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# 具有不确定性的分数阶时滞复值神经网络无源性*

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**摘要：** 该文研究了一类具有不确定性和时滞的分数阶复值神经网络无源性问题,未将复值神经网络模型拆分成两个实值系统,而是将复值系统当成一个整体直接进行处理.通过构造恰当的 Lyapunov 函数,并利用矩阵不等式技巧,建立了网络无源性的线性矩阵不等式判据.给出的数值例子和仿真验证了获得结论的可行性和有效性.

**关　键　词：** 复值神经网络； 分数阶； 无源性； 时滞； 不确定性

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# Passivity of Fractional-Order Delayed Complex-Valued Neural Networks With Uncertainties

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**Abstract**: The passivity for a class of fractional-order delayed complex-valued neural networks with uncertainties was studied. The complex-valued neural network was not divided into 2 real-valued neural networks, but treated as a whole. Through construction of the appropriate Lyapunov function and application of the inequality technique, the sufficient criterion in the form of the linear matrix inequality was established to ensure the passivity of the considered neural networks. Numerical examples and simulations verify the feasibility and effectiveness of the obtained conclusion.

**Key words**: complex-valued neural network; fractional-order; passivity; delay; uncertainty

## 引　言

自 1982 年美国科学院院士、加州理工学院生物物理学家 Hopfield 建立了神经网络的数学模型后[1]，各

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种神经网络模型相继被提出.由于神经网络具有自学习功能、联想记忆功能和鲁棒性强等特点,现已广泛应用于模式识别、信号传输、联想记忆、图像处理、保密通讯和优化计算等诸多领域[2].在神经网络的硬件实现过程中,由于放大器转换速度的限制,时滞是不可避免的[3].因此,将时滞引入神经网络,建立时滞神经网络模型并研究其动力学行为具有重要意义[4].另一方面,由于系统建模的误差和工作环境的变化,系统往往存在着参数不确定性[5].参数不确定性是许多物理系统的固有特性,是引起系统不稳定、振荡甚至混沌等复杂动力学行为的主要因素[6].因此,研究具有参数不确定的时滞神经网络动力学行为具有重要的理论和实用价值[7].众所周知,无源性理论是非线性系统稳定性分析的有效工具之一,其主要思想是系统的无源性能保证系统内部稳定[8].近年来,时滞神经网络的无源性已经被广泛的研究[9-12].

以上文献研究的神经网络,其神经元的状态、输出、权值和激活函数都取实数值,人们称之为实值神经网络.虽然实值神经网络已在诸多领域得到了广泛应用,但也有其局限性[13].例如,在信号处理中,当需要处理的数据是复值时,通常的做法是,先将复值数据的实部和虚部提取出来,得到两组实值数据,然后设计两个实值神经网络来进行处理.虽然这种处理方式有一定的合理性,但遇到了两个问题:其一,模型的维数会成倍增加,可能会导致分析和计算的复杂性;其二,由于复值信号既携带了信号的振幅信息,也携带了信号的相位信息,用两个实值神经网络加以处理的结果,可能会丢失与信号的振幅和相位有内在关联的一些信息[14].自然地,复值神经网络模型被提出[15].由于复值神经网络的神经元状态、输出、权值和激活函数都取复数值,因此它能直接处理复值数据.近年来,时滞复值神经网络的无源性得到了一些研究[16-18].

上述文献研究的神经网络,无论是实值神经网络,还是复值神经网络,其模型都是用整数阶导数描述的.与整数阶微积分相比,分数阶微积分最主要的优点是能够描述系统的记忆性和遗传性,具有整数阶微积分所不能替代的功能,能更好地揭示系统的本质特性[19].因此,一些学者借助分数阶微积分在模型刻画上的优势,将分数阶微积分理论引入到神经网络,建立了分数阶神经网络模型[20].研究发现,分数阶神经网络能够提升神经元的记忆性与遗传性,具有更加有效的计算能力和信息处理能力[21].近年来,一些学者对分数阶复值时滞神经网络的稳定性[19]、同步性[20]、分岔[21]和状态估计[22]等动力学行为进行了研究.据我们所知,分数阶复值时滞神经网络的无源性研究还未见报道.鉴于此,本文研究了具有参数不确定性的分数阶时滞复值神经网络无源性问题,建立了网络无源性的充分条件,并给出数值例子和仿真验证获得了结论的可行性和有效性.

## 1 预备知识

分数阶微积分的定义有多种形式,本文采用的是 Riemann-Liouville 定义和 Caputo 定义.

**定义 1**[23]　设 $f(t)$ 为定义在 $[a,b]$ 上的连续函数,$\alpha \in (0,1)$,则 $f(t)$ 的 Riemann-Liouville 型 $\alpha$ 阶积分定义为

$$ {}_0\mathrm{D}_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau, \qquad t \in [a,b], $$

其中 $\Gamma(\cdot)$ 是 Gamma 函数,即 $\Gamma(s) = \int_0^\infty \mathrm{e}^{-t} t^{s-1}\mathrm{d}t$.

