

# 半渗透涨缩管道内微极性流动解析求解\*

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**摘要:** 分析了半渗透涨缩管道内的微极性流体的流动. 应用合适的相似变换, 将控制方程转化为常微分方程组. 为了得到该问题的解析解, 应用同伦分析方法得到该问题的速度表达式. 并且用图形分析了各个不同参数, 特别是膨胀系数对速度场和微旋转角速度的影响.

**关键词:** 同伦分析方法; 微极性流体; 涨缩壁面; 多孔管道

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## 引 言

由于在脉动隔膜模型、过滤、血液流动、人工透析、同位素分离、呼吸系统的气体 and 血液循环模型等生物物理领域的广泛应用, 可膨胀或收缩的多孔管道流动引起了科研工作者的注意. Uchida 和 Aoki<sup>[1]</sup> 首先对壁面可径向收缩、不可渗透管道内, 不可压缩流体的非稳态流动进行了研究. Ohki<sup>[2]</sup> 讨论了考虑壁面吸附喷注情况下, 半无限长、轴向随时间变化而径向不变的圆形管道中流体流动. 为了模拟固体发动机内的层流流动, Goto 和 Uchida<sup>[3]</sup> 分析了壁面随时间径向变化半无限长圆管的不可压缩流体的层流流动. Bujurke 等<sup>[4]</sup> 得到了胀缩圆形管道中非稳态流动的级数解. Majdalani 等<sup>[5]</sup> 应用相似变换, 获得了小膨胀系数和小渗透 Reynolds 数下的胀缩管道内的摄动解. Dauenhauer 和 Majdalani<sup>[6]</sup> 得到膨胀壁面可渗透管道内的数值解. Majdalani 和 Zhou<sup>[7]</sup> 求出中等到大的 Reynolds 数下的数值和渐近解.

显然, 工业和科技中很多重要的流体都体现出非 Newton 流的特征. Eringen<sup>[8-9]</sup> 首先提出了微极性流体的理论并且推导出微结构流体的本构定律. 该理论为一些具有非 Newton 行为的流体, 像聚合物、胶状悬浮液、动物血液、液晶等, 提供了数学模型. Ariman 等<sup>[10-11]</sup> 和 Eringen<sup>[12]</sup> 对微极性流体力学及其应用作了详细的论述. Ramachandran 等<sup>[13]</sup>, Takhar 等<sup>[14]</sup>, Kelson 和 Farrell<sup>[15]</sup>, Ashraf, Kamal 和 Syed<sup>[16-17]</sup> 等多人数值计算了吸附或喷注管道内的微极性流动. 然而,

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以上的所有工作很少涉及到膨胀或收缩的壁面。

本文的主要目的是对具有半渗透涨缩壁面的管道流动进行研究.应用 Liao<sup>[18-19]</sup>提出的同伦分析方法 (HAM), 计算该问题中的速度场. Hayat 等<sup>[20-22]</sup>、Abbas 等<sup>[23]</sup> 和 Sajid 等<sup>[24-25]</sup> 多人已经成功地应用 HAM 解决了一些非线性问题. 本文讨论了不同的参数, 特别是膨胀率, 对速度场和微旋转角速度的影响, 并且以图形的形式表示出来画图进行分析.

## 1 问题模型

考虑一半无限长可渗透涨缩管道中的非稳态微极性流体. 两壁面之间的距离为  $a(t)$ , 远小于其宽度和长度. 管道的一端被复杂的固体薄膜所封闭. 下壁面为静止壁面, 而上壁面渗透速度为  $v_w$ , 并且其膨胀或者收缩的速率为  $\dot{a}(t)$ . 如图 1 所示, 建立坐标系,  $x, y$  轴分别平行、垂直于下壁面.  $u, v$  和  $N$  分别表示  $x, y$  方向的速度分量和微旋转角速度.

