

线载和弹性支承作用面内运动薄板 磁固耦合双重共振*

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摘要: 针对磁场环境中具有线载荷和弹性支承作用的面内运动薄板, 给出了系统的势能、动能及电磁力表达式, 应用 Hamilton 变分原理, 推得面内运动条形板的磁固耦合非线性振动方程. 考虑边界为夹支-铰支的约束条件, 利用变量分离法和 Galerkin 积分法, 得到了含简谐线载力和电磁阻尼力项的两自由度非线性振动微分方程组. 应用多尺度法对主-内联合共振问题进行解析求解, 得到了双重联合共振下系统的一阶状态方程和共振响应特征方程. 通过算例, 得到了面内运动薄板的一阶和二阶共振幅值变化规律曲线图, 分析了不同作用量和载荷位置对系统振动特性的影响. 结果表明: 系统发生主-内双重共振时, 解的多值性和跳跃现象明显, 弹性支承和线载荷位置对共振现象影响显著; 一阶和二阶的共振多值解区域同时出现同时消失, 体现了明显的内共振特征.

关键词: 薄板; 磁固耦合共振; 面内运动; 线载荷; 弹性支承; 多尺度法
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Double Resonance of Magnetism-Solid Coupling of in-Plane Moving Thin Plates With Linear Loads and Elastic Supports

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Abstract: For the in-plane moving thin plates with linear loads and elastic supports in magnetic field, the potential energy, the kinetic energy and the electromagnetic force expressions of the system were given. Based on the Hamiltonian variational principle, the magnetism-solid coupling nonlinear vibration equation for the in-plane moving strip plate was deduced. For the clamped-hinged boundary condition, the variable separation method and the Galerkin method were employed to obtain the 2DOF nonlinear vibration differential equations containing

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the simple harmonic linear load and the electromagnetic damping force terms. The multiscale method was used to analytically solve the principal-internal resonance problem, and the 1st-order state equation and the resonance response characteristic equation for the system under the double joint resonance were obtained. Through numerical examples, the 1st- and 2nd-order resonance amplitude curves of the in-plane moving thin plate were obtained. The effects of different parameters and load positions on vibration characteristics of the system were analyzed. The results show that, for the principal-internal resonance occurring in the system, the multivaluedness and jumping phenomenon of the solution are obvious, and the effects of the elastic support and the linear load position on the resonance are significant. Additionally, the 1st- and 2nd-order resonance multivalued solution areas appear and disappear simultaneously, which reflects obvious internal resonance characteristics.

Key words: thin plate; magnetism-solid coupling resonance; in-plane movement; line load; elastic support; multiscale method

引 言

在工程领域中,梁、板和圆柱壳在弹性条件下振动普遍存在,如建筑结构、机械构件和电器设备等在弹性约束条件下的振动具有广泛应用前景,因此,得到了国内外学者的大量关注.Ding 等^[1]对具有不对称弹性支撑梁的非线性振动问题进行了研究.Zhang 等^[2]研究了具有弹性约束边缘的正交异性矩形板的面内振动问题.Gorman^[3]在简单支撑边界条件下研究了矩形板的面内自由振动问题.Chen 等^[4]采用 Chebyshev-Lagrange 法分析了弹性约束下带有切口弹性板的弯曲和振动问题.Dozio^[5]对具有任意弹性边界的矩形板的自由面内振动问题进行了研究.Zhang 等^[6]用改进的 Fourier 级数方法研究了矩形板的振动问题.闫维明等^[7]研究了完全弹性支承变截面梁的动力特性问题.同时,在对面内和轴向运动结构的振动问题及结构耦合动力学问题的研究中,胡宇达^[8]针对磁场环境中,面内轴向运动导电薄板的动力学理论建模问题进行了研究.Ding 等^[9]在 3:1 内部共振条件下,分析了黏弹性梁强迫振动的稳态周期响应.Hu 等^[10-11]对磁场及移动载荷作用下轴向运动载流梁的内共振和参强联合共振问题进行了研究.Yang 等^[12]研究了 FGM 圆锥壳的内共振及非线性动力学特性.胡宇达等^[13]研究了轴向运动导电薄板的磁弹性非线性动力学及分岔特性.Tang 和 Chen^[14]研究了轴向运动黏弹性板在平面应力下的参数共振及稳定性问题.Chen 等^[15]研究了具有内部共振特性轴向移动梁的非线性振动问题.Li 等^[16]对浸没在流体中轴向运动板的非线性强迫振动和稳定性进行了分析.Krzysztof 等^[17]研究了轴向运动板在热载荷作用下的动力学问题,分析了热载荷、速度和轴向张力对系统动力学行为的影响.Jarosław 和 Jerzy^[18]对旋转薄壁复合压电梁的非线性振动问题进行了研究.Murillo 等^[19]研究了弹塑性锥体结构的非线性振动和动力稳定性问题.

可见,针对轴向运动结构动力学特性的研究已引起研究者的重视并取得了相应研究成果,但对于具有复杂约束作用条件,且处在多场环境中复杂运动结构耦合共振问题的研究相对较少.本文考虑弹性支承和线载荷的共同作用,对面内运动薄板的磁固耦合双重共振问题进行了研究.首先应用 Galerkin 法对偏微分振动方程进行积分离散,然后应用多尺度法对离散后的非线性常微分方程组进行求解,得到系统的双重共振响应解析解;通过算例给出了共振特性随不同参量变化规律.

1 面内运动板磁弹性振动方程

研究图 1 所示横向磁场中沿 x 轴方向面内运动的导电材料条形板,建立直角坐标系 $Oxyz$, 其中 xOy 面为板的中面, z 为法向坐标, V_{0x} 为沿 x 方向的轴向运动速度, N_{0x} 为轴向拉力, $\mathbf{B}_0(0, 0, B_{0z})$ 为外加磁感应强度矢量, 条形板沿 x 方向的有界边长为 l_1 , 板厚为 h .