**定义 2**[23]　设 $f(t)$ 为定义在 $[a,b]$ 上的连续可导函数,$\alpha \in (0,1)$,则 $f(t)$ 的 Caputo 型 $\alpha$ 阶导数定义为

$$ {}_0^{\mathrm{C}}\mathrm{D}_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-\tau)^{-\alpha} f'(\tau)\,\mathrm{d}\tau, \qquad t \in [a,b]. $$

由定义 1 和定义 2,容易得到下面的性质.

**性质 1**　当 $0 < \alpha < 1$ 时,有

$$ {}_0\mathrm{D}_t^{-\alpha}({}_0^{\mathrm{C}}\mathrm{D}_t^{\alpha} x(t)) = x(t) - x(0). $$

**性质 2**　当 $p,q \in (0,1)$ 且 $p > q$ 时,有

$$ {}_0^{\mathrm{C}}\mathrm{D}_t^{p}\, {}_0\mathrm{D}_t^{-q} x(t) = {}_0^{\mathrm{C}}\mathrm{D}_t^{p-q} x(t). $$

本文考虑如下一类具有参数不确定性的分数阶时滞复值神经网络：

$$\begin{cases} {}_0\mathrm{D}_t^{\alpha}\boldsymbol{x}(t) = -(\boldsymbol{D} + \Delta\boldsymbol{D}(t))\boldsymbol{x}(t) + (\boldsymbol{A} + \Delta\boldsymbol{A}(t))\boldsymbol{f}(\boldsymbol{x}(t)) + \\ \qquad (\boldsymbol{B} + \Delta\boldsymbol{B}(t))\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + \boldsymbol{W}\boldsymbol{\omega}(t), \\ \boldsymbol{y}(t) = \boldsymbol{M}\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{C}\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + \boldsymbol{N}\boldsymbol{\omega}(t), \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \qquad s \in [-\tau, 0], \end{cases} \tag{1}$$

其中 $t \geqslant 0, 0 < \alpha < 1$，$n$ 是神经元的数量；$\boldsymbol{x}(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}} \in \mathbb{C}^n$ 是网络在 $t$ 时刻的状态变量；$\boldsymbol{f}(\boldsymbol{x}(t)) = (f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t)))^{\mathrm{T}} \in \mathbb{C}^n$ 表示激活函数；$\boldsymbol{\omega}(t) \in \mathbb{C}^m$ 是扰动输入；$\boldsymbol{y}(t) \in \mathbb{C}^p$ 是输出变量；$\boldsymbol{D} = \mathrm{diag}\{d_1, d_2, \cdots, d_n\} \in \mathbb{R}^{n\times n}$ 是正对角阵，$\boldsymbol{A}, \boldsymbol{B} \in \mathbb{C}^{n\times n}$，$\boldsymbol{W} \in \mathbb{C}^{n\times m}$，$\boldsymbol{M}, \boldsymbol{C} \in \mathbb{C}^{p\times n}$，$\boldsymbol{N} \in \mathbb{C}^{p\times m}$；$\Delta\boldsymbol{D}(t)$，$\Delta\boldsymbol{A}(t)$ 和 $\Delta\boldsymbol{B}(t)$ 是时变参数不确定性；$\boldsymbol{\phi}(s)$ 是初始函数.

类似于文献[24]，我们给出网络(1)的无源性定义.

**定义 3** 在零初始值条件下，若存在一个参数 $\gamma > 0$，使得

$$\int_0^t [\boldsymbol{y}^*(s)\boldsymbol{\omega}(s) + \boldsymbol{\omega}^*(s)\boldsymbol{y}(s)]\mathrm{d}s \geqslant -\gamma\int_0^t \boldsymbol{\omega}^*(s)\boldsymbol{\omega}(s)\mathrm{d}s,$$

则称网络(1)是无源的.

为了建立网络(1)的无源性判据，我们对激活函数和参数不确定性做如下假设.

(H1) 存在一个正对角阵 $\boldsymbol{L} = \mathrm{diag}\{l_1, l_2, \cdots, l_n\}$，使得对于任意 $\alpha_1, \alpha_2 \in \mathbb{C}$，有

$$|f_i(\alpha_1) - f_i(\alpha_2)| \leqslant l_i|\alpha_1 - \alpha_2|, \qquad i = 1, 2, \cdots, n.$$

(H2) 参数不确定性 $\Delta\boldsymbol{D}(t)$，$\Delta\boldsymbol{A}(t)$，$\Delta\boldsymbol{B}(t)$ 满足

$$\Delta\boldsymbol{D}(t) = \boldsymbol{H}_1\boldsymbol{G}_1(t)\boldsymbol{E}_1,\ \Delta\boldsymbol{A}(t) = \boldsymbol{H}_2\boldsymbol{G}_2(t)\boldsymbol{E}_2,\ \Delta\boldsymbol{B}(t) = \boldsymbol{H}_3\boldsymbol{G}_3(t)\boldsymbol{E}_3,$$

