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一类具有转向点的三阶方程边值问题*

莫嘉琪[1,2], 温朝晖[3]

(1. 安徽师范大学 数学系,安徽 芜湖 241000;
2. 上海高校计算科学 E-研究院 SJTU 研究所, 上海 200240;
3. 安徽财经大学 统计与应用数学学院 应用数学研究所, 安徽 蚌埠 233030)

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摘要: 讨论了一类具有转向点的微分方程边值问题. 利用多重尺度等方法,构造了边值问题解一致有效的渐近展开式.

关 键 词: 转向点; 多重尺度; 边值问题
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引 言

奇摄动转向点问题是国际上学术界十分关注的一个问题[1]. 近 10 年来,许多近似方法被发展和优化,包括平均法、边界层法、匹配渐近展开法和多重尺度法. 近来,许多学者诸如 Ni 和 Wei[2], Bartier[3], Libre, Silva 和 Teixeira[4] 以及 Guarguaglini 和 Natalini[5] 做了大量的工作. 利用微分不等式等方法, Mo 等人也考虑了一类反应扩散方程[6]、激波[7]、孤子波[8-11]、激光脉冲[12-13]、海洋科学[14-16] 和大气物理[17-19] 等. 本文是用一个特殊的方法构造了一类具有转向点的奇摄动方程边值问题.

现考虑如下奇摄动边值问题:

$$L_\varepsilon y \equiv \varepsilon \frac{\mathrm{d}^3 y}{\mathrm{d}x^3} + f(x)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + g(x)\frac{\mathrm{d}y}{\mathrm{d}x} = h(x), \qquad x \in (-1,1), \tag{1}$$

$$y(-1) = A_1,\ \frac{\mathrm{d}y}{\mathrm{d}x}(-1) = A_2, \tag{2}$$

$$y(1) = B, \tag{3}$$

其中, ε 为小参数, A_1, A_2 和 B 为常数. 且 $f(0) = 0$. 显然, $x = 0$ 为方程(1)的一个转向点.

设

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作者简介: 莫嘉琪(1937—),男,浙江德清人,教授(联系人. Tel:+86-553-3869642;E-mail:mojiaqi@mail.ahnu.edu.cn).

(Ⅰ) $f(x)$, $g(x)$ 和 $h(x)$ 为在 $-1 \leqslant x \leqslant 1$ 上充分光滑的函数;

(Ⅱ) $f'(x) > 0, \int_{-\delta}^{\delta} \frac{1}{|f(x)|} \mathrm{d}x \leqslant M$,其中 $\delta \leqslant 1$, M 为正常数.

1　转向点外部解

奇摄动边值问题的退化问题为

$$f(x)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + g(x)\frac{\mathrm{d}y}{\mathrm{d}x} = h(x), \qquad x \in (-1,0), \tag{4}$$

$$y(-1) = A_1,\ y'(-1) = A_2; \tag{5}$$

$$f(x)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + g(x)\frac{\mathrm{d}y}{\mathrm{d}x} = h(x), \qquad x \in (0,1), \tag{6}$$

$$y(1) = B. \tag{7}$$

方程(4)和(6)在 $x = 0$ 有奇性. 这时退化问题(4) ~ (7) 的解 $Y_0(x)$ 为

$$Y_0(x) = B + \int_1^x \exp\left(-\int_1^t \frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(\bar{C} + \int_1^t \frac{h(t')}{f(t')}\exp\left(\int_1^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t, \quad 0 < x \leqslant 1, \tag{8}$$

$$Y_0(x) = A_1 + \int_{-1}^x \exp\left(-\int_{-1}^t \frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(A_2 + \int_{-1}^t \frac{h(t')}{f(t')}\exp\left(\int_{-1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t, \quad -1 \leqslant x < 0, \tag{9}$$

其中 $\bar{C}$ 为任意常数.

