_1(t)+\mu_2 F'_{2*}(t)]^-; \tag{57}$$

位移跳跃

$$[f_2(t)-f_{1*}(t)]^+-[f_1(t)-f_{2*}(t)]^-=w^0. \tag{58}$$

将应力函数和辅助函数代入以上两个边界条件,并将奇性主部移至方程右边,

$$\begin{aligned}&[\mu_1 F'_{10*}(t)]^+-[\mu_1 F'_{10}(t)]^-=-\mu_1\frac{R^2}{t}\bar{\Gamma}\frac{l_2\rho \mathrm{i}}{2}-\\&\mu_1\frac{Q}{2\pi\mu_1}\left(1-\frac{t}{t-\zeta_1^*}-\frac{t}{t-\zeta_2^*}-\frac{t}{t-\zeta_3^*}-\frac{t}{t-\zeta_4^*}+\frac{t}{t-\zeta_5^*}+\frac{t}{t-\zeta_6^*}\right)+\\&\mu_1\Gamma\frac{l_2\rho \mathrm{i}}{2}t-\mu_1\frac{Q}{2\pi\mu_1}\left(\frac{t}{t-\zeta_1}+\frac{t}{t-\zeta_2}+\frac{t}{t-\zeta_3}+\frac{t}{t-\zeta_4}-\frac{t}{t-\zeta_5}-\frac{t}{t-\zeta_6}-1\right)-\\&\mu_2 F'_2(t)+\mu_2 F'_{2*}(t)=C(t),\end{aligned} \tag{59}$$

$$\begin{aligned}&[-f_{10*}(t)]^+-[f_{10}(t)]^-=-\frac{R^2}{t}\bar{\Gamma}\frac{l_2\rho \mathrm{i}}{2}+\frac{Q}{2\pi\mu_1}(\ln t-\ln(t-\zeta_1^*)-\\&\ln(t-\zeta_2^*)-\ln(t-\zeta_3^*)-\ln(t-\zeta_4^*)+\ln(t-\zeta_5^*)+\ln(t-\zeta_6^*))+\\&\Gamma\frac{l_2\rho \mathrm{i}}{2}t-\frac{Q}{2\pi\mu_1}(\ln(t-\zeta_1)+\ln(t-\zeta_2)+\ln(t-\zeta_3)+\ln(t-\zeta_4)-\\&\ln(t-\zeta_5)-\ln(t-\zeta_6)-\ln(t))-f_2(t)-f_{2*}(t)+\omega(t)(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+\\&\bar{\omega}\left(\frac{R^2}{t}\right)(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0)=D(t).\end{aligned} \tag{60}$$

同样可以用 Cauchy 积分和留数定理解决以上的边值问题,区域 1 中的解析函数可表示为

$$\begin{aligned}&\mu_1 F'_{10}(\zeta)=\mu_1\frac{R^2}{\zeta}\bar{\Gamma}\frac{l_2\rho \mathrm{i}}{2}+\\&\mu_1\frac{Q}{2\pi\mu_1}\left(\frac{\zeta}{\zeta-\zeta_1}+\frac{\zeta}{\zeta-\zeta_2}+\frac{\zeta}{\zeta-\zeta_3}+\frac{\zeta}{\zeta-\zeta_4}-\frac{\zeta}{\zeta-\zeta_5}-\frac{\zeta}{\zeta-\zeta_6}-2\right)-\\&\mu_2\frac{Q}{2\pi\mu_2}\left(\frac{\zeta}{\zeta-\zeta_0}-1\right)+\mu_2\sum_{k=0}^{\infty}\bar{a}_k(k+1)\frac{R^{2(k+1)}}{\zeta^{k+1}}+\mu_2\sum_{k=0}^{\infty}b_k(-k-1)\zeta^{-k-1},\\&\qquad|\zeta|\geqslant R,\end{aligned} \tag{61}$$

