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具有收缩表面的二阶流体驻点流动的级数解*

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摘要: 研究具有收缩表面的边界层流动的解析解. 通过相似变换, 将偏微分方程简化为可用同伦分析法(HAM)求解的常微分方程. 然后讨论了具有收缩表面的二维轴对称流动.

关 键 词: 驻点流动; 二阶流体; 收缩平面; 同伦分析法
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引 言

由于非 Newton流体在工程和工业领域的广泛应用, 人们对其研究的兴趣日益增加. 至今为止提出了多种非 Newton流体的模型, 其中对二阶流体模型的研究最多[1-15]. 另外在塑料片材的空气动力挤压、玻璃纤维、造纸和高分子片材制造等领域, 非 Newton流体的边界层流动有着重要作用. 在 Sakiadis[16]前期研究的基础上, 许多作者对延伸表面上的边界层流动进行了讨论[17-24]. 现有文献表明, 研究收缩表面上的边界层流动[25-30]还不多.

本文在考虑了热传递效应的同时, 对收缩表面上二阶流体的驻点流动进行了研究. 同伦分析法是求解非线性问题的强有力的工具, 本文应用同伦分析法[31-45]给出了二维问题和轴对称问题的级数解, 并求得级数解的收敛性. 给出了图形结果并进行了讨论.

1 公式的建立

考虑收缩表面上不可压缩二阶流体的二维驻点流动. x, y, z轴上的速度分量分别用 u, v, w表示. 假设无限远处驻点流动速度为 $u = ax$, $w = -az$. 延伸(收缩)表面上驻点流动速度为 $u = b(x + c)$, $w = 0$. 其中 $b > 0$为延伸率($b < 0$为收缩率), $-c$为延伸原点位置. 流体流动和传热控制方程[30, 46]如下:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\left(u\frac{\partial^3 u}{\partial x\partial z^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2} + \frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z} + w\frac{\partial^3 u}{\partial z^3}\right), \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}\right) = k\nabla^2 T, \tag{3}$$

其中, $\nu = \mu/\rho$为动黏度, ρ为密度, α_1 为二阶参数, c_p 为比热比, T为温度, k (>0)为热传导系

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数, 方程忽略了耗散项.

2　二维驻点流动

定义[30]

$$\eta=\sqrt{(a/\nu)}z,\quad u=axf'(\eta)+bch(\eta),\quad v=0,\quad w=-\sqrt{a\nu}f(\eta),\tag{4}$$

$$\theta=(T-T_\infty)/(T_0-T_\infty).\tag{5}$$

方程 (1) 恒满足, 通过方程 (2) 和 (3) 成为

$$f'''+ff''-f'^2+1+\beta(2f'f'''-f''^2-ff^{\mathrm{IV}})=0,\tag{6}$$

$$h''+fh'-f'h+\beta(hf'''+f'h''+f''h'-fh''')=0,\tag{7}$$

$$\theta''+Prf\theta'=0,\tag{8}$$

其中　$\beta=\alpha_1 a/(\rho\nu)$.

相应的边界条件为

$$\begin{cases} f(0)=0,\ f'(0)=b/a=\alpha,\ \theta(0)=1,\\ f'(\infty)=1,\ h(0)=1,\ h(\infty)=0,\ \theta(\infty)=0,\end{cases}\tag{9}$$

式中的上撇号表示对 η 求导, 且 $Pr=\nu/k$.

