

双层网格圆底扁球壳的非线性稳定问题*

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摘要: 从基于等效夹层壳思想的双层网格扁壳,非线性弯曲理论的变分方程出发,利用坐标变换方法和驻值余能原理,导出双层网格圆底扁球壳,在均布压力作用下的轴对称大挠度方程和边界条件.采用修正迭代法,求得了两类边界条件下双层网格圆底扁球壳的非线性载荷-位移关系式和临界屈曲载荷的解析表达式,并讨论了几何参数对临界屈曲载荷的影响.

关键词: 双层网格圆底扁球壳; 非线性稳定性; 等效连续化; 修正迭代法; 驻值余能原理

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引 言

网壳结构是将杆件沿着曲面有规律地布置而组成的空间结构体系,其受力特点与薄壳结构类似,大部分荷载由网壳杆件的轴向力承受.由于具有自重轻、结构刚度大等一系列特性,这种结构可以覆盖较大的空间,不同曲面的网壳可以提供各种新颖的建筑造型,是一种越来越广泛采用的结构形式.

网壳结构的发展与建筑材料和计算理论的发展密切相关,其总的趋势是跨度越来越大,由单层网壳结构发展到双层网壳结构.自从1963年布加勒斯特国家经济展览馆93.5 m跨度的单层穹顶网壳屋盖在一场大雪彻底坍塌以后,国内外对网壳结构稳定性的研究非常重视,稳定性验算已成为网壳结构设计中的关键问题.

对网壳结构进行非线性分析主要有两种方法:等效的连续化方法和离散的有限元方法.等效连续化方法有涉及参数少、计算工作量小、物理意义明确等优点,在进行复杂网壳结构的稳定、振动分析时有较大优越性.1991年, Liu等^[1]首先采用等效连续化方法建立了单层网格扁球壳的非线性稳定理论,接着又与聂国华等^[2-6]研究了单层网格扁壳的非线性稳定和振动问题.1997年,刘人怀等^[7]又利用等效夹层壳思想将这一方法推广到双层网壳结构中,建立双层

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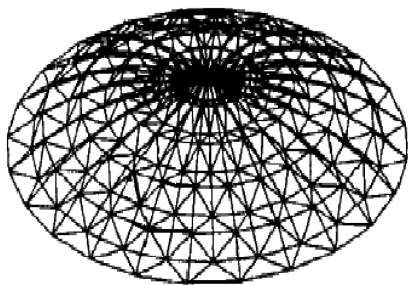
刘人怀(1940—),男,教授,中国工程院院士(联系人. E-mail: lrh@jnu.edu.cn).

网壳结构的非线性理论,将双层网格壳的弦杆等效成夹层壳的表层,腹杆等效成夹层壳的夹心。基于这一理论,刘人怀、肖潭、徐加初、卢保红和杜冰等^[8-14]对双层网格扁壳的非线性弯曲、稳定和振动等问题进行了研究。2004年,董石麟等^[15]将单层和双层球面扁网壳都等效为实体薄壳,研究了单双层球面扁网壳的非线性稳定问题。接着,他又和顾磊^[16]讨论了局部双层叉筒网壳。最近,王新志等^[17]研究了扁柱面网壳的非线性动力学行为。离散的有限元法可以对结构的荷载-位移全过程进行分析,沈世钊^[18]、赵阳^[19]、刘大卫^[20]、朱忠义^[21]、肖建春^[22]、谢志红^[23]、李春燕^[24]、何放龙^[25]等在这方面作了一些工作。网壳结构本身就是一种离散结构,采用离散法具有分析方法直观、精确度高等优点,但当网格结构比较复杂、网格分布稠密、节点数目大的情况下,离散方法显得耗时费力。

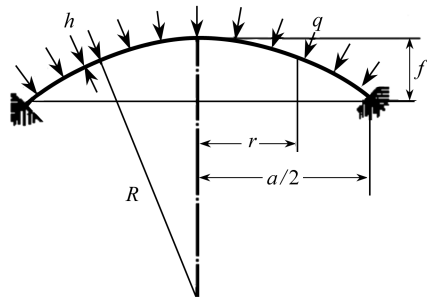
本文研究的是上下层之间的距离远小于网壳结构的曲率半径,并且网格分布稠密、网格划分均匀规则的双层网格圆底扁球壳。从双层网格扁壳非线性理论^[7]的变分方程出发,利用坐标变换导出正交正放双层圆底球面网格扁壳在均布压力作用下的轴对称平衡方程和边界条件。利用驻值余能原理^[26]导出相容方程。最后,采用修正迭代法^[27-29]求得了两类边界条件下双层网格圆底扁球壳的非线性荷载-位移关系式和临界屈曲载荷的解析表达式。据作者所知,此问题至今尚无人研究过。