针对薄板的横向振动问题,根据弹性理论并考虑条形板的变形对称性及几何非线性条件,可给出条形板单位宽度的变形势能表达式为

$$U = \int_0^{l_1} \left[\frac{D_M}{2} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{h}{2} \sigma_x + N_{0x} \right) \varepsilon_x \right] dx, \quad (1)$$

式中, $D_M = Eh^3/[12(1 - \nu^2)]$ 为弯曲刚度, E 为弹性模量, ν 为 Poisson 比; w 为横向振动挠度; $\varepsilon_x = (\partial w/\partial x)^2/2$ 为中面应变, σ_x 为平面正应力, N_{0x} 为边界轴向拉力。

对于面内运动条形薄板,系统在运动中的动能表达形式为

$$T = \frac{\rho h}{2} \int_0^{l_1} \left[V_{0x}^2 + \left(V_{0x} \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} \right)^2 \right] dx, \tag{2}$$

式中, ρ 为薄板的材料密度, t 为时间变量。

对于处于外加横向磁场环境中的面内运动良导体材料薄板,由于结构的时变运动,将会在体内出现感应电流,进而对薄板产生 Lorentz 电磁力的作用。若仅考虑薄板的横向振动位移及图 1 所示的外加横向磁场环境,则作用于薄板上的电磁力矩表达式为

$$m_x = \frac{\sigma_0 h^3 B_{0z}^2}{12} \left(\frac{\partial^2 w}{\partial x \partial t} + V_{0x} \frac{\partial^2 w}{\partial x^2} \right), \tag{3}$$

式中, σ_0 为材料电导率。

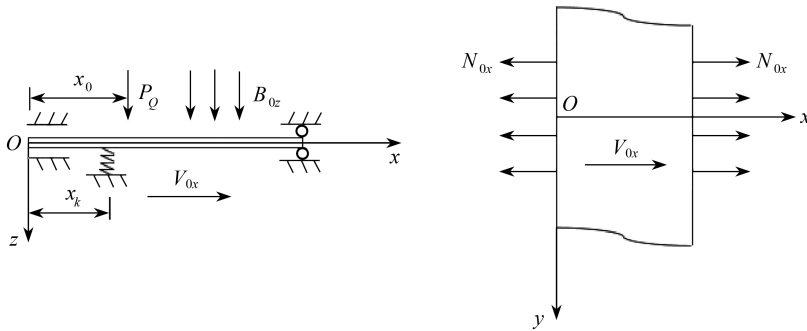


图 1 导电条形板

Fig. 1 The conductive strip plate

对于图 1 所示表面受外加横向线载荷和弹性支承作用的薄板,其作用力可分别表示为

$$P_Q = \delta(x - x_0) P \cos(\Omega_0 t), \tag{4}$$

$$F_Q = -\delta(x - x_k) k w, \tag{5}$$

式中, P 为线载荷幅值, Ω_0 为线载荷简谐频率, x_0 为线载荷位置坐标, k 为弹性支承刚度, x_k 为弹性支承位置坐标; $\delta(x - x_0)$ 和 $\delta(x - x_k)$ 为 Dirac 函数。

这样,基于以上给出的表达式并根据 Hamilton 变分原理,可推得弹性支承和线载荷作用下面内轴向运动导电条形板的磁固耦合非线性振动方程:

$$-D_M \frac{\partial^4 w}{\partial x^4} + \frac{3D_N}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} + N_{0x} \frac{\partial^2 w}{\partial x^2} + \frac{\sigma_0 h^3 B_{0z}^2}{12} \left(V_{0x} \frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x^2 \partial t} \right) + P_Q - F_Q = \rho h \left(\frac{\partial^2 w}{\partial t^2} + 2V_{0x} \frac{\partial^2 w}{\partial x \partial t} + V_{0x}^2 \frac{\partial^2 w}{\partial x^2} \right), \tag{6}$$

式中, $D_N = Eh/(1 - \nu^2)$ 为拉伸刚度。

2 主-内联合共振问题理论求解

2.1 非线性方程的 Galerkin 离散

当条形板两端受到夹支-铰支边界约束情况时,其前两阶固有频率值满足近似 3 倍的关系,从而可能激发系统的非线性内共振现象,下面针对这种边界约束情况进行分析。

为研究系统内共振特征,将满足夹支-铰支边界条件的方程(6)的位移解取为如下关于两阶模态的分离变量形式:

$$w = \sum_{n=1}^2 q_n(t) X_n, \tag{7}$$

式中

$$\begin{cases} X_n = \cosh(p_n x) - \cos(p_n x) - \xi_n (\sinh(p_n x) - \sin(p_n x)), \\ \xi_n = \frac{\cosh(p_n l_1) + \cos(p_n l_1)}{\sinh(p_n l_1) + \sin(p_n l_1)}, \end{cases} \quad p_n = \frac{(4n+1)\pi}{4l_1}.$$

将式(7)代入式(6)中,采用 Galerkin 法进行积分离散,同时考虑到系统的弱非线性振动特征,引入小参数 ε , 最终可推得面内运动板的横向两自由度非线性振动微分方程组:

$$\ddot{x}_1(\tau) + \omega_1^2 x_1(\tau) = -\varepsilon \eta_1^2 x_2(\tau) + \varepsilon \mu_{11} \dot{x}_1(\tau) + \varepsilon \mu_{21} \dot{x}_2(\tau) + \varepsilon \alpha_1 [S_{11} x_1^3(\tau) + S_{21} x_2^3(\tau) + S_{31} x_1(\tau) x_2^2(\tau) + S_{41} x_1^2(\tau) x_2(\tau)] + \varepsilon \eta_2 \cos(\Omega \tau), \quad (8)$$

$$\ddot{x}_2(\tau) + \omega_2^2 x_2(\tau) = -\varepsilon \eta_3^2 x_1(\tau) + \varepsilon \mu_{12} \dot{x}_1(\tau) + \varepsilon \mu_{22} \dot{x}_2(\tau) + \varepsilon \alpha_2 [S_{12} x_1^3(\tau) + S_{22} x_2^3(\tau) + S_{32} x_1(\tau) x_2^2(\tau) + S_{42} x_1^2(\tau) x_2(\tau)] + \varepsilon \eta_4 \cos(\Omega \tau), \quad (9)$$