根据以上假设, 控制方程表述如下<sup>[26]</sup>:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \\ - \frac{1}{\rho} \frac{\partial P}{\partial x} + \left( \nu + \frac{\kappa}{\rho} \right) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{\kappa}{\rho} \frac{\partial N}{\partial y}, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \\ - \frac{1}{\rho} \frac{\partial P}{\partial y} + \left( \nu + \frac{\kappa}{\rho} \right) \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\kappa}{\rho} \frac{\partial N}{\partial x}, \end{aligned} \quad (3)$$

$$\rho j \left( \frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = - \kappa \left( 2N + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) + \gamma \left( \frac{\partial^2 N}{\partial y^2} + \frac{\partial^2 N}{\partial x^2} \right), \quad (4)$$

这里,  $\rho, \nu$  为密度和粘性系数,  $j, \gamma, \kappa$  分别表示单位质量的微惯性、旋转梯度粘度、涡流粘度系数.  $\gamma$  可以表示为<sup>[27]</sup>

$$\gamma = \left( \mu + \frac{\kappa}{2} \right) j, \quad (5)$$

这里,  $\mu$  为动力粘性系数. 由于微元素在壁面附近的强聚集性, 我们假设靠近壁面的微元素处于静止状态, 没有旋转<sup>[28]</sup>. 相应的边界条件为<sup>[5-6, 16-17]</sup>:

$$u(x, a) = 0, \quad v(x, a) = v_w = A\dot{a}, \quad N(x, a) = 0, \quad (6)$$

$$v(x, 0) = u(x, 0) = 0, \quad N(x, 0) = 0, \quad (7)$$

$$v(0, y) = u(0, y) = 0, \quad (8)$$

这里,  $A = v_w / \dot{a}$  为喷注系数, 用来表征壁面的渗透强度.

定义

$$u = -\nu a^{-2} x F_\eta(\eta, t), \quad v = \nu a^{-1} F(\eta, t), \quad N = \nu a^{-3} x G(\eta, t), \quad \eta = \frac{y}{a(t)}. \quad (9)$$

将式(9)代入式(1)~(4), 可以得到微分方程组

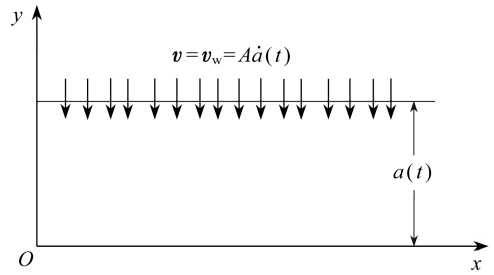


图 1 膨胀或收缩壁面的半渗透管道模型

$$(1 + K)F_{\eta\eta\eta\eta} - KG_{\eta\eta} + 3\alpha F_{\eta\eta} + \alpha\eta F_{\eta\eta\eta} + (F_{\eta}F_{\eta\eta} - FF_{\eta\eta\eta}) - \nu^{-1}a^2F_{\eta\eta t} = 0, \quad (10)$$

$$\lambda\left(\left(1 + \frac{K}{2}\right)G_{\eta\eta} + 3\alpha G + \alpha\eta G_{\eta} + (F_{\eta}G - FG_{\eta})\right) - K(2G - F_{\eta\eta}) - \lambda\nu^{-1}a^2G_t = 0, \quad (11)$$

这里,  $K = \kappa/\mu$ ,  $\lambda = j/a^2$ ,  $\alpha = a\dot{a}/\nu$  为壁面膨胀率. 当壁面膨胀时, 膨胀率为正数, 收缩时为负数.

边界条件为

$$F_{\eta} = 0, F = \frac{av_w}{\nu} = Re, G = 0, \eta = 1, \quad (12)$$

$$F_{\eta} = 0, F = 0, G = 0, \eta = 0. \quad (13)$$

令

$$f = \frac{F}{Re}, g = \frac{G}{Re}, \quad (14)$$

这里,  $Re = av_w/\nu$  为渗透 Reynolds 数. 从物理意义上讲,  $Re$  为正时, 表示吸附, 为负时表示喷注. 引入 Uchida 和 Aoki<sup>[1]</sup> 所描述的时间和空间上的相似变换, 即  $\alpha$  为常数,  $f = f(\eta)$ , 这样  $f_{\eta\eta} = 0$ . 同时我们假设  $g = g(\eta)$ . 这样原方程组(10)、(11)变为

$$(1 + K)f^{(4)} - Kg'' + 3\alpha f'' + \alpha\eta f''' + Re(f'f'' - ff''') = 0, \quad (15)$$

$$\lambda\left(\left(1 + \frac{K}{2}\right)g'' + 3\alpha g + \alpha\eta g' + Re(f'g - fg')\right) - K(2g - f'') = 0. \quad (16)$$