其中 $\boldsymbol{H}_1$，$\boldsymbol{H}_2$，$\boldsymbol{H}_3$，$\boldsymbol{E}_1$，$\boldsymbol{E}_2$，$\boldsymbol{E}_3$ 是已知的矩阵，$\boldsymbol{H}_1$，$\boldsymbol{G}_1(t)$ 和 $\boldsymbol{E}_1$ 是正对角矩阵，且 $\boldsymbol{G}_1^*(t)\boldsymbol{G}_1(t) \leqslant \boldsymbol{I}$，$\boldsymbol{G}_2^*(t)\boldsymbol{G}_2(t) \leqslant \boldsymbol{I}$，$\boldsymbol{G}_3^*(t)\boldsymbol{G}_3(t) \leqslant \boldsymbol{I}$.

**引理 1**[22] 设 $z(t) \in \mathbb{C}^n$ 是一个可微的向量值函数，$\boldsymbol{P}$ 是正 Hermite 矩阵，对任意 $t > 0$，$\alpha \in (0,1)$，恒有

$$\mathrm{D}^{\alpha}(\boldsymbol{z}^*(t)\boldsymbol{P}\boldsymbol{z}(t)) \leqslant \boldsymbol{z}^*(t)\boldsymbol{P}\mathrm{D}^{\alpha}\boldsymbol{z}(t) + \mathrm{D}^{\alpha}\boldsymbol{z}^*(t)\boldsymbol{P}\boldsymbol{z}(t).$$

**引理 2**[25] 若 $\boldsymbol{U}_i$，$\boldsymbol{V}_i$，$\boldsymbol{R}_i$ $(i = 1, 2, \cdots, m)$ 是复矩阵，且 $\boldsymbol{M}$ 满足 $\boldsymbol{M}^* = \boldsymbol{M}$，则当 $\boldsymbol{V}_i^*\boldsymbol{V}_i \leqslant \boldsymbol{I}$ 时，

$$\boldsymbol{M} + \sum_{i=1}^{m}(\boldsymbol{U}_i\boldsymbol{V}_i\boldsymbol{R}_i + \boldsymbol{R}_i^*\boldsymbol{V}_i^*\boldsymbol{U}_i^*) \leqslant \boldsymbol{0},$$

当且仅当存在 $\varepsilon_i > 0$，使得

$$\boldsymbol{M} + \sum_{i=1}^{m}(\varepsilon_i^{-1}\boldsymbol{U}_i\boldsymbol{U}_i^* + \varepsilon_i\boldsymbol{R}_i^*\boldsymbol{R}_i) \leqslant \boldsymbol{0}.$$

**引理 3**[25] 设 $\boldsymbol{U}$，$\boldsymbol{V}$，$\boldsymbol{W}$ 是常值矩阵，且 $\boldsymbol{U}^* = \boldsymbol{U}$，$\boldsymbol{V}^* = \boldsymbol{V}$，则

$$\begin{pmatrix} \boldsymbol{U} & \boldsymbol{W} \\ \boldsymbol{W}^* & -\boldsymbol{V} \end{pmatrix} < \boldsymbol{0}$$

的等价条件是

$$\boldsymbol{V} > \boldsymbol{0},\ \boldsymbol{U} + \boldsymbol{W}\boldsymbol{V}^{-1}\boldsymbol{W}^* < \boldsymbol{0}.$$

## 2 主 要 结 果

**定理 1** 在假设(H1)和假设(H2)下，若存在正定 Hermite 矩阵 $\boldsymbol{P}$ 和 $\boldsymbol{Q}$，正对角阵 $\boldsymbol{R}_1$ 和 $\boldsymbol{R}_2$，以及正常数 $\gamma$，$\varepsilon_1$，$\varepsilon_2$ 和 $\varepsilon_3$，使得下面线性矩阵不等式成立：

$$\boldsymbol{\Omega} = \begin{pmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} \\ * & \boldsymbol{\Omega}_{22} \end{pmatrix} < \boldsymbol{0}, \tag{2}$$

其中

$$\boldsymbol{\Omega}_{11}=\begin{pmatrix}\bar{\boldsymbol{\Omega}}_{11} & \mathbf{0} & \boldsymbol{PA} & \boldsymbol{PB} & \boldsymbol{PW}\\ & \bar{\boldsymbol{\Omega}}_{22} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ & & \bar{\boldsymbol{\Omega}}_{33} & \mathbf{0} & -\boldsymbol{M}^{*}\\ & & & \bar{\boldsymbol{\Omega}}_{44} & -\boldsymbol{C}^{*}\\ * & & & & \bar{\boldsymbol{\Omega}}_{55}\end{pmatrix},$$

$$\boldsymbol{\Omega}_{12}=\begin{pmatrix}\boldsymbol{PH}_1 & \boldsymbol{PH}_2 & \boldsymbol{PH}_3\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{pmatrix},\ \boldsymbol{\Omega}_{22}=\begin{pmatrix}-\varepsilon_1\boldsymbol{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\varepsilon_2\boldsymbol{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & -\varepsilon_3\boldsymbol{I}\end{pmatrix},$$