式中

$$x_n(\tau) = \frac{q_n(t)}{h}, \quad \tau = \omega_n t, \quad \Omega = \frac{\Omega_0}{\omega_n}, \quad \omega_n = \sqrt{b_1 b_2}, \quad \omega_1 = \frac{b_1}{\omega_n}, \quad \omega_2 = \frac{b_2}{\omega_n}, \quad \alpha_1 = \frac{3hD_N}{2\rho A_{11}\omega_n^2},$$

$$\alpha_2 = \frac{3hD_N}{2\rho A_{22}\omega_n^2}, \quad \mu_{11} = \frac{\sigma_0 h^3 B_{0z}^2 C_{11}}{12\rho h A_{11} \omega_n} - \frac{2V_{0x} B_{11}}{\rho h A_{11} \omega_n}, \quad \mu_{12} = \frac{\sigma_0 h^3 B_{0z}^2 C_{12}}{12\rho h A_{22} \omega_n} - \frac{2V_{0x} B_{12}}{\rho h A_{22} \omega_n},$$

$$\mu_{21} = \frac{\sigma_0 h^3 B_{0z}^2 C_{21}}{12\rho h A_{11} \omega_n} - \frac{2V_{0x} B_{21}}{\rho h A_{11} \omega_n}, \quad \mu_{22} = \frac{\sigma_0 h^3 B_{0z}^2 C_{22}}{12\rho h A_{22} \omega_n} - \frac{2V_{0x} B_{22}}{\rho h A_{22} \omega_n},$$

$$\eta_1^2 = \frac{kG_{21}}{\rho h A_{11} \omega_n^2} - \frac{N_{0x} C_{21}}{\rho h A_{11} \omega_n^2} + \frac{V_{0x}^2 C_{21}}{\rho h A_{11} \omega_n^2} - \frac{\sigma_0 h^3 B_{0z}^2 V_{0x} D_{21}}{12\rho h A_{11} \omega_n^2} + \frac{D_M E_{21}}{\rho h A_{11} \omega_n^2},$$

$$\eta_3^2 = \frac{kG_{12}}{\rho h A_{22} \omega_n^2} - \frac{N_{0x} C_{12}}{\rho h A_{22} \omega_n^2} + \frac{V_{0x}^2 C_{12}}{\rho h A_{22} \omega_n^2} - \frac{\sigma_0 h^3 B_{0z}^2 V_{0x} D_{12}}{12\rho h A_{22} \omega_n^2} + \frac{D_M E_{12}}{\rho h A_{22} \omega_n^2},$$

$$\eta_2 = \frac{P[\cosh(p_1 x_0) - \cos(p_1 x_0) - \xi_1(\sinh(p_1 x_0) - \sin(p_1 x_0))]}{\rho h A_{11} \omega_n^2},$$

$$\eta_4 = \frac{P[\cosh(p_2 x_0) - \cos(p_2 x_0) - \xi_2(\sinh(p_2 x_0) - \sin(p_2 x_0))]}{\rho h A_{22} \omega_n^2},$$

$$b_1^2 = \frac{kG_{11}}{\rho h A_{11}} - \frac{N_{0x} C_{11}}{\rho h A_{11}} + \frac{V_{0x}^2 C_{11}}{\rho h A_{11}} - \frac{\sigma_0 h^3 B_{0z}^2 V_{0x} D_{11}}{12\rho h A_{11}} + \frac{D_M E_{11}}{\rho h A_{11}},$$

$$b_2^2 = \frac{kG_{22}}{\rho h A_{22}} - \frac{N_{0x} C_{22}}{\rho h A_{22}} + \frac{V_{0x}^2 C_{22}}{\rho h A_{22}} - \frac{\sigma_0 h^3 B_{0z}^2 V_{0x} D_{22}}{12\rho h A_{22}} + \frac{D_M E_{22}}{\rho h A_{22}},$$

$$G_{11} = [\cosh(p_1 x_k) - \cos(p_1 x_k) - \xi_1(\sinh(p_1 x_k) - \sin(p_1 x_k))]^2,$$

$$G_{12} = [\cosh(p_1 x_k) - \cos(p_1 x_k) - \xi_1(\sinh(p_1 x_k) - \sin(p_1 x_k))] \times [\cosh(p_2 x_k) - \cos(p_2 x_k) - \xi_2(\sinh(p_2 x_k) - \sin(p_2 x_k))],$$

$$G_{21} = [\cosh(p_2 x_k) - \cos(p_2 x_k) - \xi_2(\sinh(p_2 x_k) - \sin(p_2 x_k))] \times [\cosh(p_1 x_k) - \cos(p_1 x_k) - \xi_1(\sinh(p_1 x_k) - \sin(p_1 x_k))],$$

$$G_{22} = [\cosh(p_2 x_k) - \cos(p_2 x_k) - \xi_2(\sinh(p_2 x_k) - \sin(p_2 x_k))]^2.$$

上面各积分式为

$$A_{ni} = \int_0^{l_1} X_n X_i dx, \quad B_{ni} = \int_0^{l_1} \frac{dX_n}{dx} X_i dx, \quad C_{ni} = \int_0^{l_1} \frac{d^2 X_n}{dx^2} X_i dx, \quad D_{ni} = \int_0^{l_1} \frac{d^3 X_n}{dx^3} X_i dx,$$

$$E_{ni} = \int_0^{l_1} \frac{d^4 X_n}{dx^4} X_i dx, \quad S_{1i} = \int_0^{l_1} \frac{d^2 X_1}{dx^2} \left(\frac{dX_1}{dx} \right)^2 X_i dx, \quad S_{2i} = \int_0^{l_1} \left(\frac{dX_2}{dx} \right)^2 \frac{d^2 X_2}{dx^2} X_i dx,$$

$$S_{3i} = \int_0^{l_1} \left[\frac{d^2 X_1}{dx^2} \left(\frac{dX_2}{dx} \right)^2 + \frac{2dX_1}{dx} \frac{dX_2}{dx} \frac{d^2 X_2}{dx^2} \right] X_i dx, \quad S_{4i} = \int_0^{l_1} \left[\left(\frac{dX_1}{dx} \right)^2 \frac{d^2 X_2}{dx^2} + \frac{2d^2 X_1}{dx^2} \frac{dX_1}{dx} \frac{dX_2}{dx} \right] X_i dx,$$

其中 $i = 1, 2, n = 1, 2$.