1 平衡方程

双层网格圆底扁球壳结构的示意图如图1所示,网壳的跨度为 a ,矢高为 f ,曲率半径为 R ,网壳的厚度为 h 。



(a) 壳体



(b) 一个截面

图1 双层网格圆底扁球壳

按照文献[7]并使用相同的符号,夹层扁壳在外力作用下的虚功方程为

$$\begin{aligned} & \iint_{\Sigma} (N_{x1} \delta \varepsilon_{x1} + N_{y1} \delta \varepsilon_{y1} + N_{xy1} \delta \gamma_{xy1} + N_{x3} \delta \varepsilon_{x3} + N_{y3} \delta \varepsilon_{y3} + N_{xy3} \delta \gamma_{xy3}) dx dy + \\ & \iint_{\Sigma} (Q_x \delta \gamma_{xz2} + Q_y \delta \gamma_{yz2}) dx dy - \oint_r (\bar{N}_{xn} \delta u + \bar{N}_{yn} \delta v + \bar{N}_z \delta w) ds - \\ & \oint_r (\bar{M}_{xn} \delta \psi_x + \bar{M}_{yn} \delta \psi_y) ds - \iint_{\Sigma} (f_x \delta u + f_y \delta v + f_z \delta w) dx dy = 0, \end{aligned} \quad (1)$$

于是,双层正交正放网壳结构在承受均布压力 q 作用下的非线性弯曲理论的变分方程为

$$\iint_{\Sigma} \left\{ \left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right) \delta u + \left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} \right) \delta v + \left[\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \left(\frac{\partial^2 w}{\partial x^2} + k_1 \right) + \right. \right.$$

$$\begin{aligned}
 & N_y \left(\frac{\partial^2 w}{\partial y^2} + k_2 \right) + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial x} \left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right) + \frac{\partial w}{\partial y} \left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} \right) + q \left] \delta w + \right. \\
 & \left. \left(\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x \right) \delta \psi_x + \left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y \right) \delta \psi_y \right\} dx dy - \\
 & \oint_{\Gamma} \left\{ (N_{xn} - \bar{N}_{xn}) \delta u + (N_{yn} - \bar{N}_{yn}) \delta v + \left[l \left(\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \right) + m \left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} \right) + \right. \right. \\
 & \left. \left. \frac{\partial w}{\partial x} N_{xn} + \frac{\partial w}{\partial y} N_{yn} - \bar{N}_z \right] \delta w + (M_{xn} - \bar{M}_{xn}) \delta \psi_x + (M_{yn} - \bar{M}_{yn}) \delta \psi_y \right\} ds = 0, \quad (2)
 \end{aligned}$$

式中, $N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y$ 为等效内力, u, v, w 分别为双层网格扁球壳中面的位移分量, ψ_x, ψ_y 分别为其中面法线在 xz 和 yz 平面内的转角, $N_{xn}, N_{yn}, M_{xn}, M_{yn}$ 分别为扁球壳单位长度外力和外力矩, $\bar{N}_{xn}, \bar{N}_{yn}, \bar{N}_z, \bar{M}_{xn}, \bar{M}_{yn}$ 为扁球壳边界 Γ 上给定的单位长度外力和外力矩,

$$\begin{cases} N_x = E_x \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - k_1 w \right], & N_y = E_y \left[\frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 - k_2 w \right], \\ N_{xy} = G_{xy} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right), \\ M_x = D_x \frac{\partial \psi_x}{\partial x}, & M_y = D_y \frac{\partial \psi_y}{\partial y}, & M_{xy} = D_{xy} \left(\frac{\partial \psi_y}{\partial x} + \frac{\partial \psi_x}{\partial y} \right), \\ Q_x = G_x \left(\psi_x + \frac{\partial w}{\partial x} \right), & Q_y = G_y \left(\psi_y + \frac{\partial w}{\partial y} \right), \end{cases} \quad (3)$$

在这些表达式中, $E_x, E_y, G_x, G_y, G_{xy}, D_x, D_y, D_{xy}$ 为等效材料常数^[7],

$$\begin{cases} E_x = \frac{2EA_1}{L_1}, & E_y = \frac{2EA_2}{L_2}, & G_x = EL_2 \left/ \left[L_1 \left(\frac{h}{A_h} + \frac{L_{c2}^3}{A_{c1} h^2} \right) \right], \right. \\ G_y = EL_1 \left/ \left[L_2 \left(\frac{h}{A_h} + \frac{L_{c1}^3}{A_{c2} h^2} \right) \right], & G_{xy} = 24E \left/ \left[L_1 L_2 \left(\frac{L_2}{I_{10}} + \frac{L_1}{I_{20}} \right) \right], \right. \\ D_x = \frac{EA_1 h^2}{2L_1}, & D_y = \frac{EA_2 h^2}{2L_2}, & D_{xy} = 64Eh^2 \left/ \left[L_1 L_2 \left(\frac{L_2}{I_{10}} + \frac{L_1}{I_{20}} \right) \right], \right. \end{cases} \quad (4)$$

式(4)中, E 为弹性模量, $L_i, A_i (i=1, 2), I_{10}$ 和 I_{20} 分别为对应网壳单元中弦杆的长度、截面面积和截面惯性矩, h 和 A_h 分别为腹杆中竖杆的长度和截面面积, L_{c1}, L_{c2} 和 A_{c1}, A_{c2} 分别为腹杆中斜杆的长度和截面面积。