并且, $\bar{\boldsymbol{\Omega}}_{11}=-\boldsymbol{DP}-\boldsymbol{PD}+\varepsilon_1\boldsymbol{E}_1^{*}\boldsymbol{E}_1+\boldsymbol{LR}_1\boldsymbol{L}+\boldsymbol{Q}$, $\bar{\boldsymbol{\Omega}}_{22}=\boldsymbol{LR}_2\boldsymbol{L}-\boldsymbol{Q}$, $\bar{\boldsymbol{\Omega}}_{33}=\varepsilon_2\boldsymbol{E}_2^{*}\boldsymbol{E}_2-\boldsymbol{R}_1$, $\bar{\boldsymbol{\Omega}}_{44}=\varepsilon_3\boldsymbol{E}_3^{*}\boldsymbol{E}_3-\boldsymbol{R}_2$, $\bar{\boldsymbol{\Omega}}_{55}=-\boldsymbol{N}^{*}-\boldsymbol{N}-\gamma\boldsymbol{I}$, 则网络(1)是无源的.

**证明**　构造以下 Lyapunov 函数:

$$V(t)=V_1(t)+V_2(t),$$

其中

$$V_1(t)={}_0\mathrm{D}_t^{-(1-\alpha)}\boldsymbol{x}^{*}(t)\boldsymbol{Px}(t),$$

$$V_2(t)=\int_{t-\tau}^{t}\boldsymbol{x}^{*}(s)\boldsymbol{Qx}(s)\,\mathrm{d}s.$$

对 $V_1(t)$ 求导,并由性质 2 和引理 1 可得

$$\begin{aligned}\dot{V}_1(t)={}&{}_0\mathrm{D}_t^{\alpha}\boldsymbol{x}^{*}(t)\boldsymbol{Px}(t)\leqslant\\ &{}_0\mathrm{D}_t^{\alpha}\boldsymbol{x}^{*}(t)\boldsymbol{Px}(t)+\boldsymbol{x}^{*}(t)\boldsymbol{P}{}_0\mathrm{D}_t^{\alpha}\boldsymbol{x}(t)=\\ &\boldsymbol{x}^{*}(t)(-\boldsymbol{DP}-\boldsymbol{PD}-\Delta\boldsymbol{D}^{*}(t)\boldsymbol{P}-\boldsymbol{P}\Delta\boldsymbol{D}(t))\boldsymbol{x}(t)+\boldsymbol{f}^{*}(\boldsymbol{x}(t))\boldsymbol{A}^{*}\boldsymbol{Px}(t)+\\ &\boldsymbol{x}^{*}(t)\boldsymbol{PAf}(\boldsymbol{x}(t))+\boldsymbol{f}^{*}(\boldsymbol{x}(t))\Delta\boldsymbol{A}^{*}(t)\boldsymbol{Px}(t)+\boldsymbol{x}^{*}(t)\boldsymbol{P}\Delta\boldsymbol{A}(t)\boldsymbol{f}(\boldsymbol{x}(t))+\\ &\boldsymbol{f}^{*}(x(t-\tau))\boldsymbol{B}^{*}\boldsymbol{Px}(t)+\boldsymbol{x}^{*}(t)\boldsymbol{PBf}(\boldsymbol{x}(t-\tau))+\boldsymbol{f}^{*}(\boldsymbol{x}(t-\tau))\Delta\boldsymbol{B}^{*}(t)\boldsymbol{Px}(t)+\\ &\boldsymbol{x}^{*}(t)\boldsymbol{P}\Delta\boldsymbol{B}(t)\boldsymbol{f}(\boldsymbol{x}(t-\tau))+\boldsymbol{\omega}^{*}(t)\boldsymbol{W}^{*}\boldsymbol{Px}(t)+\boldsymbol{x}^{*}(t)\boldsymbol{PW\omega}(t).\end{aligned}\qquad(3)$$

对 $V_2(t)$ 求导, 可得

$$\dot{V}_2(t)=\boldsymbol{x}^{*}(t)\boldsymbol{Qx}(t)-\boldsymbol{x}^{*}(t-\tau)\boldsymbol{Qx}(t-\tau).\qquad(4)$$

由假设(H1), 我们能够得到

$$\boldsymbol{x}^{*}(t)\boldsymbol{LR}_1\boldsymbol{Lx}(t)-\boldsymbol{f}^{*}(\boldsymbol{x}(t))\boldsymbol{R}_1\boldsymbol{f}(\boldsymbol{x}(t))\geqslant 0\qquad(5)$$

