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基于事件触发的时滞 Lur'e 系统主从同步研究*

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摘要： 研究了基于事件触发采样控制的时滞混沌 Lur'e 系统主从同步问题.首先考虑了系统中包含的传输时滞构造了系统时滞模型.然后,通过构造三重积分项的 Lyapunov-Krasovskii 泛函,并结合 Wirtinger 积分不等式和凸组合技术对 Lyapunov-Krasovskii 泛函的导数进行估计,给出了混沌系统主从同步的充分条件.所提出的事件触发机制应用于主从同步研究中,可以有效地减少采样数据传输,缓解网络带宽压力,提高网络带宽利用率.最后,通过时滞蔡氏电路的数值仿真,验证了所提出同步准则的有效性.

关 键 词： 混沌 Lur'e 系统； 主从同步； 事件触发； 采样数据控制

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Research on Master-Slave Synchronization of Time-Delay Lur'e Systems Based on Event Triggering

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Abstract: The master-slave synchronization of chaotic Lur'e systems with time-varying delays was investigated based on the event-triggered sampled-data control. First, the transmission time delays contained in the systems were considered, and the system time delay model was constructed. Then, the Lyapunov-Krasovskii functionals with triple integrals were constructed, the Wirtinger integral inequality and the convex combination technique were combined to estimate the derivatives of the Lyapunov-Krasovskii functionals, and the sufficient conditions for the master-slave synchronization of the chaotic system were given. Application of the proposed event-triggered mechanism to master-slave synchronization can effectively reduce the sampled data transmission, alleviate the pressure on the network bandwidth, and improve the utilization rate of the network bandwidth. Finally, the numerical simulation of the time-delay Chua's circuit verifies the effectiveness of the proposed synchronization criterion.

Key words: chaotic Lur'e system; master-slave synchronization; event-triggered; sampled-data control

引　　言

时滞广泛存在于实际工程应用中,且是不可避免的.时滞的存在会对系统的动力学行为产生重要的影

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响,往往会降低系统控制的性能,甚至破坏其稳定性,造成系统的不稳定.时滞系统动力学在航空航天、神经网络、通讯保密、生物生态学等领域得到了广泛的应用,因此,对时滞系统的研究具有理论意义和实际价值.

Lur'e 系统是一类非常典型的区间非线性系统,Chua's 电路、混沌神经网络、n- 涡旋吸引子等非线性系统都可以用 Lur'e 系统进行建模.混沌 Lur'e 系统的同步研究引起了广大学者的兴趣[1-4].已经有许多方法被提出用于实现混沌 Lur'e 系统同步,比如时滞反馈控制[5-7]、自适应控制[8-9]、脉冲控制[10]、比例微分控制[11-12]、采样数据控制[13-16].在这些控制方法中,采样数据控制是数字控制,与其他模拟控制相比,它只利用系统在采样瞬间的采样信息给控制器,减少了同步数据的传输,提高了带宽的使用率.

混沌系统的同步研究虽然得到了很大的发展,仍然有改进的空间,比如混沌系统同步的研究结果大部分都是基于时间触发的,时间触发控制虽然易于实现,但可能会造成大量冗余的采样数据传输到带宽有限的通信网络中,造成网络堵塞.为了解决这些问题,事件触发的研究应运而生.对于事件触发机制而言,只有在系统状态满足触发条件时,事件触发机制才会向通讯网络中传输数据.因此,引入事件触发机制可以显著减少采样数据传输、提高宽带利用率、节省通信资源.与时间触发控制相比,近年来事件触发控制更受学者关注.周期事件触发控制[17-18]、自触发控制[19]、自适应事件触发控制[20-21]等事件触发机制相继被提出,在网络控制系统中备受青睐.

本文研究时滞 Lur'e 系统基于事件触发控制的同步问题,当满足触发条件时,释放当前采样数据到通信信道中并传输给控制器,否则将保持控制器中的数据不变,直至满足触发条件的采样数据到来.本文的创新点如下:

1) 本文在构造 Lyapunov-Krasovskii 泛函的过程中,考虑了时变时滞 $\tau(t)$ 的信息,引入了三重积分项和增广 Lyapunov-Krasovskii 泛函,考虑了 Lur'e 系统中非线性函数的可用信息.

2) 与传统的同步方法相比,本文在混沌 Lur'e 系统同步过程中加入了事件触发机制,能够有效地减少数据的传输,缓解通信带宽的压力,提高宽带利用率.

3) 本文在估计 Lyapunov-Krasovskii 泛函导数中出现的积分时引用了基于辅助函数的积分不等式,基于辅助函数的积分不等式包含了 Jensen 不等式和 Wirtinger 积分不等式,对交叉积分项的放大程度更接近于真实值,所以泛函导数也更接近于真实值,因此本文得到的同步判据保守性更低.

1 问题描述

考虑如下的主-从混沌 Lur'e 系统同步问题:

$$M:\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{x}(t-\tau(t))+\boldsymbol{E}\boldsymbol{f}(\boldsymbol{D}\boldsymbol{x}(t)),\\ \boldsymbol{p}(t)=\boldsymbol{C}\boldsymbol{x}(t),\end{cases}\tag{1}$$

$$S:\begin{cases}\dot{\boldsymbol{y}}(t)=\boldsymbol{A}\boldsymbol{y}(t)+\boldsymbol{B}\boldsymbol{y}(t-\tau(t))+\boldsymbol{E}\boldsymbol{f}(\boldsymbol{D}\boldsymbol{y}(t))+\boldsymbol{U}(t),\\ \boldsymbol{q}(t)=\boldsymbol{C}\boldsymbol{y}(t),\end{cases}\tag{2}$$

其中,M 是主系统,S 是从系统,$\boldsymbol{x}(t)\in R^n$ 和 $\boldsymbol{y}(t)\in R^n$ 是状态向量,$\boldsymbol{p}(t)\in R^l$ 和 $\boldsymbol{q}(t)\in R^l$ 是输出向量.$\boldsymbol{A}\in R^{n\times n}$,$\boldsymbol{B}\in R^{n\times n}$,$\boldsymbol{C}\in R^{l\times n}$,$\boldsymbol{D}\in R^{m\times n}$,$\boldsymbol{E}\in R^{n\times m}$,为已知常数矩阵,时变时滞 $\tau(t)$ 满足

$$0\leqslant\tau_1\leqslant\tau(t)\leqslant\tau_2,\ u_1\leqslant\dot{\tau}(t)\leqslant u_2,\tag{3}$$

非线性函数 $\boldsymbol{f}(\cdot)\triangleq[f_1(\cdot),f_2(\cdot),\cdots,f_m(\cdot)]\in R^m$,满足 $\boldsymbol{f}(0)=\boldsymbol{0}$,假设 $f_i(\cdot)$ 属于扇形 $[k_i^-,k_i^+]$,并满足条件:

$$[f_i(s)-k_i^+s][f_i(s)-k_i^-]\leqslant0,\qquad\forall s\in\mathbf{R},\ i=1,2,\cdots,m.\tag{4}$$

本文通过采样数据反馈研究了主从同步问题,系统的输出向量 $\boldsymbol{p}(t)$,$\boldsymbol{q}(t)$ 的离散测量用于实现控制目的,在采样时刻 t_k 的离散测量为 $\boldsymbol{p}(t_k)$,$\boldsymbol{q}(t_k)$($k\in\mathbf{N}$),在传统的控制中,假设控制信号是由零阶保持器产生的,其保持时间序列为 $[t_k,t_{k+1})$.在本文中,t_k 是从传感器到采样器的传输时刻,采样控制输入可以采用如下形式表示:

$$\boldsymbol{U}(t)=\boldsymbol{K}[\boldsymbol{p}(t_k)-\boldsymbol{q}(t_k)],\qquad t\in[t_k,t_{k+1}).\tag{5}$$

误差系统定义为

$$\dot{\boldsymbol{e}}(t)=\dot{\boldsymbol{x}}(t)-\dot{\boldsymbol{y}}(t)=\boldsymbol{A}\boldsymbol{e}(t)+\boldsymbol{B}\boldsymbol{e}(t-\tau(t))+\boldsymbol{E}\boldsymbol{\eta}(\boldsymbol{D}\boldsymbol{e}(t))-\boldsymbol{U}(t). \tag{6}$$

下面分两种情况分析网络传输的时滞.

情况 1　如果 $t_k+h_1+\bar{\tau}\geqslant t_{k+1}+\tau_{k+1}$, 定义

$$h(t)=t-t_k,\qquad t\in[t_k+\tau_k,t_{k+1}+\tau_{k+1}), \tag{7}$$

显然　$\tau_k\leqslant h(t)\leqslant t_{k+1}-t_k+\tau_{k+1}\leqslant h_1+\bar{\tau}$.

情况 2　如果 $t_k+h_1+\bar{\tau}<t_{k+1}+\tau_{k+1}$, 考虑把 $[t_k+\tau_k,t_{k+1}+\tau_{k+1})$ 进行如下分段:

$$\begin{cases}I_1=[t_k+\tau_k,t_k+h_1+\bar{\tau}),\\ I_j=\left[t_k+\sum\limits_{i=1}^{j-1}h_i+\bar{\tau},t_k+\sum\limits_{i=1}^{j}h_i+\bar{\tau}\right), & j=2,3,\cdots,M-1,\\ I_{M-1}=\left[t_k+\sum\limits_{i=1}^{M-1}h_i+\bar{\tau},t_{k+1}+\tau_{k+1}\right).\end{cases} \tag{8}$$

分区网络延迟函数 $h(t)$ 定义如下:

$$h(t)=\begin{cases}t-t_k, & t\in I_1,\\ t-t_k-\sum\limits_{i=1}^{j-1}h_i, & t\in I_j,\ j=2,3,\cdots,M-2,\\ t-t_k-\sum\limits_{i=1}^{M-1}h_i, & t\in I_{M-1}.\end{cases} \tag{9}$$

由此可得, $0\leqslant\tau_k\leqslant h(t)\leqslant\bar{h}+\bar{\tau}\triangleq h,t\in[t_k+\tau_k,t_{k+1}+\tau_{k+1})$, 其中, $\bar{h}=\max\{h_1,h_2,\cdots,h_M\}$.