对于球形网格壳体, 式(2)和(3)中的 $k_1 = k_2 = 1/R$ 。研究双层网格圆底扁球壳的稳定问题, 采用极坐标更为方便, 为此, 利用直角坐标与极坐标下内力、位移变换关系, 我们有

$$\begin{cases} N_x = N_r \cos^2 \theta + N_\theta \sin^2 \theta - 2N_{r\theta} \sin \theta \cos \theta, \\ N_y = N_r \sin^2 \theta + N_\theta \cos^2 \theta + 2N_{r\theta} \sin \theta \cos \theta, \\ N_{xy} = (N_r - N_\theta) \sin \theta \cos \theta + N_{r\theta} (\cos^2 \theta - \sin^2 \theta), \\ u = u_r \cos \theta + u_\theta \sin \theta, & v = u_r \sin \theta + u_\theta \cos \theta, \\ \psi_x = \psi_r \cos \theta + \psi_\theta \sin \theta, & \psi_y = \psi_r \sin \theta + \psi_\theta \cos \theta. \end{cases} \quad (5)$$

考虑壳体只承受轴对称荷载作用, 则有理由假设非轴对称内力和非轴对称位移为 0,

$$N_{r\theta} = 0, \quad u_\theta = 0, \quad \psi_\theta = 0. \quad (6)$$

利用式(3)和(6), 将式(5)代入方程(2), 变分方程(2)在极坐标下的表达式为

$$\begin{aligned}
& \iint_{\Sigma} \left\{ \left[\frac{\partial N_r}{\partial r} + \frac{1}{r}(N_r - N_\theta) \right] \delta u_r + \left\{ G_x \left[\cos^2 \theta \left(\frac{\partial \psi_r}{\partial r} + \frac{\partial^2 w}{\partial r^2} \right) + \frac{\sin^2 \theta}{r} \left(\psi_r + \frac{\partial w}{\partial r} \right) \right] + \right. \right. \\
& G_y \left[\sin^2 \theta \left(\frac{\partial \psi_r}{\partial r} + \frac{\partial^2 w}{\partial r^2} \right) + \frac{\cos^2 \theta}{r} \left(\psi_r + \frac{\partial w}{\partial r} \right) \right] + N_r \frac{\partial^2 w}{\partial r^2} + \frac{N_\theta}{r} \frac{\partial w}{\partial r} + \\
& \left. \frac{N_r + N_\theta}{R} + \frac{\partial N_r}{\partial r} \frac{\partial w}{\partial r} + \frac{N_r - N_\theta}{r} \frac{\partial w}{\partial r} + q \right\} \delta w + \\
& \left\{ \left[D_x \cos^4 \theta + 4D_{xy} \sin^2 \theta \cos^2 \theta + D_y \sin^4 \theta \right] \frac{\partial^2 \psi_r}{\partial r^2} + \right. \\
& \left. \left[(3D_x - 8D_{xy} + 3D_y) \sin^2 \theta \cos^2 \theta + 2D_{xy} (\sin^4 \theta + \cos^4 \theta) \right] \frac{1}{r} \frac{\partial \psi_r}{\partial r} - \right. \\
& \left. \left[(3D_x - 8D_{xy} + 3D_y) \sin^2 \theta \cos^2 \theta + 2D_{xy} (\sin^4 \theta + \cos^4 \theta) \right] \frac{\psi_r}{r^2} - \right. \\
& \left. \left(\psi_r + \frac{\partial w}{\partial r} \right) \left(G_x \cos^2 \theta + G_y \sin^2 \theta \right) \right\} \delta \psi_r \Big\} r dr d\theta - \\
& \oint_{r'} \left\{ \left[(N_m - \bar{N}_m) (\cos^3 \theta + \sin^3 \theta) + (N_{\theta m} - \bar{N}_{\theta m}) (\sin \theta + \cos \theta) \sin \theta \cos \theta \right] \delta u_r + \right. \\
& \left\{ l \left[D_x \cos^3 \theta + 2D_{xy} \sin^2 \theta \cos \theta \right] + m \left[D_y \sin^3 \theta + 2D_{xy} \sin \theta \cos^2 \theta \right] \right\} \frac{\partial^2 \psi_r}{\partial r^2} + \\
& \left\{ l \left[(3D_x - 4D_{xy}) \sin^2 \theta \cos \theta + 2D_{xy} \cos^3 \theta \right] + \right. \\
& m \left[(3D_y - 4D_{xy}) \sin \theta \cos^2 \theta + 2D_{xy} \sin^3 \theta \right] \Big\} \left(\frac{1}{r} \frac{\partial \psi_r}{\partial r} - \frac{\psi_r}{r^2} \right) + \\
& \left. \left[N_m (\sin^3 \theta + \cos^3 \theta) + N_{\theta m} (\sin \theta + \cos \theta) \sin \theta \cos \theta \right] \frac{\partial w}{\partial r} - \bar{q} \right\} \delta w + \\
& \left. \left[(M_m - \bar{M}_m) (\cos^3 \theta + \sin^3 \theta) + \right. \right. \\
& \left. \left. (M_{\theta m} - \bar{M}_{\theta m}) (\sin \theta + \cos \theta) \sin \theta \cos \theta \right] \delta \psi_r \right\} ds' = 0. \tag{7}
\end{aligned}$$