2.2 多尺度法求解

对于本文所研究的边界条件可知,方程(8)、(9)中的固有频率满足关系 $\omega_2 \approx 3\omega_1$,同时考虑线载激励频率 Ω 接近于一阶频率 ω_1 的主共振情形,则将会激发非线性系统的主-内联合共振.下面采用多尺度法^[20]对这种双重联合共振问题进行理论求解.

首先,引入频率调谐参数 σ_1 和 σ_2 , 并设

$$\omega_2 = 3\omega_1 + \varepsilon\sigma_1, \quad \Omega = \omega_1 + \varepsilon\sigma_2. \quad (10)$$

在对非线性振动方程组(8)、(9)进行一次近似解析求解时,设

$$\begin{cases} x_1(\tau; \varepsilon) = x_{11}(T_0, T_1) + \varepsilon x_{12}(T_0, T_1), \\ x_2(\tau; \varepsilon) = x_{21}(T_0, T_1) + \varepsilon x_{22}(T_0, T_1), \end{cases} \quad (11)$$

式中的时间尺度 $T_0 = \varepsilon^0\tau, T_1 = \varepsilon^1\tau$.

将式(10)、(11)代入方程组(8)、(9)中,展开后令等号两侧 ε 的同次幂项系数相等,得到关于 ε^0 的近似方程:

$$\begin{cases} D_0^2 x_{11} + \omega_1^2 x_{11} = 0, \\ D_0^2 x_{21} + \omega_2^2 x_{21} = 0; \end{cases} \quad (12)$$

关于 ε^1 的近似方程:

$$\begin{cases} D_0^2 x_{12} + \omega_1^2 x_{12} = -2D_0 D_1 x_{11} - \eta_1^2 x_{21} + \mu_{11} D_0 x_{11} + \mu_{21} D_0 x_{21} + \alpha_1 (S_{11} x_{11}^3 + S_{21} x_{21}^3 + \\ S_{31} x_{11} x_{21}^2 + S_{41} x_{11}^2 x_{21}) + \eta_2 \cos(\Omega\tau), \\ D_0^2 x_{22} + \omega_2^2 x_{22} = -2D_0 D_1 x_{21} - \eta_3^2 x_{11} + \mu_{12} D_0 x_{11} + \mu_{22} D_0 x_{21} + \alpha_2 (S_{12} x_{11}^3 + S_{22} x_{21}^3 + \\ S_{32} x_{11} x_{21}^2 + S_{42} x_{11}^2 x_{21}) + \eta_4 \cos(\Omega\tau), \end{cases} \quad (13)$$

式中的微分算子 $D_0 = \frac{\partial}{\partial T_0}, D_1 = \frac{\partial}{\partial T_1}, D_0^2 = \frac{\partial^2}{\partial T_0^2}$.

将方程(13)的通解表示为如下复数形式:

$$\begin{cases} x_{11} = A_1(T_1) e^{i\omega_1 T_0} + \bar{A}_1(T_1) e^{-i\omega_1 T_0}, \\ x_{21} = A_2(T_1) e^{i\omega_2 T_0} + \bar{A}_2(T_1) e^{-i\omega_2 T_0}, \end{cases} \quad (14)$$

式中, \bar{A}_1, \bar{A}_2 分别表示 A_1, A_2 的共轭函数, $i = \sqrt{-1}$ 为单位虚数.

将式(14)代入方程(13)中,并考虑到式(10)表示的主-内联合共振频率关系,然后根据消除方程中出现永年项(长久项)的条件,推得关于变量 A_1 和 A_2 的一阶微分式:

$$\begin{cases} -2A_1' \omega_1 i + \mu_{11} A_1 \omega_1 i + 3\alpha_1 S_{11} A_1^2 \bar{A}_1 + 2\alpha_1 S_{31} A_1 A_2 \bar{A}_2 + \alpha_1 S_{41} \bar{A}_1^2 A_2 e^{i\sigma_1 T_1} + \frac{1}{2} \eta_2 e^{i\sigma_2 T_1} = 0, \\ -2A_2' \omega_2 i + \mu_{22} A_2 \omega_2 i + \alpha_2 S_{12} A_1^3 e^{-i\sigma_1 T_1} + 3\alpha_2 S_{22} A_2^2 \bar{A}_2 + 2\alpha_2 S_{42} A_1 \bar{A}_1 A_2 = 0. \end{cases} \quad (15)$$

再将式(15)中的 A_n 表示为下面复指数形式:

$$A_n(T_1) = \frac{1}{2} a_n(T_1) e^{i\beta_n(T_1)}, \quad n = 1, 2. \quad (16)$$

