应用数学和力学
APPLIED MATHEMATICS AND MECHANICS

连续综合控制系统的状态反馈广义H_2控制

孙凤琪

State Feedback Generalized H_2 Control of Continuous Integrated Control Systems

SUN Fengqi

在线阅读 View online: https://doi.org/10.21656/1000-0887.420169

---

　http://www.applmathmech.cn

连续综合控制系统的状态反馈广义 H_2 控制*

孙凤琪

（吉林师范大学 数学与计算机学院，吉林 四平 136000）

摘要： 基于 Lyapunov 稳定性理论、矩阵分析法、线性矩阵不等式等方法, 对同时带有控制输入和干扰输入的奇异摄动时变时滞不确定控制系统进行广义 H_2 控制研究. 设计一个记忆状态广义 H_2 控制器, 给出具体设计方法的判定定理. 并对时滞依赖和时滞独立两种情形下采用新的引理, 推出保守性相对更小的稳定性判据. 对所得结论进行线性化处理, 用数值样例验证了该文所得结论的有效性和可行性. 指出在零到奇异摄动上界的整个区间范围内, 闭环系统渐近稳定, 扩大了广义 H_2 稳定空间, 缩小了 L_2-L_∞的性能指标. 通过与相关文献进行稳定态指标对比, 展示出该文所得方法具有一定的优越性和较小的保守性, 并且适用于标准和非标准情形.

关　键　词： Lyapunov 稳定性；　广义 H_2 控制；　状态反馈控制器；　L_2-L_∞性能指标；　交叉项界定法
中图分类号： O231.2　　**文献标志码：** A　　DOI：10.21656/1000-0887.420169

State Feedback Generalized H_2 Control of Continuous Integrated Control Systems

SUN Fengqi
（*College of Mathematics and Computer*, *Jilin Normal University*, *Siping*, *Jilin* 136000, *P.R.China*）

Abstract: Based on the Lyapunov stability theory, the matrix analysis method and the linear matrix inequality method, etc, the generalized H_2 control of singularly perturbed uncertain-control time-varying delay systems with control input and disturbance input, was studied. A memory state generalized H_2 controller was designed, and the decision theorem for the specific design method was given. With a new lemma for delay-dependent and delay-independent cases, the relatively less conservative stability criterion was derived. The obtained results were linearized, the selected numerical examples were used to verify the effectiveness and feasibility of the derived conclusions. The results show that, the closed-loop system is asymptotically stable in the whole range from zero to the singular perturbation upper bound, which expands the generalized H_2 stability space and reduces the L_2-L_∞ performance index. The comparison of the stability state parameter index with the related literatures indicate that, the proposed method has certain advantages and less conservatism, and is suitable for standard and non-standard cases.

Key words: Lyapunov stability; generalized H_2 control; state feedback controller; L_2-L_∞ performance index; cross term definition method

引　　言

将最优控制理论应用于工业过程控制领域, 就得到了具有广泛工程背景的线性二次型 (LQG) 控制问题,

* **收稿日期：** 2021-06-22; **修订日期：** 2021-09-06
基金项目： 国家自然科学基金 (61741318)
作者简介： 孙凤琪(1968—), 女, 教授, 博士(E-mail: 1092748497@qq.com).
引用格式： 孙凤琪. 连续综合控制系统的状态反馈广义 H_2 控制[J]. 应用数学和力学, 2022, **43**(8): 901-910.

即H_2控制问题[1-3]. 广义H_2控制能使一个实际控制系统, 在含有不确定性和时变时滞的情况下保持渐近稳定, 并且满足L_2-L_∞性能指标要求. 以系统的二范数为性能指标的广义H_2控制理论, 可以获得较好的动态、稳态性能, 是现代控制理论的一个重要分支. 其源于可运用一套完整的、系统化的方法来探究非线性系统的稳定性以及控制器设计问题[4-7], 可以切实地处理系统在控制领域内存在的某些不足, 已经发展成为解决非线性不确定系统的一个有力工具. 目前已成功应用于通讯、网络控制、工业生产过程及航空航天等社会发展的各个领域.

近年来, 有很多学者对此做过深入研究[5-8]. 文献 [2, 9-10] 讨论了时滞神经网络系统广义H_2滤波器设计问题, 采用线性矩阵不等式技术推导了此类不确定系统的鲁棒L_2-L_∞状态反馈控制器存在的充分条件, 但所得结果与时滞大小无关, 当应用于小时滞系统时将具有较大的保守性. 文献 [11] 研究了一类离散状态半 Markov 跳变线性系统的异步广义H_2控制问题, 引入弱无穷小算子、松弛变量给出了等效条件, 并设计了异步控制器. 但无穷小算子技术的引入带来了计算上复杂度的增加. 文献 [12] 主要讨论了连续时间非线性系统的广义H_2控制问题, 得到了该类非线性系统存在鲁棒H_2模糊控制器的充分性条件, 但时滞的上界选取较大的数时, 得到的稳定性判据并不适用. 文献 [13] 研究了一类随机中立型时滞系统的广义H_2控制, 得出了随机中立型时滞系统广义H_2控制器的具体构造方法, 但系统中并未涉及摄动问题, 导致系统含有多个时标时并不适用.

鉴于此, 本文将研究一类带有时变时滞奇异摄动连续不确定性的综合控制系统的广义H_2控制问题, 采用线性矩阵不等式方法, 构造适当的 Lyapunov 泛函, 并结合相关引理, 推出H_2控制器的具体设计方法, 并使得闭环系统在满足一定的性能指标前提下渐近稳定.

引理 1[14] 对于适当维数的矩阵$\boldsymbol{E}_1$, $\boldsymbol{D}_1$, $\boldsymbol{E}_2$, $\boldsymbol{D}_2$, 对称矩阵 $\boldsymbol{Y}$, 不确定性矩阵$\boldsymbol{F}(t)$满足$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leqslant\boldsymbol{I}$, 则

$$\boldsymbol{Y}+\boldsymbol{E}_1\boldsymbol{F}(t)\boldsymbol{D}_1+\boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{E}_1^{\mathrm{T}}+\boldsymbol{E}_2\boldsymbol{F}(t)\boldsymbol{D}_2+\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{E}_2^{\mathrm{T}}<\boldsymbol{0}$$

的充分必要条件是存在正常数$\eta>0$, $\gamma>0$, 使得

$$\boldsymbol{Y}+\eta\boldsymbol{E}_1\boldsymbol{E}_1^{\mathrm{T}}+\eta^{-1}\boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{D}_1+\gamma\boldsymbol{E}_2\boldsymbol{E}_2^{\mathrm{T}}+\gamma^{-1}\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{D}_2<\boldsymbol{0}.$$

引理 2[14] 若$\boldsymbol{X}$, $\boldsymbol{Y}$为向量, 则文献 [14] 的引理 2.3 变为

$$2\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y}\leqslant\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{X}+\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{Y}.$$

特别地, 当$\boldsymbol{Q}_{1\times1}=\delta$时, 则

$$2\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y}\leqslant\delta^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}+\delta\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}.$$

1 问题描述

考虑如下带有控制输入和干扰输入的连续不确定时变时滞奇异摄动控制系统:

$$\begin{cases}\boldsymbol{E}(\varepsilon)\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+(\boldsymbol{A}_{\mathrm{d}}+\boldsymbol{D}_{\mathrm{d}}\boldsymbol{F}(t)\boldsymbol{E}_{\mathrm{d}})\boldsymbol{x}(t-d(t))+\boldsymbol{B}_1\boldsymbol{\omega}(t)+(\boldsymbol{B}_2+\boldsymbol{G}\boldsymbol{F}(t)\boldsymbol{H})\boldsymbol{u}(t), & t>0,\\ \boldsymbol{z}(t)=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{u}(t),\\ \boldsymbol{x}(t)=\boldsymbol{\phi}(t), & t\in[-d,0),\end{cases}\tag{1}$$

