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基于时变拓扑结构的二阶多智能体系统采样一致性*

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摘要： 基于速度一致位移差保持不变的一致性概念, 研究了二阶多智能体系统在时变拓扑下的采样一致性问题. 首先, 引入虚拟领导者, 将具有时变拓扑结构的多智能体系统的采样一致性问题转换为误差系统的采样控制稳定性问题. 其次, 通过预估采样误差, 研究采样误差对系统达到一致性的影响. 最后, 应用 Lyapunov 稳定性理论, 分析所构造的误差系统的稳定性, 并给出该误差系统最终稳定的充分条件. 数值仿真结果验证了理论分析的有效性和正确性.

关　键　词： 多智能体系统；　一致性；　采样数据；　时变拓扑

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Sampling Consensus of 2nd-Order Multi-Agent Systems Based on Time-Varying Topology

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Abstract: The sampling consensus of 2nd-order multi-agent systems with time-varying topology was investigated based on the constant position difference and the consistent speed. Firstly, the virtual leader was introduced and the sampling consensus problem of multi-agent systems was transformed into the stability problem of the corresponding error system. Secondly, with estimation of the sampling errors, the influence of sampling errors on system consistency was studied. Finally, by virtue of the Lyapunov stability theory, the stability of the constructed error system was analyzed, and a sufficient condition for the stability of the error system was given. The numerical simulation results verify the effectiveness and correctness of the theoretical analysis.

Key words: multi-agent system; consensus; sampled data; time-varying topology

引　言

随着多智能体系统的协调控制在无人驾驶飞行器、分布式小卫星群的编队飞行以及高速公路系统设计等领域的广泛应用, 多智能体系统的一致性、稳定性和时变性已经成为控制界的研究热点[1-3].

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通过区分是否存在一个领导，多智能体系统中的一致性问题可以分为两种：领导跟随一致性和无领导跟随一致性. 在文献 [4] 中，Wu 等根据不同的动力学系统，研究了具有执行器故障和外部系统干扰的领导跟随多智能体系统容错一致性跟踪控制问题. 在过去的十几年中，很多学者都专注于研究多智能体系统的二阶一致性问题. 文献 [5] 基于切换拓扑结构，研究了多智能体系统的领导跟随集群问题. 文献 [6] 以相互独立的波动偏微分方程组为模型，探讨了大型多智能体系统的编队跟踪控制问题. 文献 [7] 为了解决符号网络结构平衡问题，提出了一种适用于有向网络结构的算法，研究了具有非线性动力系统和带有有向拓扑图的奇异多智能体系统领导跟随一致性问题. 文献 [8] 中为了显著降低通信负担，提出了一种事件触发控制算法来解决二阶多智能体领导跟随一致性问题.

与具有固定拓扑结构的时不变系统相比[9-11]，带有外部扰动的时变系统的研究更具挑战性和难度. 例如，时不变系统$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$是稳定的，当且仅当矩阵 $\boldsymbol{A}$ 是 Hurwitz 矩阵. 但是，对于线性时变系统$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x}(t)$而言，这个结论就不再成立了[12]. 从过去十几年里研究时变系统稳定性的文献[13-18] 中可以看出，由于耦合项和时变节点动力学的共存，很难将时变网络的稳定性结果推广到多智能体时变网络的一致性. 总而言之，研究具有时变节点的多智能体网络动态系统比研究具有时不变节点的更具挑战性和实用性. 文献 [12] 研究了时变拓扑下带有采样的一阶多智能体系统的一致性问题，而对二阶多智能系统在时变拓扑下的采样一致性问题并没有更深一步的研究. 另一方面，在过去的一段时间内，很多学者针对切换拓扑的问题进行了研究. 文献 [19] 中最终取得一致性的充分必要条件是所采用的切换网络是联合连通的，文献 [20-23] 也使用了切换系统来模拟时变拓扑系统. 但值得注意的是，该拓扑结构由于仅在确定的切换时刻变化，所以具有一定的局限性. 由此可见，时变拓扑下二阶多智能体系统的采样一致性问题是一个值得研究的话题.

此外，现有文献中多智能体系统的研究总是假设智能体和其邻居之间是连续通信的. 与连续通信相比，采样策略因为提高了资源的有效利用从而显示出了其优越性[24-29]. 多智能体的领导跟随一致性[30-33] 已经被许多学者研究过并得出了很多的结论和成果. 但遗憾的是，具有时变拓扑结构的多智能体系统采样一致性的理论成果还比较少.

基于上述讨论，本文在固定时间间隔采样的机制下，研究了具有时变拓扑的二阶多智能体系统的一致性问题. 主要工作和贡献如下：

1) 研究了采样数据通信下带有时变特征的二阶多智能体系统的一致性问题. 首先根据 Laplace 矩阵存在一个零特征值，将具有时变特征的一致性问题等价转化为几个线性时变系统的稳定性. 然后利用 Lyapunov 方法推导出达到一致性的充分条件.

2) 通过构造虚拟领导者，将无领导跟随一致问题转化为有领导跟随一致问题，从而可以采用有领导一致问题的研究方法进行分析.

3) 给出了一个在采样数据通信下带有时变特征的二阶多智能体系统的仿真实例来验证所得结论的有效性和正确性.

符号说明：对任意向量$\boldsymbol{x} \in \mathbb{R}^n$，定义其转置和欧式范数分别为$\boldsymbol{x}^{\mathrm{T}}$和$\|\boldsymbol{x}\|$. $\boldsymbol{I}_n$为n维单位矩阵，$\boldsymbol{I}_N$为元素全为 1 的N维列向量. $\mathbb{N}$代表自然数集，$\mathbb{N}_+$代表正整数集. 令S_n为n维对称矩阵集，S_n^+是n维对称正定矩阵集，$\bar{S}_n^+$是n维对称半正定矩阵集. 对$\boldsymbol{A}, \boldsymbol{B} \in S_n$, $\boldsymbol{A} > (\geqslant) \boldsymbol{B}$意味着 $\boldsymbol{A}-\boldsymbol{B}$ 是正定(半正定)的.