最终,将式(16)代入式(15)中,进行实部和虚部分离后,得到一阶状态方程:

$$a_1' = \frac{1}{2} \mu_{11} a_1 \omega_1 + \frac{\alpha_1 S_{41}}{8\omega_1} a_1^2 a_2 \sin \gamma_1 + \frac{\eta_2}{2\omega_1} \sin \gamma_2, \quad (17a)$$

$$a_2' = \frac{1}{2} \mu_{22} a_2 - \frac{\alpha_2 S_{12}}{8\omega_2} a_1^3 \sin \gamma_1, \quad (17b)$$

$$a_1 \gamma_2' = a_1 \sigma_2 + \frac{3\alpha_1 S_{11}}{8\omega_1} a_1^3 + \frac{\alpha_1 S_{31}}{4\omega_1} a_1 a_2^2 + \frac{\alpha_1 S_{41}}{8\omega_1} a_1^2 a_2 \cos \gamma_1 + \frac{\eta_2}{2\omega_1} \cos \gamma_2, \quad (17c)$$

$$a_2 \gamma'_1 = a_2 (\sigma_1 - 3\sigma_2) + 3a_2 \gamma'_2 - \left(\frac{\alpha_2 S_{12}}{8\omega_2} a_1^3 \cos \gamma_1 + \frac{3\alpha_2 S_{22}}{8\omega_2} a_2^3 + \frac{\alpha_2 S_{42}}{4\omega_2} a_1^2 a_2 \right), \quad (17d)$$

式中 $\gamma_1 = \beta_2 - 3\beta_1 + \sigma_1 T_1$, $\gamma_2 = \sigma_2 T_1 - \beta_1$.

当研究系统稳态运动状态时,需令 $a'_1 = a'_2 = 0, \gamma'_1 = \gamma'_2 = 0$.这样,根据方程(17)并消去 γ_1 和 γ_2 , 可得到主-内联合共振下系统的幅频响应方程:

$$\frac{\alpha_2^2 S_{12}^2}{64} a_1^6 - \frac{1}{4} \mu_{22}^2 \omega_2^2 a_2^2 - \left[\omega_2 a_2 (3\sigma_2 - \sigma_1) + \frac{3\alpha_2 S_{22}}{8} a_2^3 + \frac{\alpha_2 S_{42}}{4} a_1^2 a_2 \right]^2 = 0, \quad (18)$$

$$(\mu_{11} \omega_1 \alpha_2 S_{12} a_1^2 + \mu_{22} \omega_2 \alpha_1 S_{41} a_2^2)^2 + \left[2\omega_1 \alpha_2 S_{12} \sigma_2 a_1^2 + \frac{3}{4} \alpha_1 \alpha_2 S_{11} S_{12} a_1^4 + \frac{1}{2} \alpha_1 \alpha_2 (S_{12} S_{31} - S_{41} S_{42}) a_1^2 a_2^2 - 2\omega_2 \alpha_1 S_{41} a_2^2 (3\sigma_2 - \sigma_1) - \frac{3}{4} \alpha_1 \alpha_2 S_{22} S_{41} a_2^4 \right]^2 - \alpha_2^2 S_{12}^2 \eta_2^2 a_1^2 = 0. \quad (19)$$

通过对方程组(18)和(19)的求解,即可得到条形板在弹性支承和线载荷作用下发生主-内联合共振时的两阶共振幅值及变化规律.

3 算例分析

在这一节中,我们针对面内运动条形板磁弹性主-内联合共振下的前两阶共振幅值进行计算分析.应用相关数学计算软件,通过编程对方程(18)、(19)进行求解,绘制下面共振特性曲线图(图2~9).

以铝制材料薄板为例,并给定主要参数:材料电导率 $\sigma_0 = 3.63 \times 10^7 (\Omega \cdot \text{m})^{-1}$, 密度 $\rho = 2\,670 \text{ kg/m}^3$, 弹性模量 $E = 71 \text{ GPa}$, Poisson 比 $\nu = 0.34$, 板长 $l_1 = 0.5 \text{ m}$, 板厚 $h = 0.005 \text{ m}$.

3.1 共振幅值随频率参数变化特征

图2给出了不同外加磁感应强度下系统的一阶和二阶共振幅值随频率调谐参数 $\varepsilon \sigma_2$ 变化的幅频响应曲线图(选取 $N_{0x} = 10 \text{ kN/m}$, $V_{0x} = 30 \text{ m/s}$, $P = 5 \text{ kN/m}$, $k = 100 \text{ kN/m}^2$, $x_0 = 0.15 \text{ m}$, $x_1 = 0.15 \text{ m}$).从图中可以看出,一阶共振和二阶共振的幅频响应呈现明显的非线性跳跃现象,且两者都有单值解区域和多个多值解区域.而从整体趋势看,一阶共振幅值大于二阶共振幅值.图2(a)中非零解的个数变化规律为: a_1 的非零解个数先为由1个变为3个再变为5个,接着又变为3个再变为5个,最后又变为3个; a_2 的非零解个数先由1个变为3个再变为5个,接着又变为4个再变为2个又变为4个,最后变为2个.图2(b)和图2(c)也有类似的多解性变化规律.比较三个图可知:随着磁感应强度的逐渐增大, a_1 的中部上支极值点位置变化不太明显,但 a_1 的5个非零解区域逐渐变窄;而 a_2 的最大振幅值明显变小,且多值解区域逐渐变窄.

图3给出了不同面内轴向速度下系统的一阶和二阶共振幅值变化的幅频响应曲线图(选取 $B_{0z} = 0.5 \text{ T}$, $x_0 = 0.15 \text{ m}$, $x_1 = 0.15 \text{ m}$, 其余参数与图2相同).从图中可以看出,随着速度的增大, a_1 的中部上支极值点对应的幅值明显增大,且其极值点位置有右移趋势,而 a_2 的最大幅值也有变大趋势.同时可见,系统仍存在多个多值解区域,且随速度的增大,中间部位 a_1 和 a_2 的多值解区域对应的主架曲线宽度均逐渐变大.