其中, $\boldsymbol{E}(\varepsilon)=\begin{bmatrix}\boldsymbol{I}&0\\0&\varepsilon\boldsymbol{I}\end{bmatrix}$, $\boldsymbol{x}(t)\in R^n$是系统状态向量, $\boldsymbol{\omega}(t)\in R^q$是干扰输入向量, $\boldsymbol{u}(t)\in R^p$是控制输入向量, $\boldsymbol{z}(t)\in R^m$是被调输出; $\boldsymbol{D}_{\mathrm{d}}$, $\boldsymbol{E}_{\mathrm{d}}\in R^{n\times n}$是奇异矩阵, 并且$\mathrm{rank}(\boldsymbol{E})=r<n$, $\boldsymbol{A}$, $\boldsymbol{A}_{\mathrm{d}}\in R^{n\times n}$, $\boldsymbol{B}_1\in R^{n\times q}$, $\boldsymbol{B}_2,\boldsymbol{G},\boldsymbol{H}\in R^{n\times p}$, $\boldsymbol{C}\in R^{m\times n}$, $\boldsymbol{D}\in R^{m\times p}$是常数矩阵, 其中 $\boldsymbol{A}$ 渐近稳定; $d(t)$是时变时滞可微函数, 满足

$$0\leqslant d(t)\leqslant\tau,\ \ \dot{d}(t)\leqslant\mu<1,\tag{2}$$

这里τ和μ是已知实常数; $\boldsymbol{\phi}(t)$是连续向量初始值函数; $\boldsymbol{F}(t)\in R^{i\times j}$是范数有界的不确定系统模型参数矩阵, 具有如下范数有界不确定性结构:

$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leqslant\boldsymbol{I}.\tag{3}$$

首先, 设计记忆状态反馈控制器为

$$\boldsymbol{u}(t)=\boldsymbol{K}\boldsymbol{x}(t)+\boldsymbol{K}_1\boldsymbol{x}(t-d(t)),\tag{4}$$

其中, $\boldsymbol{K}$, $\boldsymbol{K}_1$是待定的控制器增益矩阵. 将式 (4) 代入原系统 (1), 则闭环系统成为

$$\begin{cases}\boldsymbol{E}(\varepsilon)\dot{\boldsymbol{x}}(t)=(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)\boldsymbol{x}(t)+(\bar{\boldsymbol{A}}_{\rm d}+\bar{\boldsymbol{D}}_{\rm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\rm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)\boldsymbol{x}(t-d(t))+\bar{\boldsymbol{B}}_1\boldsymbol{\omega}(t),\\ z(t)=\bar{\boldsymbol{C}}\boldsymbol{x}(t)+\bar{\boldsymbol{D}}\boldsymbol{x}(t-d(t)),\end{cases} \tag{5}$$

其中

$$\bar{\boldsymbol{A}}=\boldsymbol{A},\ \bar{\boldsymbol{A}}_{\rm d}=\boldsymbol{A}_{\rm d},\ \bar{\boldsymbol{B}}_1=\boldsymbol{B}_1,\ \bar{\boldsymbol{B}}_2=\boldsymbol{B}_2\boldsymbol{K},\ \bar{\boldsymbol{B}}_3=\boldsymbol{B}_2\boldsymbol{K}_1,\ \bar{\boldsymbol{C}}=\boldsymbol{C}+\boldsymbol{D}\boldsymbol{K},$$

$$\bar{\boldsymbol{D}}=\boldsymbol{D}\boldsymbol{K}_1,\ \bar{\boldsymbol{D}}_{\rm d}=\boldsymbol{D}_{\rm d},\ \bar{\boldsymbol{E}}_{\rm d}=\boldsymbol{E}_{\rm d},\ \bar{\boldsymbol{G}}=\boldsymbol{G},\ \bar{\boldsymbol{H}}_1=\boldsymbol{H}\boldsymbol{K},\ \bar{\boldsymbol{H}}_2=\boldsymbol{H}\boldsymbol{K}_1.$$

针对系统 (1) 设计形如式 (4) 的状态反馈控制器, 对给定的标量$\gamma>0$, 设计状态反馈控制器 (4), 使得连续不确定时变时滞奇异摄动闭环系统 (5) 渐近稳定, 且满足如下L_2-L_∞性能:

$$\sup_{\omega\in L_2\text{-}0}\frac{\|z\|_\infty^2}{\|\omega\|_2^2}<\gamma^2,$$

其中$\|\omega\|_2^2=\int_0^\infty\boldsymbol{\omega}^{\rm T}(t)\boldsymbol{P}\boldsymbol{\omega}(t){\rm d}t$, $\|z\|_\infty^2=\sup_t\{z^{\rm T}(t)z(t)\}$, $\boldsymbol{P}\in R^{q\times q}$为常数矩阵.

2　广义H_2控制定理

2.1　时滞依赖情形

定理 1　给定正数$\bar{\varepsilon}>0,\gamma>0$, 对满足条件 (2) 和 (3) 的闭环系统 (5) 以及L_2-L_∞性能指标, 若存在对称正定矩阵$\boldsymbol{Q}>\boldsymbol{0}$, $\boldsymbol{M}>\boldsymbol{0}$, $\boldsymbol{P}>\boldsymbol{0}$, 矩阵$\boldsymbol{Z}_i(i=1,2,\cdots,5)$且$\boldsymbol{Z}_i=\boldsymbol{Z}_i^{\rm T}(i=1,2,3,4)$, 下列 LMIs 条件是可行的:

$$\boldsymbol{Z}_1>\boldsymbol{0}, \tag{6}$$

$$\begin{bmatrix}\boldsymbol{Z}_1+\bar{\varepsilon}\boldsymbol{Z}_3 & \bar{\varepsilon}\boldsymbol{Z}_5^{\rm T}\\ \bar{\varepsilon}\boldsymbol{Z}_5 & \bar{\varepsilon}\boldsymbol{Z}_2\end{bmatrix}>\boldsymbol{0}, \tag{7}$$

$$\begin{bmatrix}\boldsymbol{Z}_1+\bar{\varepsilon}\boldsymbol{Z}_3 & \bar{\varepsilon}\boldsymbol{Z}_5^{\rm T}\\ \bar{\varepsilon}\boldsymbol{Z}_5 & \bar{\varepsilon}\boldsymbol{Z}_2+\bar{\varepsilon}^2\boldsymbol{Z}_4\end{bmatrix}>\boldsymbol{0}, \tag{8}$$

$$\begin{bmatrix}\boldsymbol{\Omega}_{11}(0) & \boldsymbol{\Omega}_{12}(0) & \boldsymbol{\Omega}_{13}(0)\\ * & \boldsymbol{\Omega}_{22}(0) & \boldsymbol{\Omega}_{23}(0)\\ * & * & \boldsymbol{\Omega}_{33}(0)\end{bmatrix}<\boldsymbol{0}, \tag{9}$$

$$\begin{bmatrix}\boldsymbol{\Omega}_{11}(\bar{\varepsilon}) & \boldsymbol{\Omega}_{12}(\bar{\varepsilon}) & \boldsymbol{\Omega}_{13}(\bar{\varepsilon})\\ * & \boldsymbol{\Omega}_{22}(\bar{\varepsilon}) & \boldsymbol{\Omega}_{23}(\bar{\varepsilon})\\ * & * & \boldsymbol{\Omega}_{33}(\bar{\varepsilon})\end{bmatrix}<\boldsymbol{0}, \tag{10}$$

$$\begin{bmatrix}\boldsymbol{Z}^{-\rm T}(0)\boldsymbol{E}(0) & (\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})^{\rm T}\\ * & \gamma^2\boldsymbol{I}\end{bmatrix}>\boldsymbol{0}, \tag{11}$$

$$\begin{bmatrix}\boldsymbol{Z}^{-\rm T}(\bar{\varepsilon})\boldsymbol{E}(\bar{\varepsilon}) & (\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})^{\rm T}\\ * & \gamma^2\boldsymbol{I}\end{bmatrix}>\boldsymbol{0}, \tag{12}$$