和

$$\boldsymbol{x}^{*}(t-\tau)\boldsymbol{LR}_2\boldsymbol{Lx}(t-\tau)-\boldsymbol{f}^{*}(\boldsymbol{x}(t-\tau))\boldsymbol{R}_2\boldsymbol{f}(\boldsymbol{x}(t-\tau))\geqslant 0.\qquad(6)$$

由式(3)~(6),我们有

$$\begin{aligned}&\dot{V}(t)-\boldsymbol{y}^{*}(t)\boldsymbol{\omega}(t)-\boldsymbol{\omega}^{*}(t)\boldsymbol{y}(t)-\gamma\boldsymbol{\omega}^{*}(t)\boldsymbol{\omega}(t)\leqslant\\ &\quad\boldsymbol{x}^{*}(t)(-\boldsymbol{DP}-\boldsymbol{PD}-\Delta\boldsymbol{D}^{*}(t)\boldsymbol{P}-\boldsymbol{P}\Delta\boldsymbol{D}(t)+\boldsymbol{LR}_1\boldsymbol{L}+\boldsymbol{Q})\boldsymbol{x}(t)+\\ &\quad\boldsymbol{f}^{*}(\boldsymbol{x}(t))\boldsymbol{A}^{*}\boldsymbol{Px}(t)+\boldsymbol{x}^{*}(t)\boldsymbol{PAf}(\boldsymbol{x}(t))+\boldsymbol{x}^{*}(t-\tau)(\boldsymbol{LR}_2\boldsymbol{L}-\boldsymbol{Q})\boldsymbol{x}(t-\tau)-\\ &\quad\boldsymbol{f}^{*}(\boldsymbol{x}(t))\boldsymbol{R}_1\boldsymbol{f}(\boldsymbol{x}(t))-\boldsymbol{f}^{*}(\boldsymbol{x}(t-\tau))\boldsymbol{R}_2\boldsymbol{f}(\boldsymbol{x}(t-\tau))+\\ &\quad\boldsymbol{f}^{*}(\boldsymbol{x}(t))\Delta\boldsymbol{A}^{*}(t)\boldsymbol{Px}(t)+\boldsymbol{x}^{*}(t)\boldsymbol{P}\Delta\boldsymbol{A}(t)\boldsymbol{f}(\boldsymbol{x}(t))+\boldsymbol{f}^{*}(\boldsymbol{x}(t-\tau))\boldsymbol{B}^{*}\boldsymbol{Px}(t)+\\ &\quad\boldsymbol{x}^{*}(t)\boldsymbol{PBf}(\boldsymbol{x}(t-\tau))+\boldsymbol{f}^{*}(\boldsymbol{x}(t-\tau))\Delta\boldsymbol{B}^{*}(t)\boldsymbol{Px}(t)+\boldsymbol{x}^{*}(t)\boldsymbol{P}\Delta\boldsymbol{B}(t)\boldsymbol{f}(\boldsymbol{x}(t-\tau))+\\ &\quad\boldsymbol{\omega}^{*}(t)\boldsymbol{W}^{*}\boldsymbol{Px}(t)+\boldsymbol{x}^{*}(t)\boldsymbol{PW\omega}(t)-\boldsymbol{f}^{*}(\boldsymbol{x}(t))\boldsymbol{M}^{*}\boldsymbol{\omega}(t)-\boldsymbol{\omega}(t)^{*}\boldsymbol{Mf}(\boldsymbol{x}(t))-\\ &\quad\boldsymbol{f}^{*}(\boldsymbol{x}(t-\tau))\boldsymbol{C}^{*}\boldsymbol{\omega}(t)-\boldsymbol{\omega}^{*}(t)\boldsymbol{Cf}(\boldsymbol{x}(t-\tau))-\boldsymbol{\omega}^{*}(t)(\boldsymbol{N}^{*}+\boldsymbol{N}+\gamma\boldsymbol{I})\boldsymbol{\omega}(t)=\\ &\quad\boldsymbol{\xi}^{*}(t)\boldsymbol{\Pi\xi}(t),\end{aligned}\qquad(7)$$

其中

$$\boldsymbol{\xi}(t)=(\boldsymbol{x}^*(t),\ \boldsymbol{x}^*(t-\tau),\ \boldsymbol{f}^*(\boldsymbol{x}(t)),\ \boldsymbol{f}^*(\boldsymbol{x}(t-\tau)),\ \boldsymbol{\omega}^*(t))^*,$$

$$\boldsymbol{\Pi}=\begin{pmatrix}\boldsymbol{\pi}_{11} & \boldsymbol{0} & \boldsymbol{PA}+\boldsymbol{P}\Delta\boldsymbol{A}(t) & \boldsymbol{PB}+\boldsymbol{P}\Delta\boldsymbol{B}(t) & \boldsymbol{PW}\\ & \bar{\boldsymbol{\Omega}}_{22} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ & & -\boldsymbol{R}_1 & \boldsymbol{0} & -\boldsymbol{M}^*\\ & & & -\boldsymbol{R}_2 & -\boldsymbol{C}^*\\ * & & & & \bar{\boldsymbol{\Omega}}_{55}\end{pmatrix},$$

