

# 分数阶热弹理论下重力场 对二维纤维增强介质的影响\*

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**摘要:** 基于 Sherief 等提出的分数阶广义热弹性耦合理论,研究了在热冲击作用下二维纤维增强弹性体的热弹性问题.考虑了重力对二维纤维增强线性热弹性各向同性介质的影响,建立了控制方程.运用正则模态法,经过数值计算,对控制方程进行求解,得到了不同分数阶参数和不同重力场下无量纲温度、位移和应力分量的表达式,以图形的方式展示了变量的分布规律并对结果展开了讨论.研究结果为:重力场和分数阶参数对纤维增强介质的位移及应力有着重要的影响,但对温度的影响有限.

**关键词:** 二维模型; 正则模态法; 重力场; 分数阶广义热弹性理论; 热冲击  
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## Effects of the Gravity Field on 2D Fiber-Reinforced Media Under the Fractional Order Theory of Thermoelasticity

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**Abstract:** Based on the fractional order generalized thermoelastic coupling theory proposed by Sherief et al, the thermoelastic problem of 2D fiber-reinforced elastomers under thermal shock was studied. In view of the effects of gravity on 2D fiber-reinforced linearly thermoelastic isotropic media, the governing equations were established. Through the normal mode analysis and numerical calculation, the governing equations were solved, and the expressions of the dimensionless temperature, the displacement components and the stress under different fractional order parameters and different gravity fields were obtained. The distributions of variables were illustrated and the results were discussed. The results show that, the gravity field and fractional order parameters have significant impacts on the displacements and stresses of the fiber-reinforced media, but the influence on the temperature is limited.

**Key words:** 2D model; normal mode analysis; gravity field; fractional order theory of thermoelasticity; thermal shock

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## 引言

由经典 Fourier 热传导理论可知,热的传播速度是无限大的,热流矢量与温度梯度成正比.但对于一些比较极端的条件,如超高温传热、超低温传热及微尺度条件传热等,经典热传导理论就不再适用.为克服经典 Fourier 热传导理论的局限性,便衍生出了非 Fourier 热传导理论.在非 Fourier 热传导理论的发展过程中,相继出现了一些理论,这些理论认为存在以有限速度传播的热波,随后这些理论被统称为广义热弹性理论.其主要有 Lord 和 Shulman<sup>[1]</sup>提出的 L-S 理论;Green 和 Lindsay<sup>[2]</sup>提出的 G-L 理论;Green 和 Naghdi<sup>[3]</sup>提出的 G-N 理论.

由于分数阶微积分在热弹性力学中的成功应用,关于分数阶微积分的研究也越来越多.Povstenko<sup>[4]</sup>基于准静态非耦合热弹性理论,以时间分数阶导数  $0 < \alpha \leq 2$  的热传导方程为基础,研究了具有球形空腔的无限大介质中的温度分布和热应力.随后,Povstenko<sup>[5]</sup>又运用分数阶热传导方程研究了中心对称的 Cauchy 问题.Sherief 等<sup>[6]</sup>通过引入 Caputo 型分数阶导数,建立起一种新的分数阶广义热弹性耦合理论.Sherief 等<sup>[7]</sup>基于分数阶热弹性理论,研究了不同的热导率对半无限大体的影响.Ezzat 和 El-Karamany<sup>[8]</sup>运用状态空间法,基于分数阶热传导理论研究了双温磁热弹性数学模型.Ezzat 和 Fayik<sup>[9]</sup>基于分数阶热弹性扩散理论,研究了具有一个松弛因子的广义热弹扩散问题.Youssef<sup>[10]</sup>运用拉氏变换和 Fourier 变换,研究了基于分数阶广义热弹理论的二维热冲击问题.Youssef 等<sup>[11]</sup>基于分数阶广义热弹性理论,得到了斜坡加热作用的半空间问题所涉及物理量的分布规律.Ma 等<sup>[12-16]</sup>基于分数阶热弹理论,研究了二维热弹性模型问题以及在热冲击作用下含球形空腔无限大介质的动态响应问题,同时研究了具有可变材料特性的有限长热压电杆的动态响应问题.