其中

$$\bar{\boldsymbol{A}}=\boldsymbol{A},\ \bar{\boldsymbol{A}}_{\rm d}=\boldsymbol{A}_{\rm d},\ \bar{\boldsymbol{B}}_1=\boldsymbol{B},\ \bar{\boldsymbol{B}}_2=\boldsymbol{B}_2\boldsymbol{K},\ \bar{\boldsymbol{B}}_3=\boldsymbol{B}_2\boldsymbol{K}_1,\ \bar{\boldsymbol{C}}=\boldsymbol{C}+\boldsymbol{D}\boldsymbol{K},$$

$$\bar{\boldsymbol{D}}=\boldsymbol{D}\boldsymbol{K}_1,\ \bar{\boldsymbol{D}}_{\rm d}=\boldsymbol{D}_{\rm d},\ \bar{\boldsymbol{E}}_{\rm d}=\boldsymbol{E}_{\rm d},\ \bar{\boldsymbol{G}}=\boldsymbol{G},\ \bar{\boldsymbol{H}}_1=\boldsymbol{H}\boldsymbol{K},\ \bar{\boldsymbol{H}}_2=\boldsymbol{H}\boldsymbol{K}_1,$$

$$\begin{aligned}\bar{\boldsymbol{\Omega}}_{11}(0)=\ &\boldsymbol{Z}^{-\rm T}(0)(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)+(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\rm T}\boldsymbol{Z}^{-1}(0)+\\ &\boldsymbol{Z}^{-\rm T}(0)\boldsymbol{Q}\boldsymbol{Z}^{-1}(0)+\tau(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\rm T}\boldsymbol{M}(\bar{\boldsymbol{A}}^2+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1),\end{aligned}$$

$$\begin{aligned}\boldsymbol{\Omega}_{12}(0)=\ &\boldsymbol{Z}^{-\rm T}(0)(\bar{\boldsymbol{A}}_{\rm d}+\bar{\boldsymbol{D}}_{\rm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\rm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)+\\ &\tau(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\rm T}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\rm d}+\bar{\boldsymbol{D}}_{\rm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\rm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2),\end{aligned}$$

$$\bar{\boldsymbol{\Omega}}_{13}(0)=\boldsymbol{Z}^{-\rm T}(0)\bar{\boldsymbol{B}}_1+\tau(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\rm T}\boldsymbol{M}\bar{\boldsymbol{B}}_1,$$

$$\begin{aligned}\boldsymbol{\Omega}_{22}(0)=\ &-(1-\mu)\boldsymbol{Z}^{-\rm T}(0)\boldsymbol{Q}\boldsymbol{Z}^{-1}(0)+\tau(\bar{\boldsymbol{A}}_{\rm d}+\bar{\boldsymbol{D}}_{\rm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\rm d}+\bar{\boldsymbol{B}}_3+\\ &\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\rm T}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\rm d}+\bar{\boldsymbol{D}}_{\rm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\rm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2),\end{aligned}$$

$$\boldsymbol{\Omega}_{23}(\varepsilon)=\tau(\bar{\boldsymbol{A}}_{\rm d}+\bar{\boldsymbol{D}}_{\rm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\rm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\rm T}\boldsymbol{M}\bar{\boldsymbol{B}}_1,\ \boldsymbol{\Omega}_{33}(\varepsilon)=\tau\bar{\boldsymbol{B}}_1^{\rm T}\boldsymbol{M}\bar{\boldsymbol{B}}_1-\boldsymbol{P},$$

$$\bar{\boldsymbol{\Omega}}_{11}(\bar{\varepsilon}) = \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1) + (\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm{T}}\boldsymbol{Z}^{-1}(\bar{\varepsilon}) + \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{Q}\boldsymbol{Z}^{-1}(\bar{\varepsilon}) + \tau(\bar{\boldsymbol{A}}^2 + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm{T}}\boldsymbol{M}(\bar{\boldsymbol{A}}^2 + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1),$$

$$\bar{\boldsymbol{\Omega}}_{12}(\bar{\varepsilon}) = \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2) + \tau(\bar{\boldsymbol{A}}^2 + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm{T}}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2),$$

$$\bar{\boldsymbol{\Omega}}_{13}(\bar{\varepsilon}) = \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\bar{\boldsymbol{B}}_1 + \tau(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{B}}_1,$$

$$\bar{\boldsymbol{\Omega}}_{22}(\bar{\varepsilon}) = -(1-\mu)\boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{Q}\boldsymbol{Z}^{-1}(\bar{\varepsilon}) + \tau(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\mathrm{T}}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2),$$

则$\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{x}(t) + \boldsymbol{K}_1\boldsymbol{x}(t-d(t))$就是系统 (1) 的记忆状态反馈广义$H_2$控制器, 且闭环系统 (5) 是广义$H_2$稳定的.

证明 首先建立闭环系统 (5) 在$\omega(t) \equiv \boldsymbol{0}$时的渐近稳定性. 定义一个二次 L-K 泛函$V(\boldsymbol{x}(t)) = V_1(\boldsymbol{x}(t)) + V_2(\boldsymbol{x}(t)) + V_3(\boldsymbol{x}(t))$, 其中

$$V_1(\boldsymbol{x}(t)) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon)\boldsymbol{x}(t), \tag{13}$$

$$V_2(\boldsymbol{x}(t)) = \int_{t-d(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)\boldsymbol{x}(s)\mathrm{d}s, \tag{14}$$

$$V_3(\boldsymbol{x}(t)) = \int_{-\tau}^{0}\int_{t-d(t)+\theta}^{t} (\boldsymbol{E}(\varepsilon)\dot{\boldsymbol{x}}(s))^{\mathrm{T}}\boldsymbol{M}\boldsymbol{E}(\varepsilon)\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta, \tag{15}$$

其中$\boldsymbol{Z}(\varepsilon) = \begin{bmatrix} \boldsymbol{Z}_1 + \varepsilon\boldsymbol{Z}_3 & \varepsilon\boldsymbol{Z}_5^{\mathrm{T}} \\ \boldsymbol{Z}_5 & \boldsymbol{Z}_2 + \varepsilon\boldsymbol{Z}_4 \end{bmatrix}$, $\boldsymbol{Q}$, $\boldsymbol{M}$为适当维数正定对称矩阵, 即$\boldsymbol{Q}^{\mathrm{T}} = \boldsymbol{Q} > \boldsymbol{0}$, $\boldsymbol{M}^{\mathrm{T}} = \boldsymbol{M} > \boldsymbol{0}$.

由矩阵不等式条件 (6) ~ (8) 及文献 [14] 的引理 4.2, 推得

$$\boldsymbol{E}(\varepsilon)\boldsymbol{Z}(\varepsilon) = (\boldsymbol{E}(\varepsilon)\boldsymbol{Z}(\varepsilon))^{\mathrm{T}} = \boldsymbol{Z}^{\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon) > \boldsymbol{0},$$

则

$$\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon)\boldsymbol{Z}(\varepsilon) = \boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{Z}^{\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon) = \boldsymbol{E}(\varepsilon).$$

故

$$\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon) = \boldsymbol{E}(\varepsilon)\boldsymbol{Z}^{-1}(\varepsilon) > \boldsymbol{0}, \qquad \forall\varepsilon \in (0, \bar{\varepsilon}],$$

则$V(\boldsymbol{x}(t))$为正定的 L-K 泛函.