并且

$$\boldsymbol{\pi}_{11}=-\boldsymbol{DP}-\boldsymbol{PD}-\boldsymbol{P}\Delta\boldsymbol{D}(t)-\Delta\boldsymbol{D}^*(t)\boldsymbol{P}+\boldsymbol{LR}_1\boldsymbol{L}+\boldsymbol{Q},\ \bar{\boldsymbol{\Omega}}_{22}=\boldsymbol{LR}_2\boldsymbol{L}-\boldsymbol{Q},\ \bar{\boldsymbol{\Omega}}_{55}=-\boldsymbol{N}^*-\boldsymbol{N}-\gamma\boldsymbol{I}.$$

由假设（H2）可得

$$\boldsymbol{\Pi}=\begin{pmatrix}\bar{\boldsymbol{\pi}}_{11} & \boldsymbol{0} & \boldsymbol{PA} & \boldsymbol{PB} & \boldsymbol{PW}\\ & \bar{\boldsymbol{\Omega}}_{22} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ & & -\boldsymbol{R}_1 & \boldsymbol{0} & -\boldsymbol{M}^*\\ & & & -\boldsymbol{R}_2 & -\boldsymbol{C}^*\\ * & & & & \bar{\boldsymbol{\Omega}}_{22}\end{pmatrix}+$$

$$\begin{aligned}&(-\boldsymbol{H}_1^*\boldsymbol{P}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0})^*\boldsymbol{G}_1(t)(\boldsymbol{E}_1\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0})+\\ &(\boldsymbol{E}_1\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0})^*\boldsymbol{G}_1^*(t)(-\boldsymbol{H}_1^*\boldsymbol{P}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0})+\\ &(\boldsymbol{H}_2^*\boldsymbol{P}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0})^*\boldsymbol{G}_2(t)(\boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{E}_2\ \ \boldsymbol{0}\ \ \boldsymbol{0})+\\ &(\boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{E}_2\ \ \boldsymbol{0}\ \ \boldsymbol{0})^*\boldsymbol{G}_2^*(t)(\boldsymbol{H}_2^*\boldsymbol{P}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0})+\\ &(\boldsymbol{H}_3^*\boldsymbol{P}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0})^*\boldsymbol{G}_3(t)(\boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{E}_3\ \ \boldsymbol{0})+\\ &(\boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{E}_3\ \ \boldsymbol{0})^*\boldsymbol{G}_3^*(t)(\boldsymbol{H}_3^*\boldsymbol{P}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}),\end{aligned}$$

其中

$$\bar{\boldsymbol{\pi}}_{11}=-\boldsymbol{DP}-\boldsymbol{PD}+\boldsymbol{LR}_1\boldsymbol{L}+\boldsymbol{Q}.$$

由条件(2)和引理 3,我们有

$$\begin{pmatrix}\hat{\boldsymbol{\Omega}} & \boldsymbol{0} & \boldsymbol{PA} & \boldsymbol{PB} & \boldsymbol{PW}\\ & \bar{\boldsymbol{\Omega}}_{22} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ & & \bar{\boldsymbol{\Omega}}_{33} & \boldsymbol{0} & -\boldsymbol{M}^*\\ & & & \bar{\boldsymbol{\Omega}}_{44} & -\boldsymbol{C}^*\\ * & & & & \bar{\boldsymbol{\Omega}}_{55}\end{pmatrix}<\boldsymbol{0}, \tag{8}$$

其中

$$\hat{\boldsymbol{\Omega}}=\bar{\boldsymbol{\Omega}}_{11}+\varepsilon_1^{-1}\boldsymbol{PH}_1\boldsymbol{H}_1^*\boldsymbol{P}+\varepsilon_2^{-1}\boldsymbol{PH}_2\boldsymbol{H}_2^*\boldsymbol{P}+\varepsilon_3^{-1}\boldsymbol{PH}_3\boldsymbol{H}_3^*\boldsymbol{P}.$$

我们将不等式(8)改写为

$$\begin{pmatrix}\bar{\boldsymbol{\pi}}_{11} & \boldsymbol{0} & \boldsymbol{PA} & \boldsymbol{PB} & \boldsymbol{PW}\\ & \bar{\boldsymbol{\Omega}}_{22} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ & & -\boldsymbol{R}_1 & \boldsymbol{0} & -\boldsymbol{M}^*\\ & & & -\boldsymbol{R}_2 & -\boldsymbol{C}^*\\ * & & & & \bar{\boldsymbol{\Omega}}_{55}\end{pmatrix}+$$