继续用相同的方法,我们也能得到 $Y_i(x)(i = 1,2,\cdots,m)$ 和问题(1) ~ (3) 在转向点 $x = 0$ 的外部解的高阶近似

$$\bar{Y}_m(x) = \sum_{i=0}^{m} Y_i(x)\varepsilon^i.$$

2　转 向 点 层

在转向点 $x = 0$ 附近引入多重尺度变量[1]

$$\rho = \frac{\bar{h}(x)}{\varepsilon}, \qquad \bar{x} = x, \tag{10}$$

其中 $h(x)$ 为未知函数,它将在下面决定. 由式(10),有

$$\frac{\partial}{\partial x} = \frac{\bar{h}_x}{\varepsilon}\frac{\partial}{\partial \rho} + \frac{\partial}{\partial \bar{x}}, \quad \frac{\partial^2}{\partial x^2} = \frac{\bar{h}_x^2}{\varepsilon^2}\frac{\partial^2}{\partial \rho^2} + 2\frac{\bar{h}_x}{\varepsilon}\frac{\partial^2}{\partial \rho \partial \bar{x}} + \frac{\bar{h}_{xx}}{\varepsilon}\frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial \bar{x}^2},$$

$$\frac{\partial^3}{\partial x^3} = \frac{\bar{h}_x^3}{\varepsilon^3}\frac{\partial^3}{\partial \rho^3} + 3\frac{\bar{h}_x^2}{\varepsilon^2}\frac{\partial^3}{\partial \rho^2 \partial \bar{x}} + 3\frac{\bar{h}_x \bar{h}_{xx}}{\varepsilon^2}\frac{\partial^2}{\partial \rho^2} + 3\frac{\bar{h}_x}{\varepsilon}\frac{\partial^3}{\partial \rho \partial \bar{x}^2} + \frac{\bar{h}_{xx}}{\varepsilon}\frac{\partial^2}{\partial \rho \partial \bar{x}} + \frac{\bar{h}_{xxx}}{\varepsilon}\frac{\partial}{\partial \rho} + \frac{\partial^3}{\partial \bar{x}^3}.$$

为了方便起见,下面仍用 x 代替 $\bar{x}$, 这时

$$L_\varepsilon = \frac{1}{\varepsilon^2}L_0 + \frac{1}{\varepsilon}L_1 + L_2 + \varepsilon L_3, \tag{11}$$

其中

$$L_0 = \bar{h}_x^3\frac{\partial^3}{\partial \rho^3} + \bar{h}_x^2 f\frac{\partial^2}{\partial \rho^2}, \tag{12}$$

$$L_1 = 3\bar{h}_x^2 \frac{\partial^3}{\partial\rho^2 \partial x} + 3\bar{h}_x \bar{h}_{xx} \frac{\partial^2}{\partial\rho^2} + 2f\,\bar{h}_x \frac{\partial^2}{\partial\rho\partial x} + f\,\bar{h}_{xx} \frac{\partial}{\partial\rho} + g\bar{h}_x \frac{\partial}{\partial\rho}, \tag{13}$$

$$L_2 = 3\bar{h}_x \frac{\partial^3}{\partial\rho\partial x^2} + \bar{h}_{xx} \frac{\partial^2}{\partial\rho\partial x} + \bar{h}_{xxx} \frac{\partial}{\partial\rho} + f \frac{\partial^2}{\partial x^2} + g \frac{\partial}{\partial x}, \tag{14}$$

$$L_3 = \frac{\partial^3}{\partial x^3}. \tag{15}$$

令

$$\bar{h}(x) = \int_0^x f(x')\,\mathrm{d}x', \qquad 0 \leqslant x \leqslant 1. \tag{16}$$

设原问题的解 y 为

$$y = \sum_{i=0}^{\infty} y_i \varepsilon^i. \tag{17}$$

将式(17)代入式(1)并由式(11)~(16),得

$$L_0[y_1] = f^3\left(\frac{\partial^3}{\partial\rho^3} + \frac{\partial^2}{\partial\rho^2}\right), \tag{18}$$

$$L_0[y_1] = -L_1[y_0], \tag{19}$$

$$L_0[y_2] = -L_1[y_1] - L_2[y_0] + h, \tag{20}$$

$$L_0[y_i] = -L_1[y_{i-1}] - L_2[y_{i-2}] - L_3[y_{i-3}], \qquad i = 3,4,\cdots. \tag{21}$$