1　预备知识和模型构建

1.1　预备知识

这一部分，回顾了一些图论的概念、定义、引理和假设，对后续研究起到了重要作用.

用$\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \tilde{\boldsymbol{A}}\}$描述多智能体之间的无向信息交互拓扑图，其中$\mathcal{V} = \{0, 1, 2, \cdots, N\}$表示这个拓扑图的顶点集合，$\mathcal{E} = \{e_{ij} = (i, j)\} \subseteq \mathcal{V} \times \mathcal{V}$表示拓扑边集，$\tilde{\boldsymbol{A}} = [a_{ij}]_{N\times N}$为拓扑图相应的邻接矩阵. $\mathcal{V}$中的$i (i = 0, 1, 2, \cdots, N)$ 节点表示第i个智能体，当$i = 0$时，对应的多智能体为领导者. 图$\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \tilde{\boldsymbol{A}}\}$所对应的 Laplace 矩阵$\boldsymbol{L} = [l_{ij}]_{N\times N}$ 有如下定义：

$$l_{ij}=\begin{cases}\sum\limits_{j=1,j\neq i}^{N}a_{ij}, & i=j,\\ -a_{ij}, & i\neq j.\end{cases}$$

当这个网络拓扑是时变的情况下, 本文用$\mathcal{G}(t)$ 和$\boldsymbol{L}(t)$ 分别表示交互拓扑图及其对应的 Laplace 矩阵.

注 1　本文所讨论的时变拓扑图的 Laplace 矩阵$\boldsymbol{L}(t)$与已有文献中的 Laplace 矩阵一样具有列和为零的性质, 即: $\boldsymbol{1}_N^{\mathrm{T}}\boldsymbol{L}(t)=\boldsymbol{0}_N$, $\forall t\geqslant t_0$. 这一性质在式(2)的推导中有使用到.

接下来是一些有用的定义、假设、性质和引理.

定义 1[16]　对于一个连续函数$d(t)$, 如果系统$\dot{\boldsymbol{g}}(t)=d(t)\boldsymbol{g}(t)$是全局一致指数稳定的, 此时$d(t)$即被称为是一个一致指数稳定函数.

定义 2[34]　时变的 Laplace 矩阵$\boldsymbol{L}(t)$所对应的图为$\mathcal{G}(t)$, 它关于$\mathcal{T}$平均一致连通的条件是: 存在某一个$\mathcal{T}>0$, 使得 $\bar{\boldsymbol{L}}=\lim_{\mathcal{T}\to\infty}\dfrac{\int_t^{t+\mathcal{T}}\boldsymbol{L}(s)\mathrm{d}s}{\mathcal{T}}$ 有意义, 并且$t\geqslant t_0$时$\bar{\boldsymbol{L}}$对应的图$\bar{G}$是连通的. 此外, 这个极限关于t的收敛性是一致的, 即对于连续严格递减的函数$\xi:[t_0,+\infty)\to[t_0,+\infty)$有$\lim_{\mathcal{T}\to\infty}\xi(\mathcal{T})=0,\forall t\geqslant t_0$.

假设 1　假设本文涉及到的所有的时变矩阵都是有界的, 也就是说一定存在两个正常数 A 和 B 满足以下的不等式:

$$\|\boldsymbol{A}(t)\|\leqslant A,\ \|\boldsymbol{B}(t)\|\leqslant B.$$

引理 1[35]　对于任意正定矩阵$\boldsymbol{M}\in\mathbb{R}^{n\times n}$和向量$\boldsymbol{a},\boldsymbol{b}\in\mathbb{R}^n$, 有以下不等式成立:

$$2\boldsymbol{a}^{\mathrm{T}}\boldsymbol{b}\leqslant\boldsymbol{a}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{a}+\boldsymbol{b}^{\mathrm{T}}\boldsymbol{M}^{-1}\boldsymbol{b}.$$

引理 2[15]　函数$d(t)$是一致指数稳定函数的充分必要条件是: 对两个给定的正数d_1和d_2, 存在一个正数$T>0$, 使得

$$\int_t^{t+T}d(s)\mathrm{d}s\leqslant-d_1,\ \int_t^{t+\theta}d(s)\mathrm{d}s\leqslant d_2,\qquad \forall\theta\in[0,T].$$

下面, 我们将给出一个重要的引理来处理采样数据.

引理 3[18]　设存在一个可微函数$\mathcal{X}(t)$以及两个正常数α,ω满足

$$\dot{\mathcal{X}}(t)\leqslant\alpha\mathcal{X}(t)+\omega\mathcal{X}(t_k),\qquad t\in[t_k,t_{k+1}),$$

那么采样误差$E(t)=\mathcal{X}(t)-\mathcal{X}(t_k)$在当$\varDelta(h)=h(\alpha+\omega)\mathrm{e}^{\alpha h}<1$时, 满足

$$\|E(t)\|\leqslant\frac{\varDelta(h)}{1-\varDelta(h)}\|\mathcal{X}(t)\|,\qquad t\in[t_k,t_{k+1}).$$

引理 4[36]　设纯量函数$\varphi(t)$在区间$a\leqslant t<b$上连续, 右导数$\dfrac{\mathrm{d}\varphi(t)}{\mathrm{d}t}$存在且满足

$$\frac{\mathrm{d}\varphi(t)}{\mathrm{d}t}\leqslant F(t,\varphi(t)),\ \ \varphi(a)=\xi,$$