$$\varepsilon_1^{-1}(-\boldsymbol{H}_1^*\boldsymbol{P}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0})^*(-\boldsymbol{H}_1^*\boldsymbol{P}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0})+$$
$$\varepsilon_1(\boldsymbol{E}_1\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0})^*(\boldsymbol{E}_1\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0})+$$
$$\varepsilon_2^{-1}(\boldsymbol{H}_2^*\boldsymbol{P}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0})^*(\boldsymbol{H}_2^*\boldsymbol{P}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0})+$$
$$\varepsilon_2(\boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{E}_2\quad \boldsymbol{0}\quad \boldsymbol{0})^*(\boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{E}_2\quad \boldsymbol{0}\quad \boldsymbol{0})+$$
$$\varepsilon_3^{-1}(\boldsymbol{H}_3^*\boldsymbol{P}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0})^*(\boldsymbol{H}_3^*\boldsymbol{P}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0})+$$
$$\varepsilon_3(\boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{E}_3\quad \boldsymbol{0})^*(\boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{E}_3\quad \boldsymbol{0})<\boldsymbol{0}.$$

由引理 2 和不等式(8)可得

$$\boldsymbol{\Pi}<\boldsymbol{0}. \tag{9}$$

由不等式(9)和不等式(7),我们知道

$$\dot{V}(t)-\boldsymbol{y}^*(t)\boldsymbol{\omega}(t)-\boldsymbol{\omega}^*(t)\boldsymbol{y}(t)-\gamma\boldsymbol{\omega}^*(t)\boldsymbol{\omega}(t)\leqslant 0, \tag{10}$$

两边积分得

$$V(t)\leqslant V(0)+\int_0^t[\boldsymbol{y}^*(s)\boldsymbol{\omega}(s)+\boldsymbol{\omega}^*(s)\boldsymbol{y}(s)]\mathrm{d}s+\gamma\int_0^t\boldsymbol{\omega}^*(s)\boldsymbol{\omega}(s)\mathrm{d}s.$$

从 $V(t)$ 的定义可知 $V(t)\geqslant 0$,并且当初始值为零时,$V(0)=0$.这样

$$\int_0^t[\boldsymbol{y}^*(s)\boldsymbol{\omega}(s)+\boldsymbol{\omega}^*(s)\boldsymbol{y}(s)]\mathrm{d}s\geqslant -\gamma\int_0^t\boldsymbol{\omega}^*(s)\boldsymbol{\omega}(s)\mathrm{d}s.$$

因此,网络(1)是无源的.证毕.

## 3　数值仿真例子

考虑神经网络(1),其中

$$\boldsymbol{D}=\begin{pmatrix}6 & 0\\ 0 & 6.1\end{pmatrix},\ \boldsymbol{H}_1=\begin{pmatrix}0.523\,5 & 0\\ 0 & 0.523\,5\end{pmatrix},\ \boldsymbol{E}_1=\begin{pmatrix}0.346\,5 & 0\\ 0 & 0.346\,5\end{pmatrix},$$

$$\boldsymbol{A}=\begin{pmatrix}0.602\,4+0.057\,5\mathrm{i} & 0.455\,5+0.289\,8\mathrm{i}\\ 0.107\,4+0.021\,3\mathrm{i} & 0.654\,65+0.931\,5\mathrm{i}\end{pmatrix},\ \boldsymbol{H}_2=\begin{pmatrix}0.701\,6+0.320\,1\mathrm{i} & 0.001\,4+0.181\,3\mathrm{i}\\ 0.361\,4+0.393\,3\mathrm{i} & 0.384\,3+0.209\,4\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{E}_2=\begin{pmatrix}0.593\,7+0.056\,2\mathrm{i} & 0.172\,5+0.508\,3\mathrm{i}\\ 0.584\,5+0.201\,7\mathrm{i} & 0.933\,0+0.759\,2\mathrm{i}\end{pmatrix},\ \boldsymbol{B}=\begin{pmatrix}0.514\,0+0.534\,8\mathrm{i} & 0.378\,1+0.102\,8\mathrm{i}\\ 0.875\,3+0.892\,6\mathrm{i} & 0.884\,5+0.212\,0\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{H}_3=\begin{pmatrix}0.146\,1+0.674\,2\mathrm{i} & 0.421\,9+0.780\,1\mathrm{i}\\ 0.931\,1+0.279\,8\mathrm{i} & 0.056\,5+0.962\,8\mathrm{i}\end{pmatrix},\ \boldsymbol{E}_3=\begin{pmatrix}0.688\,2+0.538\,1\mathrm{i} & 0.099\,0+0.282\,8\mathrm{i}\\ 0.498\,4+0.011\,3\mathrm{i} & 0.287\,8+0.589\,6\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{W}=\begin{pmatrix}0.172\,0+0.551\,7\mathrm{i} & 0.676\,3+0.628\,4\mathrm{i}\\ 0.170\,1+0.967\,4\mathrm{i} & 0.569\,6+0.324\,5\mathrm{i}\end{pmatrix},\ \boldsymbol{M}=\begin{pmatrix}0.521\,0+0.347\,4\mathrm{i} & 0.994\,1+0.939\,4\mathrm{i}\\ 0.864\,9+0.749\,4\mathrm{i} & 0.838\,5+0.668\,1\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{C}=\begin{pmatrix}0.481\,2+0.959\,6\mathrm{i} & 0.595\,2+0.962\,0\mathrm{i}\\ 0.286\,2+0.442\,1\mathrm{i} & 0.336\,4+0.676\,4\mathrm{i}\end{pmatrix},\ \boldsymbol{N}=\begin{pmatrix}0.706\,1+0.155\,2\mathrm{i} & 0.939\,9+0.464\,1\mathrm{i}\\ 0.957\,7+0.830\,4\mathrm{i} & 0.833\,8+0.298\,7\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{G}_1(t)=\begin{pmatrix}0.1|\sin(t)| & 0\\ 0 & 0.1|\sin(t)|\end{pmatrix},$$