当前采样数据 $\boldsymbol{e}(t_s)$ 和上一次传输数据 $\boldsymbol{e}(t_k)$ 之差 $\boldsymbol{\varepsilon}(t_s)$ 定义如下:

$$\boldsymbol{\varepsilon}(t_s)=\begin{cases}\boldsymbol{0}, & t\in I_1,\\ \boldsymbol{e}\left(t_k+\sum\limits_{i=1}^{j-1}h_i\right)-\boldsymbol{e}(t_k), & t\in I_j,\ j=2,3,\cdots,M-2,\\ \boldsymbol{e}\left(t_k+\sum\limits_{i=1}^{M-1}h_i\right)-\boldsymbol{e}(t_k), & t\in I_{M-1}.\end{cases} \tag{10}$$

当前采样数据是否传输给控制器取决于事件触发条件:当前采样数据满足触发条件时,将会被传输到控制器;当前采样数据不满足触发条件时,将保持传感器的数据不变.本文所设计的事件触发条件如下:

$$\boldsymbol{\varepsilon}^{\mathrm{T}}(t_s)\boldsymbol{\Theta}_1\boldsymbol{\varepsilon}(t_s)-\sigma_1(t)\boldsymbol{e}^{\mathrm{T}}(t_s)\boldsymbol{\Theta}_2\boldsymbol{e}(t_s)-\sigma_2(t)\boldsymbol{e}^{\mathrm{T}}(t_k)\boldsymbol{\Theta}_3\boldsymbol{e}(t_k)\leqslant 0, \tag{11}$$

其中, $t_s=t_k+\sum_{i=1}^{n}h_i$ 是当前采样时刻, t_k 是最近传输时刻, $\boldsymbol{\varepsilon}(t_s)=\boldsymbol{e}(t_s)-\boldsymbol{e}(t_k)$, 矩阵 $\boldsymbol{\Theta}_i(i=1,2,3)$ 是正定对称矩阵,函数 $\sigma_1(t),\sigma_2(t)$ 满足

$$\dot{\sigma}_1(t)=\frac{1}{\sigma_1(t)}\left[\frac{1}{\sigma_1(t)}-\mu_1\right][\boldsymbol{\varepsilon}^{\mathrm{T}}(t_s)\boldsymbol{\Theta}_1\boldsymbol{\varepsilon}(t_s)-\kappa_1\boldsymbol{e}^{\mathrm{T}}(t_k)\boldsymbol{\Theta}_3\boldsymbol{e}(t_k)], \tag{12}$$

$$\dot{\sigma}_2(t)=\frac{1}{\sigma_2(t)}\left[\frac{1}{\sigma_2(t)}-\mu_2\right][\boldsymbol{\varepsilon}^{\mathrm{T}}(t_s)\boldsymbol{\Theta}_1\boldsymbol{\varepsilon}(t_s)-\kappa_2\boldsymbol{e}^{\mathrm{T}}(t_s)\boldsymbol{\Theta}_2\boldsymbol{e}(t_s)], \tag{13}$$

式中, $\mu_1,\mu_2,\kappa_1,\kappa_2$ 是给定的正常数,使得 $1/\mu_1<\sigma_1(t)<\kappa_2,1/\mu_2<\sigma_2(t)<\kappa_1$.

为了获得混沌 Lur'e 系统主从同步的判据,需要引入如下引理.

引理 1[22]　对于正定矩阵 $\boldsymbol{R}>\boldsymbol{0}$, 和可微函数 $\{\boldsymbol{\omega}(s)\mid s\in[a,b]\}$, 有如下不等式成立:

$$\int_a^b\int_\theta^b\dot{\boldsymbol{\omega}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{\omega}}(s)\mathrm{d}s\mathrm{d}\theta\geqslant 2\boldsymbol{\Omega}_3^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\Omega}_3+4\boldsymbol{\Omega}_4^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\Omega}_4,$$

$$\int_a^b\int_a^\theta\dot{\boldsymbol{\omega}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{\omega}}(s)\mathrm{d}s\mathrm{d}\theta\geqslant 2\boldsymbol{\Omega}_5^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\Omega}_5+4\boldsymbol{\Omega}_6^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\Omega}_6,$$

其中

$$\boldsymbol{\Omega}_3=\boldsymbol{\omega}(b)-\frac{1}{b-a}\int_a^b\boldsymbol{\omega}(s)\mathrm{d}s,\ \boldsymbol{\Omega}_4=\boldsymbol{\omega}(b)+\frac{2}{b-a}\int_a^b\boldsymbol{\omega}(s)\mathrm{d}s-\frac{6}{(b-a)^2}\int_a^b\int_\theta^b\boldsymbol{\omega}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$\boldsymbol{\Omega}_5=\boldsymbol{\omega}(b)-\frac{1}{b-a}\int_a^b\boldsymbol{\omega}(s)\mathrm{d}s,\ \boldsymbol{\Omega}_6=\boldsymbol{\omega}(b)-\frac{4}{b-a}\int_a^b\boldsymbol{\omega}(s)\mathrm{d}s+\frac{6}{(b-a)^2}\int_a^b\int_\theta^b\boldsymbol{\omega}(s)\mathrm{d}s\mathrm{d}\theta.$$

引理 2[23] 给定一个正定矩阵 $\boldsymbol{R}>0$，对于连续函数 $\{\boldsymbol{\omega}(s)\mid s\in[a,b]\}$，有如下不等式成立：

$$\int_a^b\boldsymbol{\omega}^{\mathrm{T}}(u)\boldsymbol{R}\boldsymbol{\omega}(u)\mathrm{d}u\geqslant\frac{1}{b-a}\boldsymbol{\Omega}_7^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\Omega}_7+\frac{3}{b-a}\boldsymbol{\Omega}_8^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\Omega}_8,$$

其中

$$\boldsymbol{\Omega}_7=\int_a^b\boldsymbol{\omega}(u)\mathrm{d}u,\ \boldsymbol{\Omega}_8=\int_a^b\boldsymbol{\omega}(u)\mathrm{d}u-\frac{2}{b-a}\int_a^b\int_a^s\boldsymbol{\omega}(r)\mathrm{d}r\mathrm{d}s.$$

引理 3[24] 对于向量 $\boldsymbol{\omega}$，标量 $a\leqslant b$，对称矩阵 $\boldsymbol{R}>\boldsymbol{0}$，有如下不等式成立：

$$\int_a^b\dot{\boldsymbol{\omega}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{\omega}}(s)\mathrm{d}s\geqslant\frac{1}{b-a}\boldsymbol{\chi}_1^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\chi}_1+\frac{3}{b-a}\boldsymbol{\chi}_2^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\chi}_2+\frac{5}{b-a}\boldsymbol{\chi}_3^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\chi}_3,$$

其中

$$\boldsymbol{\chi}_1=\boldsymbol{\omega}(b)-\boldsymbol{\omega}(a),\ \boldsymbol{\chi}_2=\boldsymbol{\omega}(b)+\boldsymbol{\omega}(a)-\frac{2}{b-a}\int_a^b\boldsymbol{\omega}(s)\mathrm{d}s,$$

$$\boldsymbol{\chi}_3=\boldsymbol{\chi}_1+\frac{6}{b-a}\int_a^b\boldsymbol{\omega}(s)\mathrm{d}s-\frac{12}{(b-a)^2}\int_a^b\int_\theta^b\boldsymbol{\omega}(s)\mathrm{d}s\mathrm{d}\theta.$$

引理 4[25] 对于一个实标量 $\alpha\in(0,1)$，对称矩阵 $\boldsymbol{X}_1>\boldsymbol{0}$ 和 $\boldsymbol{X}_2>\boldsymbol{0}$，任意矩阵 $\boldsymbol{S}_1$ 和 $\boldsymbol{S}_2$，使得以下矩阵不等式成立：

$$\begin{pmatrix}\frac{1}{\alpha}\boldsymbol{\chi}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \frac{1}{1-\alpha}\boldsymbol{\chi}_2\end{pmatrix}\geqslant\begin{pmatrix}\boldsymbol{\chi}_1+(1-\alpha)\boldsymbol{P}_1 & (1-\alpha)\boldsymbol{S}_1+\alpha\boldsymbol{S}_2\\ * & \boldsymbol{\chi}_2+\alpha\boldsymbol{P}_2\end{pmatrix},$$

其中 $\boldsymbol{P}_1=\boldsymbol{\chi}_1-\boldsymbol{S}_2\boldsymbol{X}_2^{-1}\boldsymbol{S}_2^{\mathrm{T}}$，$\boldsymbol{P}_2=\boldsymbol{\chi}_2-\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{X}_1^{-1}\boldsymbol{S}_1$.

为了简化文章的证明过程，首先给出相应的记号：

$$\begin{aligned}\boldsymbol{\xi}(t)=\mathrm{col}\Big\{&\boldsymbol{e}(t),\boldsymbol{e}(t-\tau_1),\boldsymbol{e}(t-\tau(t)),\boldsymbol{e}(t-\tau_2),\boldsymbol{e}(t-d),\boldsymbol{e}(t-h(t)),\boldsymbol{e}(t-h),\boldsymbol{\varepsilon}(t_s),\\&\dot{\boldsymbol{e}}(t),\boldsymbol{g}(\boldsymbol{D}\boldsymbol{e}(t)),\frac{1}{\tau_1}\int_{t-\tau_1}^t\boldsymbol{e}(s)\mathrm{d}s,\frac{1}{\tau(t)-\tau_1}\int_{t-\tau(t)}^{t-\tau_1}\boldsymbol{e}(s)\mathrm{d}s,\frac{1}{\tau_2-\tau(t)}\int_{t-\tau_2}^{t-\tau(t)}\boldsymbol{e}(s)\mathrm{d}s,\\&\frac{1}{d}\int_{t-d}^t\boldsymbol{e}(s)\mathrm{d}s,\frac{1}{h(t)-d}\int_{t-h(t)}^{t-d}\boldsymbol{e}(s)\mathrm{d}s,\frac{1}{h-h(t)}\int_{t-h}^{t-h(t)}\boldsymbol{e}(s)\mathrm{d}s,\frac{2}{\tau_1^2}\int_{t-\tau_1}^t\int_s^t\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,\\&\frac{2}{[\tau(t)-\tau_1]^2}\int_{t-\tau(t)}^{t-\tau_1}\int_s^{t-\tau_1}\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,\frac{2}{[\tau_2-\tau(t)]^2}\int_{t-\tau_2}^{t-\tau(t)}\int_s^{t-\tau(t)}\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,\frac{2}{d^2}\int_{t-d}^t\int_s^t\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,\\&\frac{2}{[h(t)-d]^2}\int_{t-h(t)}^{t-d}\int_s^{t-d}\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,\frac{2}{[h-h(t)]^2}\int_{t-h}^{t-h(t)}\int_s^{t-h(t)}\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,\dot{\boldsymbol{e}}(t-\tau_1),\\&\dot{\boldsymbol{e}}(t-\tau(t)),\dot{\boldsymbol{e}}(t-\tau_2),\dot{\boldsymbol{e}}(t-d),\dot{\boldsymbol{e}}(t-h)\Big\},\end{aligned}$$

$$\boldsymbol{\eta}(t)=\mathrm{col}\Big\{\boldsymbol{e}(t),\boldsymbol{e}(t-\tau_1),\boldsymbol{e}(t-\tau_2),\int_{t-\tau_2}^t\boldsymbol{e}(s)\mathrm{d}s,\boldsymbol{e}(t-d),\boldsymbol{e}(t-h),\int_{t-h}^t\boldsymbol{e}(s)\mathrm{d}s\Big\},$$

$$\dot{\boldsymbol{\eta}}(t)=\mathrm{col}\{\dot{\boldsymbol{e}}(t),\dot{\boldsymbol{e}}(t-\tau_1),\dot{\boldsymbol{e}}(t-\tau_2),\dot{\boldsymbol{e}}(t)-\dot{\boldsymbol{e}}(t-\tau_2),\dot{\boldsymbol{e}}(t-d),\dot{\boldsymbol{e}}(t-h),\dot{\boldsymbol{e}}(t)-\dot{\boldsymbol{e}}(t-h)\},$$

$$\boldsymbol{\chi}(t)=\frac{1}{\tau_1}\int_{t-\tau_1}^t\boldsymbol{e}(s)\mathrm{d}s,\ \boldsymbol{\delta}(t)=\frac{1}{\tau(t)-\tau_1}\int_{t-\tau(t)}^{t-\tau_1}\boldsymbol{e}(s)\mathrm{d}s,\ \boldsymbol{\varepsilon}(t)=\frac{1}{\tau_2-\tau(t)}\int_{t-\tau_2}^{t-\tau(t)}\boldsymbol{x}(s)\mathrm{d}s,$$

$$\boldsymbol{\phi}(t)=\frac{1}{d}\int_{t-d}^t\boldsymbol{e}(s)\mathrm{d}s,\ \boldsymbol{\varphi}(t)=\frac{1}{h(t)-d}\int_{t-h(t)}^{t-d}\boldsymbol{e}(s)\mathrm{d}s,\ \boldsymbol{\gamma}(t)=\frac{1}{h-h(t)}\int_{t-h}^{t-h(t)}\boldsymbol{e}(s)\mathrm{d}s,$$