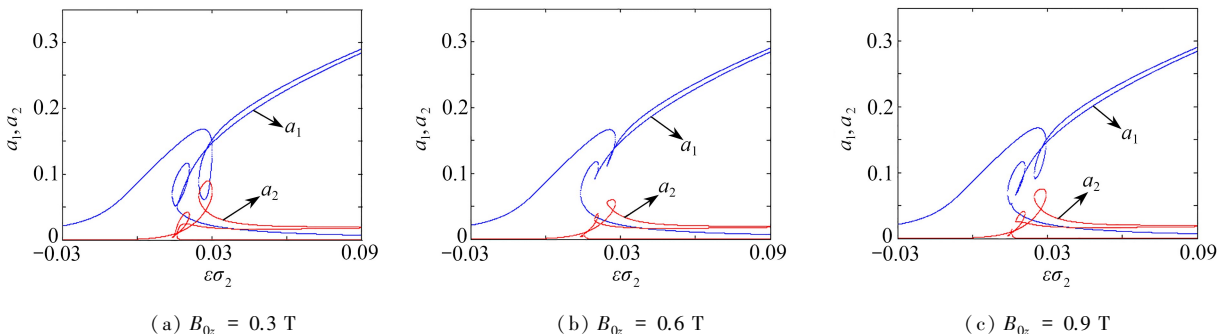


图2 不同磁感应强度下共振幅频响应图

Fig. 2 Resonance amplitude frequency response diagrams under different magnetic induction intensities

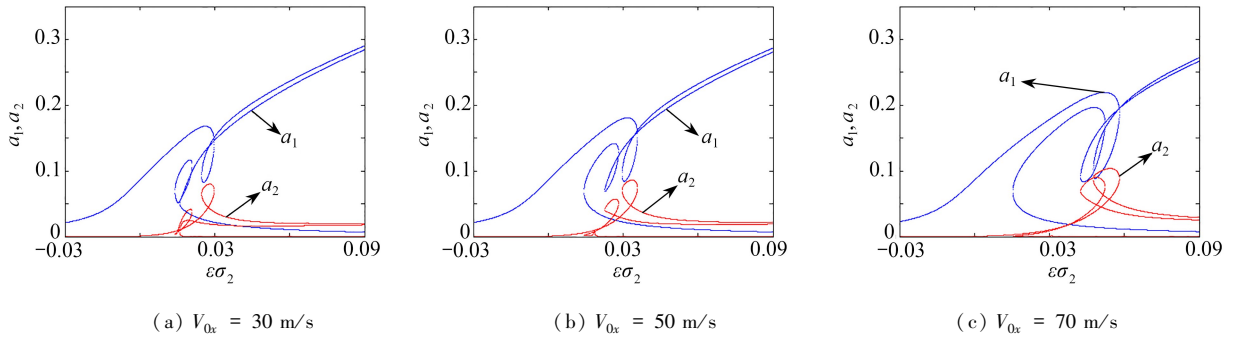


图 3 不同速度下共振幅频响应图

Fig. 3 Resonance amplitude frequency response diagrams under different speeds

图 4 给出了不同弹性支承位置下系统的一阶和二阶共振幅值变化的幅频响应曲线图(选取 $V_{0x} = 30$ m/s, $B_{0z} = 0.3$ T, $x_0 = 0.15$ m, 其余参数与图 2 相同).从图中可以看出, a_1 和 a_2 呈现多值和跳跃现象.随着弹性支承位置的右移,从图 4(a)到图 4(b), a_1 在 $\epsilon\sigma_2 = 0.09$ 处的上支解值变大,而 a_2 的最大幅值变小;从图 4(b)到图 4(c), a_1 在 $\epsilon\sigma_2 = 0.09$ 处的上支解值变化不大,而 a_2 的最大幅值变大.在图 4(a)中, a_1 和 a_2 最多有 5 个非零解,而在图 4(b)和图 4(c)中, a_1 和 a_2 最多只有 3 个非零解,可见,随着弹性支承位置的右移, a_1 和 a_2 的 5 个非零解区域逐渐消失.

图 5 给出了不同线载荷位置下系统的一阶和二阶共振幅值变化的幅频响应曲线图(选取 $V_{0x} = 30$ m/s, $B_{0z} = 0.3$ T, $x_1 = 0.15$ m, 其余参数与图 2 相同).从图中可以看出, a_1 和 a_2 的稳态解体现出了多解性特点.随着线载荷位置的右移,从图 5(a)到图 5(b), a_1 的极大值点上移, a_2 的最大幅值也变大,同时 a_1 的 5 个非零解出现的位置滞后;从图 5(b)到图 5(c), a_1 的极大值点位置和 a_2 的最大共振幅值没有明显变化.

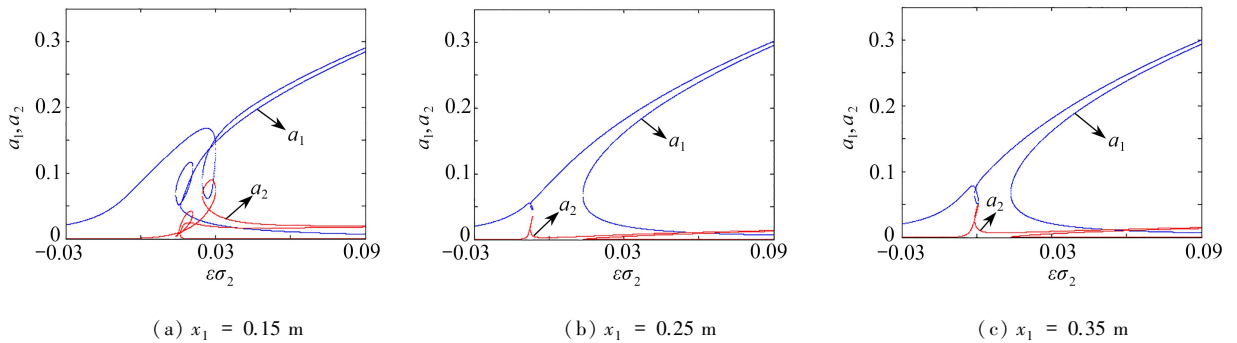


图 4 不同弹性支承位置下共振幅频响应图

Fig. 4 Resonance amplitude frequency response diagrams under different elastic support positions