2.1　收缩表面上二维驻点流动的解

以下求解收缩表面上二阶流体驻点流动. 为了用 HAM 求解, 我们选择以下初始近似值和辅助的线性算子:

$$f_{01}(\eta)=(1-\alpha)(e^{-\eta}-1)+\eta,\tag{10}$$

$$h_{01}(\eta)=e^{-\eta},\tag{11}$$

$$\theta_{01}(\eta)=e^{-\eta},\tag{12}$$

$$\mathscr{L}_{01}[\hat f(\eta;p)]=\frac{\partial^3\hat f(\eta;p)}{\partial\eta^3}+\frac{\partial^2\hat f(\eta;p)}{\partial\eta^2},\tag{13}$$

$$\mathscr{L}_{02}[\hat h(\eta;p)]=\frac{\partial^2\hat h(\eta;p)}{\partial\eta^2}+\frac{\partial\hat h(\eta;p)}{\partial\eta},\tag{14}$$

$$\mathscr{L}_{03}[\hat\theta(\eta;p)]=\frac{\partial^2\hat\theta(\eta;p)}{\partial\eta^2}+\frac{\partial\hat\theta(\eta;p)}{\partial\eta},\tag{15}$$

式中下标 01 表示第 1 种情况, 其他情况亦同. 辅助的线性算子具有属性:

$$\mathscr{L}_{01}[C_1+C_2\eta+C_3e^{-\eta}]=0,\tag{16}$$

$$\mathscr{L}_{02}[C_4+C_5e^{-\eta}]=0,\tag{17}$$

$$\mathscr{L}_{03}[C_6+C_7e^{-\eta}]=0,\tag{18}$$

其中 C_i ($i=1,2,\cdots,7$) 为任意常数. 如果 $p\in[0,1]$ 为一个嵌入参数, $\hbar_i$ ($i=1,2,3$) 为非零的辅助参数, 则 0 阶和 m 阶变形问题如下:

0 阶变形问题

$$(1-p)\mathscr{L}_{01}[\hat f(\eta;p)-f_{01}(\eta)]=p\hbar_1\mathscr{N}_1[\hat f(\eta;p)],\tag{19}$$

$$(1-p)\mathscr{L}_{02}[\hat h(\eta;p)-h_{01}(\eta)]=p\hbar_2\mathscr{N}_2[\hat h(\eta;p),\hat f(\eta;p)],\tag{20}$$

$$(1-p)\mathscr{L}_{03}[\hat\theta(\eta;p)-\theta_{01}(\eta)]=p\hbar_3\mathscr{N}_3[\hat\theta(\eta;p),\hat f(\eta;p)],\tag{21}$$

$$\hat f(0;p)=0,\quad \hat f'(0;p)=\alpha,\quad \hat f'(\infty;p)=1,\tag{22}$$

$$\hat h(0;p)=1,\quad \hat h(\infty;p)=0,\tag{23}$$

$$\hat\theta(0;p)=1,\quad \hat\theta(\infty;p)=0,\tag{24}$$

$$\mathscr{N}_1[\hat f(\eta;p)]=\frac{\partial^3\hat f(\eta;p)}{\partial\eta^3}+\hat f(\eta;p)\frac{\partial^2\hat f(\eta;p)}{\partial\eta^2}-\left(\frac{\partial\hat f(\eta;p)}{\partial\eta}\right)^2+1+$$

$$\beta\left[2\frac{\partial \hat{f}(\eta;p)}{\partial\eta}\frac{\partial^3 \hat{f}(\eta;p)}{\partial\eta^3}+\left(\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2}\right)^2-\hat{f}(\eta;p)\frac{\partial^4 \hat{f}(\eta;p)}{\partial\eta^4}\right], \tag{25}$$

$$\begin{aligned}\mathcal{N}_2[\hat{f}(\eta;p),\ \hat{h}(\eta;p)] = {} & \frac{\partial^2 \hat{h}(\eta;p)}{\partial\eta^2}+\hat{f}(\eta;p)\frac{\partial \hat{h}(\eta;p)}{\partial\eta}-\hat{h}(\eta;p)\frac{\partial \hat{f}(\eta;p)}{\partial\eta}+\\ & \beta\left[\hat{h}(\eta;p)\frac{\partial^3 \hat{f}(\eta;p)}{\partial\eta^3}+\frac{\partial \hat{f}(\eta;p)}{\partial\eta}\frac{\partial^2 \hat{h}(\eta;p)}{\partial\eta^2}\right]+\\ & \beta\left[\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2}\frac{\partial \hat{h}(\eta;p)}{\partial\eta}-\hat{f}(\eta;p)\frac{\partial^3 \hat{h}(\eta;p)}{\partial\eta^3}\right],\end{aligned} \tag{26}$$