根据文献 [14] 的引理 4.3, 可知矩阵不等式条件 (9)、(10) 蕴含下式:

$$\begin{bmatrix} \boldsymbol{\Pi}_{11}(\varepsilon) & \boldsymbol{\Pi}_{12}(\varepsilon) & \boldsymbol{\Pi}_{13}(\varepsilon) \\ * & \boldsymbol{\Pi}_{22}(\varepsilon) & \boldsymbol{\Pi}_{23}(\varepsilon) \\ * & * & \boldsymbol{\Pi}_{33}(\varepsilon) \end{bmatrix} < \boldsymbol{0}, \tag{16}$$

其中

$$\boldsymbol{\Pi}_{11}(\varepsilon) = \boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1) + (\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm{T}}\boldsymbol{Z}^{-1}(\varepsilon) + \boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon) + \tau(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm{T}}\boldsymbol{M}(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1),$$

$$\boldsymbol{\Pi}_{12}(\varepsilon) = \boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2) + \tau(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm{T}}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2),$$

$$\boldsymbol{\Pi}_{13}(\varepsilon) = \boldsymbol{Z}^{-\mathrm{T}}(\varepsilon) + \tau(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{B}}_1,$$

$$\boldsymbol{\Pi}_{22}(\varepsilon) = -(1-\mu)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon) + \tau(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\mathrm{T}}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2),$$

$$\boldsymbol{\Pi}_{23}(\varepsilon) = \tau(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{B}}_1,\ \boldsymbol{\Pi}_{33} = \tau\bar{\boldsymbol{B}}_1^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{B}}_1 - \boldsymbol{P}.$$

把$V(\boldsymbol{x}(t))$沿着闭环系统 (5) 的任意轨迹进行微分得

$$\dot{V}(\boldsymbol{x}(t)) = \dot{V}_1(\boldsymbol{x}(t)) + \dot{V}_2(\boldsymbol{x}(t)) + \dot{V}_3(\boldsymbol{x}(t)),$$

其中

$$\dot{V}_1(\boldsymbol{x}(t)) = 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)(\bar{\boldsymbol{A}}^2 + \bar{\boldsymbol{B}}_2 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)(\bar{\boldsymbol{A}}_{\mathrm{d}} + \bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}} + \bar{\boldsymbol{B}}_3 + \bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)\boldsymbol{x}(t-d(t)),$$

$$\dot V_2(\boldsymbol{x}(t))=\frac{\mathrm d}{\mathrm dt}\left(\int_{t-d(t)}^{t}\boldsymbol{x}^{\mathrm T}(s)\boldsymbol{Z}^{-\mathrm T}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)\boldsymbol{x}(s)\mathrm ds\right)=$$
$$\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{Z}^{-\mathrm T}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)\boldsymbol{x}(t)-(1-\dot d(t))\boldsymbol{x}^{\mathrm T}(t-d(t))\boldsymbol{Z}^{-\mathrm T}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)\boldsymbol{x}(t-d(t))\leqslant$$
$$\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{Z}^{-\mathrm T}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)\boldsymbol{x}(t)-(1-\mu)\boldsymbol{x}^{\mathrm T}(t-d(t))\boldsymbol{Z}^{-\mathrm T}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)\boldsymbol{x}(t-d(t)).$$

由$\boldsymbol{M}>\boldsymbol{0}$, $\dot d(t)\leqslant\mu<1$可知

$$(1-\mu)\int_{-\tau}^{0}(\boldsymbol{E}(\varepsilon)\dot{\boldsymbol{x}}(t-d(t)+\theta))^{\mathrm T}\boldsymbol{M}\boldsymbol{E}(\varepsilon)\dot{\boldsymbol{x}}(t-d(t)+\theta)\mathrm d\theta>0,$$

进而

$$\dot V_3(\boldsymbol{x}(t))\leqslant\tau\boldsymbol{x}^{\mathrm T}(t)(\bar{\boldsymbol{A}}\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm T}\boldsymbol{M}(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)\boldsymbol{x}(t)+$$
$$\tau\boldsymbol{x}^{\mathrm T}(t)(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm T}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)\boldsymbol{x}(t-d(t))+$$
$$\tau\boldsymbol{x}^{\mathrm T}(t-d(t))(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\mathrm T}\boldsymbol{M}(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)\boldsymbol{x}(t)+$$
$$\tau\boldsymbol{x}^{\mathrm T}(t-d(t))(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\mathrm T}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+$$
$$\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)\boldsymbol{x}(t-d(t)).$$

所以

$$\dot V(\boldsymbol{x}(t))\overset{\Delta}{=}\boldsymbol{\eta}^{\mathrm T}(t)\,\boldsymbol{\varXi}\,\boldsymbol{\eta}(t),$$

其中

$$\boldsymbol{\eta}(t)=[\boldsymbol{x}(t)\quad\boldsymbol{x}(t-d(t))]^{\mathrm T},$$

$$\boldsymbol{\varXi}=\begin{bmatrix}\begin{matrix}\boldsymbol{Z}^{-\mathrm T}(\varepsilon)(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)+\\(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm T}\boldsymbol{Z}^{-1}(\varepsilon)+\boldsymbol{Z}^{-\mathrm T}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)+\\\tau(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm T}\boldsymbol{M}(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)\end{matrix} & \begin{matrix}\boldsymbol{Z}^{-\mathrm T}(\varepsilon)(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}F(t)\bar{\boldsymbol{H}}_2)+\\\tau(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm T}\times\\M(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}F(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)\end{matrix}\\ * & \begin{matrix}-(1-\mu)\boldsymbol{Z}^{-\mathrm T}(\varepsilon)Q\boldsymbol{Z}^{-1}(\varepsilon)+\tau(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+\\\bar{\boldsymbol{G}}F(t)\bar{\boldsymbol{H}}_2)^{\mathrm T}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}F(t)\bar{\boldsymbol{E}}_{\mathrm d}+\\\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}F(t)\bar{\boldsymbol{H}}_2)\end{matrix}\end{bmatrix}.$$

从式 (16) 可知$\boldsymbol{\varXi}<\boldsymbol{0}$, 故推出$\boldsymbol{\eta}^{\mathrm T}(t)\boldsymbol{\varXi}\boldsymbol{\eta}(t)<0$, 于是在$\omega(t)\equiv\boldsymbol{0}$时, $\dot V(\boldsymbol{x}(t))<0$. 由 Lyapunov 稳定条件可知, 闭环系统 (5) 是渐近稳定的, $\forall\varepsilon\in(0,\bar\varepsilon]$.

为了建立闭环系统 (5) 的L_2-L_∞性能准则,假设零初始条件, 即$V(\boldsymbol{x}(t))|_{t=0}=0$, 构造新的性能指标如下:

$$J=V(\boldsymbol{x}(t))-\int_0^{t}\omega^{\mathrm T}(s)\boldsymbol{P}\omega(s)\mathrm ds,$$

其中 $\boldsymbol{P}$ 为待定的对称正定加权矩阵, 则对于任意的非零$\omega(s)\in L_2(0,\infty]$及$t\geqslant0$, 有

$$J=V(\boldsymbol{x}(t))-V(\boldsymbol{x}(t))|_{t=0}-\int_0^{t}\omega^{\mathrm T}(s)\boldsymbol{P}\omega(s)\mathrm ds=\int_0^{t}[\dot V(\boldsymbol{x}(s))-\omega^{\mathrm T}(s)\boldsymbol{P}\omega(s)]\mathrm ds\leqslant$$
$$\int_0^{t}\bar{\boldsymbol{\eta}}^{\mathrm T}(s)\boldsymbol{W}(\varepsilon)\bar{\boldsymbol{\eta}}(s)\mathrm ds,\tag{17}$$

其中

$$\bar{\boldsymbol{\eta}}(s)=[\ \boldsymbol{x}(s)\quad\boldsymbol{x}(s-d(s))\quad\omega(s)\ ]^{\mathrm T},$$

$$\boldsymbol{W}(\varepsilon)=\begin{bmatrix}\boldsymbol{W}_{11}(\varepsilon)&\boldsymbol{W}_{12}(\varepsilon)&\boldsymbol{W}_{13}(\varepsilon)\\ * &\boldsymbol{W}_{22}(\varepsilon)&\boldsymbol{W}_{23}(\varepsilon)\\ * & * &\tau\bar{\boldsymbol{B}}_1^{\mathrm T}\boldsymbol{M}_{\boldsymbol{B}}-\boldsymbol{P}\end{bmatrix},\tag{18}$$

$$\boldsymbol{W}_{11}(\varepsilon)=\boldsymbol{Z}^{-\mathrm T}(\varepsilon)(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)+(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm T}\boldsymbol{Z}^{-1}(\varepsilon)+\boldsymbol{Z}^{-\mathrm T}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)+$$
$$\tau(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm T}\boldsymbol{M}(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1),$$