$$\boldsymbol{\kappa}(t)=\frac{2}{\tau_1^2}\int_{t-\tau_1}^t\int_s^t\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,\ \boldsymbol{\lambda}(t)=\frac{2}{[\tau(t)-\tau_1]^2}\int_{t-\tau(t)}^{t-\tau_1}\int_s^{t-\tau_1}\boldsymbol{x}(\theta)\mathrm{d}\theta\mathrm{d}s,$$

$$\boldsymbol{\nu}(t)=\frac{2}{[\tau_2-\tau(t)]^2}\int_{t-\tau_2}^{t-\tau(t)}\int_{s}^{t-\tau(t)}\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,\boldsymbol{\varpi}(t)=\frac{2}{d^2}\int_{t-d}^{t}\int_{s}^{t}\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,$$

$$\boldsymbol{\theta}(t)=\frac{2}{[h(t)-d]^2}\int_{t-h(t)}^{t-d}\int_{s}^{t-d}\boldsymbol{x}(\theta)\mathrm{d}\theta\mathrm{d}s,\ \boldsymbol{\vartheta}(t)=\frac{2}{[h-h(t)]^2}\int_{t-h}^{t-h(t)}\int_{s}^{t-h(t)}\boldsymbol{e}(\theta)\mathrm{d}\theta\mathrm{d}s,$$

$$\boldsymbol{e}_i=(\boldsymbol{0}_{n\times(i-1)n}\quad \boldsymbol{I}_n\quad \boldsymbol{0}_{n\times[(26-i)n+m]}),\qquad i=1,2,\cdots,9,$$

$$\boldsymbol{e}_{10}=(\boldsymbol{0}_{m\times 9n}\quad \boldsymbol{I}_m\quad \boldsymbol{0}_{m\times 17n}),\ \boldsymbol{e}_i=(\boldsymbol{0}_{n\times[(i-2)n+m]}\quad \boldsymbol{I}_n\quad \boldsymbol{0}_{n\times(27-i)n}),\qquad i=11,12,\cdots,27,$$

$$\boldsymbol{L}=\mathrm{col}\{\boldsymbol{e}_9\quad \boldsymbol{e}_{23}\quad \boldsymbol{e}_{25}\quad \boldsymbol{e}_1-\boldsymbol{e}_4\quad \boldsymbol{e}_{26}\quad \boldsymbol{e}_{27}\quad \boldsymbol{e}_1-\boldsymbol{e}_7\},\ \boldsymbol{L}_1=(\boldsymbol{e}_1^{\mathrm{T}}\quad \boldsymbol{e}_2^{\mathrm{T}}\quad \boldsymbol{e}_4^{\mathrm{T}}\quad \tau_1\boldsymbol{e}_{11}^{\mathrm{T}}\quad \boldsymbol{e}_5^{\mathrm{T}}\quad \boldsymbol{e}_7^{\mathrm{T}}\quad d\boldsymbol{e}_{14}^{\mathrm{T}}),$$

$$\boldsymbol{L}_2=(\boldsymbol{0}_{26n+m,6n}\quad \boldsymbol{e}_{12}^{\mathrm{T}}\quad \boldsymbol{0}_{26n+m,3n}),\ \boldsymbol{L}_3=(\boldsymbol{0}_{26n+m,3n}\quad \boldsymbol{e}_{13}^{\mathrm{T}}\quad \boldsymbol{0}_{26n+m,3n}),$$

$$\boldsymbol{L}_4=(\boldsymbol{0}_{26n+m,6n}\quad \boldsymbol{e}_{15}^{\mathrm{T}}),\ \boldsymbol{L}_5=(\boldsymbol{0}_{26n+m,6n}\quad \boldsymbol{e}_{16}^{\mathrm{T}}),$$

$$\boldsymbol{H}_1=\begin{pmatrix}\boldsymbol{e}_2-\boldsymbol{e}_3\\ \boldsymbol{e}_2+\boldsymbol{e}_3-2\boldsymbol{e}_{12}\\ \boldsymbol{e}_2-\boldsymbol{e}_3+6\boldsymbol{e}_{12}-6\boldsymbol{e}_{18}\end{pmatrix},\ \boldsymbol{H}_2=\begin{pmatrix}\boldsymbol{e}_3-\boldsymbol{e}_4\\ \boldsymbol{e}_3+\boldsymbol{e}_4-2\boldsymbol{e}_{13}\\ \boldsymbol{e}_3-\boldsymbol{e}_4+6\boldsymbol{e}_{13}-6\boldsymbol{e}_{19}\end{pmatrix},$$

$$\boldsymbol{H}_3=\begin{pmatrix}\boldsymbol{e}_5-\boldsymbol{e}_6\\ \boldsymbol{e}_5+\boldsymbol{e}_6-2\boldsymbol{e}_{15}\\ \boldsymbol{e}_5-\boldsymbol{e}_6+6\boldsymbol{e}_{15}-6\boldsymbol{e}_{21}\end{pmatrix},\ \boldsymbol{H}_4=\begin{pmatrix}\boldsymbol{e}_6-\boldsymbol{e}_7\\ \boldsymbol{e}_6+\boldsymbol{e}_7-2\boldsymbol{e}_{16}\\ \boldsymbol{e}_6-\boldsymbol{e}_7+6\boldsymbol{e}_{16}-6\boldsymbol{e}_{22}\end{pmatrix}.$$

2　主要结果

本文在这一节中,给出使混沌 Lur'e 系统基于事件触发机制的采样控制器设计实现主从同步的方法,建立闭环系统(6)收敛的充分条件.

定理 1　对于给定的标量 $h>0,\tau_2\geqslant\tau_1\geqslant 0,\mu_i>0,\kappa_i>0(i=1,2),u_1,u_2$,如果存在适当维数的矩阵 $\boldsymbol{P}>\boldsymbol{0}$, $\boldsymbol{Q}_i>\boldsymbol{0}(i=1,2,\cdots,6)$, $\boldsymbol{R}_i>\boldsymbol{0}(i=1,2,\cdots,6)$, $\boldsymbol{J}_i>\boldsymbol{0}(i=1,2,\cdots,8)$, $\boldsymbol{S}_i>\boldsymbol{0}(i=1,2,3,4)$, $\boldsymbol{T}_i>\boldsymbol{0}(i=1,2,3,4)$, $\boldsymbol{\Theta}_i>\boldsymbol{0}(i=1,2,3)$, $\bar{\boldsymbol{T}}_i=\mathrm{diag}(\boldsymbol{T}_i,3\boldsymbol{T}_i,5\boldsymbol{T}_i)(i=1,2,3,4)$, $\bar{\boldsymbol{J}}_i=\mathrm{diag}(\boldsymbol{J}_i,3\boldsymbol{J}_i,5\boldsymbol{J}_i)(i=1,2,3,4)$,任意矩阵 $\boldsymbol{P}_i(i=1,2,3,4)$,使得下列线性矩阵不等式是可行的:

$$\begin{pmatrix}\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_{3,1}+\boldsymbol{\Psi}_{4,1}+\boldsymbol{\Psi}_5(\tau_1)+\boldsymbol{\Psi}_6(d) & \boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{P}_2 & \boldsymbol{H}_3^{\mathrm{T}}\boldsymbol{P}_4\\ & -(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4) & \boldsymbol{0}\\ * & & -(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8)\end{pmatrix}<\boldsymbol{0},\tag{14}$$

$$\begin{pmatrix}\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_{3,1}+\boldsymbol{\Psi}_{4,2}+\boldsymbol{\Psi}_5(\tau_1)+\boldsymbol{\Psi}_6(h) & \boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{P}_2 & \boldsymbol{H}_4^{\mathrm{T}}\boldsymbol{P}_3^{\mathrm{T}}\\ & -(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4) & \boldsymbol{0}\\ * & & -(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7)\end{pmatrix}<\boldsymbol{0},\tag{15}$$

$$\begin{pmatrix}\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_{3,2}+\boldsymbol{\Psi}_{4,1}+\boldsymbol{\Psi}_5(\tau_2)+\boldsymbol{\Psi}_6(d) & \boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{P}_1^{\mathrm{T}} & \boldsymbol{H}_3^{\mathrm{T}}\boldsymbol{P}_4\\ & -(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_3) & \boldsymbol{0}\\ * & & -(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8)\end{pmatrix}<\boldsymbol{0},\tag{16}$$

$$\begin{pmatrix}\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_{3,2}+\boldsymbol{\Psi}_{4,2}+\boldsymbol{\Psi}_5(\tau_2)+\boldsymbol{\Psi}_6(h) & \boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{P}_1^{\mathrm{T}} & \boldsymbol{H}_4^{\mathrm{T}}\boldsymbol{P}_3^{\mathrm{T}}\\ & -(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_3) & \boldsymbol{0}\\ * & & -(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7)\end{pmatrix}<\boldsymbol{0}.\tag{17}$$

$\boldsymbol{0}$ 元素除外, $\boldsymbol{\Psi}_1=[\boldsymbol{\Psi}_{ij}]_{27\times 27}$ 的剩余元素如下:

$$\boldsymbol{\Psi}_{11}=\boldsymbol{Q}_1+\boldsymbol{R}_1+\tau_1^2\boldsymbol{S}_1+(\tau_2-\tau_1)^2\boldsymbol{S}_2+d^2\boldsymbol{S}_3+(h-d)^2\boldsymbol{S}_4-9\boldsymbol{T}_1-9\boldsymbol{T}_3-6\boldsymbol{J}_1-6\boldsymbol{J}_5-$$
$$\boldsymbol{D}^{\mathrm{T}}\boldsymbol{F}\boldsymbol{\Lambda}_1\boldsymbol{D}+\mathrm{Sym}(\boldsymbol{G}^{\mathrm{T}}\boldsymbol{A})-\frac{\pi^2}{4}\boldsymbol{U},$$

$\boldsymbol{\Psi}_{12} = 3\boldsymbol{T}_1$, $\boldsymbol{\Psi}_{13} = \boldsymbol{G}^{\mathrm{T}}\boldsymbol{B}$, $\boldsymbol{\Psi}_{15} = 3\boldsymbol{T}_3$, $\boldsymbol{\Psi}_{16} = -\boldsymbol{YC} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{G} + \dfrac{\pi^2}{4}\boldsymbol{U}$,

$\boldsymbol{\Psi}_{18} = \boldsymbol{YC} - \boldsymbol{A}^{\mathrm{T}}\boldsymbol{G}$, $\boldsymbol{\Psi}_{19} = \boldsymbol{D}^{\mathrm{T}}[\boldsymbol{W}^{+}\boldsymbol{\lambda} - \boldsymbol{W}^{-}\boldsymbol{\gamma}]\boldsymbol{D} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{G} - \boldsymbol{G}^{\mathrm{T}}$,

$\boldsymbol{\Psi}_{1,10} = \boldsymbol{D}^{\mathrm{T}}\boldsymbol{\Lambda}_2\boldsymbol{F} + \boldsymbol{G}^{\mathrm{T}}\boldsymbol{E}$, $\boldsymbol{\Psi}_{1,11} = -24\boldsymbol{T}_1 - 6\boldsymbol{J}_1$, $\boldsymbol{\Psi}_{1,14} = -24\boldsymbol{T}_3 + 6\boldsymbol{J}_5$, $\boldsymbol{\Psi}_{1,17} = 30\boldsymbol{T}_1 + 12\boldsymbol{J}_1 + \boldsymbol{G}^{\mathrm{T}}$,