其中$F(t,\varphi(t))$是在含曲线$x=\varphi(t)$的某一区域$G=I\times\Omega$内定义的连续函数; 若$x=\phi(t)$在区间$a\leqslant t<b$上是微分方程

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=F(t,x(t)),\ \ x(a)=\eta\geqslant\xi=\varphi(a)$$

的G内右行最大解, 则有

$$\varphi(t)\leqslant\phi(t),\qquad a\leqslant t<b.$$

1.2　模型构建

本文考察如下带有时变拓扑的多智能体系统:

$$\begin{cases}\dot{\boldsymbol{x}}_i(t)=\boldsymbol{v}_i(t),\\ \dot{\boldsymbol{v}}_i(t)=\boldsymbol{A}(t)\boldsymbol{v}_i(t)+\boldsymbol{B}(t)\boldsymbol{u}_i(t),\qquad i=1,2,\cdots,N,\end{cases}\tag{1}$$

其中$\boldsymbol{x}_i(t),\boldsymbol{v}_i(t)\in\mathbb{R}^n,\boldsymbol{u}_i(t)\in\mathbb{R}^m$分别是第$i$个智能体的状态、速度和控制输入; $\boldsymbol{A}(t)\in\mathbb{R}^{n\times n}$, $\boldsymbol{B}(t)\in\mathbb{R}^{n\times m}$是连续矩阵值函数.

为了实现领导跟随一致性, 设计如下的采样时间分布式一致协议:

$$\boldsymbol{u}_i(t)=-\boldsymbol{D}\left(\sum_{j=1}^{N}a_{ij}(t)(\boldsymbol{v}_i(t_k)-\boldsymbol{v}_j(t_k))\right),\qquad t\in[t_k,t_{k+1}),\tag{2}$$

这里，$\{t_k\}_{k\in\mathbb{N}}=\{t_0,t_1,t_2,\cdots\}$是一个严格递增的采样序列，$t_0=0$；$\boldsymbol{D}\in\mathbb{R}^{m\times n}$是控制增益矩阵．考虑采样机制为固定间隔采样，间隔是$h=t_k-t_{k-1}$，$k\in\mathbb{N}_+$．

注 2 从式(2)中可知，本文所设计的控制协议只使用了采样时刻邻居的速度差信息，可以减少信息收集成本．

为便于对系统(1)多智能体一致性问题进行理论分析，引入虚拟领导者作为一致的目标，虚拟领导者设计为$\boldsymbol{x}_0(t)=\dfrac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_i(t)$，$\boldsymbol{v}_0(t)=\dfrac{1}{N}\sum_{i=1}^{N}\boldsymbol{v}_i(t)$，其动力学模型如下：

$$\begin{cases}\dot{\boldsymbol{x}}_0(t)=\boldsymbol{v}_0(t),\\ \dot{\boldsymbol{v}}_0(t)=\boldsymbol{A}(t)\boldsymbol{v}_0(t),\end{cases}\tag{3}$$

其中$\boldsymbol{x}_0(t),\boldsymbol{v}_0(t)\in\mathbb{R}^n$分别是虚拟领导者的位移和速度，则系统(1)无领导跟随一致问题一致转化成系统(1)与(3)的有领导跟随一致问题．

定义 3 如果

$$\begin{cases}\lim\limits_{t\to\infty}\|\boldsymbol{x}_i(t)-\boldsymbol{x}_0(t)\|=c,\\ \lim\limits_{t\to\infty}\|\boldsymbol{v}_i(t)-\boldsymbol{v}_0(t)\|=0,\end{cases}\qquad i=1,2,\cdots,N,$$

则称多智能体系统达到一致，其中c是常数位移差．

注 3 本文所设计的虚拟领导者不参与多智能体系统各节点之间的信息交互，也就是说，它不改变智能体相互连接拓扑的 Laplace 矩阵$\boldsymbol{L}(t)$．

用$\hat{\boldsymbol{x}}_i(t)=\boldsymbol{x}_i(t)-\boldsymbol{x}_0(t)$和$\hat{\boldsymbol{v}}_i(t)=\boldsymbol{v}_i(t)-\boldsymbol{v}_0(t)$来标记智能体$i$的追踪误差，则误差系统动力学模型如下：

$$\begin{cases}\dot{\hat{\boldsymbol{x}}}_i(t)=\hat{\boldsymbol{v}}_i(t),\\ \dot{\hat{\boldsymbol{v}}}_i(t)=\boldsymbol{A}(t)\hat{\boldsymbol{v}}_i(t)-\boldsymbol{B}(t)\boldsymbol{D}\left(\sum\limits_{j=1}^{N}a_{ij}(t)(\hat{\boldsymbol{v}}_i(t_k)-\hat{\boldsymbol{v}}_j(t_k))\right),\end{cases}\qquad i=1,2,\cdots,N,\ t\in[t_k,t_{k+1}).\tag{4}$$

记$\hat{\boldsymbol{x}}=[\hat{\boldsymbol{x}}_1^{\mathrm{T}},\hat{\boldsymbol{x}}_2^{\mathrm{T}},\cdots,\hat{\boldsymbol{x}}_N^{\mathrm{T}}]^{\mathrm{T}}$，$\hat{\boldsymbol{v}}=[\hat{\boldsymbol{v}}_1^{\mathrm{T}},\hat{\boldsymbol{v}}_2^{\mathrm{T}},\cdots,\hat{\boldsymbol{v}}_N^{\mathrm{T}}]^{\mathrm{T}}$，误差系统(4)可以被表述成如下的紧凑形式：

$$\begin{cases}\dot{\hat{\boldsymbol{x}}}(t)=\hat{\boldsymbol{v}}(t),\\ \dot{\hat{\boldsymbol{v}}}(t)=(\boldsymbol{I}_N\otimes\boldsymbol{A}(t))\hat{\boldsymbol{v}}(t)-(\boldsymbol{L}(t)\otimes\boldsymbol{B}(t)\boldsymbol{D})\hat{\boldsymbol{v}}(t_k),\qquad t\in[t_k,t_{k+1}).\end{cases}\tag{5}$$