$$\begin{aligned}&f_{10}(\zeta)=\frac{R^2}{\zeta}\bar{\Gamma}\frac{l_2\rho \mathrm{i}}{2}+\frac{Q}{2\pi\mu_1}(\ln(\zeta-\zeta_1)+\ln(\zeta-\zeta_2)+\\&\ln(\zeta-\zeta_3)+\ln(\zeta-\zeta_4)-\ln(\zeta-\zeta_5)-\ln(\zeta-\zeta_6)-2\ln(\zeta))-\\&\frac{Q}{2\pi\mu_2}\ln(\zeta-\zeta_0)+\frac{Q}{2\pi\mu_2}\ln\zeta+\sum_{k=0}^{\infty}b_k\zeta^{-k-1}+\sum_{k=0}^{\infty}\bar{a}_k\frac{R^{2(k+1)}}{\zeta^{k+1}}-\\&\frac{l_2\rho \mathrm{i}}{2}\left\{\frac{m}{\zeta}-\frac{1}{2\rho^2}\left(\frac{1}{\zeta+\mathrm{i}\sqrt{m}}+\frac{1}{\zeta-\mathrm{i}\sqrt{m}}\right)\right\}(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+\\&\frac{l_2\rho \mathrm{i}}{2}\frac{R^2}{\zeta}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0),\qquad|\zeta|\geqslant R,\end{aligned} \tag{62}$$

将应力函数(62)转变为新形式如下：

$$F'_{10}(\zeta)=-\frac{R^2}{\zeta}\bar{\Gamma}\frac{l_2\rho\mathrm{i}}{2}+\frac{Q}{2\pi\mu_1}\left(\frac{\zeta}{\zeta-\zeta_1}+\frac{\zeta}{\zeta-\zeta_2}+\frac{\zeta}{\zeta-\zeta_3}+\frac{\zeta}{\zeta-\zeta_4}-\frac{\zeta}{\zeta-\zeta_5}-\frac{\zeta}{\zeta-\zeta_6}-2\right)-\frac{Q}{2\pi\mu_2}\left(\frac{\zeta}{\zeta-\zeta_0}-1\right)+\sum_{k=0}^{\infty}b_k(-k-1)\zeta^{-k-1}+\sum_{k=0}^{\infty}\bar{a}_k(-k-1)\frac{R^{2(k+1)}}{\zeta^{k+1}}-\frac{l_2\rho\mathrm{i}}{2}\left\{-\frac{m}{\zeta}-\frac{1}{2\rho^2}\sum_{k=0}^{\infty}((-\mathrm{i}\sqrt{m})^k+(\mathrm{i}\sqrt{m})^k)\frac{(-k-1)}{\zeta^{k+1}}\right\}(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)-\frac{l_2\rho\mathrm{i}}{2}\frac{R^2}{\zeta}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0),\qquad |\zeta|\geqslant R. \tag{63}$$

联立方程(60)和方程(62)，可得

$$\sum_{k=0}^{\infty}\left[\left(\frac{\mu_1+\mu_2}{\mu_1}\right)\bar{a}_k(k+1)R^{2(k+1)}-\left(\frac{\mu_2-\mu_1}{\mu_1}\right)b_k(k+1)\right]\frac{1}{\zeta^{k+1}}+\frac{R^2}{\zeta}\bar{\Gamma}l_2\rho\mathrm{i}+\frac{\mu_1-\mu_2}{\mu_1}\frac{Q}{2\pi\mu_2}\left(\frac{\zeta}{\zeta-\zeta_0}-1\right)+\frac{l_2\rho\mathrm{i}}{2}\left\{-\frac{m}{\zeta}-\frac{1}{2\rho^2}\sum_{k=0}^{\infty}((-\mathrm{i}\sqrt{m})^k+(\mathrm{i}\sqrt{m})^k)\frac{(-k-1)}{\zeta^{k+1}}\right\}(\varepsilon_{xz}^0-\mathrm{i}\varepsilon_{yz}^0)+\frac{l_2\rho\mathrm{i}}{2}\frac{R^2}{\zeta}(\varepsilon_{xz}^0+\mathrm{i}\varepsilon_{yz}^0)=0,\qquad |\zeta|\geqslant R. \tag{64}$$