对式(7)第1个积分中的 θ 从 $[0, 2\pi]$ 积分,并且由于 $\delta u_r, \delta w, \delta \psi_r$ 的任意性,我们可以得到极坐标下双层网格圆底扁球壳的轴对称平衡方程:

$$N_\theta = \frac{d}{dr}(rN_r), \tag{8a}$$

$$(G_x + G_y) \frac{1}{r} \frac{d}{dr} r \left(\psi_r + \frac{dw}{dr} \right) + \frac{2}{r} \frac{d}{dr} r N_r \left(\frac{dw}{dr} + \frac{r}{R} \right) + 2q = 0, \tag{8b}$$

$$(3D_x + 4D_{xy} + 3D_y) \frac{d}{dr} \frac{1}{r} \frac{d}{dr} (r\psi_r) = 4(G_x + G_y) \left(\psi_r + \frac{dw}{dr} \right). \tag{8c}$$

将式(8b)对 r 积分一次并从中解出

$$\psi_r = -\frac{dw}{dr} - \frac{2N_r}{G_x + G_y} \left(\frac{dw}{dr} + \frac{r}{R} \right) - \frac{qr}{G_x + G_y}. \tag{9}$$

将式(9)代入式(8c),便有

$$\begin{aligned}
& (3D_x + 4D_{xy} + 3D_y) \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \left[r \frac{dw}{dr} + \frac{2rN_r}{G_x + G_y} \left(\frac{dw}{dr} + \frac{r}{R} \right) + \frac{qr^2}{G_x + G_y} \right] = \\
& 4qr + 8N_r \left(\frac{dw}{dr} + \frac{r}{R} \right). \tag{10}
\end{aligned}$$

2 相容方程的极坐标表达式

设结构只承受轴对称横向载荷作用. 引入应力函数 $\phi(x, y)$, 使得

$$N_x = \frac{\partial^2 \phi}{\partial y^2}, N_y = \frac{\partial^2 \phi}{\partial x^2}, N_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}. \quad (11)$$

为了得到相容方程, 在边界 Γ 上假定位移分量如下:

$$u_n = \bar{u}_n, u_s = 0, w = 0, \frac{\partial w}{\partial s} = 0, \frac{\partial w}{\partial n} = 0. \quad (12)$$

应用驻值余能原理^[26], 使得

$$\iint_{\Sigma} \left[\varepsilon_x \delta N_x + \varepsilon_y \delta N_y + \gamma_{xy} \delta N_{xy} - \frac{\partial^2 w}{\partial x^2} \delta M_x - 2 \frac{\partial^2 w}{\partial x \partial y} \delta M_{xy} - \frac{\partial^2 w}{\partial y^2} \delta M_y + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \delta N_x + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \delta N_{xy} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \delta N_y \right] dx dy - \oint_{\Gamma} \bar{u}_n \delta N_n d\Gamma = 0. \quad (13)$$

由变分方程(2)中 $\delta u, \delta v, \delta \psi_x, \delta \psi_y$ 和 δw 的任意性, 容易得到

$$\begin{cases} \frac{\partial \delta N_x}{\partial x} + \frac{\partial \delta N_{xy}}{\partial y} = 0, \frac{\partial \delta N_{xy}}{\partial x} + \frac{\partial \delta N_y}{\partial y} = 0, \\ \frac{\partial^2 \delta M_x}{\partial x^2} + 2 \frac{\partial^2 \delta M_{xy}}{\partial x \partial y} + \frac{\partial^2 \delta M_y}{\partial y^2} + \delta N_x \frac{\partial^2 w}{\partial x^2} + \\ 2 \delta N_y \frac{\partial^2 w}{\partial y^2} + \delta N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{1}{R} (N_x + N_y) = 0. \end{cases} \quad (14)$$

利用式(14)和条件(12), 由式(13)可以得到

$$\iint_{\Sigma} \left\{ \left[\varepsilon_x - \frac{\partial u}{\partial x} - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + \frac{w}{R} \right] \delta N_x + \left[\gamma_{xy} - \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} - \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] \delta N_{xy} + \left[\varepsilon_y - \frac{\partial v}{\partial y} - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + \frac{w}{R} \right] \delta N_y \right\} dx dy + \oint_{\Gamma} (u_n - \bar{u}_n) \delta N_n d\Gamma = 0. \quad (15)$$

将式(3)和式(11)代入式(15), 并令在边界 Γ 上

$$\delta \phi = \delta \left(\frac{\partial \phi}{\partial s} \right) = \delta \left(\frac{\partial \phi}{\partial n} \right) = 0. \quad (16)$$

经过代入, 式(15)最后成为

$$\iint_{\Sigma} \left\{ \left[\frac{1}{E_x} \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial u}{\partial x} - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + \frac{w}{R} \right] \delta \frac{\partial^2 \phi}{\partial y^2} - \left[\frac{1}{G_{xy}} \frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] \delta \left(-\frac{\partial^2 \phi}{\partial x \partial y} \right) + \left[\frac{1}{E_y} \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial v}{\partial y} - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + \frac{w}{R} \right] \delta \frac{\partial^2 \phi}{\partial x^2} \right\} dx dy + \oint_{\Gamma} (u_n - \bar{u}_n) \delta N_n d\Gamma = 0. \quad (17)$$