$\boldsymbol{\Psi}_{1,20} = 30\boldsymbol{T}_3 + 12\boldsymbol{J}_5$, $\boldsymbol{\Psi}_{22} = \boldsymbol{Q}_2 - \boldsymbol{Q}_1 - 9\boldsymbol{T}_1 - 6\boldsymbol{J}_2 - 6\boldsymbol{J}_3$, $\boldsymbol{\Psi}_{2,11} = 36\boldsymbol{T}_1 + 18\boldsymbol{J}_2$, $\boldsymbol{\Psi}_{2,12} = -6\boldsymbol{J}_3$,

$\boldsymbol{\Psi}_{2,17} = -30\boldsymbol{T}_1 - 12\boldsymbol{J}_2$, $\boldsymbol{\Psi}_{2,18} = 12\boldsymbol{J}_3$, $\boldsymbol{\Psi}_{33} = (1-u_1)\boldsymbol{Q}_3 + (u_2-1)\boldsymbol{Q}_2 - 6\boldsymbol{J}_3 - 6\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{36} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{G}$,

$\boldsymbol{\Psi}_{38} = -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{G}$, $\boldsymbol{\Psi}_{39} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{G}$, $\boldsymbol{\Psi}_{3,12} = 18\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{3,13} = -6\boldsymbol{J}_3$, $\boldsymbol{\Psi}_{3,18} = -12\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{3,19} = 12\boldsymbol{J}_3$,

$\boldsymbol{\Psi}_{44} = -\boldsymbol{Q}_3 - 6\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{4,13} = 18\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{4,19} = -12\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{55} = -9\boldsymbol{T}_3 - 6\boldsymbol{J}_6 - 6\boldsymbol{J}_7 + \boldsymbol{R}_2 - \boldsymbol{R}_1$,

$\boldsymbol{\Psi}_{5,14} = 36\boldsymbol{T}_3 + 18\boldsymbol{J}_6$, $\boldsymbol{\Psi}_{5,15} = -6\boldsymbol{J}_7$, $\boldsymbol{\Psi}_{5,20} = -30\boldsymbol{T}_3 - 12\boldsymbol{J}_6$, $\boldsymbol{\Psi}_{5,21} = 12\boldsymbol{J}_7$,

$\boldsymbol{\Psi}_{66} = (1+\kappa_1\mu_1)\boldsymbol{\Theta}_3 + (1+\kappa_2\mu_2)\boldsymbol{\Theta}_2 - 6\boldsymbol{J}_7 - 6\boldsymbol{J}_8 - \mathrm{Sym}(\boldsymbol{YC}) - \dfrac{\pi^2}{4}\boldsymbol{U}$,

$\boldsymbol{\Psi}_{68} = -(1+\kappa_1\mu_1)\boldsymbol{\Theta}_3 + \mathrm{Sym}(\boldsymbol{YC})$, $\boldsymbol{\Psi}_{69} = -\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Y}^{\mathrm{T}} - \boldsymbol{G}^{\mathrm{T}}$, $\boldsymbol{\Psi}_{6,10} = \boldsymbol{G}^{\mathrm{T}}\boldsymbol{E}$, $\boldsymbol{\Psi}_{6,15} = 18\boldsymbol{J}_8$,

$\boldsymbol{\Psi}_{6,16} = -6\boldsymbol{J}_7$, $\boldsymbol{\Psi}_{621} = -12\boldsymbol{J}_8$, $\boldsymbol{\Psi}_{622} = 12\boldsymbol{J}_7$, $\boldsymbol{\Psi}_{77} = -\boldsymbol{R}_3 - 6\boldsymbol{J}_8$, $\boldsymbol{\Psi}_{7,16} = 18\boldsymbol{J}_8$, $\boldsymbol{\Psi}_{7,22} = -12\boldsymbol{J}_8$,

$\boldsymbol{\Psi}_{88} = -(\mu_1+\mu_2)\boldsymbol{\Theta}_1 + (1+\kappa_1\mu_1)\boldsymbol{\Theta}_3 - \mathrm{Sym}(\boldsymbol{YC})$, $\boldsymbol{\Psi}_{89} = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{Y}^{\mathrm{T}} + \boldsymbol{G}^{\mathrm{T}}$, $\boldsymbol{\Psi}_{8,10} = -\boldsymbol{G}^{\mathrm{T}}\boldsymbol{E}$,

$$\boldsymbol{\Psi}_{99} = \tau_1^2\boldsymbol{T}_1 + (\tau_2-\tau_1)^2\boldsymbol{T}_2 + d^2\boldsymbol{T}_3 + (h-d)^2\boldsymbol{T}_4 + \frac{\tau_1^2}{2}(\boldsymbol{J}_1+\boldsymbol{J}_2) + \frac{(\tau_2-\tau_1)^2}{2}(\boldsymbol{J}_3+\boldsymbol{J}_4) + \frac{d_2^2}{2}(\boldsymbol{J}_5+\boldsymbol{J}_6) + \frac{(h-d)^2}{2}(\boldsymbol{J}_7+\boldsymbol{J}_8) - \mathrm{Sym}(\boldsymbol{G}^{\mathrm{T}}) + \boldsymbol{Q}_4 + \boldsymbol{R}_4 + h^2\boldsymbol{U},$$

$\boldsymbol{\Psi}_{9,10} = \boldsymbol{D}^{\mathrm{T}}(\boldsymbol{Y}-\boldsymbol{\lambda}) + \boldsymbol{G}^{\mathrm{T}}\boldsymbol{E}$, $\boldsymbol{\Psi}_{10,10} = -\boldsymbol{F}$, $\boldsymbol{\Psi}_{11,11} = -4\tau_1^2\boldsymbol{S}_1 - 192\boldsymbol{T}_1 - 66\boldsymbol{J}_2 - 18\boldsymbol{J}_1$,

$\boldsymbol{\Psi}_{11,17} = 3\tau_1^2\boldsymbol{S}_1 + 180\boldsymbol{T}_1 + 48\boldsymbol{J}_2 + 24\boldsymbol{J}_1$, $\boldsymbol{\Psi}_{12,12} = -18\boldsymbol{J}_3 - 66\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{12,18} = 24\boldsymbol{J}_3 + 48\boldsymbol{J}_4$,

$\boldsymbol{\Psi}_{13,13} = -18\boldsymbol{J}_3 - 66\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{13,19} = 24\boldsymbol{J}_3 + 48\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{14,14} = -4d^2\boldsymbol{S}_3 - 192\boldsymbol{T}_3 - 18\boldsymbol{J}_5 - 66\boldsymbol{J}_6$,

$\boldsymbol{\Psi}_{14,20} = 3d^2\boldsymbol{S}_3 + 180\boldsymbol{T}_3 + 24\boldsymbol{J}_5 + 48\boldsymbol{J}_6$, $\boldsymbol{\Psi}_{15,15} = -18\boldsymbol{J}_7 - 66\boldsymbol{J}_8$, $\boldsymbol{\Psi}_{15,21} = 24\boldsymbol{J}_7 + 48\boldsymbol{J}_8$,

$\boldsymbol{\Psi}_{16,16} = -18\boldsymbol{J}_7 - 66\boldsymbol{J}_8$, $\boldsymbol{\Psi}_{16,22} = 24\boldsymbol{J}_7 + 48\boldsymbol{J}_8$, $\boldsymbol{\Psi}_{17,17} = -3\tau_1\boldsymbol{S}_1 - 180\boldsymbol{T}_1 - 36\boldsymbol{J}_2 - 36\boldsymbol{J}_1$,

$\boldsymbol{\Psi}_{18,18} = -36\boldsymbol{J}_3 - 36\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{19,19} = -36\boldsymbol{J}_3 - 36\boldsymbol{J}_4$, $\boldsymbol{\Psi}_{20,20} = -3d^2\boldsymbol{S}_3 - 180\boldsymbol{T}_3 - 36\boldsymbol{J}_5 - 36\boldsymbol{J}_6$,

$\boldsymbol{\Psi}_{21,21} = -36\boldsymbol{J}_7 - 36\boldsymbol{J}_8$, $\boldsymbol{\Psi}_{22,22} = -36\boldsymbol{J}_7 - 36\boldsymbol{J}_8$, $\boldsymbol{\Psi}_{23,23} = \boldsymbol{Q}_5 - \boldsymbol{Q}_4$,

$\boldsymbol{\Psi}_{24,24} = (1-u_1)\boldsymbol{Q}_6 + (u_2-1)\boldsymbol{Q}_5$, $\boldsymbol{\Psi}_{25,25} = -\boldsymbol{Q}_6$, $\boldsymbol{\Psi}_{26,26} = \boldsymbol{R}_5 - \boldsymbol{R}_4$, $\boldsymbol{\Psi}_{27,27} = -\boldsymbol{R}_6$,

$\boldsymbol{\Psi}_2 = \mathrm{Sym}(\boldsymbol{L}_1\boldsymbol{PL})$,

$$\boldsymbol{\Psi}_3(\tau(t)) = -\boldsymbol{H}_1^{\mathrm{T}}(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_3)\boldsymbol{H}_1 - \boldsymbol{H}_2^{\mathrm{T}}(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4)\boldsymbol{H}_2 + \boldsymbol{H}_1^{\mathrm{T}}\bar{\boldsymbol{J}}_3\boldsymbol{H}_1 + \boldsymbol{H}_2^{\mathrm{T}}\bar{\boldsymbol{J}}_4\boldsymbol{H}_2 - \frac{\tau_2-\tau(t)}{\tau_2-\tau_1}\begin{pmatrix}\boldsymbol{H}_1\\ \boldsymbol{H}_2\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_3-\boldsymbol{P}_2(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4)^{-1}\boldsymbol{P}_2^{\mathrm{T}} & \boldsymbol{P}_1\\ * & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{H}_1\\ \boldsymbol{H}_2\end{pmatrix} - \frac{\tau(t)-\tau_1}{\tau_2-\tau_1}\begin{pmatrix}\boldsymbol{H}_1\\ \boldsymbol{H}_2\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{0} & \boldsymbol{P}_2\\ * & \bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4-\boldsymbol{P}_1^{\mathrm{T}}(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_3)^{-1}\boldsymbol{P}_1\end{pmatrix}\begin{pmatrix}\boldsymbol{H}_1\\ \boldsymbol{H}_2\end{pmatrix},$$

$$\boldsymbol{\Psi}_4(h(t)) = -\boldsymbol{H}_3^{\mathrm{T}}(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7)\boldsymbol{H}_3 - \boldsymbol{H}_4^{\mathrm{T}}(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8)\boldsymbol{H}_4 + \boldsymbol{H}_3^{\mathrm{T}}\bar{\boldsymbol{J}}_7\boldsymbol{H}_3 + \boldsymbol{H}_4^{\mathrm{T}}\bar{\boldsymbol{J}}_8\boldsymbol{H}_4 - \frac{h-h(t)}{h-d}\begin{pmatrix}\boldsymbol{H}_3\\ \boldsymbol{H}_4\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7-\boldsymbol{P}_4(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8)^{-1}\boldsymbol{P}_4^{\mathrm{T}} & \boldsymbol{P}_3\\ * & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{H}_3\\ \boldsymbol{H}_4\end{pmatrix} - \frac{h(t)-d}{h-d}\begin{pmatrix}\boldsymbol{H}_3\\ \boldsymbol{H}_4\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{0} & \boldsymbol{P}_4\\ * & \bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8-\boldsymbol{P}_3^{\mathrm{T}}(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7)^{-1}\boldsymbol{P}_3\end{pmatrix}\begin{pmatrix}\boldsymbol{H}_3\\ \boldsymbol{H}_4\end{pmatrix},$$