为找到系数 a_k 和 b_k 的关系，可通过边界 Γ_2 的边界条件求得

$$\tau_{rz2}(t)=0,\qquad |t|=1. \tag{65}$$

将自由边界条件转变为

$$[\mu_2F'_2(t)]^+-[\mu_2F'_{2**}(t)]^-=0. \tag{66}$$

将应力函数和辅助函数代入以上两个边界条件，并将奇性主部移至方程右边，

$$0^+-0^-=-\mu_2F'_2(t)+\mu_2F'_{2**}(t). \tag{67}$$

用同样的方法可得

$$0=\mu_2\sum_{k=0}^{\infty}a_k(k+1)\zeta^{k+1}-\mu_2\sum_{k=0}^{\infty}\bar{b}_k(k+1)\zeta^{k+1}-\mu_2\frac{Q}{2\pi\mu_2}\frac{\zeta}{\zeta-\zeta_0},\qquad |\zeta|\leqslant 1, \tag{68}$$

$$0=-\mu_2\sum_{k=0}^{\infty}\bar{a}_k(k+1)\frac{1}{\zeta^{k+1}}-\mu_2\sum_{k=0}^{\infty}b_k(-k-1)\zeta^{-k-1}+\mu_2\frac{Q}{2\pi\mu_2}\left(1-\frac{\zeta}{\zeta-\zeta_0^{**}}\right),\qquad |\zeta|\geqslant 1. \tag{69}$$

以上方程的解为

$$(a_k-\bar{b}_k)(k+1)=-\frac{Q}{2\pi\mu_2}\left(\frac{1}{\zeta_0}\right)^{k+1}, \tag{70}$$

$$(\bar{a}_k-b_k)(k+1)=-\frac{Q}{2\pi\mu_2}(\zeta_0^{**})^{k+1}. \tag{71}$$

将方程(71)代入方程(64)，并在区域1中 Laurent 级数展开为

$$-\frac{Q}{2\pi\mu_2}\left(\frac{\zeta}{\zeta-\zeta_0}-1\right)=-\frac{Q}{2\pi\mu_2}\sum_{k=0}^{\infty}\left(\frac{\zeta_0}{\zeta}\right)^{k+1},$$

可得

$$b_0 = \frac{-R^2\bar{\Gamma}l_2\rho\mathrm{i} - \dfrac{\mu_1-\mu_2}{\mu_1}\dfrac{Q}{2\pi\mu_2}\zeta_0 + \left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)R^2\dfrac{Q}{2\pi\mu_2}\zeta_0^{**}}{\left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)R^2 - \left(\dfrac{\mu_2-\mu_1}{\mu_1}\right)} + \frac{-\dfrac{l_2\rho\mathrm{i}}{2}\left\{-m+\dfrac{1}{\rho^2}\right\}(\varepsilon_{xz}^0 - \mathrm{i}\varepsilon_{yz}^0) - \dfrac{l_2\rho\mathrm{i}}{2}R^2(\varepsilon_{xz}^0 + \mathrm{i}\varepsilon_{yz}^0)}{\left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)R^2 - \left(\dfrac{\mu_2-\mu_1}{\mu_1}\right)}, \tag{72}$$

$$a_0 = -\frac{Q}{2\pi\mu_2}\frac{1}{\zeta_0} + \bar{b}_0, \tag{73}$$

$$b_k = \frac{-\dfrac{\mu_1-\mu_2}{\mu_1}\dfrac{Q}{2\pi\mu_2}\zeta_0^{k+1} + \left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)R^{2(k+1)}\dfrac{Q}{2\pi\mu_2}(\zeta_0^{**})^{(k+1)}}{\left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)(k+1)R^{2(k+1)} - \left(\dfrac{\mu_2-\mu_1}{\mu_1}\right)(k+1)} + \frac{-\dfrac{l_2\rho\mathrm{i}}{2}\left\{-\dfrac{1}{2\rho^2}\left((-\mathrm{i}\sqrt{m})^k + (\mathrm{i}\sqrt{m})^k\right)(-k-1)\right\}(\varepsilon_{xz}^0 - \mathrm{i}\varepsilon_{yz}^0)}{\left(\dfrac{\mu_1+\mu_2}{\mu_1}\right)(k+1)R^{2(k+1)} - \left(\dfrac{\mu_2-\mu_1}{\mu_1}\right)(k+1)}, \tag{74}$$

$$a_k = -\frac{Q}{(k+1)2\pi\mu_2}\left(\frac{1}{\zeta_0}\right)^{k+1} + \bar{b}_k, \qquad k \geqslant 1. \tag{75}$$

将以上 4 个系数代入方程(6)和(8)可得应力函数.