因此，原系统(1)和(2)实现领导跟随一致性等价于误差系统(5)的稳定性问题．

2 主要结论

令$\hat{\boldsymbol{L}}(t)=\bar{\boldsymbol{L}}-\boldsymbol{L}(t)$，系统(5)可以写成

$$\begin{cases}\dot{\hat{\boldsymbol{x}}}(t)=\hat{\boldsymbol{v}}(t),\\ \dot{\hat{\boldsymbol{v}}}(t)=(\boldsymbol{I}_N\otimes\boldsymbol{A}(t))\hat{\boldsymbol{v}}(t)-(\bar{\boldsymbol{L}}(t)\otimes\boldsymbol{B}(t)\boldsymbol{D})\hat{\boldsymbol{v}}(t_k)+(\hat{\boldsymbol{L}}(t)\otimes\boldsymbol{B}(t)\boldsymbol{D})\hat{\boldsymbol{v}}(t_k),\qquad t\in[t_k,t_{k+1}).\end{cases}\tag{6}$$

由于无向图$\bar{\mathcal{G}}$是连通的，因此它所对应的 Laplace 矩阵$\bar{\boldsymbol{L}}$的特征值是实数，且均不小于 0．令正交阵$\boldsymbol{P}=\left[\dfrac{\boldsymbol{1}_N}{\sqrt{N}},\tilde{\boldsymbol{P}}\right]$，其满足以下转换$\boldsymbol{P}^{-1}\bar{\boldsymbol{L}}\boldsymbol{P}=\begin{pmatrix}0 & \\ & \bar{\boldsymbol{J}}\end{pmatrix}$，且$\boldsymbol{P}^{-1}\bar{\boldsymbol{L}}\boldsymbol{P}=\begin{pmatrix}0 & \\ & \boldsymbol{K}(t)\end{pmatrix}$，其中$\bar{\boldsymbol{J}}=\begin{bmatrix}\lambda_2 & & \\ & \ddots & \\ & & \lambda_N\end{bmatrix}_{(N-1)\times(N-1)}$，$0<\lambda_2\leqslant\lambda_3\leqslant\cdots\leqslant\lambda_N$是$\bar{\boldsymbol{L}}$的非零特征值，$\boldsymbol{K}(t)$是$N-1$维的方阵．

令$\begin{cases}\tilde{\boldsymbol{x}}(t)=(\boldsymbol{P}^{-1}\otimes\boldsymbol{I}_n)\hat{\boldsymbol{x}}(t)=[\tilde{\boldsymbol{x}}_1^{\mathrm{T}}(t),\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)]^{\mathrm{T}}\\ \tilde{\boldsymbol{v}}(t)=(\boldsymbol{P}^{-1}\otimes\boldsymbol{I}_n)\hat{\boldsymbol{v}}(t)=[\tilde{\boldsymbol{v}}_1^{\mathrm{T}}(t),\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)]^{\mathrm{T}}\end{cases}$，则系统(6)可化为

$$\begin{cases}\dot{\tilde{\boldsymbol{x}}}_1(t)=\tilde{\boldsymbol{v}}_1(t),\\ \dot{\tilde{\boldsymbol{v}}}_1(t)=\boldsymbol{A}(t)\tilde{\boldsymbol{v}}_1(t)\end{cases}\tag{7}$$

和

$$\begin{cases} \dot{\tilde{\boldsymbol{x}}}_2(t) = \tilde{\boldsymbol{v}}_2(t), \\ \dot{\tilde{\boldsymbol{v}}}_2(t) = (\boldsymbol{I}_{N-1} \otimes \boldsymbol{A}(t))\tilde{\boldsymbol{v}}_2(t) - (\bar{\boldsymbol{J}} \otimes \boldsymbol{B}(t)\boldsymbol{D})\tilde{\boldsymbol{v}}_2(t_k) + (\boldsymbol{K}(t) \otimes \boldsymbol{B}(t)\boldsymbol{D})\tilde{\boldsymbol{v}}_2(t_k), \qquad t \in [t_k, t_{k+1}), \end{cases} \tag{8}$$

其中　$\tilde{\boldsymbol{x}}_1(t), \tilde{\boldsymbol{v}}_1(t) \in \mathbb{R}^n$, $\tilde{\boldsymbol{x}}_2(t), \tilde{\boldsymbol{v}}_2(t) \in \mathbb{R}^{(N-1)n}$.

那么, 系统(5)稳定的充要条件是系统(8)达到稳定. 因此只要找到系统(8)稳定的充分条件, 就可以得到具有时变特征的二阶多智能体系统(1)的一致性结论.