边界条件为<sup>[17]</sup>:

$$f'(\eta) = 0, f(\eta) = 0, g(\eta) = 0, \eta = 0, \quad (17)$$

$$f'(\eta) = 0, f(\eta) = 1, g(\eta) = 0, \eta = 1, \quad (18)$$

当  $\alpha = 0$ , 这就是 Ashraf 等<sup>[17]</sup> 曾讨论过的情况.

## 2 速度和角速度的同伦分析解

选择初始函数

$$f_0(\eta) = 3\eta^2 - 2\eta^3, g_0(\eta) = 0 \quad (19)$$

和辅助线性算子

$$\mathcal{L}_1(f) = f^{(4)}, \mathcal{L}_2(g) = g'', \quad (20)$$

辅助算子满足

$$\mathcal{L}_1(C_1 + C_2\eta + C_3\eta^2 + C_4\eta^3) = 0, \mathcal{L}_2(C_5 + C_6\eta) = 0, \quad (21)$$

这里  $C_i (i = 1, 2, \dots, 6)$  为任意常数.

由以上定义, 可得零阶形变方程

$$(1 - p)\mathcal{L}_1(\hat{f} - f_0) = ph\mathfrak{N}_1(\hat{f}, \hat{g}), \quad (22)$$

$$\hat{f}'(0, p) = 0, \hat{f}(0, p) = 0, \hat{f}'(1, p) = 0, \hat{f}(1, p) = 1, \quad (23)$$

$$(1 - p)\mathcal{L}_2(\hat{g} - g_0) = ph\mathfrak{N}_2(\hat{f}, \hat{g}), \quad (24)$$

$$\hat{g}'(0, p) = 0, \hat{g}(1, p) = 0, \quad (25)$$

这里

$$\mathfrak{N}_1(\hat{f}, \hat{g}) = (1 + K) \frac{\partial^4 \hat{f}(\eta, p)}{\partial \eta^4} + Re \left( \frac{\partial \hat{f}(\eta, p)}{\partial \eta} \frac{\partial^2 \hat{f}(\eta, p)}{\partial \eta^2} \right) -$$

$$\hat{f}(\eta, p) \frac{\partial^3 \hat{f}(\eta, p)}{\partial \eta^3} + 3\alpha \frac{\partial^2 \hat{f}(\eta, p)}{\partial \eta^2} + \alpha \eta \frac{\partial^3 \hat{f}(\eta, p)}{\partial \eta^3} - K \frac{\partial^2 \hat{g}(\eta, p)}{\partial \eta^2}, \quad (26)$$

$$\begin{aligned} \mathfrak{N}_2(\hat{f}, \hat{g}) = & \lambda \left( \left( 1 + \frac{K}{2} \right) \frac{\partial^2 \hat{g}(\eta, p)}{\partial \eta^2} + \operatorname{Re} \left( \frac{\partial \hat{f}(\eta, p)}{\partial \eta} \hat{g}(\eta, p) - \hat{f}(\eta, p) \frac{\partial \hat{g}(\eta, p)}{\partial \eta} \right) + \right. \\ & \left. 3\alpha \hat{g}(\eta, p) + \alpha \eta \frac{\partial \hat{g}(\eta, p)}{\partial \eta} \right) - K \left( 2\hat{g}(\eta, p) - \frac{\partial^2 \hat{f}(\eta, p)}{\partial \eta^2} \right), \end{aligned} \quad (27)$$