$$\boldsymbol{W}_{12}(\varepsilon)=\boldsymbol{Z}^{-\mathrm T}(\varepsilon)(\bar{\boldsymbol{A}}_{\mathrm d}+\bar{\boldsymbol{D}}_{\mathrm d}F(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)+\tau(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\mathrm T}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm d}+$$
$$\bar{\boldsymbol{D}}_{\mathrm d}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm d}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1),$$

$$\boldsymbol{W}_{13}(\varepsilon)=\boldsymbol{Z}^{-\mathrm T}(\varepsilon)\bar{\boldsymbol{B}}_1+\tau(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}_2+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_1)^{\mathrm T}\boldsymbol{M}\bar{\boldsymbol{B}},$$

$$\boldsymbol{W}_{22}(\varepsilon)=-(1-\mu)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)+\tau(\bar{\boldsymbol{A}}_{\mathrm{d}}+\bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\mathrm{T}}\boldsymbol{M}(\bar{\boldsymbol{A}}_{\mathrm{d}}+\bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2),$$

$$\boldsymbol{W}_{23}(\varepsilon)=\tau(\bar{\boldsymbol{A}}_{\mathrm{d}}+\bar{\boldsymbol{D}}_{\mathrm{d}}\boldsymbol{F}(t)\bar{\boldsymbol{E}}_{\mathrm{d}}+\bar{\boldsymbol{B}}_3+\bar{\boldsymbol{G}}\boldsymbol{F}(t)\bar{\boldsymbol{H}}_2)^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{B}}_1.$$

由条件 (11) 和 (12) 可知, $\boldsymbol{W}(0)<\boldsymbol{0}$, $\boldsymbol{W}(\bar{\varepsilon})<\boldsymbol{0}$, 则$\boldsymbol{W}(\varepsilon)<\boldsymbol{0}$. 由 Lyapunov 稳定性理论可知, 闭环系统 (5) 是渐近稳定的, $\forall\varepsilon\in(0,\bar{\varepsilon}]$.

因此$J\leqslant 0$, 由式 (13) ~ (15)、(17) 可得

$$V_1(\boldsymbol{x}(t))=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon)\boldsymbol{x}(t)\leqslant V(\boldsymbol{x}(t))-0\leqslant\int_0^t\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{P}\boldsymbol{\omega}(s)\mathrm{d}s. \tag{19}$$

另一方面, 由 Schur 补引理可知, 式 (9) 等价于

$$(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})^{\mathrm{T}}(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})<\gamma^2\boldsymbol{Z}^{-\mathrm{T}}(0)\boldsymbol{E}(0),$$

式 (10) 等价于

$$(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})^{\mathrm{T}}(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})<\gamma^2\boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{E}(\bar{\varepsilon}).$$

由文献 [12] 的引理 4.3 可知, $\forall\varepsilon\in(0,\bar{\varepsilon}]$有

$$(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})^{\mathrm{T}}(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})<\gamma^2\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon). \tag{20}$$

由式 (19)、(20) 可得

$$\begin{aligned}\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)&=\boldsymbol{x}^{\mathrm{T}}(t)(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})^{\mathrm{T}}(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}})\boldsymbol{x}(t)<\gamma^2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon)\boldsymbol{x}(t)\leqslant\\&\quad\gamma^2\int_0^t\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{P}\boldsymbol{\omega}(s)\mathrm{d}s\leqslant\gamma^2\int_0^\infty\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{\omega}(t)\mathrm{d}t.\end{aligned}$$

对所有的$t\geqslant 0$取最大值, 则对于任意的非零$\boldsymbol{\omega}(t)\in L_2[0,\infty)$, 有

$$\|z(t)\|_\infty^2<\gamma^2\|\omega(t)\|_2^2,$$

其中

$$\|z(t)\|_\infty^2=\sup_t\{z^{\mathrm{T}}(t)z(t)\}.$$

上述不等式的两边同时除以$\|\omega(t)\|_2^2$, 并对所有的非零$\boldsymbol{\omega}(t)\in L_2[0,\infty)$取最大值, 可得$\sup_{\omega\in L_2\text{-}0}\dfrac{\|z\|_\infty^2}{\|\omega\|_2^2}<\gamma^2$, 定理得证.

定理 1 得出了闭环统满足L_2-L_∞性能指标的广义H_2控制器成立的条件, 但是控制器的参数是未知的, 上式关于变量 $\boldsymbol{K}$, $\boldsymbol{K}_1$, $\boldsymbol{M}$, $\boldsymbol{P}$, $\boldsymbol{Q}$ 和$\boldsymbol{Z}(\varepsilon)$是非线性的, 进行线性化处理, 即得如下定理, 具体推证略.

定理 2 给定正数$\bar{\varepsilon}>0$, $\gamma>0$, 对满足条件 (2) 和 (3) 的闭环系统系统 (5) 以及L_2-L_∞性能指标 (8), 若存在适当维数的矩阵$\tilde{\boldsymbol{K}}$和$\tilde{\boldsymbol{K}}_1$, 正数$\lambda_1>0$, $\lambda_2>0$, 对称正定矩阵$\boldsymbol{Q}>\boldsymbol{0}$, $\tilde{\boldsymbol{M}}>\boldsymbol{0}$, $\boldsymbol{P}>\boldsymbol{0}$, 以及矩阵$\boldsymbol{Z}_i(i=1,2,\cdots,5)$且$\boldsymbol{Z}_i=\boldsymbol{Z}_i^{\mathrm{T}}(i=1,2,3,4)$, 使得在满足线性矩阵不等式条件 (6) ~ (8) 时, 下列 LMIs 条件是可行的:

$$\begin{bmatrix}\boldsymbol{\varLambda}_{11}(0) & \boldsymbol{\varLambda}_{12}(0) & \boldsymbol{B}_1 & \boldsymbol{\varLambda}_{14}(0) & \tilde{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}} & 0\\ * & -(1-\mu)\boldsymbol{Q} & 0 & \boldsymbol{\varLambda}_{24}(0) & \tilde{\boldsymbol{K}}_1\boldsymbol{H}^{\mathrm{T}} & \boldsymbol{\varLambda}_{26}(0)\\ * & * & -\boldsymbol{P} & \boldsymbol{B}_1^{\mathrm{T}} & 0 & 0\\ * & * & * & -\tau^{-1}\tilde{\boldsymbol{M}}+\lambda_1\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}}+\lambda_2\boldsymbol{D}_{\mathrm{d}}\boldsymbol{D}_{\mathrm{d}}^{\mathrm{T}} & 0 & 0\\ * & * & * & * & -\lambda_1\boldsymbol{I} & 0\\ * & * & * & * & * & -\lambda_2\boldsymbol{I}\end{bmatrix}<\boldsymbol{0},$$

$$\begin{bmatrix}\boldsymbol{\varLambda}_{11}(\bar{\varepsilon}) & \boldsymbol{\varLambda}_{12}(\bar{\varepsilon}) & \boldsymbol{B}_1 & \boldsymbol{\varLambda}_{14}(\bar{\varepsilon}) & \tilde{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}} & 0\\ * & -(1-\mu)\boldsymbol{Q} & 0 & \boldsymbol{\varLambda}_{24}(\bar{\varepsilon}) & \tilde{\boldsymbol{K}}_1\boldsymbol{H}^{\mathrm{T}} & \boldsymbol{\varLambda}_{26}(\bar{\varepsilon})\\ * & * & -\boldsymbol{P} & \boldsymbol{B}_1^{\mathrm{T}} & 0 & 0\\ * & * & * & -\tau^{-1}\tilde{\boldsymbol{M}}+\lambda_1\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}}+\lambda_2\boldsymbol{D}_{\mathrm{d}}\boldsymbol{D}_{\mathrm{d}}^{\mathrm{T}} & 0 & 0\\ * & * & * & * & -\lambda_1\boldsymbol{I} & 0\\ * & * & * & * & * & -\lambda_2\boldsymbol{I}\end{bmatrix}<\boldsymbol{0},$$

$$\begin{bmatrix}\boldsymbol{Z}^{-\mathrm{T}}(0)\boldsymbol{E}(0) & (\boldsymbol{C}+\boldsymbol{D}\boldsymbol{K}+\boldsymbol{D}\boldsymbol{K}_1)^{\mathrm{T}}\\ * & \gamma^2\boldsymbol{I}\end{bmatrix}>\boldsymbol{0},$$

$$\begin{bmatrix} \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{E}(\bar{\varepsilon}) & (\boldsymbol{C}+\boldsymbol{D}\boldsymbol{K}+\boldsymbol{D}\boldsymbol{K}_1)^{\mathrm{T}} \\ * & \gamma^2\boldsymbol{I} \end{bmatrix} > \boldsymbol{0},$$