定理 1　已知一个连续一致有界函数$\varphi(t)$, 两个可微函数$\boldsymbol{M}(t)$: $[0,+\infty) \to S_n^+$, $\boldsymbol{N}(t)$: $[0,+\infty) \to \bar{S}_n^+$和两个正的常数$m_1 \leqslant m_2$, 它们满足以下条件:

$$\dot{\boldsymbol{M}}(t) + \boldsymbol{M}(t)\boldsymbol{A}(t) + \boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{M}(t) + \boldsymbol{I}_{N-1} - \lambda_i \boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D} - \lambda_i \boldsymbol{D}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}(t)\boldsymbol{M}(t) \leqslant \varphi(t)\boldsymbol{M}(t), \qquad i = 1, 2, \cdots, N, \tag{9}$$

$$\dot{\boldsymbol{N}}(t) + \boldsymbol{N}(t)\boldsymbol{N}^{\mathrm{T}}(t) \leqslant \varphi(t)\boldsymbol{N}(t), \tag{10}$$

$$m_1 \boldsymbol{I}_n \leqslant \boldsymbol{M}(t) \leqslant m_2 \boldsymbol{I}_n. \tag{11}$$

如果$\dfrac{\varphi(t)}{2} + \varDelta(t)$ 是一个全局一致指数稳定函数, 其中

$$\varDelta(t) = [\lambda_N \hat{\rho}(h) + \xi(t)(1 + \hat{\rho}(h))]\sqrt{\frac{m_2}{m_1}}\|\boldsymbol{B}(t)\boldsymbol{D}\|,\ \hat{\rho}(h) = \frac{\rho(h)}{1 - \rho(h)},\ \rho(h) = [A + (\xi(t) + \lambda_2)B]h\mathrm{e}^{Ah} < 1,$$

则时变系统(5)能够达到一致.

证明　构造 Lyapunov 函数

$$\boldsymbol{V}(t) = \sqrt{\boldsymbol{V}_1(t) + \boldsymbol{V}_2(t)}, \tag{12}$$

其中　$\boldsymbol{V}_1(t) = \tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{N}}(t)\tilde{\boldsymbol{x}}_2(t)$, $\boldsymbol{V}_2(t) = \tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{M}}(t)\tilde{\boldsymbol{v}}_2(t)$, $\hat{\boldsymbol{M}}(t) = \boldsymbol{I}_{N-1} \otimes M(t)$, $\hat{\boldsymbol{N}}(t) = \boldsymbol{I}_{N-1} \otimes \boldsymbol{N}(t)$.

则式(12)两边关于t求导得

$$\dot{\boldsymbol{V}}(t) = \frac{1}{2\boldsymbol{V}(t)}[2\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{N}}(t)\dot{\tilde{\boldsymbol{x}}}_2(t) + \tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\dot{\hat{\boldsymbol{N}}}(t)\tilde{\boldsymbol{x}}_2(t) + 2\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{M}}(t)\dot{\tilde{\boldsymbol{v}}}_2(t) + \tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)\dot{\hat{\boldsymbol{M}}}(t)\tilde{\boldsymbol{v}}_2(t)]. \tag{13}$$

将式(8)代入式(13), 即可得式(12)中的 Lyapunov 函数沿着式(8)两边关于t的导函数:

$$\begin{aligned} \dot{\boldsymbol{V}}(t) = {} & \frac{1}{2\boldsymbol{V}(t)}\{2\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{N}}(t)\tilde{\boldsymbol{v}}_2(t) + \tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\dot{\hat{\boldsymbol{N}}}(t)\tilde{\boldsymbol{x}}_2(t) + \\ & 2\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{M}}(t)[(\boldsymbol{I}_{N-1} \otimes \boldsymbol{A}(t))\tilde{\boldsymbol{v}}_2(t) - (\bar{\boldsymbol{J}} \otimes \boldsymbol{B}(t)\boldsymbol{D})\tilde{\boldsymbol{v}}_2(t_k) + (\boldsymbol{K}(t) \otimes \boldsymbol{B}(t)\boldsymbol{D})\tilde{\boldsymbol{v}}_2(t_k)] + \tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)\dot{\hat{\boldsymbol{M}}}(t)\tilde{\boldsymbol{v}}_2(t)\} = \\ & \frac{1}{2\boldsymbol{V}(t)}\{\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\dot{\hat{\boldsymbol{N}}}(t)\tilde{\boldsymbol{x}}_2(t) + 2\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{N}}(t)\tilde{\boldsymbol{v}}_2(t) + \\ & \tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)[\dot{\hat{\boldsymbol{M}}}(t) + \boldsymbol{I}_{N-1} \otimes \boldsymbol{M}(t)\boldsymbol{A}(t) + \boldsymbol{I}_{N-1} \otimes \boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{M}(t)]\tilde{\boldsymbol{v}}_2(t) - \\ & 2\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)(\bar{\boldsymbol{J}} \otimes \boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D})\tilde{\boldsymbol{v}}_2(t_k) + 2\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)(\boldsymbol{K}(t) \otimes \boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D})\tilde{\boldsymbol{v}}_2(t_k)\} = \\ & \frac{1}{2\boldsymbol{V}(t)}\{\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\dot{\hat{\boldsymbol{N}}}(t)\tilde{\boldsymbol{x}}_2(t) + 2\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{N}}(t)\tilde{\boldsymbol{v}}_2(t) + \\ & \tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)[\dot{\hat{\boldsymbol{M}}}(t) + \boldsymbol{I}_{N-1} \otimes \boldsymbol{M}(t)\boldsymbol{A}(t) + \boldsymbol{I}_{N-1} \otimes \boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{M}(t) - \bar{\boldsymbol{J}} \otimes \boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D} - \bar{\boldsymbol{J}} \otimes \boldsymbol{D}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}(t)\boldsymbol{M}(t)]\tilde{\boldsymbol{v}}_2(t) + \\ & 2\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)(\bar{\boldsymbol{J}} \otimes \boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D})(\tilde{\boldsymbol{v}}_2(t) - \tilde{\boldsymbol{v}}_2(t_k)) + 2\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)(\boldsymbol{K}(t) \otimes \boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D})\tilde{\boldsymbol{v}}_2(t_k)\}. \end{aligned} \tag{14}$$