纤维增强复合材料是由增强纤维材料,如玻璃纤维、碳纤维、芳纶纤维等,与基体材料经过缠绕、模压或拉挤等成型工艺形成的复合材料.纤维增强复合材料由于其重量轻、强度高,被广泛应用到各个领域.Singh<sup>[17]</sup>研究了纤维增强各向异性热弹性介质中平面波的传播问题.Othman 和 Said 等<sup>[18]</sup>研究了分数阶、静水初始应力和重力场对线性纤维增强各向同性热弹性介质中平面波的影响.对于经典弹性理论,重力场的影响常常被忽略.Bromwich<sup>[19]</sup>首次考虑了重力对弹性固体介质中波传播的影响.Ailawalia 和 Narah<sup>[20]</sup>研究了在重力作用下广义热弹性固体的旋转变形问题.Othman 等<sup>[21]</sup>建立了一个新的纤维增强模型,在不同理论下讨论了重力场对弹性介质变形的影响.

我们在大量查阅有关分数阶热弹性理论的文献后可以发现,学者就分数阶广义热弹性耦合问题做了很多研究,并取得了一定的成果,但基于 Sherief 型分数阶广义热弹性耦合理论,对二维纤维增强材料的研究相对较少.

综上所述,本文将基于 Sherief 型分数阶广义热弹性耦合理论,研究在热冲击作用下二维纤维增强弹性体的热弹性问题.在考虑重力对二维纤维增强线性热弹性各向同性介质的影响下,建立了控制方程.运用正则模态法,经过数值计算,对控制方程进行求解,得到了不同分数阶参数和不同重力场下无量纲温度、位移和应力分量的表达式,以图形的方式展示了变量的分布规律并对结果展开了讨论.

## 1 基本方程及问题描述

1983 年,Belfied 等<sup>[22]</sup>提出的纤维增强线性热弹性各向异性介质相对于增强方向  $\mathbf{a}$  的本构方程为

$$\begin{aligned} \sigma_{ij} = & \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \xi (a_k a_m e_{km} \delta_{ij} + a_i a_j e_{kk}) + \\ & 2(\mu_L - \mu_T) (a_i a_k e_{kj} + a_j a_k e_{ki}) + \zeta a_k a_m e_{km} a_i a_j - \gamma (T - T_0) \delta_{ij}, \end{aligned} \quad (1)$$

式中,  $\sigma_{ij}$  为应力分量;  $e_{ij}$  为应变分量;  $\lambda, \mu_T$  为弹性常数;  $\xi, \zeta, \mu_L - \mu_T$  为增强参数;  $\gamma = (3\lambda + 2\mu)\alpha_1, \alpha_1$  为热膨胀系数;  $\delta_{ij}$  为 Kronecker 函数;  $T$  为温度,  $T_0$  为参考温度;  $\mathbf{a} \equiv (a_1, a_2, a_3), a_1^2 + a_2^2 + a_3^2 = 1$ .我们选择的增强方向  $\mathbf{a} \equiv (1, 0, 0)$ .

几何方程为

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}). \quad (2)$$

位移分量  $u, v, w$  在二维问题中假设为  $u = u(x, y, t), v = v(x, y, t), w = 0$ , 代入方程(2)可得

$$e_{xx} = \frac{\partial u}{\partial x}, e_{yy} = \frac{\partial v}{\partial y}, e_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), e_{xz} = e_{yz} = e_{zz} = 0. \quad (3)$$

将方程(3)代入方程(1)得

$$\sigma_{xx} = A_{11} \frac{\partial u}{\partial x} + A_{12} \frac{\partial v}{\partial y} - \gamma(T - T_0), \quad (4)$$

$$\sigma_{yy} = A_{12} \frac{\partial u}{\partial x} + A_{22} \frac{\partial v}{\partial y} - \gamma(T - T_0), \quad (5)$$

$$\sigma_{zz} = A_{12} \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} - \gamma(T - T_0), \quad (6)$$

$$\sigma_{xy} = \mu_L \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \sigma_{zx} = \sigma_{zy} = 0, \quad (7)$$

式中  $A_{11} = \lambda + 2(\xi + \mu_T) + 4(\mu_L - \mu_T) + \zeta$ ,  $A_{12} = \lambda + \xi$ ,  $A_{22} = \lambda + 2\mu_T$ .