由式(18),有

$$y_0 = A_0(x) + B_0(x)\rho + C_0(x)\exp(-\rho). \tag{22}$$

将式(22)代入式(19),得

$$L_0[y_1] \equiv f^3\left(\frac{\partial^2}{\partial\rho^2} + \frac{\partial^2}{\partial\rho^2}\right) y_1 =$$

$$-\left(3\bar{h}_x^2 \frac{\partial^3}{\partial\rho^2 \partial x} + 3\bar{h}_x \bar{h}_{xx} \frac{\partial^2}{\partial\rho^2} + 2f\bar{h}_x \frac{\partial^2}{\partial\rho\partial x} + f\bar{h}_{xx} \frac{\partial}{\partial\rho} + g\bar{h}_x \frac{\partial}{\partial\rho}\right) y_0.$$

为了 y_1 和 y_0 一样是转向点层性质. 令 $L_0[y_1] = 0$, 我们有

$$\left(3\bar{h}_x^2 \frac{\partial^3}{\partial\rho^2 \partial x} + 3\bar{h}_x \bar{h}_{xx} \frac{\partial^2}{\partial\rho^2} + 2\,f\bar{h}_x \frac{\partial^2}{\partial\rho\partial x} + f\,\bar{h}_{xx} \frac{\partial}{\partial\rho} + g\bar{h}_x \frac{\partial}{\partial\rho}\right) y_0 = 0.$$

由式(22),得

$$f\left(2f\frac{\mathrm{d}B_0}{\mathrm{d}x} + (f_x + g)B_0\right) + f\left(f\frac{\mathrm{d}C_0}{\mathrm{d}x} + (2f_x - g)C_0\right)\exp(-\rho) = 0$$

即

$$2f\frac{\mathrm{d}B_0}{\mathrm{d}x} + (f_x + g)B_0 = 0, \tag{23}$$

$$f\frac{\mathrm{d}C_0}{\mathrm{d}x} + (2f_x - g)C_0 = 0. \tag{24}$$

由式(23)和(24),有

$$B_0(x) = C_1\exp\left(-\int_0^x \frac{f_x(x') + g(x')}{2f(x')}\,\mathrm{d}x'\right), \qquad -1 \leqslant x \leqslant 1, \tag{25}$$

$$C_0(x) = C_2\exp\left(-\int_0^x \frac{2f(x') - g(x')}{f(x')}\,\mathrm{d}x'\right), \qquad -1 \leqslant x \leqslant 1. \tag{26}$$

同样,为了 y_2 和 y_0,y_1 一样是转向点层性质. 令 $L_0[y_2] = 0$, 我们有

$$L_1[y_1]+L_2[y_0]-h=0.$$

由上面可知,能够得到

$$f\frac{\mathrm{d}^2A_0}{\mathrm{d}x^2}+g\frac{\mathrm{d}A_0}{\mathrm{d}x}=h(x).$$

于是有

$$A_0(x)=C_3+\int_0^x\exp\left(-\int_0^t\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(C_4+\int_0^t\frac{h(t')}{f(t')}\exp\left(\int_0^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t,\quad -1\leqslant x\leqslant 1. \tag{27}$$

将式(25)~(27)代入式(22),有

$$y_0(x)=C_5+\int_0^x\exp\left(-\int_0^t\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(C_4+\int_0^t\frac{h(t')}{f(t')}\exp\left(\int_0^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t+\frac{C_1}{\varepsilon}\exp\left(-\int_0^x\frac{f_x(x')+g(x')}{2f(x')}\mathrm{d}x'\right)\int_0^x f(x')\mathrm{d}x'+C_2\exp\left(-\int_0^x\frac{2f(x')-g(x')}{f(x')}\mathrm{d}x'\right)\exp\left(-\frac{1}{\varepsilon}\int_0^x f(x')\mathrm{d}x'\right),\quad -1\leqslant x\leqslant 1. \tag{28}$$