对(17)式的第1积分进行分部积分并利用式(16), 使得

$$\iint_{\Sigma} \left[\frac{1}{E_x} \frac{\partial^4 \phi}{\partial y^4} + \frac{1}{E_y} \frac{\partial^4 \phi}{\partial x^4} + \frac{1}{G_{xy}} \frac{\partial^4 \phi}{\partial x^2 \partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \frac{1}{R} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \right] dx dy + \oint_{\Gamma} (u_n - \bar{u}_n) \delta N_n d\Gamma = 0. \quad (18)$$

利用坐标变换方法将式(18)的第1个积分表达式转换成极坐标的形式, 然后对 ϕ 和 w 作

轴对称近似,即取 $\phi = \phi(r), w = w(r)$. 对式(18)的极坐标形式积分表达式中的 θ 从 0 到 2π 进行积分,最后得到极坐标下双层网格圆底扁球壳的相容方程:

$$\left(\frac{3}{E_x} + \frac{3}{E_y} + \frac{1}{G_{xy}}\right) \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} r \frac{d\phi}{dr} + \frac{8}{r} \frac{d^2 w}{dr^2} \frac{dw}{dr} + \frac{8}{R} \frac{1}{r} \left(\frac{d}{dr} r \frac{dw}{dr}\right) = 0. \tag{19}$$

在轴对称情况下,薄膜力函数为

$$N_r = \frac{1}{r} \frac{d\phi}{dr}, N_\theta = \frac{d^2\phi}{dr^2}. \tag{20}$$

对式(19)积分一次并利用式(20),便得

$$\left(\frac{3}{E_x} + \frac{3}{E_y} + \frac{1}{G_{xy}}\right) r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} r^2 N_r + 4 \left(\frac{dw}{dr}\right)^2 + 8 \frac{r}{R} \frac{dw}{dr} = 0. \tag{21}$$

于是,方程(10)和(21)便成为双层网格圆底扁球壳的轴对称大挠度弯曲方程.

3 边界条件

考虑两种支承情况:

1) 边缘滑动固定

$$\text{当 } r = \frac{a}{2} \text{ 时, } w = 0, \psi_r = 0, N_r = 0, \tag{22}$$

$$\text{当 } r = 0 \text{ 时, } \psi_r = 0, N_r \text{ 有限}; \tag{23}$$

2) 边缘夹紧固定

$$\text{当 } r = \frac{a}{2} \text{ 时, } w = 0, \psi_r = 0, \int_0^{2\pi} u_r d\theta = 0, \tag{24}$$

$$\text{当 } r = 0 \text{ 时, } \psi_r = 0, N_r \text{ 有限}; \tag{25}$$

其中 ψ_r 由式(9)给出, u_r 表示径向位移,可由如下方程导出.

在极坐标系下,利用假设的位移场和应力场,我们得到

$$\varepsilon_\theta = \frac{u_r}{r} - \frac{w}{R}. \tag{26}$$

注意到当 $r = a/2$ 时, $w = 0$, 因而当 $r = a/2$ 时,

$$u_r = (a/2) \varepsilon_\theta. \tag{27}$$

根据直角坐标和极坐标的应变变换关系,得到

$$\varepsilon_\theta = \varepsilon_{x0} \sin^2\theta + \varepsilon_{y0} \cos^2\theta - \gamma_{xy0} \sin\theta \cos\theta. \tag{28}$$

而由 Hooke 定律,得到

$$\varepsilon_{x0} = \frac{N_x}{E_x}, \varepsilon_{y0} = \frac{N_y}{E_y}, \gamma_{xy0} = \frac{N_{xy}}{G_{xy}}. \tag{29}$$

利用式(6)和(8a)及轴对称条件,最后得到

$$u_r = \left(\frac{1}{E_x} + \frac{1}{E_y} - \frac{1}{G_{xy}}\right) N_r \sin^2\theta \cos^2\theta + \left(\frac{\sin^4\theta}{E_x} + \frac{\cos^4\theta}{E_y} + \frac{\sin^2\theta \cos^2\theta}{G_{xy}}\right) \frac{d}{dr} (r N_r). \tag{30}$$

4 非线性边值问题的求解

这里我们对边缘滑动固定和边缘夹紧固定两种支承下的稳定问题进行研究,均布压力 q 作用下的双层网格圆底扁球壳如图 2 所示. 为了分析简便,我们引入下列无量纲量:

$$\left\{ \begin{aligned} \rho &= \frac{2r}{a}, W = \frac{w}{h}, \Phi = \frac{dW}{d\rho} + \lambda_1 \rho, S = \frac{4arN_r}{3D_x + 4D_{xy} + 3D_y}, \\ P &= \frac{a^4 q}{4(3D_x + 4D_{xy} + 3D_y)h}, \\ \lambda_1 &= \frac{a^2}{4Rh}, \lambda_2 = \frac{3D_x + 4D_{xy} + 3D_y}{(G_x + G_y)a^2}, \\ \lambda_3 &= \frac{32h^2}{(3D_x + 4D_{xy} + 3D_y)(3/E_x + 3/E_y + 1/G_{xy})}, \\ \lambda_4 &= \left(\frac{1}{E_x} + \frac{1}{E_y} - \frac{1}{G_{xy}} \right) \left/ \left(\frac{3}{E_x} + \frac{3}{E_y} + \frac{1}{G_{xy}} \right) \right. \end{aligned} \right. \quad (31)$$