$$\mathcal{N}_3[\hat{\theta}(\eta;p),\ \hat{f}(\eta;p)] = \frac{\partial^2 \hat{\theta}(\eta;p)}{\partial\eta^2}+Pr\,\hat{f}(\eta;p)\frac{\partial \hat{\theta}(\eta;p)}{\partial\eta}, \tag{27}$$

m 阶变形问题

$$\mathcal{L}_{01}[f_m(\eta)-\chi_m f_{m-1}(\eta)] = \hbar_1\mathcal{R}_{1m}(\eta), \tag{28}$$

$$\mathcal{L}_{02}[h_m(\eta)-\chi_m h_{m-1}(\eta)] = \hbar_2\mathcal{R}_{2m}(\eta), \tag{29}$$

$$\mathcal{L}_{03}[\theta_m(\eta)-\chi_m \theta_{m-1}(\eta)] = \hbar_3\mathcal{R}_{3m}(\eta), \tag{30}$$

$$\begin{cases} f_m(0)=0,\quad f'_m(0)=0,\quad f'_m(\infty)=0, \\ h_m(0)=0,\quad h_m(\infty)=0,\quad \theta_m(0)=0,\quad \theta_m(\infty)=0, \end{cases} \tag{31}$$

$$\begin{aligned}\mathcal{R}_{1m}(\eta) = {} & f'''_{m-1}(\eta)+(1-\chi_m)+\sum_{k=0}^{m-1}[f_{m-1-k}f''_k-f'_{m-1-k}f'_k]+\\ & \beta\sum_{k=0}^{m-1}[2f'_{m-1-k}f'''_k+f''_{m-1-k}f''_k-f_{m-1-k}f^{\mathrm{IV}}_k],\end{aligned} \tag{32}$$

$$\begin{aligned}\mathcal{R}_{2m}(\eta) = {} & h''_{m-1}(\eta)+\sum_{k=0}^{m-1}[h'_{m-1-k}f_k-h_{m-1-k}f'_k]+\\ & \beta\sum_{k=0}^{m-1}[h_{m-1-k}f'''_k+f'_{m-1-k}h''_k+h'_{m-1-k}f''_k-f_{m-1-k}h'''_k],\end{aligned} \tag{33}$$

$$\mathcal{R}_{3m}(\eta) = \theta''_{m-1}(\eta)+Pr\sum_{k=0}^{m-1}\theta'_{m-1-k}f_k, \tag{34}$$

$$\chi_m = \begin{cases} 0, & m \leqslant 1, \\ 1, & m > 1, \end{cases} \tag{35}$$

通过 MATHEMATICA 符号计算软件, 可以得到方程 (28) ~ (30)的前几阶近似解. 得到 f, h, θ 的解

$$f(\eta) = \sum_{m=0}^{\infty}f_m(\eta) = \lim_{M\to\infty}\left[\sum_{m=0}^{M}a^0_{m,0}+\sum_{n=1}^{2M+1}e^{-(n+1)\eta}\left(\sum_{m=n-1}^{2M}\sum_{k=1}^{2m+1-n}a^k_{m,n}\eta^k\right)\right], \tag{36}$$

$$h(\eta) = \sum_{m=0}^{\infty}h_m(\eta) = \lim_{M\to\infty}\left[\sum_{n=1}^{2M+1}e^{-(n+1)\eta}\left(\sum_{m=n-1}^{2M}\sum_{k=0}^{2m+1-n}c^k_{m,n}\eta^k\right)\right], \tag{37}$$

$$\theta(\eta) = \sum_{m=0}^{\infty}\theta_m(\eta) = \lim_{M\to\infty}\left[\sum_{n=1}^{2M+1}e^{-(n+1)\eta}\left(\sum_{m=n-1}^{2M}\sum_{k=0}^{2m+1-n}b^k_{m,n}\eta^k\right)\right], \tag{38}$$