3　数值分析

在界面 Γ_1 上由无穷远处荷载、集中荷载和本征应变产生的界面应力可定义为

$$\tau_{yz}(\zeta)\big|_{|\zeta|=R} = -\mathrm{Im}\left[\frac{\mu_2 f_2'(\zeta)}{\omega'(\zeta)}\right]\Bigg|_{|\zeta|=R}, \tag{76}$$

$$\tau_{xz}(\zeta)\big|_{|\zeta|=R} = \mathrm{Re}\left[\frac{\mu_2 f_2'(\zeta)}{\omega'(\zeta)}\right]\Bigg|_{|\zeta|=R}. \tag{77}$$

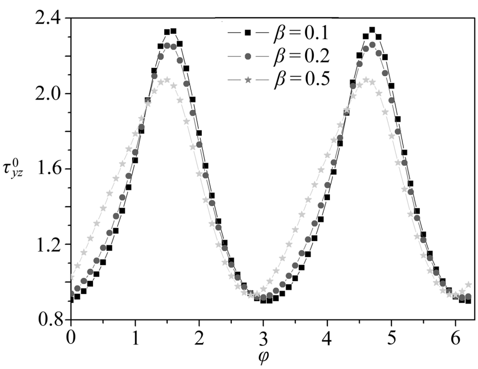


图 3　τ_{yz}^0-φ **曲线**($\eta=2, \alpha=1, R=2$, β 取不同值)

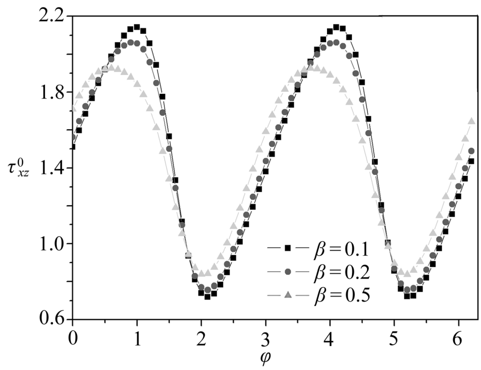


图 4　τ_{xz}^0-φ **曲线**($\eta=2, \alpha=1, R=2$, β 取不同值)

在不同参数比下,界面应力随裂纹几何尺寸变化的曲线如图 3 至图 18 所示. 所有不同形

式荷载作用产生的界面剪切应力随剪切模量比 η 的减少而减少，这是很显然的，因为临近的刚度更高或更低的材料可以集中或削弱界面应力。另外，除了本征应变产生的剪切应力 τ_{yz} 外，剪切应力会随着裂纹几何比率 β 的增大而减小。