借助上述无量纲量,方程(10)和(21)可表示成

$$L(\Phi) = (\rho^2 - \lambda_2 L)(S\Phi/\rho) + P\rho^3, L(S) = -\lambda_3 \rho(\Phi^2 - \lambda_1^2 \rho^2), \quad (32)$$

其中 L 是微分算子,

$$L(\dots) = \rho^2 \frac{d}{d\rho} \frac{1}{\rho} \frac{d}{d\rho} \rho(\dots). \quad (33)$$

无量纲化后的边界条件为

1) 边缘滑动固定

$$\text{当 } \rho = 1 \text{ 时, } W = 0, \Phi = \lambda_1 - \lambda_2 S\Phi - \lambda_2 P, S = 0, \quad (34)$$

$$\text{当 } \rho = 0 \text{ 时, } \Phi = -\lambda_2 \frac{S\Phi}{\rho}, S = 0; \quad (35)$$

2) 边缘夹紧固定

$$\text{当 } \rho = 1 \text{ 时, } W = 0, \Phi = \lambda_1 - \lambda_2 S\Phi - \lambda_2 P, \frac{dS}{d\rho} + \lambda_4 \frac{S}{\rho} = 0, \quad (36)$$

$$\text{当 } \rho = 0 \text{ 时, } \Phi = -\lambda_2 \frac{S\Phi}{\rho}, S = 0. \quad (37)$$

首先采用修正迭代法求解边缘滑动固定下的非线性边值问题(32)和(34)及(35).

选取壳体无量纲中心挠度 $W_0 = W|_{\rho=0}$ 作为迭代参数,利用式(32)的第1式和式(31)的第

3式,得

$$W_0 = - \int_0^1 (\Phi - \lambda_1 \rho) d\rho. \quad (38)$$

对于第一次近似,得到下列线性边值问题:

$$L(\Phi_1) = P\rho^3, L(S_1) = -\lambda_3 \rho(\Phi_1^2 - \lambda_1^2 \rho^2). \quad (39a,b)$$

$$\text{当 } \rho = 1 \text{ 时, } \Phi_1 = \lambda_1 - \lambda_2 P, S_1 = 0, \quad (40a,b)$$

$$\text{当 } \rho = 0 \text{ 时, } \Phi_1 = 0, S_1 = 0. \quad (41a,b)$$

于是满足方程(39a)和边界条件(40a)和(41a)的解是

$$\Phi_1 = P(a_1 \rho^3 + a_2 \rho) + \lambda_1 \rho, \quad (42)$$

其中

$$a_1 = \frac{1}{8}, a_2 = -\frac{1}{8} - \lambda_2. \quad (43)$$

将式(42)代入式(38),得

$$P = \alpha_0 W_0, \quad (44)$$

其中

$$\alpha_0 = -\frac{4}{a_1 + 2a_2}. \quad (45)$$

将式(44)代入式(42),得

$$\Phi_1 = (a_1 \rho^3 + a_2 \rho) \alpha_0 W_0 + \lambda_1 \rho. \quad (46)$$

利用已求得的解(46),方程(39b)在边界条件(40b)和(41b)下的解是

$$S_1 = (b_1 \rho^7 + b_2 \rho^5 + b_3 \rho^3 + b_4 \rho) \alpha_0^2 W_0^2 + \lambda_1 \alpha_0 W_0 (b_5 \rho^5 + b_6 \rho^3 + b_7 \rho), \quad (47)$$

其中各系数为

$$\begin{cases} b_1 = -\frac{\lambda_3 a_1^2}{48}, & b_2 = -\frac{\lambda_3 a_1 a_2}{12}, & b_3 = -\frac{\lambda_3 a_2^2}{8}, & b_4 = -(b_1 + b_2 + b_3), \\ b_5 = -\frac{\lambda_3 a_1}{12}, & b_6 = -\frac{\lambda_3 a_2}{4}, & b_7 = -(b_5 + b_6). \end{cases} \quad (48)$$

关于二次近似,可得到关于 Φ 的下列边值问题:

$$L(\Phi_2) = (\rho^2 - \lambda_2 L) \left(\frac{1}{\rho} S_1 \Phi_1 \right) + P \rho^3, \quad (49)$$

当 $\rho = 1$ 时,

$$\Phi_2 = \lambda_1 \rho - \lambda_2 \frac{S_1 \Phi_1}{\rho} - \lambda_2 \rho P, \quad (50)$$