纤维增强热弹性介质在重力场影响下的运动方程为<sup>[21]</sup>

$$\rho \frac{\partial^2 u}{\partial t^2} = A_{11} \frac{\partial^2 u}{\partial x^2} + B_2 \frac{\partial^2 v}{\partial x \partial y} + B_1 \frac{\partial^2 u}{\partial y^2} - \gamma \frac{\partial T}{\partial x} + \rho g \frac{\partial v}{\partial x}, \quad (8)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = A_{22} \frac{\partial^2 v}{\partial y^2} + B_2 \frac{\partial^2 u}{\partial x \partial y} + B_1 \frac{\partial^2 v}{\partial x^2} - \gamma \frac{\partial T}{\partial y} - \rho g \frac{\partial u}{\partial x}, \quad (9)$$

式中  $B_1 = \mu_L$ ,  $B_2 = \xi + \lambda + \mu_L$ .

Sherief 等<sup>[6]</sup>在 2010 年提出的分数阶广义热弹性理论中的热传导方程为

$$\kappa T_{,ii} = \left( 1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha} \right) (\rho C_E \dot{T} + \gamma T_0 \dot{u}_{i,i}), \quad (10)$$

式中,  $\kappa$  是导热张量;  $\rho$  是密度;  $\tau_0$  是热松弛时间;  $\alpha$  是分数阶参数,  $0 < \alpha \leq 1$ ;  $C_E$  是恒定应变下的比热. 在上面的方程中, 参数上方的一个点表示对时间的导数, 参数右下标的逗号后表示对相应坐标的部分偏导.

使用以下无量纲变量, 将上述方程转换为无量纲形式:

$$\begin{cases} x' = c_1 \eta x, y' = c_1 \eta y, u' = c_1 \eta u, v' = c_1 \eta v, t' = c_1^2 \eta t, \\ \tau'_0 = c_1^2 \eta \tau_0, g' = \frac{g}{c_1^3 \eta}, \theta = \frac{\gamma(T - T_0)}{\lambda + 2\mu_T}, \sigma'_{ij} = \frac{\sigma_{ij}}{\mu_T}, \quad i, j = 1, 2, \end{cases} \quad (11)$$

式中  $\eta = \frac{\rho C_E}{\kappa}$ ,  $c_1^2 = \frac{\lambda + 2\mu_T}{\rho}$ .

应用上述无量纲变量, 对方程(8)~(10)进行无量纲化, 得(为方便起见, 去掉右上角的撇号)

$$\frac{\partial^2 u}{\partial t'^2} = h_{11} \frac{\partial^2 u}{\partial x'^2} + h_2 \frac{\partial^2 v}{\partial x' \partial y'} + h_1 \frac{\partial^2 u}{\partial y'^2} - \frac{\partial \theta}{\partial x'} + g' \frac{\partial v}{\partial x'}, \quad (12)$$

$$\frac{\partial^2 v}{\partial t'^2} = h_{22} \frac{\partial^2 v}{\partial y'^2} + h_2 \frac{\partial^2 u}{\partial x' \partial y'} + h_1 \frac{\partial^2 v}{\partial x'^2} - \frac{\partial \theta}{\partial y'} - g' \frac{\partial u}{\partial x'}, \quad (13)$$

$$\frac{\partial^2 \theta}{\partial x'^2} + \frac{\partial^2 \theta}{\partial y'^2} = \left[ 1 + (c_1^2 \eta)^{\alpha-1} \tau_0 \frac{\partial^\alpha}{\partial t'^\alpha} \right] \left( \frac{\partial \theta}{\partial t'} + \varepsilon \frac{\partial \theta}{\partial t'} \right), \quad 0 < \alpha \leq 1, \quad (14)$$

式中  $(h_1, h_2, h_{11}, h_{22}) = \frac{(B_1, B_2, A_{11}, A_{22})}{\rho c_1^2}$ ,  $\varepsilon = \frac{\gamma^2 T_0}{\rho C_E (\lambda + 2\mu_T)}$ .