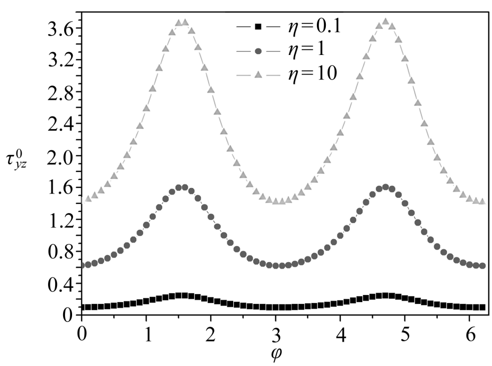


图5 τ_{yz}^0-φ 曲线（$\beta = 0.1, \alpha = 1, R = 2, \eta$ 取不同值）

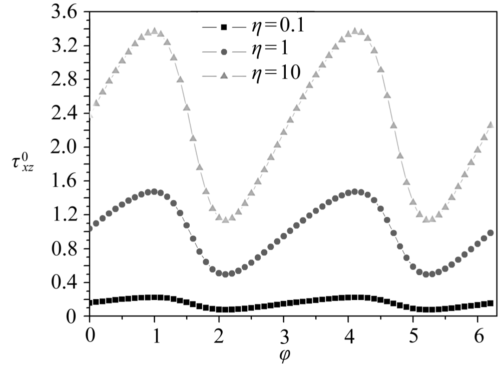


图6 τ_{xz}^0-φ 曲线（$\beta = 0.1, \alpha = 1, R = 2, \eta$ 取不同值）

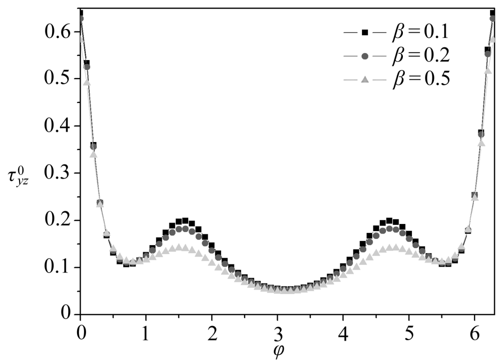


图7 τ_{yz}^0-φ 曲线（$\eta = 2, \varphi_0 = 0, r_0 = 2.5, R = 2, \beta$ 取不同值）

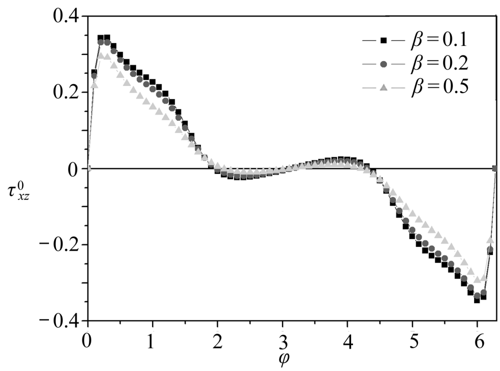


图8 τ_{xz}^0-φ 曲线（$\eta = 2, \varphi_0 = 0, r_0 = 2.5, R = 2, \beta$ 取不同值）

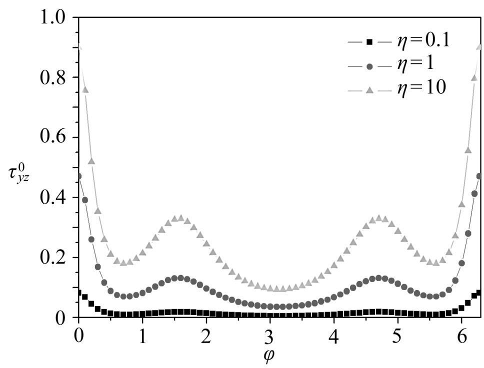


图9 τ_{yz}^0-φ 曲线（$\beta = 0.1, \varphi_0 = 0, r_0 = 2.5, R = 2, \eta$ 取不同值）

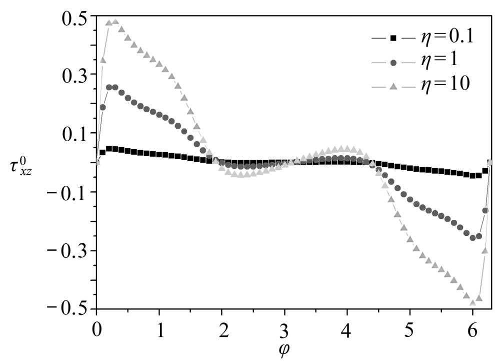


图10 τ_{xz}^0-φ 曲线（$\beta = 0.1, \varphi_0 = 0, r_0 = 2.5, R = 2, \eta$ 取不同值）

无穷远荷载产生的界面的应力可以无量纲化为 $\tau_{yz}^0 = \tau_{yz}/\tau_{yz}^\infty$ 和 $\tau_{xz}^0 = \tau_{xz}/\tau_{yz}^\infty$。在不同的参数比下，无穷远荷载产生的界面应力随着角度 φ 变化的曲线如图3至图6所示。

作用在基体上的集中荷载产生的界面应力可以无量纲化为 $\tau_{yz}^0 = \tau_{yz} l_2/Q$ 和 $\tau_{xz}^0 = \tau_{xz} l_2/Q$。

不同参数比下,作用在基体上的集中荷载产生的界面应力随角度 φ 变化的曲线如图 7 至图 10 所示. 不同的参数比下,作用在增强涂层中的集中荷载产生的界面应力随角度 φ 变化曲线如图 11 至图 14 所示.