$p \in [0, 1]$  为嵌入变量,  $h$  为非零辅助参数. 当  $p$  由 0 连续变到 1,  $\hat{f}(\eta, p)$ ,  $\hat{g}(\eta, p)$  分别由初始函数  $f_0(\eta)$ ,  $g_0(\eta)$  变成  $f(\eta)$ ,  $g(\eta)$ . 应用 Taylor 公式,  $\hat{f}(\eta, p)$ ,  $\hat{g}(\eta, p)$  可以写为

$$\hat{f}(\eta, p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m, \quad f_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \hat{f}(\eta, p)}{\partial p^m} \right|_{p=0}, \quad (28)$$

$$\hat{g}(\eta, p) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta) p^m, \quad g_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \hat{g}(\eta, p)}{\partial p^m} \right|_{p=0}, \quad (29)$$

这两个级数的收敛性强烈依赖于  $h$ . 一旦选定了合适的  $h$ , 级数(28)和(29)在  $p=1$  收敛. 由级数(28)和(29)可得

$$f = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \quad (30)$$

$$g = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta). \quad (31)$$

对方程(22)和(24)关于  $p$  求  $m$  次导数, 然后令  $p=0$ , 除以  $m!$ , 可以得到  $m$  阶形变方程

$$\mathcal{L}_1(f_m(\eta) - \chi_m f_{m-1}(\eta)) = h \mathfrak{N}_m^f(\eta), \quad (32)$$

$$f'_m(0) = 0, \quad f_m(0) = 0, \quad f'_m(1) = 0, \quad f_m(1) = 0, \quad (33)$$

$$\begin{aligned} \mathfrak{N}_m^f(\eta) = & (1 + K) f_{m-1}^{(4)} + 3\alpha f_{m-1}'' + \alpha \eta f_{m-1}''' - K g_{m-1}'' + \\ & \sum_{k=0}^{m-1} \operatorname{Re}(f'_{m-k-1} f_k'' - f_{m-k-1} f_k'''), \end{aligned} \quad (34)$$

$$\mathcal{L}_2(g_m(\eta) - \chi_m g_{m-1}(\eta)) = h \mathfrak{N}_m^g(\eta), \quad (35)$$

$$g_m(0) = 0, \quad g_m(1) = 0, \quad (36)$$

$$\begin{aligned} \mathfrak{N}_m^g(\eta) = & \lambda \left( \left( 1 + \frac{K}{2} \right) g_{m-1}'' + 3\alpha g_{m-1} + \alpha \eta g_{m-1}' \right) - \\ & K(2g_{m-1} - f_{m-1}'') + \lambda \sum_{k=0}^{m-1} \operatorname{Re}(f'_{m-k-1} g_k - f_{m-k-1} g_k'), \end{aligned} \quad (37)$$

这里

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases} \quad (38)$$

方程(32)和(35)的通解为

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 \eta + C_3 \eta^2 + C_4 \eta^3, \quad (39)$$

$$g_m(\eta) = g_m^*(\eta) + C_5 + C_6 \eta, \quad (40)$$

这里,  $f_m^*(\eta)$ ,  $g_m^*(\eta)$  表示方程(32)和(35)的特解, 利用边界条件(33)和(36), 可以确定积分常数  $C_i$  ( $i=1, 2, \dots, 6$ ). 应用 Maple, 可以很容易地对方程(32)和(35)逐次求解. 两个方程的第 1 项可以求出:

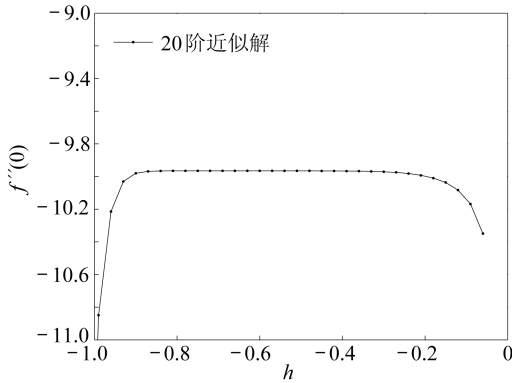
$$f_1(\eta) = \frac{h \operatorname{Re}(4\eta^7 - 14\eta^6 + 21\eta^5 + 16\eta^2 - 27\eta^3)}{70} +$$