当 $\rho = 0$ 时,

$$\Phi_2 = -\lambda_2 \frac{S_1 \Phi_1}{\rho}. \quad (51)$$

利用已得到的解(46)和(47),方程(49)满足条件(50)和(51)的解是

$$\begin{aligned} \Phi_2 = & \lambda_1 \rho + (c_1 \rho^3 + c_2 \rho) P + \alpha_0^3 W_0^3 (c_3 \rho^{11} + c_4 \rho^9 + c_5 \rho^7 + c_6 \rho^5 + c_7 \rho^3 + c_8 \rho) + \\ & \lambda_1 \alpha_0^2 W_0^2 (c_9 \rho^9 + c_{10} \rho^7 + c_{11} \rho^5 + c_{12} \rho^3 + c_{13} \rho) + \\ & \lambda_1^2 \alpha_0 W_0 (c_{14} \rho^7 + c_{15} \rho^5 + c_{16} \rho^3 + c_{17} \rho), \end{aligned} \quad (52)$$

其中

$$\begin{cases} c_1 = \frac{1}{8}, & c_2 = -\left(\lambda_2 + \frac{1}{8}\right), & c_3 = \frac{a_1 b_1}{120}, & c_4 = \frac{a_1 b_2 + a_2 b_1}{80} - \lambda_2 a_1 b_1, \\ c_5 = \frac{a_2 b_2 + a_1 b_3}{48} - \lambda_2 (a_1 b_2 + a_2 b_1), \\ c_6 = \frac{a_2 b_3 + a_1 b_4}{24} - \lambda_2 (a_2 b_2 + a_1 b_3), \\ c_7 = \frac{a_2 b_4}{8} - \lambda_2 (a_2 b_3 + a_1 b_4), & c_8 = -\left(\sum_{i=3}^7 c_i + \lambda_2 \sum_{j=1}^4 \sum_{i=1}^2 a_i b_j\right), \\ c_9 = \frac{a_1 b_5 + b_1}{80}, & c_{10} = \frac{a_2 b_5 + a_1 b_6 + b_2}{48} - \lambda_2 (a_1 b_5 + b_1), \\ c_{11} = \frac{a_2 b_6 + a_1 b_7 + b_3}{24} - \lambda_2 (a_2 b_5 + a_1 b_6 + b_2), \end{cases} \quad (53a)$$

$$\begin{cases} c_{12} = \frac{a_2 b_7 + b_4}{8} - \lambda_2 (a_2 b_6 + a_1 b_7 + b_3), \\ c_{13} = - \left[\sum_{i=9}^{12} c_i + \lambda_2 \left(\sum_{j=5}^7 \sum_{i=1}^2 a_i b_j + \sum_{i=1}^4 b_i \right) \right], c_{14} = \frac{b_5}{48}, \\ c_{15} = \frac{b_6}{24} - \lambda_2 b_5, c_{16} = \frac{b_7}{8} - \lambda_2 b_6, c_{17} = - \left(\sum_{i=14}^{16} c_i + \lambda_2 \sum_{i=5}^7 b_i \right). \end{cases} \quad (53b)$$

将式(52)代入式(38),得到

$$P = (\alpha_0 + \lambda_1^2 \alpha_1) W_0 + \lambda_1 \alpha_2 W_0^2 + \alpha_3 W_0^3, \quad (54)$$

其中

$$\begin{cases} \alpha_1 = \alpha_0^2 \left(\frac{1}{8} c_{14} + \frac{1}{6} c_{15} + \frac{1}{4} c_{16} + \frac{1}{2} c_{17} \right), \\ \alpha_2 = \alpha_0^3 \left(\frac{1}{10} c_9 + \frac{1}{8} c_{10} + \frac{1}{6} c_{11} + \frac{1}{4} c_{12} + \frac{1}{2} c_{13} \right), \\ \alpha_3 = \alpha_0^4 \left(\frac{1}{12} c_3 + \frac{1}{10} c_4 + \frac{1}{8} c_5 + \frac{1}{6} c_6 + \frac{1}{4} c_7 + \frac{1}{2} c_8 \right). \end{cases} \quad (55)$$

更高次近似可以用类似方法得到,然而,对于工程实际问题,二次近似足够精确。

为了得到临界载荷,我们使用极值条件

$$\frac{dP}{dW_0} = 0. \quad (56)$$

由此,方程便得到如下形式的无量纲临界屈曲载荷表达式:

$$P^* = (\alpha_0 + \lambda_1^2 \alpha_1) W_0^* + \lambda_1 \alpha_2 W_0^{*2} + \alpha_3 W_0^{*3}, \quad (57)$$

其中对应于式(58)中正、负号的 P^* 分别是下、上临界载荷。对于实际工程,只有上临界载荷才有意义。 W_0^* 表示双层网格圆底扁球壳失稳时的无量纲临界挠度,

$$W_0^* = \frac{-\lambda_1 \alpha_2 \pm \sqrt{(\alpha_2^2 - 3\alpha_1 \alpha_3) \lambda_1^2 - 3\alpha_0 \alpha_3}}{3\alpha_3}. \quad (58)$$

令方程(56)中的判别式为0,得到壳体临界几何参数

$$(\lambda_1)_0 = \sqrt{\frac{3\alpha_0 \alpha_3}{\alpha_2^2 - 3\alpha_1 \alpha_3}}. \quad (59)$$

于是, $(\lambda_1)_0$ 被用来区分壳体屈曲与否。当 $\lambda_1 \geq (\lambda_1)_0$ 时,壳体将会出现屈曲;而当 $\lambda_1 < (\lambda_1)_0$ 时,壳体不会发生屈曲。