## 2 正则模态分析

正则模态分析应用于不同的领域, 给出了不受位移、温度、应力等假定变量限制的解. 本文所考虑变量的解可以按正则模态法以如下形式给出:

$$[u, v, \theta, \sigma'_{ij}](x, y, t) = [u^*(x), v^*(x), \theta^*(x), \sigma'_{ij}^*(x)] \exp(\omega t + iby), \quad (15)$$

式中,  $\omega$  为时间常数;  $i = \sqrt{-1}$ ;  $b$  为  $y$  方向上的波数;  $u^*(x), v^*(x), \theta^*(x)$  和  $\sigma_{ij}^*$  为场量的振幅。

用 Taylor 级数将  $e^{\omega t}$  展开, 得

$$e^{\omega t} = \sum_{n=1}^{\infty} \frac{(\omega t)^n}{n!}. \quad (16)$$

运用以下公式:

$$\frac{d^\alpha t^n}{dt^\alpha} = \frac{\Gamma(n+1)}{\Gamma(n+1-\alpha)} t^{n-\alpha}, \quad n \geq 1, \quad (17)$$

可得

$$\frac{d^\alpha e^{\omega t}}{dt^\alpha} = t^{-\alpha} \sum_{n=1}^{\infty} \frac{(\omega t)^n}{\Gamma(n+1-\alpha)}. \quad (18)$$

将方程(15)和方程(18)代入方程(12)~(14), 得到

$$(h_{11}D^2 - A_1)u^* + (ibh_2 + g)Dv^* = D\theta^*, \quad (19)$$

$$(h_1D^2 - A_2)v^* + (ibh_2 - g)Du^* = ib\theta^*, \quad (20)$$

$$A_4Du^* + ibA_4v^* = (D^2 - A_3)\theta^*, \quad (21)$$

式中

$$A_1 = \omega^2 + h_1b^2, \quad A_2 = \omega^2 + h_{22}b^2, \quad A_3 = b^2 + \omega[1 + (c_1^2\eta)^{\alpha-1}\tau_0L^*],$$

$$A_4 = \omega\varepsilon[1 + (c_1^2\eta)^{\alpha-1}\tau_0L^*], \quad D = \frac{d}{dx}, \quad L^* = t^{-\alpha} \sum_{n=1}^{\infty} \frac{(\omega t)^n}{\Gamma(n+1-\alpha)}.$$

将方程(19)~(21)中的  $\theta^*(x)$  和  $v^*(x)$  消除, 得到了如下满足  $u^*(x)$  的偏微分方程:

$$(D^6 - AD^4 + BD^2 - C)u^*(x) = 0, \quad (22)$$

式中

$$A = \frac{1}{h_1h_{11}}(h_1A_4 + h_{11}A_2 + h_1A_1 - h_2^2b^2 - g^2 + h_1h_{11}A_3), \quad (23)$$

$$B = \frac{1}{h_1h_{11}}(A_2A_4 + b^2h_{11}A_4 + h_{11}A_2A_3 + A_1A_2 + h_1A_1A_3 - 2h_2b^2A_4 - h_2^2b^2A_3 - A_3g^2), \quad (24)$$

$$C = \frac{1}{h_1h_{11}}(A_1A_2A_3 + b^2A_1A_4). \quad (25)$$

方程(22)可被分解为如下形式:

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)u^*(x) = 0, \quad (26)$$

式中,  $k_n^2 (n = 1, 2, 3)$  是方程(27)的特征根,

$$k^6 - Ak^4 + Bk^2 - C = 0. \quad (27)$$

当  $x \rightarrow \infty$  时, 方程(22)的解为

$$u^*(x) = \sum_{n=1}^3 M_n \exp(-k_n x); \quad (28)$$

同理, 可得到

$$v^*(x) = \sum_{n=1}^3 M'_n \exp(-k_n x), \quad (29)$$

$$\theta^*(x) = \sum_{n=1}^3 M''_n \exp(-k_n x), \quad (30)$$

式中,  $M_n, M'_n$  和  $M''_n$  是依赖于  $b$  和  $\omega$  的函数。

将方程(28)~(30)代入方程(19)~(21), 得到

$$M'_n = R_{1n}M_n, \quad (31)$$

$$M''_n = R_{2n}M_n, \quad (32)$$

式中

$$\begin{cases} R_{1n} = \frac{ibA_1 + ibh_2k_n^2 - ibh_{11}k_n^2 - gk_n^2}{h_1k_n^3 + b^2h_2k_n - A_2k_n - ibgk_n}, \\ R_{2n} = \frac{(h_{11}k_n^2 - A_1) - (ibh_2 + g)k_nR_{1n}}{k_n}, \end{cases} \quad n = 1, 2, 3.$$