$$\boldsymbol{\Psi}_5(\tau(t)) = -(\tau_2-\tau_1)[\tau(t)-\tau_1][\boldsymbol{e}_{12}^{\mathrm{T}}\boldsymbol{S}_2\boldsymbol{e}_{12} - 3(-\boldsymbol{e}_{12}+\boldsymbol{e}_{18})^{\mathrm{T}}\boldsymbol{S}_2(-\boldsymbol{e}_{12}+\boldsymbol{e}_{18}) + \mathrm{Sym}(\boldsymbol{L}_2\boldsymbol{PL})] - (\tau_2-\tau_1)[\tau_2-\tau(t)][\boldsymbol{e}_{13}^{\mathrm{T}}\boldsymbol{S}_2\boldsymbol{e}_{13} - 3(-\boldsymbol{e}_{13}+\boldsymbol{e}_{19})^{\mathrm{T}}\boldsymbol{S}_2(-\boldsymbol{e}_{13}+\boldsymbol{e}_{19}) + \mathrm{Sym}(\boldsymbol{L}_3\boldsymbol{PL})],$$

$$\boldsymbol{\Psi}_6(h(t)) = -(h-d)[h(t)-d][\boldsymbol{e}_{15}^{\mathrm{T}}\boldsymbol{S}_4\boldsymbol{e}_{15} - 3(-\boldsymbol{e}_{15}+\boldsymbol{e}_{21})^{\mathrm{T}}\boldsymbol{S}_4(-\boldsymbol{e}_{15}+\boldsymbol{e}_{21}) + \mathrm{Sym}(\boldsymbol{L}_4\boldsymbol{PL})] -$$

$$(h-d)[h-h(t)][\boldsymbol{e}_{16}^{\mathrm{T}}\boldsymbol{S}_4\boldsymbol{e}_{16}-3(-\boldsymbol{e}_{16}+\boldsymbol{e}_{22})^{\mathrm{T}}\boldsymbol{S}_4(-\boldsymbol{e}_{16}+\boldsymbol{e}_{22})+\mathrm{Sym}(\boldsymbol{L}_5\boldsymbol{P}\boldsymbol{L})],$$

$$\boldsymbol{\Psi}_{3,1}=\begin{pmatrix}2\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_3 & \boldsymbol{P}_1\\ * & \bar{\boldsymbol{T}}_2\end{pmatrix},\ \boldsymbol{\Psi}_{3,2}=\begin{pmatrix}\bar{\boldsymbol{T}}_2 & \boldsymbol{P}_2\\ * & 2\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4\end{pmatrix},\ \boldsymbol{\Psi}_{4,1}=\begin{pmatrix}2\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7 & \boldsymbol{P}_3\\ * & \bar{\boldsymbol{T}}_4\end{pmatrix},\ \boldsymbol{\Psi}_{4,2}=\begin{pmatrix}\bar{\boldsymbol{T}}_4 & \boldsymbol{P}_4\\ * & 2\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8\end{pmatrix}.$$

则误差系统(6)是全局渐近稳定的,且时滞反馈控制增益矩阵为

$$\boldsymbol{K}=\boldsymbol{G}^{-1}\boldsymbol{Y}.$$

证明　构造的 Lyapunov-Krasovskii 泛函如下:

$$V(t)=\sum_{i=1}^{7}V_i(t),\tag{18}$$

$$V_1(t)=\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{\eta}(t),$$

$$\begin{aligned}V_2(t)=&\int_{t-\tau_1}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{e}(s)\mathrm{d}s+\int_{t-\tau(t)}^{t-\tau_1}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{Q}_2\boldsymbol{e}(s)\mathrm{d}s+\int_{t-\tau_2}^{t-\tau(t)}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{Q}_3\boldsymbol{e}(s)\mathrm{d}s+\\&\int_{t-\tau_1}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Q}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s+\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Q}_5\dot{\boldsymbol{e}}(s)\mathrm{d}s\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Q}_6\dot{\boldsymbol{e}}(s)\mathrm{d}s+\\&\int_{t-d}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{R}_1\boldsymbol{e}(s)\mathrm{d}s+\int_{t-h(t)}^{t-d}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{e}(s)\mathrm{d}s+\int_{t-h}^{t-h(t)}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{R}_3\boldsymbol{e}(s)\mathrm{d}s+\\&\int_{t-d}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{R}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s+\int_{t-h(t)}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{R}_5\dot{\boldsymbol{e}}(s)\mathrm{d}s+\int_{t-h}^{t-h(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{R}_6\dot{\boldsymbol{e}}(s)\mathrm{d}s,\end{aligned}$$

$$\begin{aligned}V_3(t)=&\tau_1\int_{t-\tau_1}^{t}\int_{\theta}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_1\boldsymbol{e}(s)\mathrm{d}s\mathrm{d}\theta+\tau_1\int_{t-\tau_1}^{t}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta+\\&(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\int_{\theta}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_2\boldsymbol{e}(s)\mathrm{d}s\mathrm{d}\theta+(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta+\\&d\int_{t-d}^{t}\int_{\theta}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_3\boldsymbol{e}(s)\mathrm{d}s\mathrm{d}\theta+d\int_{t-d}^{t}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta+\\&(h-d)\int_{t-h}^{t-d}\int_{\theta}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_4\boldsymbol{e}(s)\mathrm{d}s\mathrm{d}\theta+(h-d)\int_{t-h}^{t-d}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta,\end{aligned}$$

$$\begin{aligned}V_4(t)=&\int_{t-\tau_1}^{t}\int_{\rho}^{t}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\rho+\int_{t-\tau_1}^{t}\int_{t-\tau_1}^{\rho}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\rho+\\&\int_{t-\tau_2}^{t-\tau_1}\int_{\rho}^{t-\tau_1}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\rho+\int_{t-\tau_2}^{t-\tau_1}\int_{t-\tau_2}^{\rho}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\rho+\\&\int_{t-d}^{t}\int_{\rho}^{t}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_5\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\rho+\int_{t-d}^{t}\int_{t-d}^{\rho}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_6\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\rho\mathrm{d}\theta+\\&\int_{t-h}^{t-d}\int_{\rho}^{t-d}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_7\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\rho+\int_{t-h}^{t-d}\int_{t-h}^{\rho}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_8\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}\rho,\end{aligned}$$

$$V_5(t)=h^2\int_{t_s}^{t}\dot{\boldsymbol{e}}(s)\boldsymbol{U}\dot{\boldsymbol{e}}(s)\mathrm{d}s-\frac{\pi^2}{4}\int_{t_s}^{t}[\boldsymbol{e}(s)-\boldsymbol{e}(t_k)]^{\mathrm{T}}\boldsymbol{U}[\boldsymbol{e}(s)-\boldsymbol{e}(t_k)]\mathrm{d}s,$$

$$V_6(t)=2\sum_{i=1}^{m}\left\{\int_0^{\boldsymbol{d}_i^{\mathrm{T}}\boldsymbol{e}}[\boldsymbol{\lambda}_i(\omega_i^{+}s-g_i(s))]\mathrm{d}s+\int_0^{\boldsymbol{d}_i^{\mathrm{T}}\boldsymbol{e}}[\boldsymbol{\gamma}_i(g_i(s)-\omega_i^{-}s)]\mathrm{d}s\right\},$$

$$V_7(t)=\frac{1}{2}\sigma_1^2(t)+\frac{1}{2}\sigma_2^2(t).$$

沿着误差系统式(6)的轨线,求 $V(t)$ 关于时间的导数:

$$\dot{V}_1(t)=2\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{\eta}}(t),\tag{19}$$

$$\begin{aligned}\dot{V}_2(t)=&\boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{Q}_1+\boldsymbol{R}_1)\boldsymbol{e}(t)+\boldsymbol{e}^{\mathrm{T}}(t-\tau_1)(\boldsymbol{Q}_2-\boldsymbol{Q}_1)\boldsymbol{e}(t-\tau_1)-\boldsymbol{e}^{\mathrm{T}}(t-\tau_2)\boldsymbol{Q}_3\boldsymbol{e}(t-\tau_2)+\\&(1-\dot{\tau}(t))\dot{\boldsymbol{e}}^{\mathrm{T}}(t-\tau(t))(\boldsymbol{Q}_5-\boldsymbol{Q}_6)\dot{\boldsymbol{e}}(t-\tau(t))+\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{Q}_4\dot{\boldsymbol{x}}(t)+\\&\dot{\boldsymbol{e}}^{\mathrm{T}}(t-\tau_1)(\boldsymbol{Q}_5-\boldsymbol{Q}_6)\dot{\boldsymbol{e}}(t-\tau_1)-\dot{\boldsymbol{e}}^{\mathrm{T}}(t-\tau_2)\boldsymbol{Q}_6\dot{\boldsymbol{e}}(t-\tau_2)+\\&\boldsymbol{e}^{\mathrm{T}}(t-d)(\boldsymbol{R}_2-\boldsymbol{R}_1)\boldsymbol{e}(t-d)-\boldsymbol{e}^{\mathrm{T}}(t-h)\boldsymbol{R}_3\boldsymbol{e}(t-h)+\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{R}_4\dot{\boldsymbol{e}}(t)+\\&\dot{\boldsymbol{e}}^{\mathrm{T}}(t-d)(\boldsymbol{R}_5-\boldsymbol{R}_4)\dot{\boldsymbol{e}}(t-d)-\dot{\boldsymbol{e}}^{\mathrm{T}}(t-h)\boldsymbol{R}_6\dot{\boldsymbol{e}}(t-h),\end{aligned}\tag{20}$$

$$\begin{aligned}\dot{V}_3(t)=&\boldsymbol{e}^{\mathrm{T}}(t)[\tau_1^2\boldsymbol{S}_1+(\tau_2-\tau_1)^2\boldsymbol{S}_2+d^2\boldsymbol{S}_3+(h-d)^2\boldsymbol{S}_4]\boldsymbol{e}(t)+\\&\dot{\boldsymbol{e}}^{\mathrm{T}}(t)(\tau_1^2\boldsymbol{T}_1+(\tau_2-\tau_1)^2\boldsymbol{T}_2+d^2\boldsymbol{T}_3+(h-d)^2\boldsymbol{T}_4)\dot{\boldsymbol{e}}(t)-\\&\tau_1\int_{t-\tau_1}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_1\boldsymbol{e}(s)\mathrm{d}s-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_2\boldsymbol{e}(s)\mathrm{d}s-\tau_1\int_{t-\tau_1}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s-\\&(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s-d\int_{t-d}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_3\boldsymbol{e}(s)\mathrm{d}s-(h-d)\int_{t-h}^{t-d}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_4\boldsymbol{e}(s)\mathrm{d}s-\\&d\int_{t-d}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s-(h-d)\int_{t-h}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s,\end{aligned}\tag{21}$$

$$\begin{aligned}\dot{V}_4(t)=&\frac{\tau_1^2}{2}\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\Bigg[\frac{\tau_1^2}{2}(\boldsymbol{J}_1+\boldsymbol{J}_2)+\frac{(\tau_2-\tau_1)^2}{2}(\boldsymbol{J}_3+\boldsymbol{J}_4)+\frac{d^2}{2}(\boldsymbol{J}_5+\boldsymbol{J}_6)+\\&\frac{(h-d)^2}{2}(\boldsymbol{J}_7+\boldsymbol{J}_8)\Bigg]\dot{\boldsymbol{e}}(t)-\\&\int_{t-\tau_1}^{t}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta-\int_{t-\tau_1}^{t}\int_{t-\tau_1}^{\theta}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta-\int_{t-\tau_2}^{t-\tau_1}\int_{t-\tau_2}^{\theta}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s-\\&\int_{t-\tau_2}^{t-\tau_1}\int_{t-\tau_2}^{\theta}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta-\int_{t-d}^{t}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_5\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta-\int_{t-d}^{t}\int_{t-d}^{\theta}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_6\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta-\\&\int_{t-h}^{t-d}\int_{\theta}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_5\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta-\int_{t-h}^{t-d}\int_{t-h}^{\theta}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_8\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta,\end{aligned}\tag{22}$$