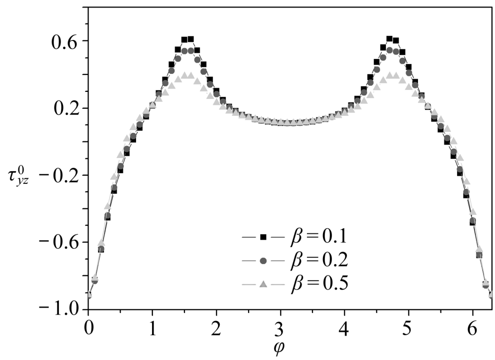


图 11　τ_{yz}^0-φ 曲线($\eta = 2, \varphi_0 = 0, r_0 = 1.1, R = 1.5$, β 取不同值)

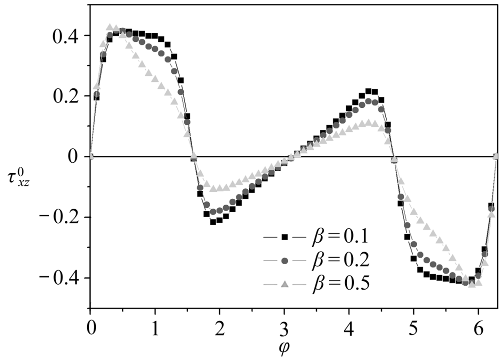


图 12　τ_{xz}^0-φ 曲线($\eta = 2, \varphi_0 = 0, r_0 = 1.1, R = 1.5$, β 取不同值)

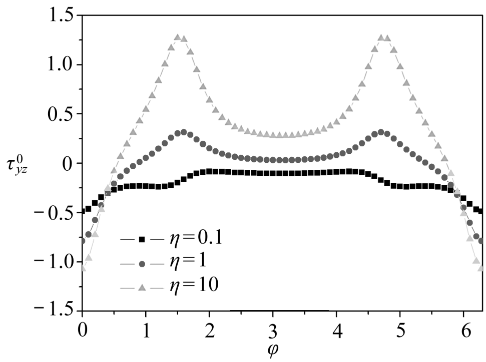


图 13　τ_{yz}^0-φ 曲线($\beta = 0.1, \varphi_0 = 0, r_0 = 1.1, R = 1.5, \eta$ 取不同值)

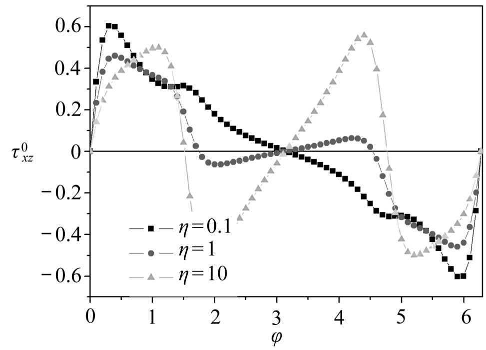


图 14　τ_{xz}^0-φ 曲线($\beta = 0.1, \varphi_0 = 0, r_0 = 1.1, R = 1.5, \eta$ 取不同值)

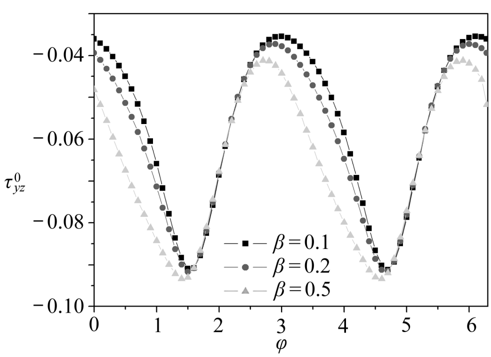


图 15　τ_{yz}^0-φ 曲线($\eta = 2, R = 2$, $\varepsilon_{yz}^0 = -0.1$, β 取不同值)