注意到转向点层的性质,当 $\rho\to+\infty$ 时,$y_0\to 0$. 由式(28),能够得到

$$C_1=0,\ C_5=-\int_0^x\exp\left(-\int_0^t\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(C_4+\int_0^t\frac{h(t')}{f(t')}\exp\left(\int_0^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t.$$

于是,我们有首次近似转向点层

$$y_0=C_2\exp\left(-\int_0^x\frac{2f(x')-g(x')}{f(x')}\mathrm{d}x'\right)\exp\left(-\frac{1}{\varepsilon}\int_0^x f(x')\mathrm{d}x'\right),\quad -1\leqslant x\leqslant 1, \tag{29}$$

其中 C_2 为任意常数.

继续由式(18)~(21),利用相同的方法,我们也能得到 $y_i(i=1,2,\cdots,m)$ 和有原问题(1)~(3)的转向点层的高阶近似

$$\bar{y}_m=\sum_{i=0}^m y_i\varepsilon^i.$$

3 首次近似解的匹配

现将解 Y_0 和 y_0 匹配.

将 $x=h^{-1}(\varepsilon\rho)$ 代入式(8)和(9),固定 ρ 且取 $\varepsilon\to 0$, 得

$$Y_0^{\mathrm{o}}=B+\int_1^0\exp\left(-\int_1^t\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(\bar{C}+\int_1^t\frac{h(t')}{f(t')}\exp\left(\int_1^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t,\quad 0\leqslant x\leqslant 1, \tag{30}$$

$$Y_0^{\mathrm{o}}=A_1+\int_{-1}^0\exp\left(-\int_{-1}^t\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(A_2+\int_{-1}^t\frac{h(t')}{f(t')}\exp\left(\int_{-1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t,\quad -1\leqslant x\leqslant 0. \tag{31}$$

将 $\rho=h(x)/\varepsilon$ 代入式(29),固定 x 且取 $\varepsilon\to 0$, 得

$$y_0^{\mathrm{i}}=C_2\exp\left(-\int_0^1\frac{2f(x')-g(x')}{f(x')}\mathrm{d}x'\right). \tag{32}$$

由匹配原则[1]，$Y_0^{o}=y_0^{i}$，并由式(30)~(32)，有

$$C_2=\left[A_1+\int_{-1}^{0}\exp\left(-\int_{-1}^{t}\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(A_2+\int_{-1}^{t}\frac{h(t')}{f(t')}\exp\left(\int_{-1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t\right]\times \left(\exp\left(-\int_{0}^{1}\frac{2f(x')-g(x')}{f(x')}\mathrm{d}x'\right)\right)^{-1}, \tag{33}$$

$$\bar{C}=\left[A_1-B+\int_{-1}^{0}\exp\left(-\int_{-1}^{t}\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(A_2+\int_{-1}^{t}\frac{h(t')}{f(t')}\exp\left(\int_{-1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t-\int_{1}^{0}\exp\left(-\int_{1}^{t}\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(\int_{1}^{t}\frac{h(t')}{f(t')}\exp\left(\int_{1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t\right]\times \left(\int_{0}^{1}\exp\left(-\int_{1}^{t}\frac{g(t')}{f(t')}\mathrm{d}t'\right)\mathrm{d}t\right)^{-1}. \tag{34}$$