因此,可得

$$v^*(x) = \sum_{n=1}^3 R_{1n} M_n \exp(-k_n x), \quad (33)$$

$$\theta^*(x) = \sum_{n=1}^3 R_{2n} M_n \exp(-k_n x). \quad (34)$$

将方程(15)代入方程(4)~(7),可得

$$\mu_T \sigma_{xx}^* = A_{11} Du^* + ibA_{12} v^* - A_{22} \theta^*, \quad (35)$$

$$\mu_T \sigma_{yy}^* = A_{12} Du^* + ibA_{22} v^* - A_{22} \theta^*, \quad (36)$$

$$\mu_T \sigma_{zz}^* = A_{12} Du^* + ib\lambda v^* - A_{22} \theta^*, \quad (37)$$

$$\mu_T \sigma_{xy}^* = \mu_L (ibu^* + Dv^*), \quad \sigma_{zx}^* = \sigma_{zy}^*. \quad (38)$$

将方程(28)、(33)和(34)代入方程(35)~(38),得到

$$\sigma_{xx}^* = \sum_{n=1}^3 R_{3n} M_n \exp(-k_n x), \quad (39)$$

$$\sigma_{yy}^* = \sum_{n=1}^3 R_{4n} M_n \exp(-k_n x), \quad (40)$$

$$\sigma_{zz}^* = \sum_{n=1}^3 R_{5n} M_n \exp(-k_n x), \quad (41)$$

$$\sigma_{xy}^* = \sum_{n=1}^3 R_{6n} M_n \exp(-k_n x), \quad (42)$$

式中

$$\begin{cases} R_{3n} = \frac{1}{\mu_T} (-A_{11}k_n + ibA_{12}R_{1n} - A_{22}R_{2n}), \\ R_{4n} = \frac{1}{\mu_T} (-A_{12}k_n + ibA_{22}R_{1n} - A_{22}R_{2n}), \\ R_{5n} = \frac{1}{\mu_T} (-A_{12}k_n + ib\lambda R_{1n} - A_{22}R_{2n}), \\ R_{6n} = \frac{\mu_L}{\mu_T} (ib - k_n R_{1n}), \end{cases} \quad n = 1, 2, 3.$$

该问题的边界条件为

$$\begin{cases} \sigma_{xx}(x, y, t) = 0, \\ \sigma_{xy}(x, y, t) = 0, \\ \theta(x, y, t) = \theta_0 H(t) H(L - |y|), \end{cases} \quad (43)$$

式中,  $H(\cdot)$ 代表 Heaviside 单位阶跃函数,  $\theta_0$ 代表一个常数。

将方程(34)、(39)和(42)代入上述边界条件中,可得到如下方程:

$$\begin{cases} \sum_{n=1}^3 R_{3n} M_n = 0, \\ \sum_{n=1}^3 R_{6n} M_n = 0, \\ \sum_{n=1}^3 R_{2n} M_n = \theta_0 H(t) H(L - |y|). \end{cases} \quad (44)$$

方程(44)结合边界条件(43)在  $x = 0$  时,可得到以下方程:

$$\begin{cases} R_{31}M_1 + R_{32}M_2 + R_{33}M_3 = 0, \\ R_{61}M_1 + R_{62}M_2 + R_{63}M_3 = 0, \\ R_{21}M_1 + R_{22}M_2 + R_{23}M_3 = \theta_0. \end{cases} \quad (45)$$

通过对方程(45)求解,得到  $M_n (n = 1, 2, 3)$  的值为

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} R_{31} & R_{32} & R_{33} \\ R_{61} & R_{62} & R_{63} \\ R_{21} & R_{22} & R_{23} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ \theta_0 \end{pmatrix}. \quad (46)$$