其中

$$\boldsymbol{\Lambda}_{11}(0)=\boldsymbol{A}\boldsymbol{Z}(0)+\boldsymbol{B}_2\tilde{\boldsymbol{K}}+\boldsymbol{Z}^{\mathrm{T}}(0)\boldsymbol{A}^{\mathrm{T}}+\tilde{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}+\boldsymbol{Q}+\lambda_1\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}}+\lambda_2\boldsymbol{D}_{\mathrm{d}}\boldsymbol{D}_{\mathrm{d}}^{\mathrm{T}},$$

$$\boldsymbol{\Lambda}_{12}(0)=\boldsymbol{A}_{\mathrm{d}}\boldsymbol{Z}(0)+\boldsymbol{B}_2\tilde{\boldsymbol{K}}_1,\ \boldsymbol{\Lambda}_{14}(0)=\boldsymbol{Z}^{\mathrm{T}}(0)\boldsymbol{A}^{\mathrm{T}}+\tilde{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}+\lambda_1\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}}+\lambda_2\boldsymbol{D}_{\mathrm{d}}\boldsymbol{D}_{\mathrm{d}}^{\mathrm{T}},$$

$$\boldsymbol{\Lambda}_{24}(0)=\boldsymbol{Z}^{\mathrm{T}}(0)\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}+\tilde{\boldsymbol{K}}_1^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}},\ \boldsymbol{\Lambda}_{26}(0)=\boldsymbol{Z}^{\mathrm{T}}(0)\boldsymbol{E}_{\mathrm{d}}^{\mathrm{T}},$$

$$\boldsymbol{\Lambda}_{11}(\bar{\varepsilon})=\boldsymbol{A}\boldsymbol{Z}(\bar{\varepsilon})+\boldsymbol{B}_2\tilde{\boldsymbol{K}}+\boldsymbol{Z}^{\mathrm{T}}(\bar{\varepsilon})\boldsymbol{A}^{\mathrm{T}}+\tilde{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}+\boldsymbol{Q}+\lambda_1\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}}+\lambda_2\boldsymbol{D}_{\mathrm{d}}\boldsymbol{D}_{\mathrm{d}}^{\mathrm{T}},$$

$$\boldsymbol{\Lambda}_{12}(\bar{\varepsilon})=\boldsymbol{A}_{\mathrm{d}}\boldsymbol{Z}(\bar{\varepsilon})+\boldsymbol{B}_2\tilde{\boldsymbol{K}}_1,\ \boldsymbol{\Lambda}_{14}(\bar{\varepsilon})=\boldsymbol{Z}^{\mathrm{T}}(\bar{\varepsilon})\boldsymbol{A}^{\mathrm{T}}+\tilde{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}+\lambda_1\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}}+\lambda_2\boldsymbol{D}_{\mathrm{d}}\boldsymbol{D}_{\mathrm{d}}^{\mathrm{T}},$$

$$\boldsymbol{\Lambda}_{24}(\bar{\varepsilon})=\boldsymbol{Z}^{\mathrm{T}}(\bar{\varepsilon})\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}+\tilde{\boldsymbol{K}}_1^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}},\ \boldsymbol{\Lambda}_{26}(\bar{\varepsilon})=\boldsymbol{Z}^{\mathrm{T}}(\bar{\varepsilon})\boldsymbol{E}_{\mathrm{d}}^{\mathrm{T}},$$

则$\boldsymbol{u}(t)=\boldsymbol{K}\boldsymbol{x}(t)+\boldsymbol{K}_1\boldsymbol{x}(t-d(t))$，其中$\boldsymbol{K}=\tilde{\boldsymbol{K}}\boldsymbol{Z}^{-1}(\varepsilon)$, $\boldsymbol{K}_1=\tilde{\boldsymbol{K}}_1\boldsymbol{Z}^{-1}(\varepsilon)$，为系统 (1) 的记忆状态反馈广义$H_2$控制器，且闭环系统 (5) 是广义$H_2$稳定的.

注 1　系统控制器亦可以设计成输出状态反馈控制器, 理论推导均与定理 1 方法类似, 同时$d(t)$也可以推广到无限时滞情形, 此略.

2.2 时滞独立情形

定理 3　给定正数$\bar{\varepsilon}>0$, $\gamma>0$, 对满足条件 (2) 和 (3) 的闭环系统 (5) 以及L_2-L_∞性能指标, 若存在对称正定矩阵$\boldsymbol{Q}>\boldsymbol{0}$, $\boldsymbol{P}>\boldsymbol{0}$，正数$\delta_1>0$，$\delta_2>0$，$\eta_1>0$, $\eta_2>0$, $\tilde{\eta}_3>0$，矩阵$\boldsymbol{Z}_i(i=1,2,\cdots,5)$且$\boldsymbol{Z}_i=\boldsymbol{Z}_i^{\mathrm{T}}(i=1,2,3,4)$, 使得在满足线性矩阵不等式条件 (9) ~ (11) 时, 下列 LMIs 条件是可行的:

$$\begin{bmatrix}
\boldsymbol{\Gamma}_{11}(0) & 0 & 0 & \tilde{\boldsymbol{\Gamma}}_{14}(\varepsilon) & 0 & 0 & 0 & \boldsymbol{I} & \boldsymbol{I} & \boldsymbol{G} & \boldsymbol{\Gamma}_{1,11}(0) & \tilde{\boldsymbol{\Gamma}}_{1,12}(\varepsilon) \\
* & -(1-\mu)\boldsymbol{Q} & 0 & 0 & \boldsymbol{\Gamma}_{25}(0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \delta_2\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1-\boldsymbol{P} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\eta_1\boldsymbol{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\tilde{\delta}_1\boldsymbol{I} & \boldsymbol{D}_{\mathrm{d}} & \boldsymbol{G} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\eta_2\boldsymbol{I} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\eta_3\boldsymbol{I} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\delta_1\boldsymbol{I} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\delta_2\boldsymbol{I} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\tilde{\eta}_1\boldsymbol{I} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\tilde{\eta}_2\boldsymbol{I} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\tilde{\eta}_3\boldsymbol{I}
\end{bmatrix} < \boldsymbol{0},$$

$$\begin{bmatrix}
\boldsymbol{\Gamma}_{11}(\bar{\varepsilon}) & 0 & 0 & \tilde{\boldsymbol{\Gamma}}_{14}(\varepsilon) & 0 & 0 & 0 & \boldsymbol{I} & \boldsymbol{I} & \boldsymbol{G} & \boldsymbol{\Gamma}_{1,11}(\bar{\varepsilon}) & \tilde{\boldsymbol{\Gamma}}_{1,12}(\varepsilon) \\
* & -(1-\mu)\boldsymbol{Q} & 0 & 0 & \boldsymbol{\Gamma}_{25}(\bar{\varepsilon}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \delta_2\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1-\boldsymbol{P} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\eta_1\boldsymbol{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\tilde{\delta}_1\boldsymbol{I} & \boldsymbol{D}_{\mathrm{d}} & \boldsymbol{G} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\eta_2\boldsymbol{I} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\eta_3\boldsymbol{I} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\delta_1\boldsymbol{I} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\delta_2\boldsymbol{I} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\tilde{\eta}_1\boldsymbol{I} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\tilde{\eta}_2\boldsymbol{I} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\tilde{\eta}_3\boldsymbol{I}
\end{bmatrix} < \boldsymbol{0},$$

$$\begin{bmatrix} \boldsymbol{Z}^{\mathrm{T}}(0)\boldsymbol{E}(0) & (\boldsymbol{C}+\boldsymbol{D}\boldsymbol{K}+\boldsymbol{D}\boldsymbol{K}_1)^{\mathrm{T}} \\ * & \gamma^2\boldsymbol{I} \end{bmatrix} > \boldsymbol{0},$$

$$\begin{bmatrix} \boldsymbol{Z}^{\mathrm{T}}(\bar{\varepsilon})\boldsymbol{E}(\bar{\varepsilon}) & (\boldsymbol{C}+\boldsymbol{D}\boldsymbol{K}+\boldsymbol{D}\boldsymbol{K}_1)^{\mathrm{T}} \\ * & \gamma^2\boldsymbol{I} \end{bmatrix} > \boldsymbol{0},$$