$$\dot{V}_5(t)=h^2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{U}\dot{\boldsymbol{e}}(t)-\frac{\pi^2}{4}[\boldsymbol{e}(t)-\boldsymbol{e}(t_s)]^{\mathrm{T}}\boldsymbol{U}[\boldsymbol{e}(t)-\boldsymbol{e}(t_s)],\tag{23}$$

$$\dot{V}_6(t)=2\{[\boldsymbol{W}^{+}\boldsymbol{De}(t)-\boldsymbol{g}(\boldsymbol{De}(t))]^{\mathrm{T}}\boldsymbol{\lambda}D\dot{\boldsymbol{e}}(t)+[\boldsymbol{g}(\boldsymbol{De}(t)-\boldsymbol{W}^{-}\boldsymbol{De}(t)\boldsymbol{\gamma}D\dot{\boldsymbol{e}}(t))]\},\tag{24}$$

$$\begin{aligned}\dot{V}_7(t)=&\sigma_1(t)\dot{\sigma}_1(t)+\sigma_2(t)\dot{\sigma}_2(t)=\\&\left[\frac{1}{\sigma_1(t)}-\mu_1\right][\boldsymbol{\varepsilon}^{\mathrm{T}}(t_s)\boldsymbol{\Theta}_1\boldsymbol{\varepsilon}(t_s)-\kappa_1\boldsymbol{e}^{\mathrm{T}}(t_k)\boldsymbol{\Theta}_3\boldsymbol{e}(t_k)]+\\&\left[\frac{1}{\sigma_2(t)}-\mu_2\right][\boldsymbol{\varepsilon}^{\mathrm{T}}(t_s)\boldsymbol{\Theta}_1\boldsymbol{\varepsilon}(t_s)-\kappa_2\boldsymbol{e}^{\mathrm{T}}(t_s)\boldsymbol{\Theta}_2\boldsymbol{e}(t_s)].\end{aligned}\tag{25}$$

由引理 2 可得

$$-\int_{t-\tau_1}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_1\boldsymbol{e}(s)\mathrm{d}s\leqslant-\tau_1\boldsymbol{\chi}^{\mathrm{T}}(t)\boldsymbol{S}_1\boldsymbol{\chi}(t)-3\tau_1[-\boldsymbol{\chi}(t)+\boldsymbol{\kappa}(t)]^{\mathrm{T}}\boldsymbol{S}_1[-\boldsymbol{\chi}(t)+\boldsymbol{\kappa}(t)],\tag{26}$$

$$\begin{aligned}-\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_2\boldsymbol{e}(s)\mathrm{d}s=&-\int_{t-\tau(t)}^{t-\tau_1}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_2\boldsymbol{e}(s)\mathrm{d}s-\int_{t-\tau_2}^{t-\tau(t)}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_2\boldsymbol{e}(s)\mathrm{d}s\leqslant\\&-(\tau(t)-\tau_1)\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{S}_2\boldsymbol{\delta}(t)-3(\tau(t)-\tau_1)[-\boldsymbol{\delta}(t)+\boldsymbol{\lambda}(t)]^{\mathrm{T}}\boldsymbol{S}_2[-\boldsymbol{\delta}(t)+\boldsymbol{\lambda}(t)]-\\&(\tau_2-\tau(t))\boldsymbol{\varepsilon}^{\mathrm{T}}(t)\boldsymbol{S}_2\boldsymbol{\varepsilon}(t)-3(\tau_2-\tau(t))[-\boldsymbol{\varepsilon}(t)+\boldsymbol{\nu}(t)]^{\mathrm{T}}\boldsymbol{S}_2[-\boldsymbol{\varepsilon}(t)+\boldsymbol{\nu}(t)],\end{aligned}\tag{27}$$

$$-\int_{t-d}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_3\boldsymbol{e}(s)\mathrm{d}s\leqslant-d\boldsymbol{\phi}^{\mathrm{T}}(t)\boldsymbol{S}_3\boldsymbol{\phi}(t)-3d[\boldsymbol{\phi}(t)-\boldsymbol{\varpi}(t)]^{\mathrm{T}}\boldsymbol{S}_3[\boldsymbol{\phi}(t)-\boldsymbol{\varpi}(t)],\tag{28}$$

$$\begin{aligned}-\int_{t-h}^{t-d}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_4\boldsymbol{e}(s)\mathrm{d}s=&-\int_{t-h(t)}^{t-d}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_4\boldsymbol{e}(s)\mathrm{d}s-\int_{t-h}^{t-h(t)}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{S}_4\boldsymbol{e}(s)\mathrm{d}s\leqslant\\&-(h(t)-d)\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{S}_4\boldsymbol{\varphi}(t)-3(h(t)-d)[-\boldsymbol{\varphi}(t)+\boldsymbol{\theta}(t)]^{\mathrm{T}}\boldsymbol{S}_4[-\boldsymbol{\varphi}(t)+\boldsymbol{\theta}(t)]-\\&(h-h(t))\boldsymbol{\gamma}^{\mathrm{T}}(t)\boldsymbol{S}_4\boldsymbol{\gamma}(t)-3(h-h(t))[-\boldsymbol{\varphi}(t)+\boldsymbol{\vartheta}(t)]^{\mathrm{T}}\boldsymbol{S}_4[-\boldsymbol{\varphi}(t)+\boldsymbol{\vartheta}(t)].\end{aligned}\tag{29}$$

由引理 3 可得

$$\begin{aligned}-\tau_1\int_{t-\tau_1}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\leqslant&-[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_1)]^{\mathrm{T}}\boldsymbol{T}_1[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_1)]-\\&3[\boldsymbol{e}(t)+\boldsymbol{e}(t-\tau_1)-2\boldsymbol{\chi}(t)]^{\mathrm{T}}\boldsymbol{T}_1[\boldsymbol{e}(t)+\boldsymbol{e}(t-\tau_1)-2\boldsymbol{\chi}(t)]-\\&5[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_1)+6\boldsymbol{\chi}(t)-6\boldsymbol{\kappa}(t)]^{\mathrm{T}}\boldsymbol{T}_1[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_1)+6\boldsymbol{\chi}(t)-6\boldsymbol{\kappa}(t)],\end{aligned}\tag{30}$$

$$-d\int_{t-d}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s\leqslant-[\boldsymbol{e}(t)-\boldsymbol{e}(t-d)]^{\mathrm{T}}\boldsymbol{T}_3[\boldsymbol{e}(t)-\boldsymbol{e}(t-d)]-$$

$$[\boldsymbol{e}(t)+\boldsymbol{e}(t-d)-2\boldsymbol{\phi}(t)]^{\mathrm{T}}\boldsymbol{T}_3[\boldsymbol{e}(t)+\boldsymbol{e}(t-d)-2\boldsymbol{\phi}(t)]-$$
$$[\boldsymbol{e}(t)-\boldsymbol{e}(t-d)+6\boldsymbol{\phi}(t)-6\boldsymbol{\varpi}(t)]^{\mathrm{T}}\boldsymbol{T}_3[\boldsymbol{e}(t)-\boldsymbol{e}(t-d)+6\boldsymbol{\phi}(t)-6\boldsymbol{\varpi}(t)],\tag{31}$$

$$-(\tau_2-\tau(t))\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s-(\tau(t)-\tau_1)\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s=$$
$$-\frac{1-\rho_1}{\rho_1}[\tau(t)-\tau_1]\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s-\frac{1-\rho_2}{\rho_2}[\tau_2-\tau(t)]\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s,\tag{32}$$

$$-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s=$$
$$-(\tau_2-\tau_1)\left[\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s+\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\right]=$$
$$-\frac{1}{\rho_1}(\tau(t)-\tau_1)\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s-\frac{1}{\rho_2}(\tau_2-\tau(t))\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s.\tag{33}$$

由引理 4 和引理 1 可得

$$-\frac{1}{\rho_1}(\tau(t)-\tau_1)\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)(\boldsymbol{T}_2+\boldsymbol{J}_3)\dot{\boldsymbol{e}}(s)\mathrm{d}s-\frac{1}{\rho_2}(\tau_2-\tau(t))\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)(\boldsymbol{T}_2+\boldsymbol{J}_4)\dot{\boldsymbol{e}}(s)\mathrm{d}s+$$
$$(\tau(t)-\tau_1)\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)(\boldsymbol{T}_2+\boldsymbol{J}_3)\dot{\boldsymbol{e}}(s)\mathrm{d}s+(\tau_2-\tau(t))\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)(\boldsymbol{T}_2+\boldsymbol{J}_4)\dot{\boldsymbol{e}}(s)\mathrm{d}s\leqslant$$
$$\boldsymbol{\xi}^{\mathrm{T}}(t)[-\boldsymbol{H}_1^{\mathrm{T}}(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_3)\boldsymbol{H}_1-\boldsymbol{H}_2^{\mathrm{T}}(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4)\boldsymbol{H}_2+\boldsymbol{H}_1^{\mathrm{T}}\bar{\boldsymbol{J}}_3\boldsymbol{H}_1+\boldsymbol{H}_2^{\mathrm{T}}\bar{\boldsymbol{J}}_4\boldsymbol{H}_2]\boldsymbol{\xi}(t)+$$
$$\boldsymbol{\xi}^{\mathrm{T}}(t)\left[-\rho_2\begin{pmatrix}\boldsymbol{H}_1\\\boldsymbol{H}_2\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\bar{T}+\bar{\boldsymbol{J}}_3-\boldsymbol{P}_2(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4)^{-1}\boldsymbol{P}_2^{\mathrm{T}} & \boldsymbol{P}_1\\ * & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{H}_1\\\boldsymbol{H}_2\end{pmatrix}\right]\boldsymbol{\xi}(t)+$$
$$\boldsymbol{\xi}^{\mathrm{T}}(t)\left[-\rho_1\begin{pmatrix}\boldsymbol{H}_1\\\boldsymbol{H}_2\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{0} & \boldsymbol{P}_2\\ * & \bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_4-\boldsymbol{P}_1^{\mathrm{T}}(\bar{\boldsymbol{T}}_2+\bar{\boldsymbol{J}}_3)^{-1}\boldsymbol{P}_1\end{pmatrix}\begin{pmatrix}\boldsymbol{H}_1\\\boldsymbol{H}_2\end{pmatrix}\right]\boldsymbol{\xi}(t),\tag{34}$$

$$-\int_{t-\tau_1}^{t}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\leqslant-[\boldsymbol{e}(t)-\boldsymbol{\chi}_1(t)]^{\mathrm{T}}\boldsymbol{J}_1[\boldsymbol{e}(t)-\boldsymbol{\chi}_1(t)]-$$
$$2[\boldsymbol{e}(t)+2\boldsymbol{\chi}_1(t)-3\boldsymbol{\kappa}(t)]^{\mathrm{T}}\boldsymbol{J}_1[\boldsymbol{e}(t)+2\boldsymbol{\chi}_1(t)-3\boldsymbol{\kappa}(t)],\tag{35}$$