### 3 数值结果及讨论

为了研究分数阶参数和重力场对纤维增强材料的影响,采用以下物理常数的数值结果:

$$\begin{aligned} \lambda &= 7.59 \times 10^9 \text{ N/m}^2, \mu_T = 1.89 \times 10^9 \text{ N/m}^2, \mu_L = 2.45 \times 10^9 \text{ N/m}^2, b = 1, \\ T_0 &= 293 \text{ K}, g = 9.8 \text{ m/s}^2, \xi = -1.28 \times 10^9 \text{ N/m}^2, \zeta = 3.2 \times 10^8 \text{ N/m}^2, \\ \rho &= 7800 \text{ kg/m}^3, C_E = 383.1 \text{ J/(kg}\cdot\text{K)}, \alpha_1 = 1.78 \times 10^{-5} \text{ K}^{-1}, \tau_0 = 0.02, \\ \mu &= 3.86 \times 10^{10} \text{ kg/(m}\cdot\text{s}^2), \kappa = 386 \text{ W/(m}\cdot\text{K)}, \omega = \omega_0 + i\xi_0, \omega_0 = 2, \xi_0 = 1, \theta_0 = 1. \end{aligned}$$

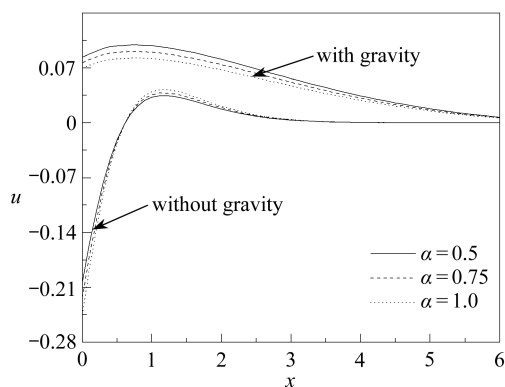


图 1 存在和不存在重力场时水平位移的分布

Fig. 1 The horizontal displacement distribution with and without the gravity field

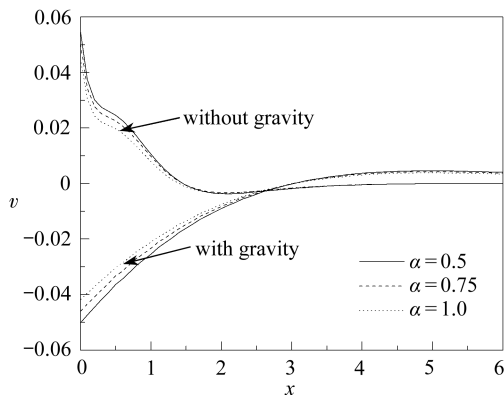


图 2 存在和不存在重力场时垂直位移的分布

Fig. 2 The vertical displacement distribution with and without the gravity field

基于分数阶广义热弹性理论,讨论重力场和分数阶参数对所考虑的物理变量的影响,这里取  $t = 0.1$  时,分数阶参数  $\alpha = 0.5, 0.75, 1.0$ ,重力值  $g = 0, 9.8$ .图 1~5 显示了在不同分数阶参数下,有重力场和无重力场时,位移、温度和应力数值变化间的一些比较.无量纲位移的分布规律如图 1、2 所示,无量纲温度的分布如图 3 所示,无量纲应力的分布规律如图 4、5 所示.

图 1 描绘了无量纲水平位移  $u$  的分布规律.从图中可以看出,当重力场存在时,水平位移  $u$  的峰值随着分数阶参数值的增加而减小;当重力场不存在时,水平位移  $u$  的峰值随分数阶参数值的增加而增加.当分数阶参数值相等时,重力场的存在使水平位移  $u$  的值增大.