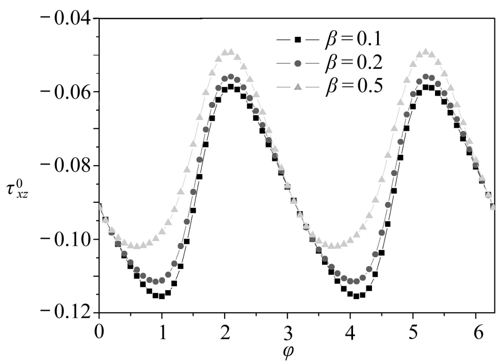


图 16　τ_{xz}^0-φ 曲线($\eta = 2, R = 2$, $\varepsilon_{yz}^0 = -0.1$, β 取不同值)

本征应变产生的界面应力可以无量纲化为 $\tau_{yz}^0 = \tau_{yz}/\mu_1$ 和 $\tau_{xz}^0 = \tau_{xz}/\mu_1$. 不同参数比下,本征应变产生的界面应力随角度 φ 变化的曲线如图 15 至图 18 所示.

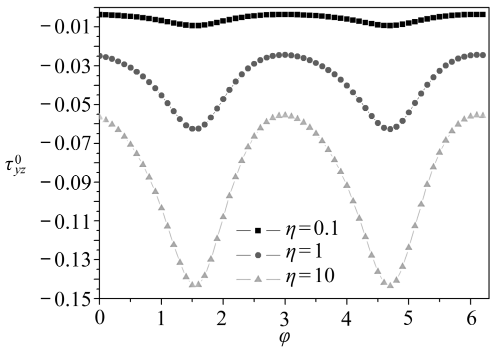


图 17 τ_{yz}^0-φ 曲线(β = 0.1, ε_{yz}^0 = −0.1, R = 2, η 取不同值)

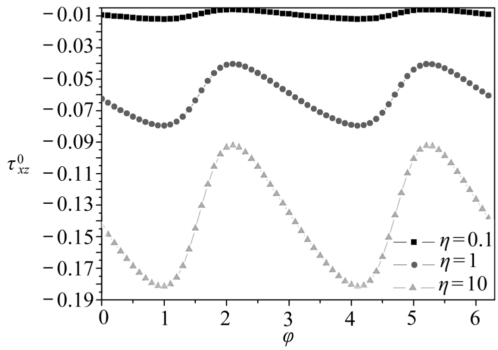


图 18 τ_{xz}^0-φ 曲线(β = 0.1, ε_{yz}^0 = −0.1, R = 2, η 取不同值)

4 结 论

本文运用保角映射及解析延拓技术,分析了Ⅲ型无穷远处荷载和集中荷载作用下,连接在唇形裂纹面上的内埋应变加强层模型,获得了涂层和裂纹界面上的界面应力数值曲线.结果表明,在所有不同的荷载作用下产生的界面剪切应力随剪切模量比 η 的减少而减少,因为周围刚度更高或更低的材料可以集中或削弱界面应力.另外,除了本征应变产生的剪切应力 τ_{yz} 外,剪切应力会随着裂纹几何比率 β 的增大而减小.

通过以上分析可知,对于不同形式的荷载,合理的材料匹配、连接及几何特征能够有效地减少应力集中和界面应力.

本文的分析方法也适用于Ⅰ型和Ⅱ型平面断裂问题.

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Analytical Solution for a Strained Reinforcement Layer Bonded to a Lip-Shaped Crack Under Remote Mode Ⅲ Uniform Load and a Concentrated Load

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Abstract: The analytical solution of stress field for a strained reinforcement layer bonded to a lip-shaped crack under a remote mode Ⅲ uniform load and a concentrated load was obtained explicitly in the series form by using the technique of conformal mapping and the method of analytic continuation. The effect of material combinations, bond of interface and geometric configurations on interfacial stresses generated by eigenstrain, remote load and concentrated load were studied. The results show that the stress concentration and interfacial stresses could be reduced by rational material combinations and geometric configurations designs for different load forms.

Key words: lip-shaped crack; strained reinforcement layer; concentrated load; remote load; eigenstrain; interfacial stress