根据引理 1, 可以得到

$$2\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{N}}(t)\tilde{\boldsymbol{v}}_2(t) \leqslant \tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\hat{\boldsymbol{N}}(t)\hat{\boldsymbol{N}}^{\mathrm{T}}(t)\tilde{\boldsymbol{x}}_2(t) + \tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)(\boldsymbol{I}_{N-1} \otimes \boldsymbol{I}_n)\tilde{\boldsymbol{v}}_2(t). \tag{15}$$

应用引理 3 可从式(8)中推导出

$$\|\tilde{\boldsymbol{v}}_2(t) - \tilde{\boldsymbol{v}}_2(t_k)\| \leqslant \hat{\rho}(h)\|\tilde{\boldsymbol{v}}_2(t)\|. \tag{16}$$

由式(16), 易得

$$\|\tilde{\boldsymbol{v}}_2(t_k)\| = \|\tilde{\boldsymbol{v}}_2(t) + (\tilde{\boldsymbol{v}}_2(t_k) - \tilde{\boldsymbol{v}}_2(t))\| \leqslant \|\tilde{\boldsymbol{v}}_2(t)\| + \|\tilde{\boldsymbol{v}}_2(t_k) - \tilde{\boldsymbol{v}}_2(t)\| \leqslant (1 + \hat{\rho}(h))\|\tilde{\boldsymbol{v}}_2(t)\|. \tag{17}$$

于是从式(16)和(17)可得到如下不等式:

$$\begin{aligned}&2\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)(\bar{\boldsymbol{J}}\otimes\boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D})(\tilde{\boldsymbol{v}}_2(t)-\tilde{\boldsymbol{v}}_2(t_k))\leqslant 2\sqrt{\boldsymbol{V}_2(t)}\left\|\sqrt{\hat{\boldsymbol{M}}(t)}\bar{\boldsymbol{J}}\otimes\boldsymbol{B}(t)\boldsymbol{D}(\tilde{\boldsymbol{v}}_2(t)-\tilde{\boldsymbol{v}}_2(t_k))\right\|\leqslant\\&\quad 2\sqrt{\boldsymbol{V}_2(t)}\sqrt{m_2}\lambda_N\|\boldsymbol{B}(t)\boldsymbol{D}\|\hat{\rho}(h)\|\tilde{\boldsymbol{v}}_2(t)\|\leqslant\\&\quad 2\lambda_N\sqrt{\frac{m_2}{m_1}}\hat{\rho}(h)\|\boldsymbol{B}(t)\boldsymbol{D}\|\boldsymbol{V}_2(t),\end{aligned}\tag{18}$$

其中最后一步是由于$\boldsymbol{V}_2(t)\geqslant m_1\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)\tilde{\boldsymbol{v}}_2(t)=m_1\|\tilde{\boldsymbol{v}}_2(t)\|^2$. 同理可得

$$2\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)(\boldsymbol{K}(t)\otimes\boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D})\tilde{\boldsymbol{v}}_2(t_k)\leqslant 2\sqrt{\frac{m_2}{m_1}}(1+\hat{\rho}(h))\|\boldsymbol{K}(t)\otimes\boldsymbol{B}(t)\boldsymbol{D}\|\boldsymbol{V}_2(t).\tag{19}$$

注意到$\hat{\boldsymbol{M}}(t)=\boldsymbol{I}_{N-1}\otimes\boldsymbol{M}(t)$, $\hat{\boldsymbol{N}}(t)=\boldsymbol{I}_{N-1}\otimes\boldsymbol{N}(t)$, 再结合条件(9)~(11)可知

$$\begin{cases}\dot{\hat{\boldsymbol{M}}}(t)+\boldsymbol{I}_{N-1}\otimes\boldsymbol{M}(t)\boldsymbol{A}(t)+\boldsymbol{I}_{N-1}\otimes\boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{M}(t)+\boldsymbol{I}_{N-1}\otimes\boldsymbol{I}_n-\\\quad\bar{\boldsymbol{J}}\otimes\boldsymbol{M}(t)\boldsymbol{B}(t)\boldsymbol{D}-\bar{\boldsymbol{J}}\otimes\boldsymbol{D}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}(t)\boldsymbol{M}(t)\leqslant\varphi(t)\hat{\boldsymbol{M}}(t),\qquad i=1,2,\cdots,N,\\\dot{\hat{\boldsymbol{N}}}(t)+\hat{\boldsymbol{N}}(t)\hat{\boldsymbol{N}}^{\mathrm{T}}(t)\leqslant\varphi(t)\hat{\boldsymbol{N}}(t).\end{cases}\tag{20}$$

将式(15)~(20)代入式(14), 即可得

$$\begin{aligned}&\dot{\boldsymbol{V}}(t)\leqslant\frac{1}{2\boldsymbol{V}(t)}\Big\{\tilde{\boldsymbol{x}}_2^{\mathrm{T}}(t)\varphi(t)\hat{\boldsymbol{N}}(t)\dot{\tilde{\boldsymbol{x}}}_2(t)+\tilde{\boldsymbol{v}}_2^{\mathrm{T}}(t)\varphi(t)\hat{\boldsymbol{M}}(t)\tilde{\boldsymbol{v}}_2(t)+\\&\quad 2\lambda_N\sqrt{\frac{m_2}{m_1}}[\hat{\rho}(h)\|\boldsymbol{B}(t)\boldsymbol{D}\|+(1+\hat{\rho}(h))\|\boldsymbol{K}(t)\otimes\boldsymbol{B}(t)\boldsymbol{D}\|]\boldsymbol{V}_2(t)\Big\}\leqslant\\&\quad\frac{1}{2\boldsymbol{V}(t)}\left\{\varphi(t)+2\lambda_N\sqrt{\frac{m_2}{m_1}}[\hat{\rho}(h)\|\boldsymbol{B}(t)\boldsymbol{D}\|+(1+\hat{\rho}(h))\|\boldsymbol{K}(t)\otimes\boldsymbol{B}(t)\boldsymbol{D}\|]\right\}\boldsymbol{V}^2(t)\leqslant\\&\quad\left(\frac{\varphi(t)}{2}+\varDelta(t)\right)\boldsymbol{V}(t),\end{aligned}\tag{21}$$