3 轴对称收缩表面上的轴对称驻点流动

通过变换[30]

$$\begin{cases} \eta(x,y) = \sqrt{(a/\nu)}\,z,\quad u = axg'(\eta)+bcl(\eta),\quad v = ayg'(\eta), \\ w = -2\sqrt{a\nu}\,g(\eta),\quad \theta = (T-T_\infty)/(T_0-T_\infty), \end{cases} \tag{39}$$

由方程 (2)和 (3)得到

$$g''' + 2gg'' - g'^2 + 1 + \beta\left(2g'g''' + g''^2 - 2gg^{\mathrm{iv}}\right) = 0, \tag{40}$$

$$l'' + 2gl' - g'l + \beta\left(lg''' + g'l'' + g''l' - 2gl'''\right) = 0, \tag{41}$$

$$\theta'' + 2Pr\,g\theta' = 0, \tag{42}$$

相应的边界条件[30]为

$$\begin{cases} g(0) = 0,\quad g'(0) = b/a = \alpha,\quad \theta(0) = 1, \\ g'(\infty) = 1,\quad l(0) = 1,\quad l(\infty) = 0,\quad \theta(\infty) = 0, \end{cases} \tag{43}$$

采用前小节的推导过程,得到轴对称收缩表面上的轴对称驻点流动解:

$$g(\eta) = \sum_{m=0}^{\infty} g_m(\eta) = \lim_{M\to\infty}\left[\sum_{m=0}^{M} a_{m,0}^{0} + \sum_{n=1}^{2M+1} \mathrm{e}^{-(n+1)\eta}\left(\sum_{m=n-1}^{2M}\sum_{k=0}^{2m+1-n} a_{m,n}^{k}\eta^{k}\right)\right], \tag{44}$$

$$l(\eta) = \sum_{m=0}^{\infty} l_m(\eta) = \lim_{M\to\infty}\left[\sum_{n=1}^{2M+1} \mathrm{e}^{-(n+1)\eta}\left(\sum_{m=n-1}^{2M}\sum_{k=0}^{2m+1-n} c_{m,n}^{k}\eta^{k}\right)\right], \tag{45}$$

$$\theta(\eta) = \sum_{m=0}^{\infty} \theta_m(\eta) = \lim_{M\to\infty}\left[\sum_{n=1}^{2M+1} \mathrm{e}^{-(n+1)\eta}\left(\sum_{m=n-1}^{2M}\sum_{k=0}^{2m+1-n} b_{m,n}^{k}\eta^{k}\right)\right]. \tag{46}$$