$$\frac{h\alpha(15\eta^4 - 6\eta^3 - \eta^2 - 8\eta^5)}{20}, \quad (41)$$

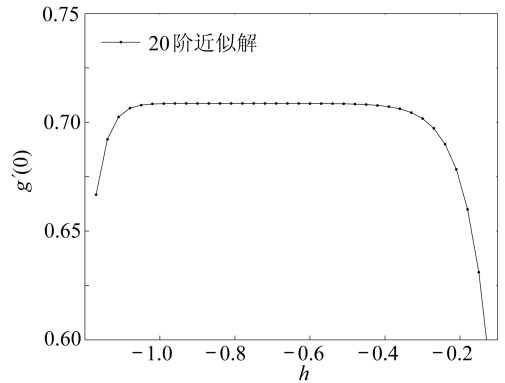
$$g_1(\eta) = \lambda hK(3\eta^2 - 2\eta^3 - \eta). \quad (42)$$

### 3 结果讨论

如 Liao<sup>[18-19]</sup> 所指出, 级数(30)和(31)的收敛性依赖于  $h$ .  $h$  的值确定了 HAM 的收敛区间和收敛速度. 图 2 为  $h$  曲线,  $h$  允许的取值范围分别为  $-0.9 < h < -0.3$  和  $-1 < h < -0.4$ .

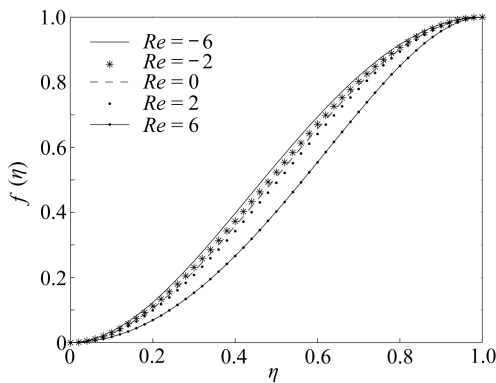


(a) 对于  $f''$

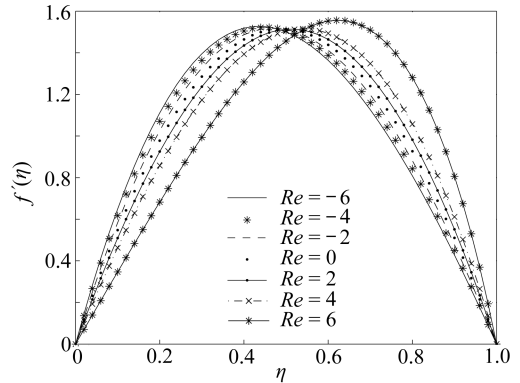


(b) 对于  $g'$

图 2 当  $\alpha = 0.5, K = 1, Re = 1, \lambda = 0.5$  时对应的  $h$  曲线



(a) 对于  $f$



(b) 对于  $f'$

图 3 当  $\alpha = 1, K = 1, \lambda = 0.5$  时,  $f'$  和  $f$  随  $Re$  的变化曲线

为了分析无量纲量  $Re, \alpha, K, \lambda$  在速度场和角速度场的影响, 选择  $h = -0.6$ , 我们可以得到 HAM 级数的 20 阶近似解. 对于不同的参数  $Re, \alpha, K$  和  $\lambda$ , 我们也可以应用 OHAM<sup>[29]</sup> 去选择最优的收敛控制参数  $h$ .

图 3 给出了 Reynolds 数  $Re$  对速度的影响. 由图 3(a) 可以看出, 从下壁面到上壁面,  $f$  曲线由 0 变到 1; 而且在中间位置  $\eta = 1/2$  附近, 曲线的凹凸性发生改变. 由图 3(b) 可以看出, 当  $Re = 0$  (即喷注速度为 0), 曲线  $f'(\eta)$  为抛物线并且关于  $\eta = 1/2$  对称; 当上壁面有流体注入时,  $f'(\eta)$  的最大值向下壁面方向移动, 而且其最大值随着  $Re$  的绝对值的增加而增加; 当  $Re > 0$  时, 即当吸附速度作用在上壁面时, 最大值点向上壁面移动.