其中 $\varDelta(t)=[\lambda_N\hat{\rho}(h)+\xi(t)(1+\hat{\rho}(h))]\sqrt{\dfrac{m_2}{m_1}}\|\boldsymbol{B}(t)\boldsymbol{D}\|.$

根据定理所给条件可知 $\dfrac{\varphi(t)}{2}+\varDelta(t)$是一个全局一致指数稳定函数, 由式(20)再结合引理 2 可得, 存在一系列间隔为给定正常数T的时刻序列$\{T_m\}_{m=0}^{\infty}$, 即$T_{m+1}-T_m=T, m=0,1,\cdots$, 且$T_0=t_0$使得

$$\begin{aligned}&\int_{T_m}^{T_{m+1}}\left(\frac{\varphi(s)}{2}+\varDelta(s)\right)\mathrm{d}s\leqslant -d_1,\\&\int_{T_m}^{T_m+\beta}\left(\frac{\varphi(s)}{2}+\varDelta(s)\right)\mathrm{d}s\leqslant d_2,\qquad\forall\beta\in[0,T],\end{aligned}\tag{22}$$

其中d_1,d_2为任意给定的正常数. 于是对任意$t\in[t_k,t_{k+1})$, 都存在一个常数$\bar{\beta}(0<\bar{\beta}\leqslant T)$ 使得$t=\bar{\beta}+t_0+T_m$, 进而由引理 4 和式(22)可知

$$\begin{aligned}&\boldsymbol{V}(t)\leqslant\mathrm{e}^{\int_{t_0}^{t}\varrho(s)\mathrm{d}s}\boldsymbol{V}(t_0)\leqslant\\&\quad\mathrm{e}^{\int_{T_m}^{t}\varrho(s)\mathrm{d}s}\mathrm{e}^{\int_{T_{m-1}}^{T_m}\varrho(s)\mathrm{d}s}\cdots\mathrm{e}^{\int_{T_0}^{T_1}\varrho(s)\mathrm{d}s}\boldsymbol{V}(t_0)\leqslant\mathrm{e}^{d_2}\mathrm{e}^{-md_1}\boldsymbol{V}(t_0).\end{aligned}\tag{23}$$

又由于$mT\leqslant t-t_0\leqslant(m+1)T$, 结合式(22), 可得结论

$$\boldsymbol{V}(t)\leqslant\mathrm{e}^{d_1+d_2}\mathrm{e}^{-\frac{d_1}{T}(t-t_0)}\boldsymbol{V}(t_0).$$

最终由初值的有界性可得$\boldsymbol{V}(t)\to\boldsymbol{0}$, $t\to\infty$, 即$\tilde{\boldsymbol{v}}_2(t)\to\boldsymbol{0}$, $t\to\infty$, 又因为$\dot{\tilde{\boldsymbol{x}}}_2(t)=\tilde{\boldsymbol{v}}_2(t)$, 所以当$t\to\infty$时, $\tilde{\boldsymbol{x}}_2(t)$收敛到一个常数, 这个常数保证了智能体之间位移差保持不变.

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注 4 从定理的证明中可以发现, 各个智能体与虚拟领导者的位移差是一个常数, 这个常数的大小与哪些因素有关, 是我们下一步的研究目标.

3 仿真实验

本节以时变网络拓扑结构为例验证所提结果的有效性. 这里多智能体系统(1)中含有三个智能体, 每个智

能体有两个维度, 它所对应的 Laplace 矩阵如下:

$$\boldsymbol{L}(t)=\frac{1}{2}\begin{pmatrix}1+\cos(2\pi t) & -1-\cos(2\pi t) & 0\\ -1-\cos(2\pi t) & 1+\cos(2\pi t) & 0\\ 0 & 0 & 0\end{pmatrix},\qquad 0<\operatorname{mod}(t,2)\leqslant 1,$$

$$\boldsymbol{L}(t)=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 1+\sin(2\pi t) & -1-\sin(2\pi t)\\ 0 & -1-\sin(2\pi t) & 1+\sin(2\pi t)\end{pmatrix},\qquad 1<\operatorname{mod}(t,2)\leqslant 2,$$

又由于$\mathcal{G}(t)$关于$\mathcal{T}=2$是平均一致连通的, 所以

$$\bar{\boldsymbol{L}}=\begin{pmatrix}0.5 & -0.5 & 0\\ -0.5 & 1 & -0.5\\ 0 & -0.5 & 0.5\end{pmatrix}.$$