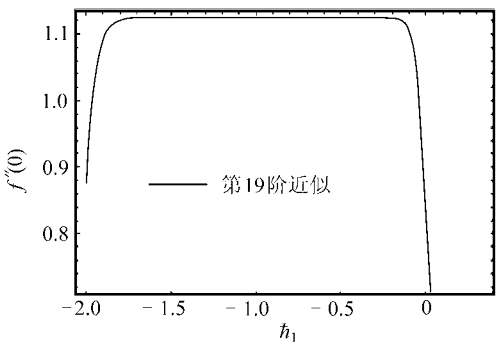

图 1　二维驻点流中 f 的 $\hbar$ 曲线

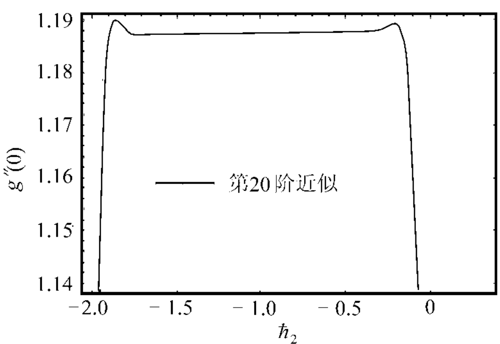

图 2　轴对称收缩表面上轴对称驻点流中 g 的 $\hbar$ 曲线

4　结果与讨论

通过方程(36)~(38)和方程(44)~(46)的计算,得到两个问题的解析解,包含非零的辅助参数 $\hbar_i$ $(i=1,2)$,调整该参数用来控制解的收敛性. $\hbar$ 曲线[31]定义为:在 $\hbar$ 所有可能取值的有效区域内呈一条水平线段. 图 1和图 2分别给出了本文中第 19阶和第 20阶的 $\hbar$ 曲线,从图中可以看出, $\hbar$ 值的允许范围分别为 $-1.8\leqslant\hbar_1\leqslant-0.3$,和 $-1.9\leqslant\hbar_2\leqslant-0.3$.

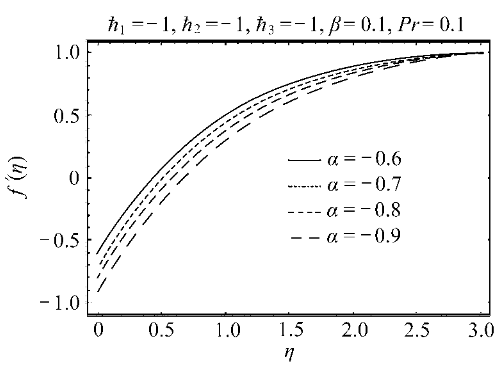

图 3　二维驻点流动中收缩参数 α 对 f' 的影响

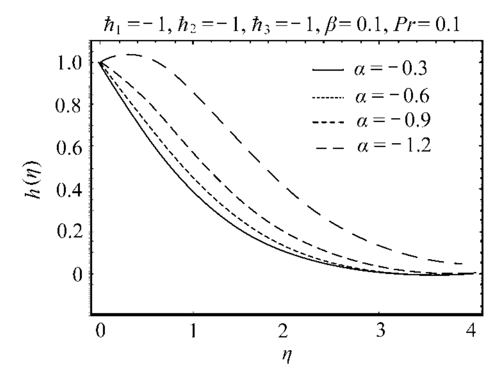

图 4　二维驻点流动中收缩参数 α 对 h 的影响

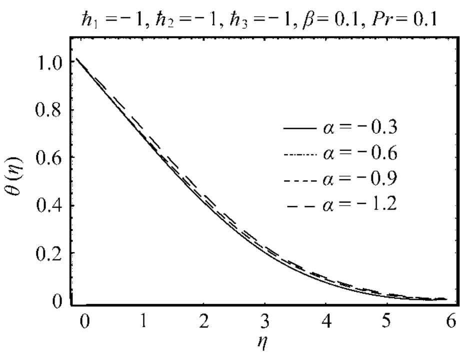

图 5　二维驻点流动中收缩参数 α 对 θ 的影响

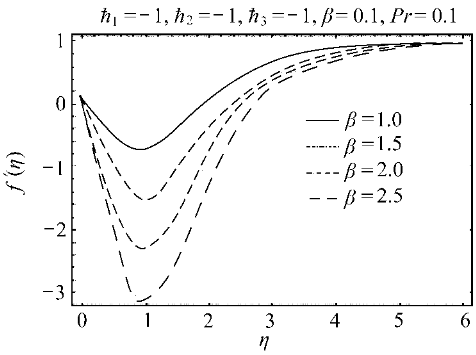

图 6　二维驻点流动中二阶参数 β 对 f' 的影响

图 3~图 7给出了二维驻点流动的速度 f', h和温度 θ. 其中图 3~图 5显示了不同收缩参数 α < 0时, η与无量纲速度 f', h和温度 θ之间的关系. f'随 α的增大而减小, 而 h和 θ随 α的增大而增大.