对于边缘夹紧固定边界,只须将式(48)的系数 b_4 和 b_7 改为

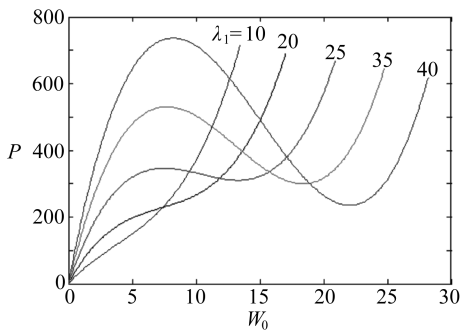
$$\begin{cases} b_4 = - \frac{(7 + \lambda_4) b_1 + (5 + \lambda_4) b_2 + (3 + \lambda_4) b_3}{1 + \lambda_4}, \\ b_7 = - \frac{(5 + \lambda_4) b_5 + (3 + \lambda_4) b_6}{1 + \lambda_4}, \end{cases} \quad (60)$$

其余跟边缘滑动固定边界情况相同。

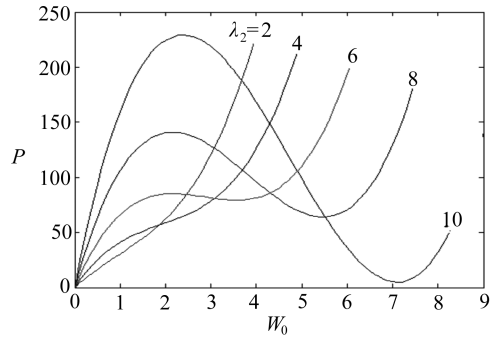
5 数值结果与讨论

我们对承受均布横向载荷且外边缘滑动固定或外边缘夹紧固定的双层网格圆底扁球壳结构的非线性稳定性问题进行了数值计算。网壳的有关参数如下:跨度 $a = 60$ m,厚度 $h = 0.6$ m,

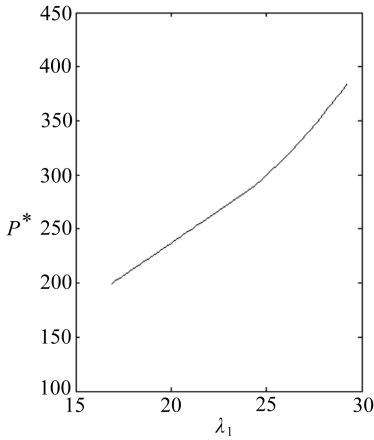
网格的长度和宽度均为 $L_1 = L_2 = L = 0.6$ m, 网格表层杆件采用 $\phi 102 \times 7$ mm 的圆钢管, 腹杆和斜杆采用 $\phi 60 \times 3.5$ mm 的圆钢管, 弹性模量为 $E = 1.9 \times 10^5$ MPa.



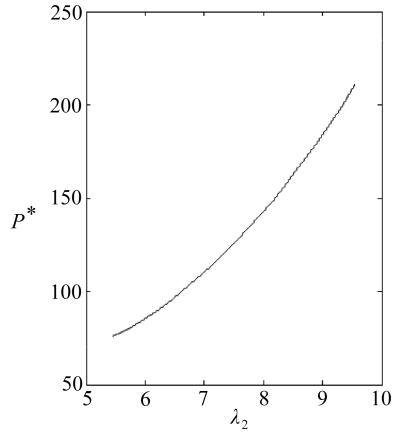
(a) 滑动固定



(b) 夹紧固定

图2 不同 λ_1 值下的特征曲线

(a) 滑动固定



(b) 夹紧固定

图3 上临界屈曲压力 P^* 随 λ_1 值的变化曲线

将上述数据代入式(54), 则外边缘滑动固定和外边缘夹紧固定的数值结果分别在图2(a)和2(b)中给出. 从图2可以看出, 当壳体几何参数 λ_1 较小, 壳体比较扁平时, 壳体不会出现失稳现象; 当 λ_1 值增大时, 壳体发生屈曲. 并且从图2不难看出, 当几何参数 λ_1 偏小时, 壳体在夹紧固定边界下更容易发生屈曲.

将上述数据代入式(57), 则在外边缘滑动固定和外边缘夹紧固定条件下, 上临界屈曲压力 P^* 随几何参数 λ_1 的变化曲线在图3中给出. 结果表明, 上临界屈曲压力 P^* 随着 λ_1 值的增加而增大.

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Nonlinear Stability of a Double-Deck Reticulated Circular Shallow Spherical Shell

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Abstract: Based on the variational equation of the nonlinear bending theory of double-deck reticulated shallow shells, equations of large deflection and boundary conditions for a double-deck reticulated circular shallow spherical shell under a uniformly distributed pressure were derived by using coordinate transformation means and stationary complementary energy principle. The characteristic relationship and critical buckling pressure for the shell with two types of boundary conditions were obtained by taking the modified iteration method. The effects of geometrical parameters on the buckling behavior were also discussed.

Key words: double-deck reticulated circular shallow spherical shell; nonlinear stability; equivalent continuum method; modified iteration method; stationary complementary energy principle