$$-\int_{t-\tau_1}^{t}\int_{t-\tau_1}^{\theta}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\leqslant-[\boldsymbol{e}(t-\tau_1)+\boldsymbol{\chi}_1(t)]^{\mathrm{T}}\boldsymbol{J}_2[\boldsymbol{e}(t-\tau_1)+\boldsymbol{\chi}_1(t)]-$$
$$2[\boldsymbol{e}(t-\tau_1)-4\boldsymbol{\chi}_1(t)+3\boldsymbol{\kappa}(t)]^{\mathrm{T}}\boldsymbol{J}_2[\boldsymbol{e}(t-\tau_1)-4\boldsymbol{\chi}_1(t)+3\boldsymbol{\kappa}(t)],\tag{36}$$

$$-\int_{t-\tau_2}^{t-\tau_1}\int_{\theta}^{t-\tau_1}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta=-\left(\int_{t-\tau(t)}^{t-\tau_1}\int_{\theta}^{t-\tau_1}+\int_{t-\tau_2}^{t-\tau(t)}\int_{t-\tau(t)}^{t-\tau_1}+\int_{t-\tau_2}^{t-\tau(t)}\int_{\theta}^{t-\tau(t)}\right)\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\leqslant$$
$$-2[\boldsymbol{e}(t-\tau_1)-\boldsymbol{\delta}(t)]^{\mathrm{T}}\boldsymbol{J}_3[\boldsymbol{e}(t-\tau_1)-\boldsymbol{\delta}(t)]-$$
$$4[\boldsymbol{e}(t-\tau_1)+2\boldsymbol{\delta}(t)-3\boldsymbol{\lambda}(t)]^{\mathrm{T}}\boldsymbol{J}_3[\boldsymbol{e}(t-\tau_1)+2\boldsymbol{\delta}(t)-3\boldsymbol{\lambda}(t)]-$$
$$(\tau_2-\tau(t))\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{J}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s-$$
$$2[\boldsymbol{e}(t-\tau(t))-\boldsymbol{\varepsilon}(t)]^{\mathrm{T}}\boldsymbol{J}_3[\boldsymbol{e}(t-\tau(t))-\boldsymbol{\varepsilon}(t)]-$$
$$4[\boldsymbol{e}(t-\tau(t))+2\boldsymbol{\varepsilon}(t)-3\boldsymbol{\nu}(t)]^{\mathrm{T}}\boldsymbol{J}_3[\boldsymbol{e}(t-\tau(t))+2\boldsymbol{\varepsilon}(t)-3\boldsymbol{\nu}(t)],\tag{37}$$

$$-\int_{t-\tau_2}^{t-\tau_1}\int_{t-\tau_2}^{\theta}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{J}_4\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta=-\left(\int_{t-\tau(t)}^{t-\tau_1}\int_{t-\tau_2}^{t-\tau(t)}+\int_{t-\tau(t)}^{t-\tau_1}\int_{t-\tau(t)}^{t-\theta}+\int_{t-\tau_2}^{t-\tau(t)}\int_{t-\tau_2}^{\theta}\right)\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta\leqslant$$
$$-(\tau(t)-\tau_1)\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s-$$
$$2[\boldsymbol{e}(t-\tau(t))-\boldsymbol{\delta}(t)]^{\mathrm{T}}\boldsymbol{J}_4[\boldsymbol{e}(t-\tau(t))-\boldsymbol{\delta}(t)]-$$
$$4[\boldsymbol{e}(t-\tau(t))-4\boldsymbol{\delta}(t)+3\boldsymbol{\lambda}(t)]^{\mathrm{T}}\boldsymbol{J}_4[\boldsymbol{e}(t-\tau(t))-4\boldsymbol{\delta}(t)+3\boldsymbol{\lambda}(t)]-$$
$$2[-\boldsymbol{e}(t-\tau_2)+\boldsymbol{\varepsilon}(t)]^{\mathrm{T}}\boldsymbol{J}_4[-\boldsymbol{e}(t-\tau_2)+\boldsymbol{\varepsilon}(t)]-$$
$$4[\boldsymbol{e}(t-\tau_2)-4\boldsymbol{\varepsilon}(t)+3\boldsymbol{\nu}(t)]^{\mathrm{T}}\boldsymbol{J}_4[\boldsymbol{e}(t-\tau_2)-4\boldsymbol{\varepsilon}(t)+3\boldsymbol{\nu}(t)],\tag{38}$$

$$
-\int_{t-d}^{t}\int_{\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_5\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta \leqslant -[\boldsymbol{e}(t)-\boldsymbol{\phi}(t)]^{\mathrm{T}}\boldsymbol{J}_5[\boldsymbol{e}(t)-\boldsymbol{\phi}(t)] - 2[-\boldsymbol{e}(t)-2\boldsymbol{\phi}(t)+3\boldsymbol{\varpi}(t)]^{\mathrm{T}}\boldsymbol{J}_5[-\boldsymbol{e}(t)-2\boldsymbol{\phi}(t)+3\boldsymbol{\varpi}(t)], \tag{39}
$$

$$
-\int_{t-d}^{t}\int_{t-d}^{\theta}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_6\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta \leqslant -[-\boldsymbol{e}(t-d)-\boldsymbol{\phi}(t)]^{\mathrm{T}}\boldsymbol{J}_6[-\boldsymbol{e}(t-d)-\boldsymbol{\phi}(t)] - 2[\boldsymbol{e}(t-d)-4\boldsymbol{\phi}(t)+3\boldsymbol{\varpi}(t)]^{\mathrm{T}}\boldsymbol{J}_6[\boldsymbol{e}(t-d)-4\boldsymbol{\phi}(t)+3\boldsymbol{\varpi}(t)], \tag{40}
$$

$$
\begin{aligned}
-\int_{t-h}^{t-d}\int_{\theta}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_7\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta ={}& -\left(\int_{t-h(t)}^{t-d}\int_{\theta}^{t-d}+\int_{t-h}^{t-h(t)}\int_{t-h(t)}^{t-d}+\int_{t-h}^{t-h(t)}\int_{\theta}^{t-h(t)}\right)\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_7\dot{\boldsymbol{e}}(s)\mathrm{d}s \leqslant \\
& -2[\boldsymbol{e}(t-d)-\boldsymbol{\varphi}(t)]^{\mathrm{T}}\boldsymbol{J}_7[\boldsymbol{e}(t-d)-\boldsymbol{\varphi}(t)] - \\
& 4[\boldsymbol{e}(t-d)+2\boldsymbol{\varphi}(t)-3\boldsymbol{\theta}(t)]^{\mathrm{T}}\boldsymbol{J}_7[\boldsymbol{e}(t-d)+2\boldsymbol{\varphi}(t)-3\boldsymbol{\theta}(t)] - \\
& [h-h(t)]\int_{t-h(t)}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_7\dot{\boldsymbol{e}}(s)\mathrm{d}s - \\
& 2[\boldsymbol{e}(t-h(t))-\boldsymbol{\gamma}(t)]^{\mathrm{T}}\boldsymbol{J}_7[\boldsymbol{e}(t-h(t))-\boldsymbol{\gamma}(t)] - \\
& 4[\boldsymbol{e}(t-h(t))+2\boldsymbol{\gamma}(t)-3\boldsymbol{\vartheta}(t)]^{\mathrm{T}}\boldsymbol{J}_7[\boldsymbol{e}(t-h(t))+2\boldsymbol{\gamma}(t)-3\boldsymbol{\vartheta}(t)],
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
-\int_{t-h}^{t-d}\int_{t-h}^{\theta}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_8\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta ={}& -\left(\int_{t-h(t)}^{t-d}\int_{t-h}^{t-h(t)}+\int_{t-h(t)}^{t-d}\int_{t-h(t)}^{\theta}+\int_{t-h}^{t-h(t)}\int_{t-h}^{\theta}\right)\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_8\dot{\boldsymbol{e}}(s)\mathrm{d}s \leqslant \\
& -[h(t)-d]\int_{t-h}^{t-h(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_8\dot{\boldsymbol{e}}(s)\mathrm{d}s - \\
& 2[-\boldsymbol{e}(t-h(t))+\boldsymbol{\varphi}(t)]^{\mathrm{T}}\boldsymbol{J}_8[-\boldsymbol{e}(t-h(t))+\boldsymbol{\varphi}(t)] - \\
& 4[\boldsymbol{e}(t-h(t))-4\boldsymbol{\varphi}(t)+3\boldsymbol{\theta}(t)]^{\mathrm{T}}\boldsymbol{J}_8[\boldsymbol{e}(t-h(t))-4\boldsymbol{\varphi}(t)+3\boldsymbol{\theta}(t)] - \\
& 2[-\boldsymbol{e}(t-h)+\boldsymbol{\gamma}(t)]^{\mathrm{T}}\boldsymbol{J}_8[-\boldsymbol{e}(t-h)+\boldsymbol{\gamma}(t)] - \\
& 4[\boldsymbol{e}(t-h)-4\boldsymbol{\gamma}(t)+3\boldsymbol{\vartheta}(t)]^{\mathrm{T}}\boldsymbol{J}_8[\boldsymbol{e}(t-h)-4\boldsymbol{\gamma}(t)+3\boldsymbol{\vartheta}(t)],
\end{aligned} \tag{42}
$$

$$
\begin{aligned}
& -[h-h(t)]\int_{t-h(t)}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_7\dot{\boldsymbol{e}}(s)\mathrm{d}s-[h(t)-d]\int_{t-h}^{t-h(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_8\dot{\boldsymbol{e}}(s)\mathrm{d}s = \\
& \quad -\frac{1-\rho_3}{\rho_3}[h(t)-d]\int_{t-h(t)}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_7\dot{\boldsymbol{e}}(s)\mathrm{d}s-\frac{1-\rho_4}{\rho_4}[h-h(t)]\int_{t-h}^{t-h(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_8\dot{\boldsymbol{e}}(s)\mathrm{d}s,
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
& -(h-d)\int_{t-h}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s = \\
& \quad -(h-d)\int_{t-h(t)}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s-(h-d)\int_{t-h}^{t-h(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s = \\
& \quad -\frac{1}{\rho_3}[h(t)-d]\int_{t-h(t)}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s-\frac{1}{\rho_4}[h-h(t)]\int_{t-h}^{t-h(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_4\dot{\boldsymbol{e}}(s)\mathrm{d}s,
\end{aligned} \tag{44}
$$

$$
\begin{aligned}
& -\frac{1}{\rho_3}[h(t)-d]\int_{t-h(t)}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)(\boldsymbol{T}_4+\boldsymbol{J}_7)\dot{\boldsymbol{e}}(s)\mathrm{d}s-\frac{1}{\rho_4}[h-h(t)]\int_{t-h}^{t-h(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)(\boldsymbol{T}_4+\boldsymbol{J}_8)\dot{\boldsymbol{e}}(s)\mathrm{d}s + \\
& \quad [h(t)-d]\int_{t-h(t)}^{t-d}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_7\dot{\boldsymbol{e}}(s)\mathrm{d}s+[h-h(t)]\int_{t-h}^{t-h(t)}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{J}_8\dot{\boldsymbol{e}}(s)\mathrm{d}s \leqslant \\
& \quad \boldsymbol{\xi}^{\mathrm{T}}(t)[-\boldsymbol{H}_3^{\mathrm{T}}(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7)\boldsymbol{H}_3-\boldsymbol{H}_4^{\mathrm{T}}(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8)\boldsymbol{H}_4+\boldsymbol{H}_3^{\mathrm{T}}\bar{\boldsymbol{J}}_7\boldsymbol{H}_3+\boldsymbol{H}_4^{\mathrm{T}}\bar{\boldsymbol{J}}_8\boldsymbol{H}_4]\boldsymbol{\xi}(t) + \\
& \quad \boldsymbol{\xi}^{\mathrm{T}}(t)\left[-\rho_4\begin{pmatrix}\boldsymbol{H}_3\\ \boldsymbol{H}_4\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7-\boldsymbol{P}_4(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8)^{-1}\boldsymbol{P}_4^{\mathrm{T}} & \boldsymbol{P}_3\\ * & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{H}_3\\ \boldsymbol{H}_4\end{pmatrix}\right]\boldsymbol{\xi}(t) + \\
& \quad \boldsymbol{\xi}^{\mathrm{T}}(t)\left[-\rho_3\begin{pmatrix}\boldsymbol{H}_3\\ \boldsymbol{H}_4\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{0} & \boldsymbol{P}_4\\ * & \bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_8-\boldsymbol{P}_3^{\mathrm{T}}(\bar{\boldsymbol{T}}_4+\bar{\boldsymbol{J}}_7)^{-1}\boldsymbol{P}_3\end{pmatrix}\begin{pmatrix}\boldsymbol{H}_3\\ \boldsymbol{H}_4\end{pmatrix}\right]\boldsymbol{\xi}(t),
\end{aligned} \tag{45}
$$