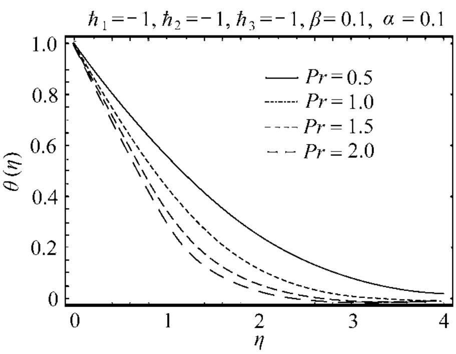

图 7　二维驻点流动中 Prandtl数对 θ 的影响

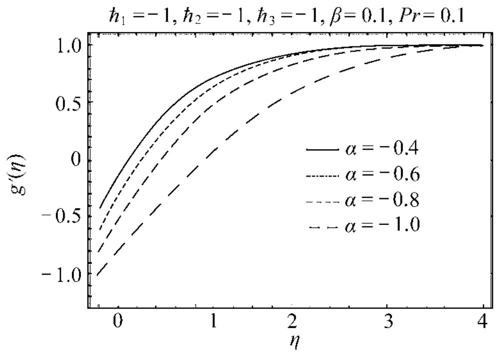

图 8　轴对称收缩表面上的轴对称驻点流动中收缩参数 α 对 g' 的影响

图 6给出了二阶参数 β对 f' 的影响, f' 随 β的增大而减小; 图 7给出了 Prandtl数对 θ的影响, θ随 Prandtl数的增大而减小.

图 8~图 10给出了轴对称收缩表面上轴对称驻点流动时, 对于不同的 α, β和 Pr值, g', l, θ和 η之间的关系. 从图中可以看出, g'随收缩参数 α的增大而增大, 而 l和 θ随 α的增大而减小.

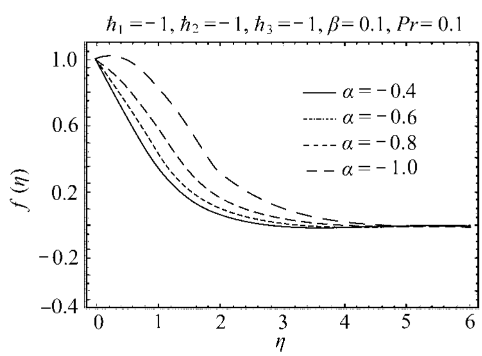

图 9　轴对称收缩表面上的轴对称驻点流动中收缩参数 α 对 l的影响

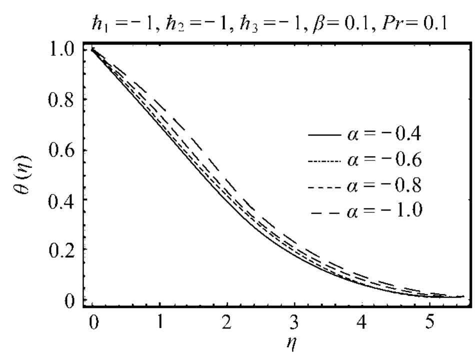

图 10　轴对称收缩表面上的轴对称驻点流动中收缩参数 α 对 θ 的影响

5 总　　结

本文对收缩表面上二阶流体的驻点流动进行了研究. 通过应用同伦分析法, 得到二维驻点

流动和轴对称收缩表面两种情况的有效级数解, 并给出了结果的收敛性. 最后给出本文的图形结果, 并分析了不同参数对结果的影响. 从本文的分析可以看出, 对于强非线性问题, 同伦分析法对控制和调整问题的收敛区域, 提供了一个简易的方法, 该方法可应用于很多高度非线性的问题.

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Series Solutions for the Stagnation Flow of a Second Grade Fluid Over a Shrinking Sheet

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Abstract: The analytic solutions of boundary layer flows bounded by a shrinking sheet are derived. Using similarity transformations, the partial differential equations were reduced into the ordinary differential equations which were then solved by homotopy analysis method (HAM). Two-dimensional and axisymmetric shrinking flow cases were discussed.

Key words: stagnation flow; second grade fluid; shrinking sheet; homotopy analysis method