由图 4 和图 5 可以看出,  $f'(\eta)$  的最大值是  $\alpha$  的增函数,  $K$  的减函数.

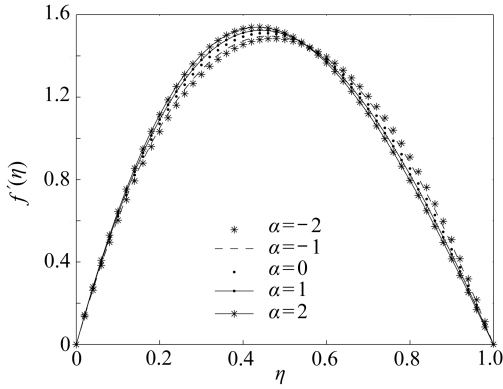


图4 当  $Re = -5, K = 1, \lambda = 0.5$  时,  $f'$  随  $\alpha$  的变化曲线

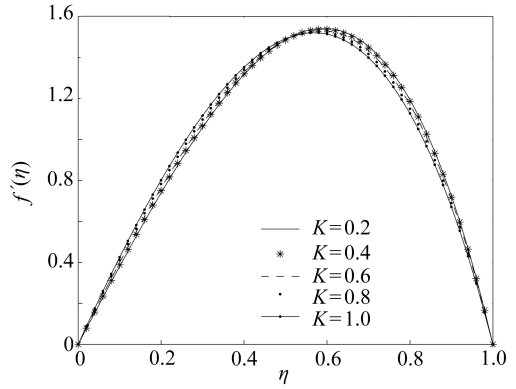


图5 当  $Re = 5, \alpha = 1, \lambda = 0.5$  时,  $f'$  随  $K$  的变化曲线

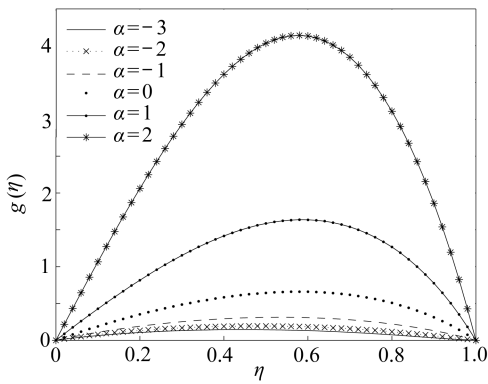


图6 当  $K = 1, Re = 5, \lambda = 0.5$  时,  $g$  随  $\alpha$  的变化曲线

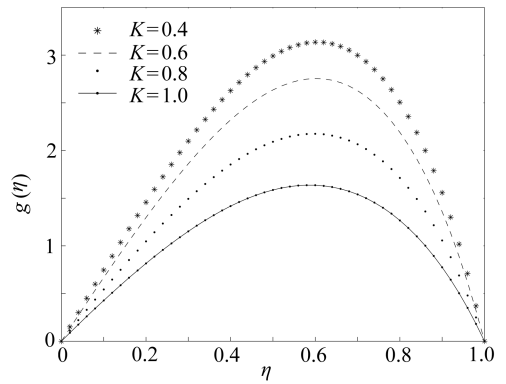


图7 当  $\alpha = 1, Re = 5, \lambda = 0.5$  时,  $g$  随  $K$  的变化曲线

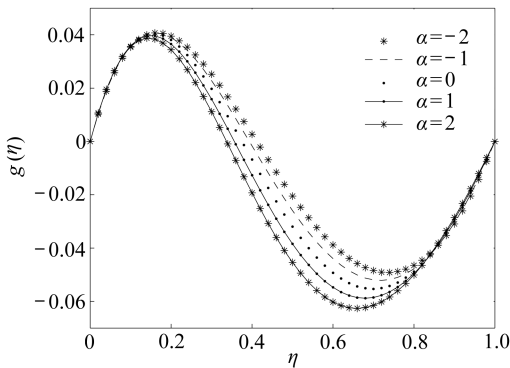


图8 当  $Re = -5, K = 1, \lambda = 0.5$  时,  $g$  随  $\alpha$  的变化曲线

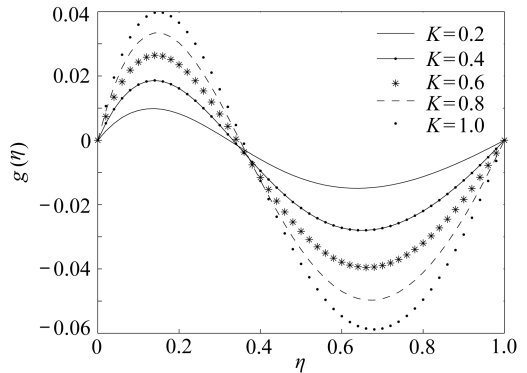


图9 当  $\alpha = 1, Re = -5, \lambda = 0.5$  时,  $g$  随  $K$  的变化曲线

图6~图9描述了 Reynolds 数  $Re$ , 壁面膨胀率  $\alpha$ ,  $K$  对微旋转角速度的影响。当  $Re$  为正数时,  $g$  为变量  $\alpha$  的增函数,  $K$  的减函数。当  $Re$  为负时, 微旋转速度有一个零值点, 并且在两壁面附近有不同的符号值。符号的不同是基于这样一个事实, 即两壁面的剪切力朝相反的方向旋转, 而且可以看出, 壁面膨胀率  $\alpha$ ,  $K$  对微旋转角速度有较强的影响。

由图10和图11可以看出,  $\lambda$  对  $g$  的影响与  $K$  相似。

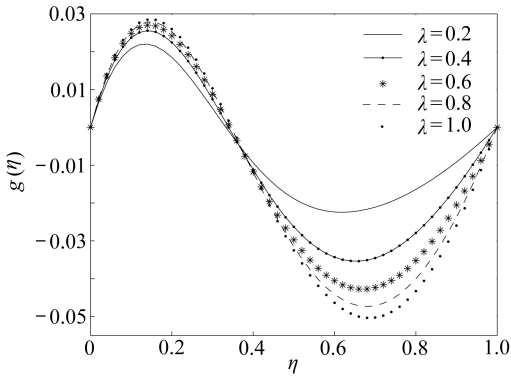


图 10 当  $\alpha = 1, K = 0.6, Re = -5$  时,  