$$\boldsymbol{G}_2(t)=\begin{pmatrix}0.1\sin(t)+0.1\cos(t)\mathrm{i} & 0.2\cos(t)+0.2\cos(t)\mathrm{i}\\ 0.2\sin(2t)+0.1\cos(t)\mathrm{i} & 0.2\sin(2t)+0.1\cos(t)\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{G}_3(t)=\begin{pmatrix}0.1\sin(t)+0.1\cos(t)\mathrm{i} & 0.2\cos(t)+0.2\cos(t)\mathrm{i}\\ 0.2\sin(2t)+0.1\cos(t)\mathrm{i} & 0.2\sin(2t)+0.1\cos(t)\mathrm{i}\end{pmatrix},$$

$$\tau=0.5,\ f_1(x_1(t))=f_2(x_2(t))=0.5\tanh(x(t)),$$

$$\omega_1(t)=0.1\cos(t)+0.1\cos(t)\mathrm{i},\ \omega_2(t)=0.1\sin(t)+0.1\sin(t)\mathrm{i}.$$

容易验证假设(H1)和(H2)成立,并且 $\boldsymbol{L}=\mathrm{diag}\{0.5,0.5\}$.求解线性矩阵不等式(2),得

$$\boldsymbol{P}=\begin{pmatrix}7.557\,6 & -0.484\,8+0.002\,1\mathrm{i}\\ -0.484\,8-0.002\,1\mathrm{i} & 7.362\,5\end{pmatrix},$$

$$\boldsymbol{Q}=\begin{pmatrix}38.006\ 6 & -2.782+0.128\ 7\mathrm{i}\\ -2.782-0.128\ 7\mathrm{i} & 33.887\ 3\end{pmatrix},$$
$$\boldsymbol{R}_1=\begin{pmatrix}55.255\ 9 & 0\\ 0 & 74.463\ 4\end{pmatrix},\ \boldsymbol{R}_2=\begin{pmatrix}59.926\ 1 & 0\\ 0 & 47.014\ 3\end{pmatrix},$$
$$\varepsilon_1=34.535\ 5,\ \varepsilon_2=19.630\ 6,\ \varepsilon_3=24.656\ 3,\ \gamma=34.098\ 6.$$

因此定理1的条件被满足,从而分数阶复值时滞神经网络(1)是无源的.图1进一步验证了所得结果的有效性.

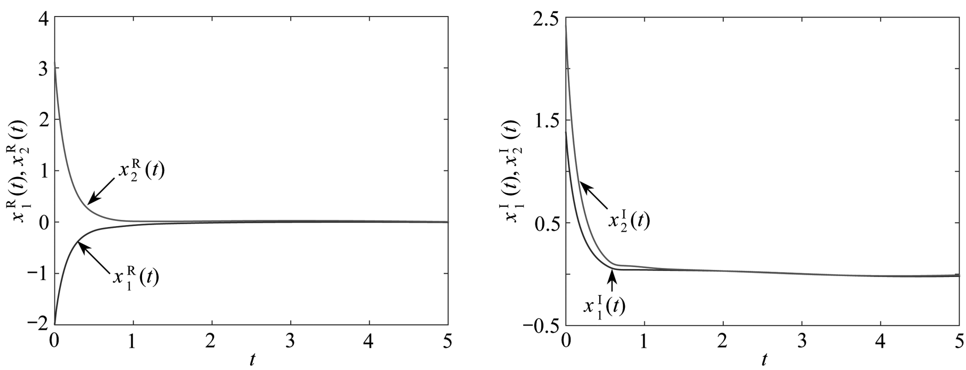


**图1** 状态变量实部和虚部的时间响应轨线

Fig. 1 The response trajectories of real and imaginary parts of state variables

# 4 结　　论

本文研究了具有不确定参数的分数阶时滞复值神经网络的无源性问题,在不分离激活函数实部和虚部的情形下,通过构造恰当的 Lyapunov 函数,得到了网络无源性的充分判据.然后给出了一个仿真实例,验证了获得结论的有效性.

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