智能体系统(1)中的$\boldsymbol{A}(t)$, $\boldsymbol{B}(t)$是随着时间变化的, 令$\boldsymbol{A}(t)=\begin{pmatrix}0 & 1\\ \frac{4}{5}\sin(t)-1 & \frac{4}{5}\cos(t)\end{pmatrix}$, $\boldsymbol{B}(t)=\begin{pmatrix}1\\1\end{pmatrix}$, 我们有$\boldsymbol{M}(t)=\begin{pmatrix}0.5\sin(t)+1 & 0\\ 0 & 0.5\sin(t)+1\end{pmatrix}$, $\boldsymbol{N}(t)=\begin{pmatrix}0.5\sin(t)+0.6 & 0\\ 0 & 0\end{pmatrix}$. 根据控制协议取耦合强度$\boldsymbol{D}=(1.998\,9\quad 1.214\,3)$, 计算得$A=1.799\,8$, $B=1.414\,2$. 通过定理 1, 选择$\xi(t)=0.05/t$, $\tau=t_k-t_{k-1}=0.1$, $\varphi(t)=26\mathrm{e}^{-0.01t}-2\varDelta(t)$, 则$\varphi(t)/2+\varDelta(t)=13\mathrm{e}^{-0.01t}$是全局一致指数稳定函数, 且$\rho(h)=0.304\,4<1$. 初始条件为$\boldsymbol{x}_1(0)=[3;4]^{\mathrm{T}}$, $\boldsymbol{x}_2(0)=[5;4]^{\mathrm{T}}$; $\boldsymbol{x}_3(0)=[3;10]^{\mathrm{T}}$, $\boldsymbol{v}_1(0)=[1;2]^{\mathrm{T}}$, $\boldsymbol{v}_2(0)=[2;3]^{\mathrm{T}}$, $\boldsymbol{v}_3(0)=[8;4]^{\mathrm{T}}$. 通过使用 MATLAB 得出下列图像, 图 1 展示了所有智能体的位置状态以及位移差保持不变, 图 2 展示了所有智能体的速度状态达到一致. 由图 1 和图 2 可以看出节点与虚拟领导者的位移差趋向于常数, 速度差趋向于零. 数值模拟验证了理论分析在时变拓扑下的多智能体系统采样一致性的正确性.

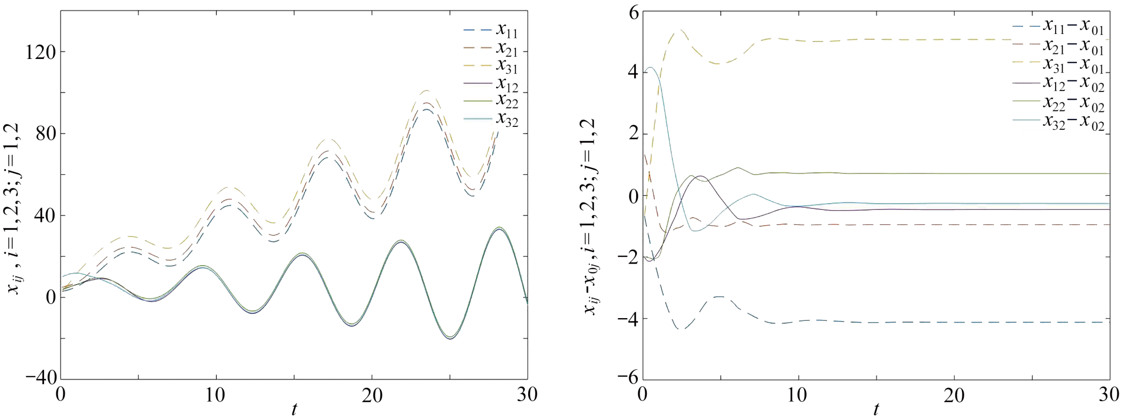

图 1　采样数据控制协议下三个智能体的位移轨迹及其与虚拟领导者轨迹的误差

Fig. 1　The position trajectories of 3 agents under the sampled-data control protocol and the errors from their virtual leader trajectories

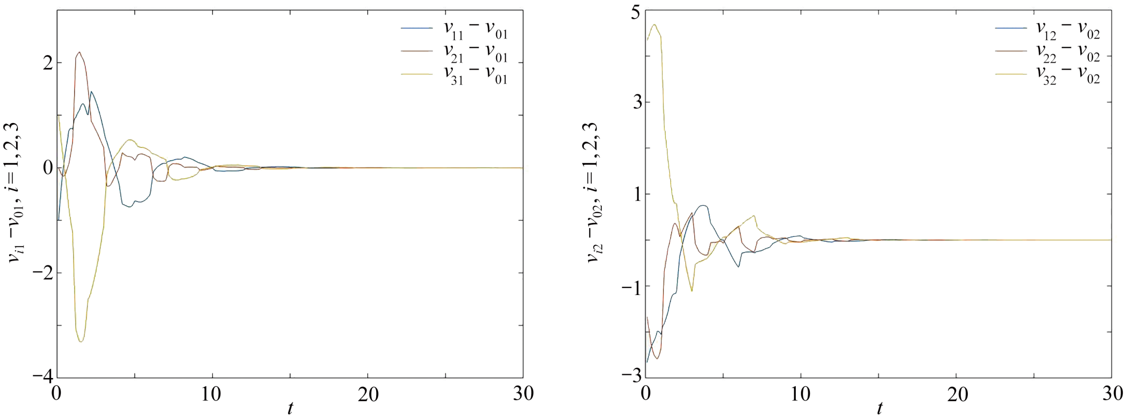

图 2　采样数据控制协议下三个智能体与其虚拟领导者速度的误差

Fig. 2　Errors of speeds between 3 agents and their virtual leader under the sampled data control protocol

4 结 论

基于采样控制方法，分析了时变拓扑下二阶多智能体的采样一致性．通过引入虚拟领导者构造误差系统，将一致性问题转化为稳定性问题，并设计了合适的 Lyapunov 函数来证明误差系统达到稳定的充分条件．最后的数值实验验证了多智能体系统一致性分析的准确性和有效性．各个多智能体与虚拟领导者的固定位移差大小与哪些因素有关这一问题尚未得到解决，将是我们下一步的研究目标．

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