 $g$  随  $\lambda$  的变化曲线

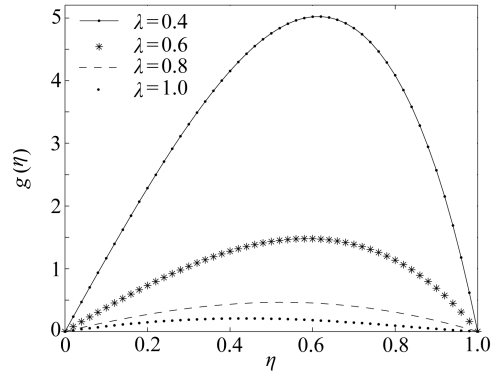


图 11 当  $\alpha = 1, K = 0.6, Re = 5$  时,  
 $g$  随  $\lambda$  的变化曲线

## 4 结 论

本文分析了胀缩半渗透管道中的微极性流体的速度场和角速度场.应用 HAM,对控制方程进行求解,将不同的参数对速度和角速度的影响通过图形的方式表示出来,并且加以讨论.

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## Analytic Solution for the Flow of a Micropolar Fluid Through a Semi-Porous Channel With an Expanding or Contracting Wall

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**Abstract:** The flow of a micropolar fluid in a semi-porous channel with an expanding or contracting wall was investigated. The governing equations were reduced to ordinary ones by using similar transformations. In order to get the analytic solution of the problem, homotopy analysis method (HAM) was employed to obtain the expressions for velocity fields. Graphs are sketched and discussed for the effects of various parameters, especially the expansion ratio, on velocity and micro-rotation fields in detail.

**Key words:** homotopy analysis method; micropolar fluid; expanding or contracting wall; porous channel