由式(8)、(9)、(29)以及合成展开法[1]，我们便得到边值问题(1)~(3)的首次近似 $\bar{y}_0(x)$：

$$\bar{y}_0(x)=\int_{0}^{x}\exp\left(-\int_{1}^{t}\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(\bar{C}+\int_{1}^{t}\frac{h(t')}{f(t')}\exp\left(\int_{1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t+ C_2\exp\left(-\int_{0}^{x}\frac{2f(x')-g(x')}{f(x')}\mathrm{d}x'\right)\exp\left(-\frac{1}{\varepsilon}\int_{0}^{x}f(x')\mathrm{d}x'\right),\quad 0\leqslant x\leqslant 1, \tag{35}$$

$$\bar{y}_0(x)=\int_{0}^{x}\exp\left(-\int_{-1}^{t}\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(A_2+\int_{-1}^{t}\frac{h(t')}{f(t')}\exp\left(\int_{-1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t+ C_2\exp\left(-\int_{0}^{x}\frac{2f(x')-g(x')}{f(x')}\mathrm{d}x'\right)\exp\left(-\frac{1}{\varepsilon}\int_{0}^{x}f(x')\mathrm{d}x'\right),\quad -1\leqslant x\leqslant 0, \tag{36}$$

其中，C_2、$\bar{C}$ 由式(33)和(34)表示.

最后，令

$$y_\varepsilon(x)=\bar{y}_0(x)+R_\varepsilon(x). \tag{37}$$

由式(35)~(37)，我们可得

$$L(R_\varepsilon)=L(y_\varepsilon)-L(\bar{y}_0)=O(\varepsilon),\qquad -1\leqslant x\leqslant 1,\ 0<\varepsilon\ll 1,$$

$$R_\varepsilon(-1)=O(\varepsilon),\ R_\varepsilon'(-1)=O(\varepsilon),\ R_\varepsilon(1)=O(\varepsilon),\qquad 0<\varepsilon\ll 1.$$

于是，由极值原理[1]，便有

$$R(x)=O(\varepsilon),\qquad -1\leqslant x\leqslant 1,\ 0<\varepsilon\ll 1.$$

这时有如下定理：

定理 在假设(Ⅰ)和(Ⅱ)下，具有转向点 $x=0$ 的奇摄动问题(1)~(3)的解 $y_\varepsilon(x)$ 有一致有效的渐近展开式：

$$\bar{y}_0(x)=\int_{0}^{x}\exp\left(-\int_{1}^{t}\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(\bar{C}+\int_{1}^{t}\frac{h(t')}{f(t')}\exp\left(\int_{1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t+ C_2\exp\left(-\int_{0}^{x}\frac{2f(x')-g(x')}{f(x')}\mathrm{d}x'\right)\exp\left(-\frac{1}{\varepsilon}\int_{0}^{x}f(x')\mathrm{d}x'\right)+O(\varepsilon),\quad 0\leqslant x\leqslant 1,\ 0<\varepsilon\ll 1,$$

$$\bar{y}_0(x)=\int_{0}^{x}\exp\left(-\int_{1}^{t}\frac{g(t')}{f(t')}\mathrm{d}t'\right)\left(A_2+\int_{1}^{t}\frac{h(t')}{f(t')}\exp\left(\int_{1}^{t'}\frac{g(t'')}{f(t'')}\mathrm{d}t''\right)\mathrm{d}t'\right)\mathrm{d}t+ C_2\exp\left(-\int_{0}^{x}\frac{2f(x')-g(x')}{f(x')}\mathrm{d}x'\right)\exp\left(-\frac{1}{\varepsilon}\int_{0}^{x}f(x')\mathrm{d}x'\right)+O(\varepsilon),\quad -1\leqslant x\leqslant 0,\ 0<\varepsilon\ll 1,$$

其中，C_2，$\bar{C}$ 由式(33)、(34)表示。

参考文献：

[1] de Jager E M, JIANG Fu-ru. *The Theory of Singular Perturbation*[M]. Amsterdam: North-Holland Publishing Co,1996.

[2] NI Wei-ming, WEI Jun-cheng. On positive solution concentrating on spheres for the Gierer-Meinhardt system[J]. *J Differ Equations*, 2006, **221**(1): 158-189.

[3] Bartier Jean-Philippe. Global behavior of solutions of a reaction-diffusion equation with gradient absorption in unbounded domains[J]. *Asymptotic Anal*, 2006, **46**(3/4): 325-347.