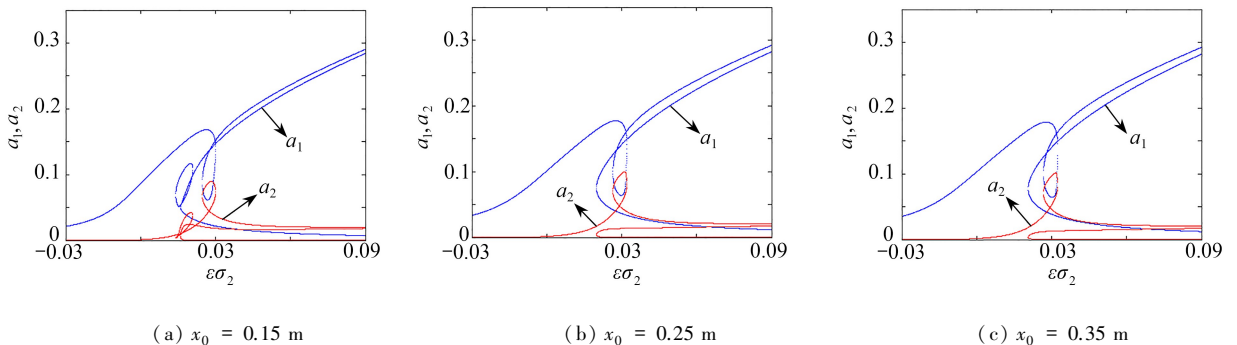


图 5 不同线载荷位置下共振幅频响应图

Fig. 5 Resonance amplitude frequency response diagrams under different line load positions

3.2 共振幅值随激励力变化特征

图6给出了不同磁感应强度下一阶和二阶共振幅值的振幅-外激励力曲线图(选取 $N_{0x} = 10 \text{ kN/m}$, $V_{0x} = 30 \text{ m/s}$, $\Omega = 0.6$, $k = 100 \text{ kN/m}^2$, $x_0 = 0.15 \text{ m}$, $x_1 = 0.15 \text{ m}$)。从图中可以看出,一阶共振和二阶共振的曲线图都有单值解和多值解区域,都存在跳跃现象,且从整体趋势看,一阶共振幅值大于二阶共振幅值。随着磁感应强度的增大, a_1 的5个非零解出现的位置逐渐右移,且5个非零解的区域逐渐变窄,同时 a_2 的4个非零解区域也随之右移并逐渐变窄。可见,由于内共振现象的影响,一阶和二阶的共振多值解区域同时出现,同时消失。

图7给出了不同面内运动速度下,一阶和二阶共振幅值的振幅-外激励力曲线图(选取 $B_{0z} = 0.5 \text{ T}$, $x_0 = 0.15 \text{ m}$, $x_1 = 0.15 \text{ m}$, 其余参数与图6相同)。从图中可以看出,两阶共振均被激发,曲线存在着多解值和跳跃现象。随着轴向速度的增大, a_1 的5个非零解出现的位置逐渐左移,即产生5个共振解的激励力值变小,同时 a_2 的4个非零解出现的位置也随之逐渐左移,系统内共振特征明显。

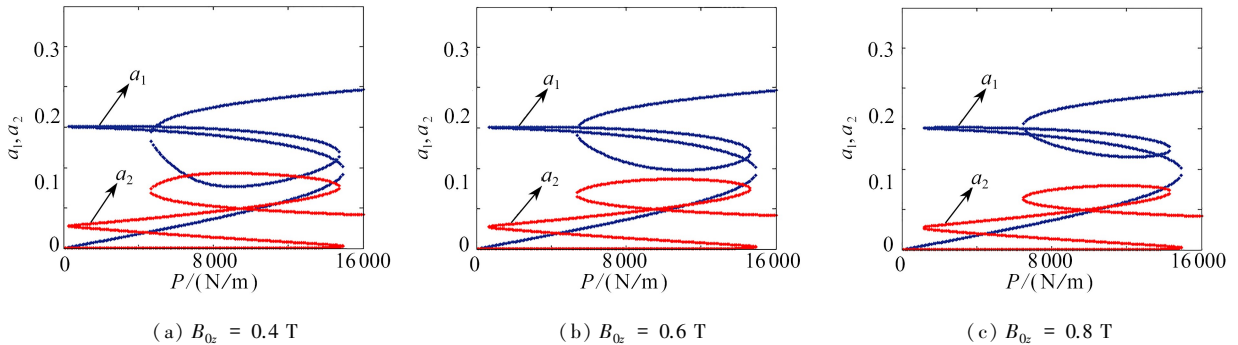


图6 不同磁感应强度下振幅随激励力变化图

Fig. 6 amplitude diagrams changing with excitation forces under different magnetic induction intensities

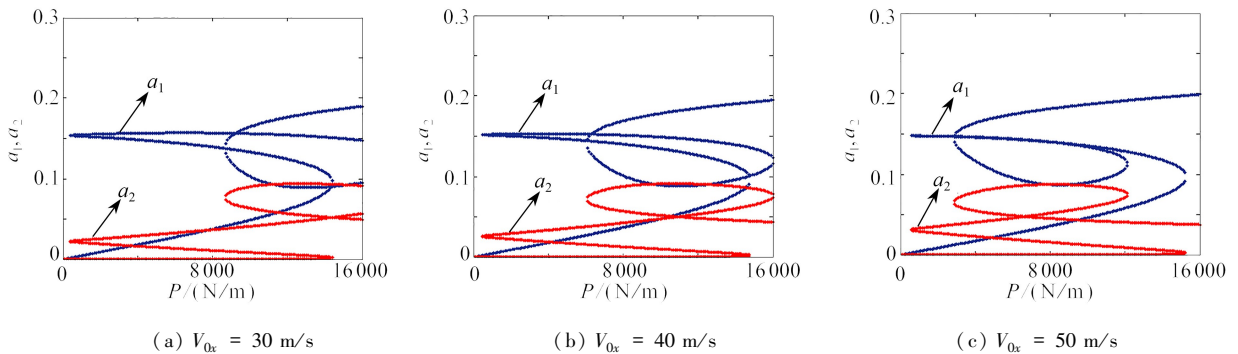


图7 不同速度下振幅随激励力变化图

Fig. 7 Amplitude diagrams amplitude changing with excitation forces under different speeds