图 2 描绘了无量纲垂直位移  $v$  的分布规律.从图中可以看出,当重力场存在时,垂直位移  $v$  的峰值随着分数阶参数值的增加而减小;当重力场不存在时,垂直位移  $v$

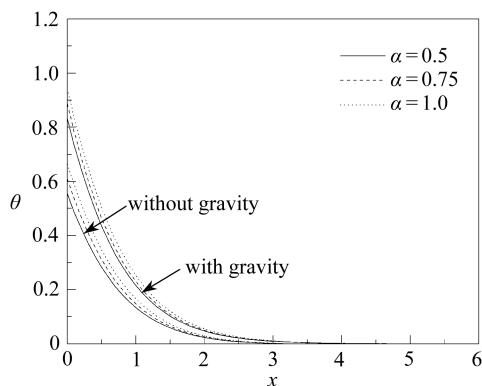


图 3 存在和不存在重力场时温度的分布  
Fig. 3 The temperature distribution with and without the gravity field



的值整体随分数阶参数值的增加而减小.当分数阶参数值相等时,重力场对垂直位移  $v$  的影响较明显.

图3描绘了无量纲温度  $\theta$  的分布规律.从图中可以看出,当重力场存在时,分数阶参数值的增加会导致无量纲温度  $\theta$  值的增大;当重力场不存在时,温度  $\theta$  的分布趋势与重力场存在时的分布趋势相同.当分数阶参数值相等时,重力场的存在使温度  $\theta$  的值增大.

图4、5描绘了无量纲应力分量  $\sigma_{xx}$  和  $\sigma_{xy}$  的分布规律.从图4中可以看出,当重力场存在时,应力分量  $\sigma_{xx}$  的峰值随着分数阶参数值的增加而增大;当重力场不存在时,应力分量  $\sigma_{xx}$  的峰值随着分数阶参数值的增加而减小.当分数阶参数值相等时,重力场的存在会增大应力分量  $\sigma_{xx}$  的振幅.从图5中可以看出,应力分量  $\sigma_{xy}$  的分布趋势与图4中应力分量  $\sigma_{xx}$  的分布趋势相同.

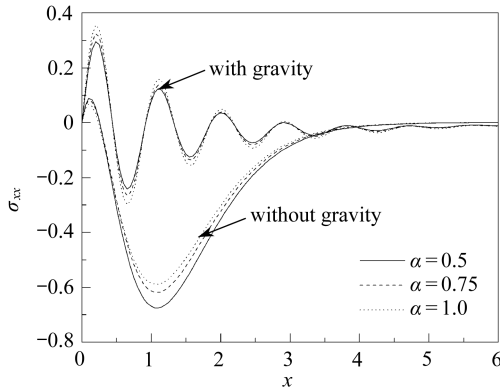


图4 存在和不存在重力场时  
应力  $\sigma_{xx}$  的分布

Fig. 4 The strain component  $\sigma_{xx}$  distribution  
with and without the gravity field

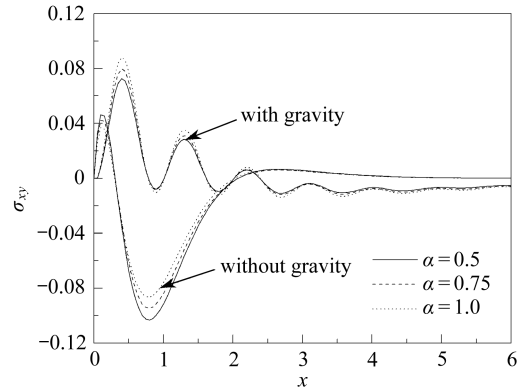


图5 存在和不存在重力场时  
应力  $\sigma_{xy}$  的分布

Fig. 5 The strain component  $\sigma_{xy}$  distribution  
with and without the gravity field

## 4 结 论

本文在分数阶广义热弹性理论的框架下,利用正则模态分析方法,分析了无量纲位移、温度和应力的分布情况,考虑了不同参数如重力场、分数阶参数对它们的影响.根据以上结果,我们得到了一些结论:

- 1) 分数阶参数对无量纲位移、温度和应力有一定的影响.所考虑的物理量的值随分数阶参数值的增大而增大,如温度、应力分量等.
- 2) 所有函数都是连续的,所考虑的物理量的值都随距离  $x$  的增大而最终趋于零.
- 3) 重力场的存在对所考虑的物理量有很大的影响,如位移、应力分量等.当重力场存在时,应力分量的振幅增大.但是,研究结果也表明重力场对温度的影响很小.
- 4) 无量纲位移、温度和应力不仅取决于  $x, y$  和  $t$ , 还取决于分数阶参数.

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