其中

$$\boldsymbol{\varGamma}_{11}(0) = \boldsymbol{A}\boldsymbol{Z}^{-1}(0) + \boldsymbol{B}_2\tilde{\boldsymbol{K}} + \boldsymbol{Z}^{-\mathrm{T}}(0)\boldsymbol{A}^{\mathrm{T}} + \tilde{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}},$$

$$\boldsymbol{\varGamma}_{1,11}(0) = \boldsymbol{Z}^{-\mathrm{T}}(0)\boldsymbol{E}_{\mathrm{d}},\ \boldsymbol{\varGamma}_{25}(0) = \boldsymbol{Z}^{-\mathrm{T}}(0)\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}} + \boldsymbol{Z}^{-\mathrm{T}}(0)\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}},$$

$$\boldsymbol{\varGamma}_{1,12}(0) = \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}},$$

$$\boldsymbol{\varGamma}_{11}(\bar{\varepsilon}) = \boldsymbol{A}\boldsymbol{Z}^{-1}(\bar{\varepsilon}) + \boldsymbol{B}_2\tilde{\boldsymbol{K}} + \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{A}^{\mathrm{T}} + \tilde{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}},$$

$$\boldsymbol{\varGamma}_{1,11}(\bar{\varepsilon}) = \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{E}_{\mathrm{d}},\ \boldsymbol{\varGamma}_{25}(\bar{\varepsilon}) = \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}} + \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}},$$

$$\boldsymbol{\varGamma}_{1,12}(\bar{\varepsilon}) = \boldsymbol{Z}^{-\mathrm{T}}(\bar{\varepsilon})\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}},$$

则$\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{x}(t) + \boldsymbol{K}_1\boldsymbol{x}(t-d(t))$, 其中$\boldsymbol{K} = \tilde{\boldsymbol{K}}\boldsymbol{Z}(\varepsilon)$, $\boldsymbol{K}_1 = \tilde{\boldsymbol{K}}_1\boldsymbol{Z}(\varepsilon)$, 为系统 (1) 的记忆状态反馈广义$H_2$控制器, 且闭环系统 (5) 是广义$H_2$稳定的.

可以定义一个二次 L-K 泛函$V(\boldsymbol{x}(t)) = V_1(\boldsymbol{x}(t)) + V_2(\boldsymbol{x}(t))$, 其中

$$V_1(\boldsymbol{x}(t)) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{E}(\varepsilon)\boldsymbol{x}(t),$$

$$V_2(\boldsymbol{x}(t)) = \int_{t-d(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Z}^{-\mathrm{T}}(\varepsilon)\boldsymbol{Q}\boldsymbol{Z}^{-1}(\varepsilon)\boldsymbol{x}(s)\mathrm{d}s,$$

其中$\boldsymbol{Q}$为适当维数的正定对称矩阵, 即$\boldsymbol{Q}^{\mathrm{T}} = \boldsymbol{Q} > \boldsymbol{0}$.

证略.

注 2 用类似定理 1 的方法可讨论如下系统的广义H_2控制问题:

$$\begin{cases} \boldsymbol{E}(\varepsilon)\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + (\boldsymbol{A}_{\mathrm{d}} + \boldsymbol{D}_{\mathrm{d}}\boldsymbol{F}(t)\boldsymbol{E}_{\mathrm{d}})\boldsymbol{x}(t-d(t)) + \boldsymbol{B}\boldsymbol{u}(t), & t > 0, \\ z(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t), & t \in [-d,\ 0). \end{cases}$$

详略.

3 算 例

考虑如下带有控制输入和干扰输入的时变时滞不确定控制系统:

$$\begin{cases} \boldsymbol{E}(\varepsilon)\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + (\boldsymbol{A}_{\mathrm{d}} + \boldsymbol{D}_{\mathrm{d}}\boldsymbol{F}(t)\boldsymbol{E}_{\mathrm{d}})\boldsymbol{x}(t-d(t)) + \boldsymbol{B}_1\omega(t) + (\boldsymbol{B}_2 + \boldsymbol{G}\boldsymbol{F}(t)\boldsymbol{H})\boldsymbol{u}(t), & t > 0, \\ z(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t), & t \in [-d,\ 0), \end{cases}$$

其中

$$\boldsymbol{E}(\varepsilon) = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix},\ \boldsymbol{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}},\ d(t) = 0.5,\ \tau = 1,\ \mu = 0.8,\ \boldsymbol{A} = \begin{bmatrix} -0.112\,5 & -0.02 \\ 1 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_{\mathrm{d}} = \begin{bmatrix} -0.112\,5 & -0.005 \\ 0 & 0 \end{bmatrix},\ \boldsymbol{D}_{\mathrm{d}} = \begin{bmatrix} -0.112\,5 & -0.23 \\ 0 & 0 \end{bmatrix},\ \boldsymbol{E}_{\mathrm{d}} = \begin{bmatrix} -0.112\,5 & -1.527 \\ 1 & 0 \end{bmatrix},$$

$$\boldsymbol{G} = \begin{bmatrix} 0.2 & 1 \end{bmatrix},\ \boldsymbol{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \boldsymbol{C} = \begin{bmatrix} 1 & 0 \end{bmatrix},\ \boldsymbol{D} = \begin{bmatrix} 0.2 \\ 1 \end{bmatrix}.$$

初始条件: $\boldsymbol{x}(0) = \begin{bmatrix} -1 \\ -1.2 \end{bmatrix}$, 令$\bar{\varepsilon} = 0.35$, 由定理 2 及 MATLAB 工具箱中 LMI 工具箱函数, 求解 LMIs 条件可得

$$\boldsymbol{P} = \begin{bmatrix} 0.104\,2 & 0.695\,4 \\ 0.695\,4 & 0.105\,6 \end{bmatrix},\ \boldsymbol{Q} = \begin{bmatrix} 0.289\,6 & 0.003\,8 \\ 0.003\,8 & -0.065\,0 \end{bmatrix},\ \boldsymbol{M} = \begin{bmatrix} 4.069\,5 & 0.011\,9 \\ 0.011\,9 & 4.067\,4 \end{bmatrix},\ \lambda_1 = 0.520\,6,$$

$$\lambda_2 = 9.198\,0,\ Z_1 = -0.003\,9,\ Z_2 = 0.277\,8,\ Z_3 = 0.006\,3,\ Z_4 = 4.773\,8,\ Z_5 = 4.588\,6.$$

给定$\gamma = 1.5$, 应用定理 2可得该系统的记忆状态反馈控制$\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{x}(t) + \boldsymbol{K}_1\boldsymbol{x}(t-d(t))$, 其中

$\boldsymbol{K} = [-2.252\,4 \quad 0.023\,6]$, $\boldsymbol{K}_1 = [0.305\,7 \quad 0.044\,3]$, 求解 LMIs, 最优值$\gamma^* = 1.126\,5$完全符合设计要求.

由表 1 知, 定理 2 所给的控制器存在, 并且具有较大的奇异摄动参数上界值 0.35, 同时广义H_2控制区间在 (0, 0.35] 区间内, 大于文献 [12] 的 (0, 0.3], 扩大了闭环系统的渐近稳定范围, 最优L_2-L_∞性能指标 1.1265 也相对更小. 验证了本文所得结论的有效性和可行性, 得到的控制器比文献 [12] 控制效果更具有一定的优越性, 并且适用于标准和非标准情形.