[4] Libre J, da Silva P R, Teixeira M A. Regularization of discontinuous vector fields on R^3 via singular perturbation[J]. *J Dyn Differ Equations*, 2007, **19**(2): 309-331.

[5] Guarguaglini F R, Natalini R. Fast reaction limit and large time behavior of solutions to a nonlinear model of sulphation phenomena[J]. *Commun Partial Differ Equations*, 2007, **32**(2): 163-189.

[6] MO Jia-qi, LIN Wan-tao. Asymptotic solution of activator inhibitor systems for nonlinear reaction diffusion equations[J]. *J Sys Sci Complexity*, 2008, **20**(1): 119-128.

[7] MO Jia-qi, WANG Hui. Nonlinear singularly perturbed approximate solution for generalized Lotke-Volterra ecological model[J]. *Acta Ecologica Sinica*, 2007, **27**(10): 4366-4370.

[8] 莫嘉琪,陈丽华. 一类 Landau-Ginzburg-Higgs 扰动方程孤子的近似解[J]. 物理学报,2008, **57**(8): 4646-4648.

[9] MO Jia-qi. Variational iteration solving method for a class of generalized Boussinesq equation[J]. *Chin Phys Lett*, 2009, **26**(6): 060202.

[10] 莫嘉琪. 一类广义 Sine-Gordon 扰动方程的解析解[J]. 物理学报,2009, **58**(5): 2930-2933.

[11] 莫嘉琪,程燕. 广义 Boussinesq 方程的同伦映射近似解[J]. 物理学报,2009, **58**(7): 4379-4382.

[12] MO Jia-qi. Homotopiv mapping solving method for gain fluency of a laser pulse amplifier[J]. *Science in China*,*Ser G*, 2009, **52**(7): 1007-1070.

[13] 莫嘉琪, 张伟江, 何铭. 激光脉冲放大器传输波的计算[J]. 物理学报,2006, **55**(7): 3233-3236.

[14] 莫嘉琪, 林万涛, 林一骅. 一个赤道太平洋西部边界潜流广义扰动机理的近似解法[J]. 物理学报,2007, **56**(6): 3127-3131.

[15] MO Jia-qi, LIN Wan-tao, WANG Hui. Asymptotic solution of a sea-air oscillator for ENSO mechanism[J]. *Chin Phys*, 2007, **16**(3) 578-581.

[16] MO Jia-qi, LIN Wan-tao, WANG Hui. Variational iteration solving method of a sea-air oscillator model for the ENSO[J]. *Prog Nat Sci*, 2007, **17**(2): 230-232.

[17] MO Jia-qi, LIN Wan-tao, WANG Hui. Singularly perturbed solution of coupled model in atmosphere-ocean for global climate[J]. *Chin Geographical Sci*, 2008, **18**(2): 193-195.

[18] MO Jia-qi, LIN Wan-tao. Asymptotic solution for a class of sea-air oscillator model for El-Nino-southern oscillation[J]. *Chin Phys*, 2008, **17**(2): 370-372.

[19] MO Jia-qi, LIN Wan-tao. The homotopic method of travelling wave solution for El Nino tropic sea-air coupled oscillator[J]. *Chin Phys*, 2008, **17**(3): 743-746.

A Class of Boundary Value Problems for Third-Order Differential Equation With a Turning Point

MO Jia-qi[1,2], WEN Zhao-hui[3]

(1. *Department of Mathematics*, *Anhui Normal University*, *Wuhu*, *Anhui* 241000, *P. R. China*;
2. *Division of Computational Science*, *E-Institutes of Shanghai Universities at SJTU*, *Shanghai* 200240, *P. R. China*;
3. *Institute of Applied Mathematics*, *School of Statistics and Applied Mathematics*, *Anhui University of Finance and Economics*, *Bengbu*, *Anhui* 233030, *P. R. China*)

Abstract: A class of boundary value problem for differential equation with a turning point was considered. Using the method of multiple scales and others, the uniformly valid asymptotic expansion of solution for the boundary value problem was constructed.

Key words: turning point; multiple scales; boundary value problem