$$
\begin{aligned}
\boldsymbol{0} ={}& 2[\boldsymbol{e}^{\mathrm{T}}(t)+\dot{\boldsymbol{e}}^{\mathrm{T}}(t)+\boldsymbol{e}^{\mathrm{T}}(t_k)]\boldsymbol{G}^{\mathrm{T}}\times \\
& [-\dot{\boldsymbol{e}}(t)+\boldsymbol{A}\boldsymbol{e}(t)+\boldsymbol{B}\boldsymbol{e}(t-\tau(t))+\boldsymbol{E}\boldsymbol{g}(\boldsymbol{D}\boldsymbol{e}(t))-\boldsymbol{K}\boldsymbol{C}\boldsymbol{e}(t-h(t))+\boldsymbol{K}\boldsymbol{C}\boldsymbol{\varepsilon}(t_s)] = \\
& 2[\boldsymbol{e}^{\mathrm{T}}(t)+\dot{\boldsymbol{e}}^{\mathrm{T}}(t)+\boldsymbol{e}^{\mathrm{T}}(t-h(t))-\boldsymbol{\varepsilon}^{\mathrm{T}}(t_s)]\boldsymbol{G}^{\mathrm{T}}\times
\end{aligned}
$$

$$[-\dot{\boldsymbol{e}}(t)+\boldsymbol{A}\boldsymbol{e}(t)+\boldsymbol{B}\boldsymbol{e}(t-\tau(t))+\boldsymbol{E}\boldsymbol{g}(\boldsymbol{D}\boldsymbol{e}(t))-\boldsymbol{K}\boldsymbol{C}\boldsymbol{e}(t-h(t))+\boldsymbol{K}\boldsymbol{C}\boldsymbol{\varepsilon}(t_s)], \tag{46}$$

$$\boldsymbol{0}\leqslant-[\boldsymbol{g}(\boldsymbol{D}\boldsymbol{e}(t))-\boldsymbol{W}^{+}\boldsymbol{D}\boldsymbol{e}(t)]^{\mathrm{T}}\boldsymbol{F}[\boldsymbol{g}(\boldsymbol{D}\boldsymbol{e}(t))-\boldsymbol{W}^{-}\boldsymbol{D}\boldsymbol{e}(t)], \tag{47}$$

$$\dot{V}(t)\leqslant\boldsymbol{\xi}^{\mathrm{T}}(t)[\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_3(\tau(t))+\boldsymbol{\Psi}_4(h(t))+\boldsymbol{\Psi}_5(\tau(t))+\boldsymbol{\Psi}_6(h(t))]\boldsymbol{\xi}(t). \tag{48}$$

基于凸组合技术, $\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_3(\tau(t))+\boldsymbol{\Psi}_4(h(t))+\boldsymbol{\Psi}_5(\tau(t))+\boldsymbol{\Psi}_6(h(t))\leqslant\boldsymbol{0}$ 成立,当且仅当以下四式成立:

$$\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_3(\tau_1)+\boldsymbol{\Psi}_4(d)+\boldsymbol{\Psi}_5(\tau_1)+\boldsymbol{\Psi}_6(d)<\boldsymbol{0}, \tag{49}$$

$$\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_3(\tau_1)+\boldsymbol{\Psi}_4(h)+\boldsymbol{\Psi}_5(\tau_1)+\boldsymbol{\Psi}_6(h)<\boldsymbol{0}, \tag{50}$$

$$\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_3(\tau_2)+\boldsymbol{\Psi}_4(d)+\boldsymbol{\Psi}_5(\tau_2)+\boldsymbol{\Psi}_6(d)<\boldsymbol{0}, \tag{51}$$

$$\boldsymbol{\Psi}_1+\boldsymbol{\Psi}_2+\boldsymbol{\Psi}_3(\tau_2)+\boldsymbol{\Psi}_4(h)+\boldsymbol{\Psi}_5(\tau_2)+\boldsymbol{\Psi}_6(h)<\boldsymbol{0}. \tag{52}$$

则有 $\dot{V}(t)<0$, 根据 Schur 补引理, $\dot{V}(t)<0$, 等价于线性矩阵不等式(14)~(17),所以误差系统(6)是渐近稳定的,证明结束.

3 数值仿真

为了验证本文所提方法的有效性,做如下的仿真:

$$\begin{cases}\dot{x}_1(t)=a(x_2(t)-m_1x_1(t)+g(x_1(t)))-cx_1(t-d(t)),\\ \dot{x}_2(t)=x_1(t)-x_2(t)+x_3(t)-cx_1(t-d(t)),\\ \dot{x}_3(t)=-bx_2(t)+c(2x_1(t-d(t))-x_3(t-d(t))),\end{cases}$$

其中,非线性函数 $g(x_1(t))=(1/2)(m_1-m_0)(|x_1(t)+1|-|x_1(t)-1|)$, 其余参数 $m_0=-1/7, m_1=2/7$, $a=9, b=14.28, c=0.1$, 时变时滞 $d(t)=|\sin(t)|$.

系统(1)和(2)可表示成 Lur' e 系统的形式:

$$\boldsymbol{A}=\begin{bmatrix}-am_1 & a & 0\\ 1 & -1 & 1\\ 0 & -b & 0\end{bmatrix}, \boldsymbol{B}=\begin{bmatrix}-c & 0 & 0\\ -c & 0 & 0\\ 2c & 0 & -c\end{bmatrix}, \boldsymbol{E}=\begin{bmatrix}-a(m_0-m_1)\\ 0\\ 0\end{bmatrix}, \boldsymbol{C}=\boldsymbol{D}=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}^{\mathrm{T}},$$

非线性函数 $f_1(x_1(t))=\frac{1}{2}(|x_1(t)+1|-|x_1(t)-1|)\in[0,1]$, $f_2(x_2(t))=f_3(x_3(t))=0$.

主从系统的初始值分别取为 $\boldsymbol{x}(0)=[0.2\quad 0.3\quad 0.2]^{\mathrm{T}}$, $\boldsymbol{y}(0)=[-0.3\quad -0.1\quad 0.4]^{\mathrm{T}}$.取 $\mu_1=\mu_2=5, \kappa_1=\kappa_2=2, u_1=u_2=0, \tau_2=1, \sigma_1(0)=0.454, \sigma_2(0)=0.456$, 应用 MATLAB 求解线性矩阵不等式(14)~(17),根据定理 1 计算所得到的最大采样区间的数值如表 1 所示.由表 1 可知,与文献[1,4,13,16]相比,本文所得到的最大采样区间的数值更优.当 $h=0.5629$ 时,求解线性矩阵不等式(14)~(17),所得控制器增益矩阵和触发参数如下:

$$\boldsymbol{W}_1=\begin{bmatrix}0.8592 & 0.1502 & -0.2593\\ 0.1502 & 0.8432 & -0.1215\\ -0.2593 & -0.1215 & 1.1246\end{bmatrix}, \boldsymbol{W}_2=\begin{bmatrix}0.7442 & -0.2495 & 0.3496\\ -0.2495 & 0.9036 & 0.2496\\ 0.3496 & 0.2496 & 0.9733\end{bmatrix},$$

$$\boldsymbol{W}_3=\begin{bmatrix}0.7245 & -0.2505 & 0.3512\\ -0.2505 & 0.8986 & 0.2484\\ 0.3512 & 0.2484 & 0.9816\end{bmatrix}, \boldsymbol{K}=\begin{bmatrix}3.3827\\ 0.3015\\ -2.9116\end{bmatrix}.$$

表 1 最大采样区间

Table 1 The maximum sampling period h

reference	[1]	[4]	[13]	[16]	theorem
h	0.482 4	0.435 5	0.478 9	0.437 1	0.562 9

MATLAB 仿真得到的误差向量和传输间隔的响应曲线,如图 1 和图 2 所示.从图 1 可以看出,在最大采样区间 $h=0.5629$ 时,误差系统的状态轨迹在有限时间内收敛于零,即时滞 Lur' e 系统在短时间内能够实现主从系统同步.从图 2 可以看出,在最大采样区间 $h=0.5629$ 时,主从同步过程中加入事件触发机制能够减

少信号的传输次数,提高通信利用率.

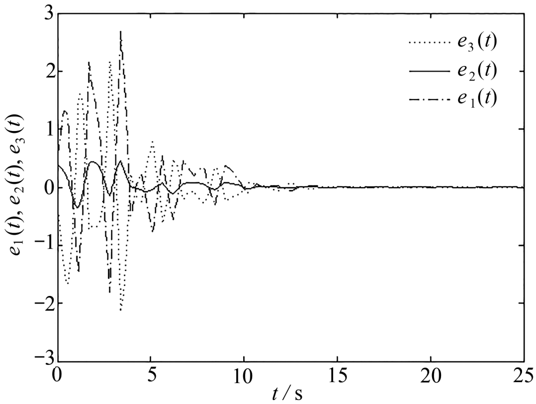

图 1 采样周期为 0.562 9 时, 误差 $\boldsymbol{e}(t)$ 的响应曲线

Fig. 1 Responses of $\boldsymbol{e}(t)$ with a sampling period of $h = 0.562\ 9$

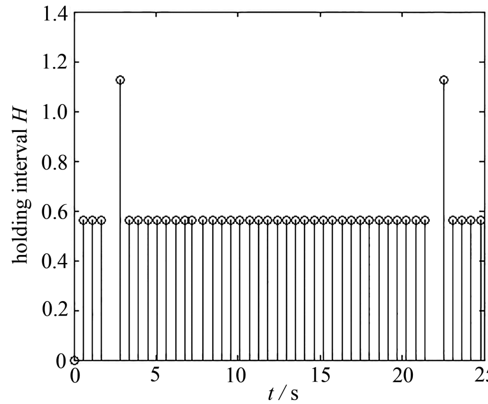

图 2 事件触发传输瞬间及其释放间隔

Fig. 2 Transmission instants and releasing intervals in the case of event triggering

4 结 论

本文研究了基于事件触发机制的混沌 Lur'e 系统主从同步问题,为了减少保守性,在构造 Lyapunov-Krasovskii 泛函时考虑了非线性函数的扇区约束条件,引进了基于辅助函数的积分不等式和增广 Lyapunov-Krasovskii 泛函,结合 Lyapunov 稳定性理论和线性矩阵不等式技术,给出了混沌 Lur'e 系统主从同步的充分条件.通过设置触发条件能够降低系统的状态信息在通信网络中的传输次数,缓解网络带宽压力.最后,通过数值仿真验证了所提方法的有效性.

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