表 1　广义 H_2 稳定性能指标对比

Table 1　Comparison of generalized H_2 stability performance indicators

method	generalized H_2 control gain matrix $\boldsymbol{K}$	optimal L_2-L_∞ performance index γ^*	perturbation upper bound $\bar{\varepsilon}$
ref. [12]	$\boldsymbol{K}_1 = [-22.046\,7 \quad -14.167\,5]$ $\boldsymbol{K}_2 = [-22.037\,6 \quad -12.656\,2]$	1.3255	0.3
theorem 2	$\boldsymbol{K} = [-2.252\,4 \quad 0.023\,6]$ $\boldsymbol{K}_1 = [0.305\,7 \quad 0.044\,3]$	1.1265	0.35

4　结　　语

1) 本文引入新的广义H_2性能指标的定义, 选取新的依赖于时滞和摄动参数的二次型 Lyapunov 泛函. 同时借助新的引理及交叉项界定方法, 对记忆情形推出在时滞依赖和时滞独立两种情况下的系统广义H_2稳定的充分性判据, 且在$(0, \bar{\varepsilon}]$区间内, 扩大了稳定界, 使记忆状态反馈广义H_2控制器具有更小的保守性.

2) 本文研究了状态反馈广义H_2控制问题, 如何将结论推广到输出反馈广义H_2控制问题中, 还有待进一步探讨.

3) 若加深理论高度, 将连续奇异摄动系统拓展到离散奇异摄动系统, 怎样将连续奇异摄动系统广义H_2控制理论应用到离散奇异摄动系统中, 解决时变时滞离散奇异摄动系统的广义H_2控制问题, 这将是一个极具挑战的课题.

4) 在推得时滞依赖和时滞独立相关的稳定性结果的过程中, 可以对其中的某一变量作进一步的限定, 减小计算量, 使过程和结论简单化, 这样用较大一点的保守性, 换取实际控制系统的简便性, 也是一个具有实际意义的理论方向. 因篇幅所限, 将继续在今后的学习中予以研究. 作为一个新的研究领域, 广义H_2控制理论仍处于不断改进, 不断发展之中[15-18], 在它广泛的工程背景下, 无论是理论本身, 还是工程的实际应用, 都必将会取得更有意义的成果.

参考文献(References):

[1] 王德进. H_2和H_∞优化控制理论[M]. 哈尔滨: 哈尔滨工业大学出版社, 2001. (WANG Dejin. *H_2 and H_∞ Optimal Control Theory*[M]. Harbin: Harbin Institute of Technology Press, 2001. (in Chinese))

[2] 高会军, 王常虹, 李艳辉. 时滞不确定系统的广义H_2控制[J]. 电机与控制学报, 2002, **6**(4): 328-332. (GAO Huijun, WANG Changhong, LI Yanhui. Generalized H_2 control for uncertain time-delayed systems[J]. *Electric Machines and Control*, 2002, **6**(4): 328-332.(in Chinese))

[3] 房庆祥. 线性奇异系统的H_2控制问题[D]. 硕士学位论文. 济南: 山东大学, 2002. (FANG Qingxiang. H_2 control problems of linear singular systems[D]. Master Thesis. Jinan: Shandong University, 2002. (in Chinese))

[4] 张庆灵, 张雪峰, 翟丁. 控制理论基础[M]. 北京: 高等教育出版社, 2008. (ZHANG Qingling, ZHANG Xuefeng, ZHAI Ding. *Fundamentals of Control Theory*[M]. Beijing: Higher Education Press, 2008. (in Chinese))

[5] 王婕. 离散时间随机乘积噪声系统的若干问题研究[D]. 硕士学位论文. 泰安: 山东科技大学, 2017. (WANG Jie. Study on some problems of discrete-time stochastic systems with multiplicative noises[D]. Master Thesis. Taian: Shandong University of Science and Technology, 2017. (in Chinese))

[6] KATAYAMA T, WASHIKITA Y. Solutions to Wiener filtering and stationary LQG problems via H_2 control theory, part 1: continuous-time systems[J]. *Memoirs of the Faculty of Engineering, Kyoto University*, 1990, **52**(3): 219-244.

[7] WILSON D A. Convolution and Hankel operator norms for linear systems[J]. *IEEE Transactions on Automatic Control*, 1989, **34**(1): 94-97.

[8] 赵春燕. 时滞系统的非脆弱广义H_2控制[J]. 哈尔滨师范大学自然科学学报, 2008, **24**(4): 40-43. (ZHAO Chunyan. Non-fragile generalized H_2 control for linear time-delay systems[J]. *Natural Science Journal of Harbin Normal University*, 2008, **24**(4): 40-43.(in Chinese))

[9] 郭祥贵, 王武, 杨富文, 等. 凸多面体不确定系统的鲁棒L_2-L_∞控制[J]. 中南大学学报(自然科学版), 2007, **38**(1): 273-277. (GUO Xianggui, WANG Wu, YANG Fuwen, et al. Robust L_2-L_∞ control for polyhedral continuous-time uncertain systems[J]. *Journal of Central South University* (*Science and Technology*), 2007, **38**(1): 273-277.(in Chinese))

[10] 郭杨, 姚郁, 王仕成, 等. 基于有限时间H_2性能指标的导弹机动突防策略设计[J]. 宇航学报, 2010, **31**(10): 2289-2294. (GUO Yang, YAO Yu, WANG Shicheng, et al. Finite-time H_2 performance measure-based strategy design[J]. *Journal of Astronautics*, 2010, **31**(10): 2289-2294.(in Chinese))

[11] XIN B, ZHAO D L. Generalized H_2 control of the linear system with semi-Markov jumps[J]. *International Journal of Robust and Nonlinear Control*, 2021, **31**(3): 1005-1020.

[12] 赵海英, 奥顿, 吴忠强. 不确定非线性时滞系统的广义H_2控制[J]. 自动化仪表, 2010, **31**(8): 18-22. (ZHAO Haiying, AO Dun, WU Zhongqiang. Generalized H_2 control for uncertain nonlinear systems with time-delay[J]. *Process Automation Instrumentation*, 2010, **31**(8): 18-22.(in Chinese))

[13] 史玉英, 吴保卫. 不确定性中立型时滞系统的鲁棒稳定性[J]. 云南师范大学学报, 2007, **27**(2): 1-5. (SHI Yuying, WU Baowei. Robust stability of uncertain neutral system with time delays[J]. *Journal of Yunnan Normal University*, 2007, **27**(2): 1-5.(in Chinese))

[14] 孙凤琪. 时滞奇异摄动不确定系统的稳定性分析与控制[M]. 北京: 科学出版社, 2018. (SUN Fengqi. *Stability Analysis and Control of Singularly Perturbed Uncertain Systems With Time Delay*[M]. Beijing: Science Press, 2018. (in Chinese))

[15] 李江荣. 模糊控制系统的可达集分析与广义H_2控制[D]. 博士学位论文. 西安: 西安电子科技大学, 2012. (LI Jiangrong. Reachable set analysis and generalized H_2 control of fuzzy control systems[D]. PhD Thesis. Xi'an: Xidian University, 2012. (in Chinese))

[16] HUANG H, FENG G. Delay-dependent H_∞ and generalized H_2 filtering for delayed neural networks[J]. *IEEE Transactions on Circuits and Systems I*: *Regular Papers*, 2009, **56**(4): 846-857.

[17] 孙凤琪. 不确定时滞摄动滤波误差动态系统的稳定性分析[J]. 应用数学和力学, 2020, **41**(8): 899-911. (SUN Fengqi. Stability analysis of uncertain singularly perturbed filter error dynamic systems with time delays[J]. *Applied Mathematics and Mechanics*, 2020, **41**(8): 899-911.(in Chinese))

[18] 孙凤琪. 离散时滞奇异摄动控制系统的稳定性分析[J]. 应用数学和力学, 2021, **42**(7): 696-703. (SUN Fengqi. Stability analysis of discrete time-delay singularly perturbed uncertainty control systems[J]. *Applied Mathematics and Mechanics*, 2021, **42**(7): 696-703.(in Chinese))