图8给出了不同弹性支承位置下一阶和二阶共振幅值的振幅-外激励力曲线图(选取 $B_{0z} = 0.4 \text{ T}$, $V_{0x} = 30 \text{ m/s}$, $x_0 = 0.15 \text{ m}$, 其余参数与图6相同)。从图中可以看出,一阶共振和二阶共振的曲线图都有单值解和多值解区域。随着弹性支承位置的右移,从图8(a)到图8(b), a_1 和 a_2 的曲线形状发生变化, a_1 的5个非零解区域和 a_2 的4个非零解区域消失,且 a_2 的共振幅值明显变小;从图8(b)到图8(c), a_1 由3个解过渡到1个解的分岔点右移,3个解的区域也变大, a_2 幅值则略有增大。同样, a_1 和 a_2 的几个多解值区域是相互对应的,解的个数发生突变所对应的分岔点相同。

图9给出了不同线载荷位置下一阶和二阶共振幅值的振幅-外激励力曲线图(选取 $B_{0z} = 0.4 \text{ T}$, $V_{0x} = 30 \text{ m/s}$, $x_1 = 0.15 \text{ m}$, 其余参数与图6相同)。从图中可以看出,相对于图9(a),图9(b)中 a_1 的5个非零解和 a_2 的4个非零解区域同时左移并变窄;而相对于图9(b),图9(c)中 a_1 和 a_2 的非零多值解区域有变大趋势,但不明显。同时表明,由于内共振的存在,一阶和二阶共振曲线对应的多解值区域同时出现同时消失,具有多个相同的分岔点。

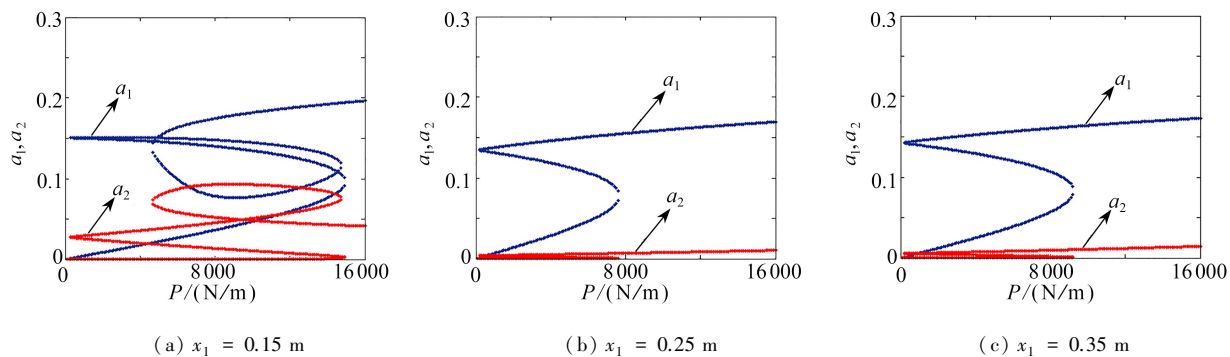


图 8 不同弹性支承位置下振幅随激励力变化图

Fig. 8 Amplitude diagrams changing with excitation forces under different elastic support positions

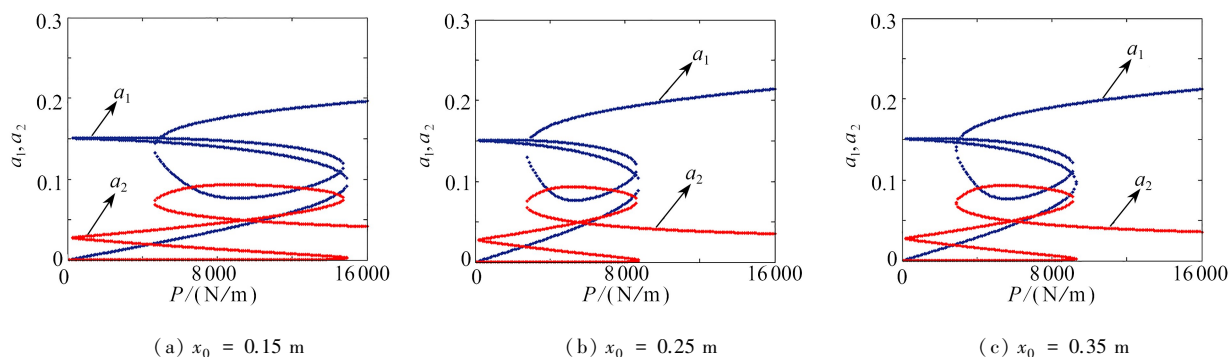


图 9 不同线载荷位置下振幅随激励力变化图

Fig. 9 Amplitude diagrams changing with excitation forces under different line load positions

4 结 论

本文研究了弹性支承和线载荷作用下面内运动薄板的磁弹性联合共振问题,应用 Galerkin 法和多尺度法进行求解,推得系统的两自由度非线性振动方程组和联合共振响应特征方程.计算结果表明:

- 1) 弹性支承下,面内运动条形板的共振响应曲线具有单值解区域和多值解区域,多值解区域最多出现 5 个非零解,并存在明显跳跃现象,呈现复杂的非线性振动特征;
- 2) 系统发生联合共振时,一阶共振幅值明显大于二阶共振幅值,且一阶和二阶的共振多解值区域同时出现和消失,体现明显的内共振特征,且弹性支承和线载荷的位置对共振现象影响显著.

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